DIFFERENTIAL CALCULUS

Every question, eight papers

DBE · National Senior Certificate · Mathematics Paper 1

November 2021 - 2024 and May/June 2021, 2022, 2024, 2025

22 questions · 283 marks · 8 papers

Each question is a clip of the original DBE page. The wording, the mark allocations and the diagrams are exactly as they were printed - nothing has been retyped or redrawn. Only the blank answer space has been removed, so work in a separate book.

How it is arranged

One chapter per topic, so you can work through a whole skill at a time.

Inside a chapter the questions run from fewest marks to most. That is an order by size, not by difficulty: unlike the IEB, the DBE does not split its papers into an easier and a harder section, and its marking guidelines carry no cognitive-level tags. Nothing in these papers ranks one question as harder than another, so this booklet does not pretend otherwise.

Contents

Differentiating: first principles, the rules, and tangents — 8 questions, 110 marks
11 to 18 marks each

Cubic graphs, and the graph of f' — 9 questions, 136 marks
8 to 23 marks each

Optimisation and rates of change — 5 questions, 37 marks
6 to 8 marks each

Differentiating: first principles, the rules, and tangents

The limit definition written out in full, which DBE asks for in every paper for 5 marks, then the rules applied to whatever awkward form the question chooses - surds, fractions, products multiplied out first. Then the tangent: its gradient is the derivative at that point, and most of these questions end by using that fact backwards to find a missing coefficient.

8 questions · 110 marks

11 to 18 marks each

1. First principles on minus x squared, then two by the rules

DBE NSC Paper 1 · May/June 2022 · Q8 · 11 marks

QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given that $f(x) = -x^2$. (5)

8.2 Determine:

8.2.1 $f'(x)$, if it is given that $f(x) = 4x^3 - 5x^2$ (2)

8.2.2 $D_x\left[\frac{-6\sqrt[3]{x}+2}{x^4}\right]$ (4)

[11]

2. First principles, then differentiating a product of surds

DBE NSC Paper 1 · May/June 2021 · Q8 · 12 marks

QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given that $f(x) = 3x^2$. (5)

8.2 Determine:

8.2.1 $f'(x)$ if $f(x) = x^2 - 3 + \frac{9}{x^2}$ (3)

8.2.2 $g'(x)$ if $g(x) = \left(\sqrt{x}+3\right)\left(\sqrt{x}-1\right)$ (4)

[12]

3. First principles, then dy/dx of a quartic and of a surd

DBE NSC Paper 1 · November 2021 · Q9 · 12 marks

QUESTION 9

9.1 Determine $f'(x)$ from first principles if it is given that $f(x) = 2x^2 - 3x$. (5)

9.2 Determine:

9.2.1 $\frac{dy}{dx}$ if $y = 4x^5 - 6x^4 + 3x$ (3)

9.2.2 $D_x\left[-\frac{\sqrt[3]{x}}{2} + \left(\frac{1}{3x}\right)^2\right]$ (4)

[12]

4. First principles, then the tangent with the least gradient

DBE NSC Paper 1 · November 2022 · Q7 · 12 marks

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = x^2 + x$. (5)

7.2 Determine $f'(x)$ if $f(x) = 2x^5 - 3x^4 + 8x$. (3)

7.3 The tangent to $g(x) = ax^3 + 3x^2 + bx + c$ has a minimum gradient at the point $(-1 ; -7)$. For which values of x will g be concave up? (4)

[12]

5. First principles, then where a tangent's gradient stays positive

DBE NSC Paper 1 · November 2023 · Q7 · 13 marks

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = -4x^2$ (5)

7.2 Determine:

7.2.1 $f'(x)$ if $f(x) = 2x^3 - 3x$ (2)

7.2.2 $D_x\left(7\sqrt[3]{x^2} + 2x^{-5}\right)$ (3)

7.3 For which values of x will the tangent to $f(x) = -2x^3 + 8x$ have a positive gradient? (3)

[13]

6. First principles, then b and c recovered from a given tangent

DBE NSC Paper 1 · May/June 2024 · Q8 · 15 marks

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x)=\frac{1}{x}$. (5)

8.2 Determine:

8.2.1 $\frac{d}{dx}\left(\sqrt{4x^6}+\sqrt{2}.x^2\right)$ (3)

8.2.2 $g'(x)$ if $g(x)=\frac{3x^4-4x^2+6}{x^2}$ (3)

8.3 The equation of the tangent to $f(x)=3x^2+bx+c$ at $x=1$ is given by $y=9x-9$. Determine the values of b and c. (4)

[15]

7. First principles, then a tangent common to two curves

DBE NSC Paper 1 · May/June 2025 · Q8 · 17 marks

QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given that $f(x)=x^2-2$ (5)

8.2 Determine:

8.2.1 $\frac{d}{dx}\left[3x^2-4x\right]$ (2)

8.2.2 $g'(x)$ if $g(x)=-2\sqrt{x}(x-1)^2$ (4)

8.3 Given that $y=4x-14$ is a common tangent to $f(x)=2x^2-4x-6$ and $g(x)=ax^2+bx-18$, calculate the values of a and b. (6)

[17]

8. The rules, a tangent at x = 2, then first principles

DBE NSC Paper 1 · November 2024 · Q8 · 18 marks

QUESTION 8

8.1 Determine:

8.1.1 $\frac{d}{dx}\left[3x-5x^2\right]$ (2)

8.1.2 $g'(x)$ if $g(x)=\frac{2}{x^2}-\sqrt[3]{x^7}$ (4)

8.2 Determine the equation of the tangent to $f(x)=x^3-4x^2+2x+3$ at $x=2$. (3)

8.3 Given: $f(x)=-6x^2$

8.3.1 Determine $f'(x)$ from first principles. (5)

8.3.2 Write down how you will restrict the domain of f such that f^{-1}, the inverse of f, is a function. (1)

8.3.3 Determine the equation of f^{-1} for $f^{-1}(x)\leq 0$. Write your answer in the form $y=\ldots$ (3)

[18]

Cubic graphs, and the graph of f'

A cubic and its derivative, read together. The derivative gives the turning points, its sign gives where the graph rises and falls, and the second derivative gives concavity and the point of inflection. Most questions walk from the equation to the sketch; at least one walks back the other way.

9 questions · 136 marks

8 to 23 marks each

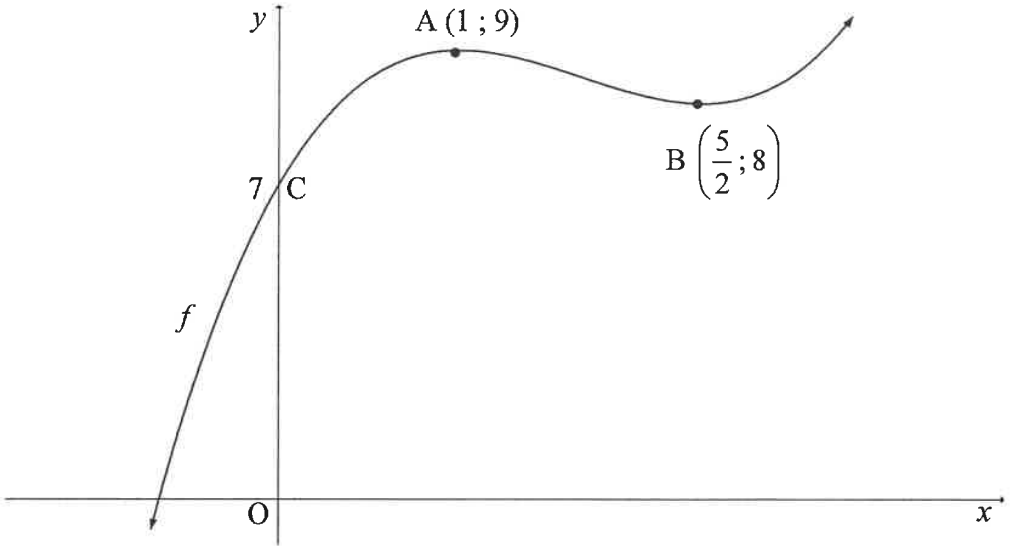
9. Reading f' off the turning points of f

DBE NSC Paper 1 · November 2024 · Q9 · 8 marks

QUESTION 9

A(1 ; 9) and $B\left(\frac{5}{2};8\right)$ are the turning points of graph f below.

C(0 ; 7) is the y-intercept of f.



9.1 For which values of x is f decreasing? (2)

9.2 Write down the x-intercepts of f', the derivative of f. (2)

9.3 For which values of x will f be concave up? (2)

9.4 Determine the value of k for which $y = f(x) + k$ will have THREE positive x-intercepts. (2)

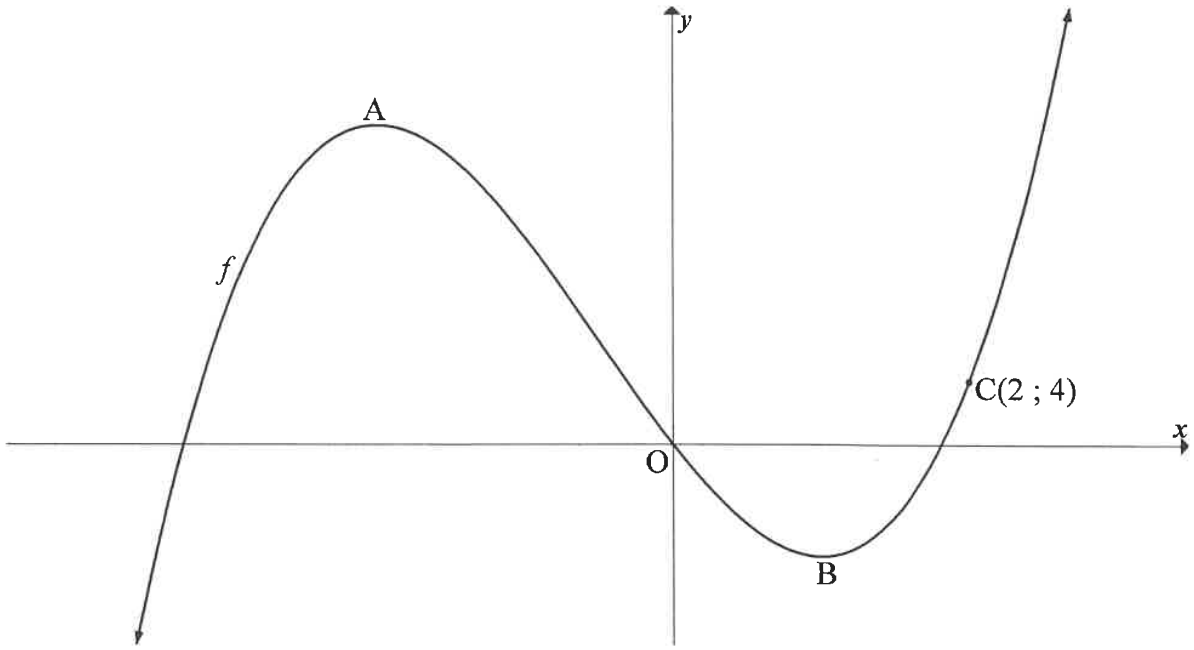
[8]

10. Turning points, then where the cubic is concave up

DBE NSC Paper 1 · May/June 2021 · Q9 · 11 marks

QUESTION 9

The graph of $f(x) = 2x^3 + 3x^2 - 12x$ is sketched below.
A and B are the turning points of f. C(2 ; 4) is a point on f.



9.1 Determine the coordinates of A and B. (5)

9.2 For which values of x will f be concave up? (3)

9.3 Determine the equation of the tangent to f at C(2 ; 4). (3)

[11]

11. Sketch a cubic from four facts, without ever finding its equation

DBE NSC Paper 1 · May/June 2021 · Q10 · 13 marks

QUESTION 10

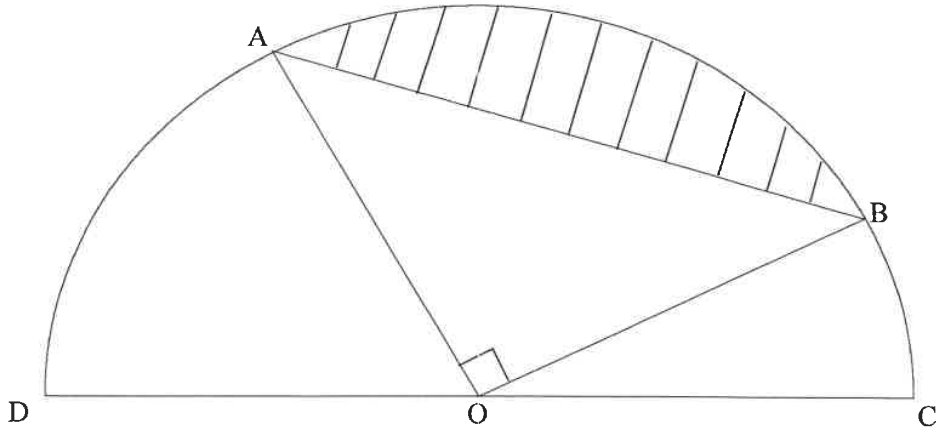
10.1 The graph of $f(x) = ax^3 + bx^2 + cx + d$ has two turning points.

The following information about f is also given:

- $f(2) = 0$
- The x-axis is a tangent to the graph of f at $x = -1$
- $f'(1) = 0$
- $f'\left(\frac{1}{2}\right) > 0$

Without calculating the equation of f, use this information to draw a sketch graph of f, only indicating the x-coordinates of the x-intercepts and turning points. (4)

10.2 O is the centre of a semicircle passing through A, B, C and D. The radius of the semicircle is $\left(x - x^2\right)$ units for $0 < x < 1$. ΔAOB is right-angled at O.



10.2.1 Show that the area of the shaded part is given by:

$$\text{Area} = \left(\frac{\pi - 2}{4}\right)\left(x^4 - 2x^3 + x^2\right)$$

(5)

10.2.2 Determine the value of x for which the shaded area will be a maximum. (4)

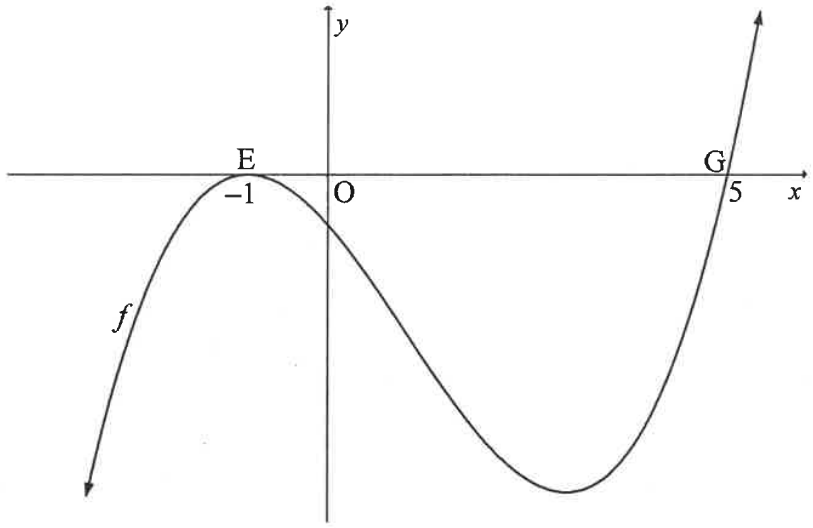
[13]

12. a, b and c from two intercepts, then the local minimum

DBE NSC Paper 1 · May/June 2024 · Q9 · 13 marks

QUESTION 9

The graph of $f(x) = ax^3 + bx^2 + cx - 5$ is drawn below. E(– 1 ; 0) and G(5 ; 0) are the x-intercepts of f.



9.1 Show that $a = 1$, $b = -3$ and $c = -9$. (3)

9.2 Calculate the value of x for which f has a local minimum value. (4)

9.3 Use the graph to determine the values of x for which $f''(x).f(x) > 0$. (3)

9.4 For which values of t will the graph of $p(x) = f(x) + t$ have two distinct positive roots and one negative root? (3)

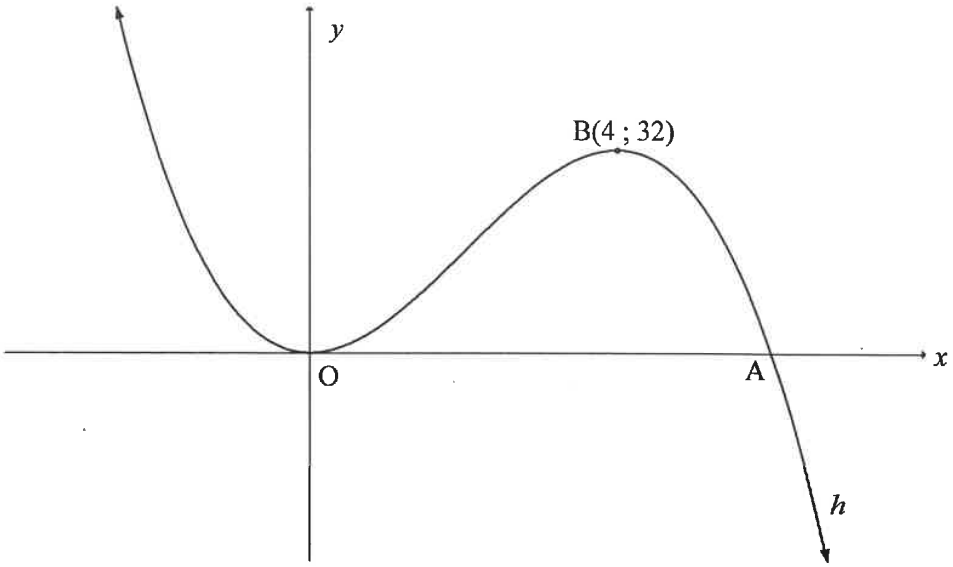
[13]

13. a and b from two turning points, then the x-intercept

DBE NSC Paper 1 · November 2021 · Q10 · 15 marks

QUESTION 10

The graph of $h(x) = ax^3 + bx^2$ is drawn.
The graph has turning points at the origin, O(0 ; 0) and B(4 ; 32).
A is an x-intercept of h.



10.1 Show that $a = -1$ and $b = 6$. (5)

10.2 Calculate the coordinates of A. (3)

10.3 Write down the values of x for which h is:

10.3.1 Increasing (2)

10.3.2 Concave down (2)

10.4 For which values of k will $-(x-1)^3 + 6(x-1)^2 - k = 0$ have one negative and two distinct positive roots? (3)

[15]

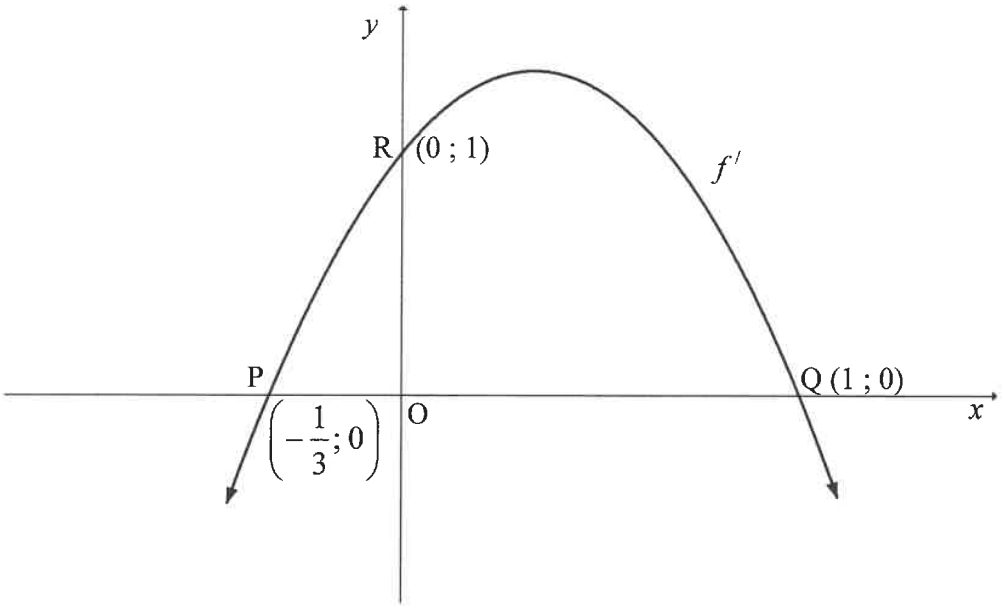
14. Given the graph of f', work back to f

DBE NSC Paper 1 · November 2022 · Q8 · 17 marks

QUESTION 8

The graph of $y = f'(x) = mx^2 + nx + k$ is drawn below.

The graph passes the points $P\left(-\frac{1}{3}; 0\right)$, $Q(1; 0)$ and $R(0; 1)$.



8.1 Determine the values of m, n and k. (6)

8.2 If it is further given that $f(x) = -x^3 + x^2 + x + 2$:

8.2.1 Determine the coordinates of the turning points of f. (3)

8.2.2 Draw the graph of f. Indicate on your graph the coordinates of the turning points and the intercepts with the axes. (5)

8.3 Points E and W are two variable points on f' and are on the same horizontal line.

- h is a tangent to f' at E.
- g is a tangent to f' at W.
- h and g intersect at D(a ; b).

8.3.1 Write down the value of a. (1)

8.3.2 Determine the value(s) of b for which h and g will no longer be tangents to f'. (2)

[17]

15. Turning points, a sketch, then reading answers off it

DBE NSC Paper 1 · November 2023 · Q8 · 18 marks

QUESTION 8

Given: $f(x) = -x^3 + 6x^2 - 9x + 4 = (x-1)^2(-x+4)$

8.1 Determine the coordinates of the turning points of f. (4)

8.2 Draw a sketch graph of f. Clearly label all the intercepts with the axes and any turning points. (4)

8.3 Use the graph to determine the value(s) of k for which $-x^3 + 6x^2 - 9x + 4 = k$ will have three real and unequal roots. (2)

8.4 The line $g(x) = ax + b$ is the tangent to f at the point of inflection of f. Determine the equation of g. (6)

8.5 Calculate the value of θ, the acute angle formed between g and the x-axis in the first quadrant. (2)

[18]

16. Show k = 1, then the turning points, the concavity and the sketch

DBE NSC Paper 1 · May/June 2025 · Q9 · 18 marks

QUESTION 9

Given: $f(x) = x^3 - 6x^2 + 9x - 4 = (x-4)(x-k)^2$

9.1 Show that $k = 1$. (2)

9.2 Calculate the coordinates of the turning points of f. (4)

9.3 Describe the concavity of f at $x = -3$. (2)

9.4 Draw the graph of f. Label ALL turning points and intercepts with the axes. (4)

9.5 Calculate the maximum vertical distance between f and h for $1 < x < 3$, if $h(x) = -2f'(x)$. (6)

[18]

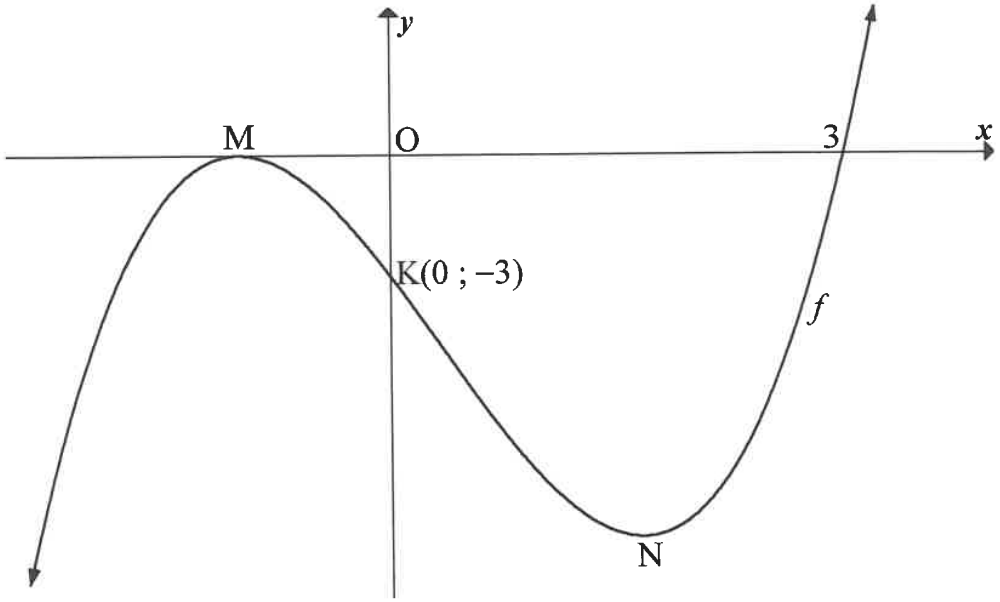
17. A cubic from its intercepts, and everything that then follows

DBE NSC Paper 1 · May/June 2022 · Q9 · 23 marks

QUESTION 9

Sketched below is the graph of $f(x) = x^3 + ax^2 + bx + c$.
The x-intercepts of f are at $(3\ ;\ 0)$ and M, where M lies on the negative x-axis.

K$(0\ ;\ -3)$ is the y-intercept of f.
M and N are the turning points of f.



9.1 Show that the equation of f is given by $f(x) = x^3 - x^2 - 5x - 3$. (5)

9.2 Calculate the coordinates of N. (5)

9.3 For which values of x will:

9.3.1 $f(x) < 0$ (2)

9.3.2 f be increasing (2)

9.3.3 f be concave up (3)

9.4 Determine the maximum vertical distance between the graphs of f and f' in the interval $-1 < x < 0$. (6)

[23]

Optimisation and rates of change

A quantity to be made as large or as small as possible, wrapped in a situation - a poster, a rectangle, the cost of a journey. The differentiation is short and always the same: build the expression, differentiate it, set it to zero. Building the expression is the part worth practising.

5 questions · 37 marks

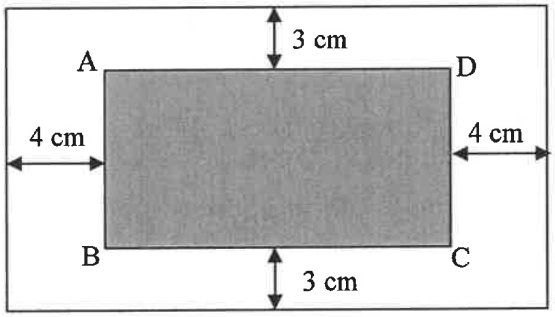
6 to 8 marks each

18. A printed poster, and the size that wastes the least page

DBE NSC Paper 1 · November 2023 · Q9 · 6 marks

QUESTION 9

The diagram below represents a printed poster. Rectangle ABCD is the part on which the text is printed. This shaded area ABCD is $432\ \text{cm}^2$ and $AD = x$ cm. ABCD is 4 cm from the left and right edges of the page and 3 cm from the top and bottom of the page.



9.1 Show that the total area of the page is given by:

$$A(x) = \frac{3\,456}{x} + 6x + 480$$

(3)

9.2 Determine the value of x such that the total area of the page is a minimum. (3)

[6]

19. The speed that keeps a journey's cost as low as possible

DBE NSC Paper 1 · November 2021 · Q11 · 7 marks

QUESTION 11

After travelling a distance of 20 km from home, a person suddenly remembers that he did not close a tap in his garden. He decides to turn around immediately and return home to close the tap.

The cost of the water, at the rate at which water is flowing out of the tap, is R1,60 per hour.

The cost of petrol is $\left(1{,}2 + \frac{x}{4000}\right)$ rands per km, where x is the average speed in km/h.

Calculate the average speed at which the person must travel home to keep his cost as low as possible.

[7]

20. The shortest distance from a point to a parabola

DBE NSC Paper 1 · November 2022 · Q9 · 8 marks

QUESTION 9

Given $f(x) = x^2$.

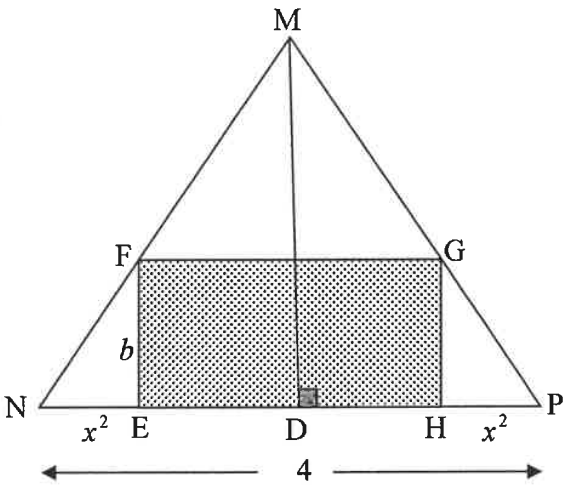
Determine the minimum distance between the point $(10\ ;\ 2)$ and a point on f. **[8]**

21. Show the area is 6x squared minus 3x to the fourth, then maximise it

DBE NSC Paper 1 · May/June 2024 · Q10 · 8 marks

QUESTION 10

EHGF is a rectangle. HE is produced x^2 cm to N and EH is produced x^2 cm to P. NF produced intersects PG produced at M to form an isosceles triangle MNP with NM = MP. D lies on NP where MD $\perp$ NP. NP = 4 cm and MD = 3 cm.



10.1 Show that the area of EFGH is given by $A(x) = 6x^2 - 3x^4$. (4)

10.2 Calculate the maximum area of rectangle EFGH. (4)

[8]

22. A cyclist's speed, given as a rate of change

DBE NSC Paper 1 · November 2024 · Q10 · 8 marks

QUESTION 10

A cyclist rode from town P and stopped at town T. The speed (in km/h) at which this cyclist rode, is represented by the equation $s'(t) = -3t^2 + 18t$.

NOTE: Speed is the rate of change in distance with respect to time.

10.1 Calculate the maximum speed that the cyclist reached on this ride. (3)

10.2 Calculate the distance between town P and town T. (5)

[8]

Index by paper

Every calculus question in each paper, and where it sits in this booklet.

May/June 2021		**3 questions, 36 of 150 marks**	
Q8	First principles, then differentiating a product of surds	12 marks	p3
Q9	Turning points, then where the cubic is concave up	11 marks	p9
Q10	Sketch a cubic from four facts, without ever finding its equation	13 marks	p10
November 2021		**3 questions, 34 of 150 marks**	
Q9	First principles, then dy/dx of a quartic and of a surd	12 marks	p3
Q10	a and b from two turning points, then the x-intercept	15 marks	p12
Q11	The speed that keeps a journey's cost as low as possible	7 marks	p17
May/June 2022		**2 questions, 34 of 150 marks**	
Q8	First principles on minus x squared, then two by the rules	11 marks	p3
Q9	A cubic from its intercepts, and everything that then follows	23 marks	p15
November 2022		**3 questions, 37 of 150 marks**	
Q7	First principles, then the tangent with the least gradient	12 marks	p4
Q8	Given the graph of f', work back to f	17 marks	p13
Q9	The shortest distance from a point to a parabola	8 marks	p18
November 2023		**3 questions, 37 of 150 marks**	
Q7	First principles, then where a tangent's gradient stays positive	13 marks	p4
Q8	Turning points, a sketch, then reading answers off it	18 marks	p14
Q9	A printed poster, and the size that wastes the least page	6 marks	p17
May/June 2024		**3 questions, 36 of 150 marks**	
Q8	First principles, then b and c recovered from a given tangent	15 marks	p5
Q9	a, b and c from two intercepts, then the local minimum	13 marks	p11
Q10	Show the area is 6x squared minus 3x to the fourth, then maximise it	8 marks	p18
November 2024		**3 questions, 34 of 150 marks**	
Q8	The rules, a tangent at x = 2, then first principles	18 marks	p6
Q9	Reading f' off the turning points of f	8 marks	p8
Q10	A cyclist's speed, given as a rate of change	8 marks	p19
May/June 2025		**2 questions, 35 of 150 marks**	
Q8	First principles, then a tangent common to two curves	17 marks	p5
Q9	Show k = 1, then the turning points, the concavity and the sketch	18 marks	p14

Index by topic

A question appears under every topic it uses, so it can be found even when it is filed in another chapter.

Differentiating from first principles — 8 questions

1. First principles on minus x squared, then two by the rules p3
2. First principles, then differentiating a product of surds p3
3. First principles, then dy/dx of a quartic and of a surd p3
4. First principles, then the tangent with the least gradient p4
5. First principles, then where a tangent's gradient stays positive p4
6. First principles, then b and c recovered from a given tangent p5
7. First principles, then a tangent common to two curves p5
8. The rules, a tangent at x = 2, then first principles p6

The rules of differentiation — 8 questions

1. First principles on minus x squared, then two by the rules p3
2. First principles, then differentiating a product of surds p3
3. First principles, then dy/dx of a quartic and of a surd p3
4. First principles, then the tangent with the least gradient p4
5. First principles, then where a tangent's gradient stays positive p4
6. First principles, then b and c recovered from a given tangent p5
7. First principles, then a tangent common to two curves p5
8. The rules, a tangent at x = 2, then first principles p6

Tangents to a curve — 5 questions

4. First principles, then the tangent with the least gradient p4
5. First principles, then where a tangent's gradient stays positive p4
6. First principles, then b and c recovered from a given tangent p5
7. First principles, then a tangent common to two curves p5
8. The rules, a tangent at x = 2, then first principles p6

The cubic graph — 8 questions

10. Turning points, then where the cubic is concave up p9
11. Sketch a cubic from four facts, without ever finding its equation p10
12. a, b and c from two intercepts, then the local minimum p11
13. a and b from two turning points, then the x-intercept p12
14. Given the graph of f', work back to f p13
15. Turning points, a sketch, then reading answers off it p14
16. Show k = 1, then the turning points, the concavity and the sketch p14
17. A cubic from its intercepts, and everything that then follows p15

The graph of f' — 2 questions

9. Reading f' off the turning points of f p8
14. Given the graph of f', work back to f p13

Turning points — 8 questions

9. Reading f' off the turning points of f p8
10. Turning points, then where the cubic is concave up p9
11. Sketch a cubic from four facts, without ever finding its equation p10
12. a, b and c from two intercepts, then the local minimum p11
13. a and b from two turning points, then the x-intercept p12
15. Turning points, a sketch, then reading answers off it p14
16. Show k = 1, then the turning points, the concavity and the sketch p14

17. A cubic from its intercepts, and everything that then follows p15

Concavity and the point of inflection — 5 questions

10. Turning points, then where the cubic is concave up p9
13. a and b from two turning points, then the x-intercept p12
14. Given the graph of f', work back to f p13
16. Show k = 1, then the turning points, the concavity and the sketch p14
17. A cubic from its intercepts, and everything that then follows p15

Sketching the graph — 5 questions

11. Sketch a cubic from four facts, without ever finding its equation p10
12. a, b and c from two intercepts, then the local minimum p11
15. Turning points, a sketch, then reading answers off it p14
16. Show k = 1, then the turning points, the concavity and the sketch p14
17. A cubic from its intercepts, and everything that then follows p15

Optimisation: maximum and minimum — 5 questions

18. A printed poster, and the size that wastes the least page p17
19. The speed that keeps a journey's cost as low as possible p17
20. The shortest distance from a point to a parabola p18
21. Show the area is 6x squared minus 3x to the fourth, then maximise it p18
22. A cyclist's speed, given as a rate of change p19

Rates of change — 1 question

22. A cyclist's speed, given as a rate of change p19

Where these came from

So that "every calculus question" is a claim you can check.

Eight DBE NSC Mathematics Paper 1 papers were read end to end:
November 2021, 2022, 2023, 2024 and May/June 2021, 2022, 2024, 2025.

All 338 questions in them were classified. These papers are scans - they carry no text layer at all - so the structure was rebuilt by reading them with OCR, and then checked three ways: every paper adds back up to exactly 150, the Afrikaans paper matches the English question for question, and the official marking guideline, which IS ordinary text, agrees with the marks read off every question.

DBE publishes an official marking guideline for every one of these papers. The companion file "CAPS_P1_Calculus_Solutions.pdf" carries the guideline for each question here, in the same order and under the same numbers.