DIFFERENTIAL CALCULUS

Solutions

DBE's own marking guidelines, in booklet order

Every solution here is a clip of the official DBE marking guideline published with the paper the question came from. The working, the mark ticks and the accepted alternatives are DBE's. Nothing has been rewritten or re-derived.

The numbers match the question booklet: solution 34 answers question 34, and the chapters are in the same order.

DBE writes one guideline for both languages, so these pages are bilingual and their marking notes are largely in English. That is DBE's own page, reproduced as printed.

A marking guideline is written for markers, not learners. It shows the shortest route to the marks, so where a step looks compressed, that is the guideline being terse rather than a step missing.

1. First principles on minus x squared, then two by the rules

Official DBE marking guideline · May/June 2022 · Q8 · 11 marks

QUESTION/*VRAAG* 8

8.1	$f(x)=-x^2$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-(x+h)^2+x^2}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-x^2-2xh-h^2+x^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{-2xh-h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(-2x-h)}{h}$ $=\lim\limits_{h\to 0}(-2x-h)$ $\therefore f'(x)=-2x$	✓ substitution into formula ✓ $-(x^2+2xh+h^2)$ ✓ $-2xh-h^2$ ✓ $-2x-h$ ✓answer (5)
	OR/*OF* $f(x)=-x^2$ $f(x+h)=-(x+h)^2=-x^2-2xh-h^2$ $f(x+h)-f(x)=-x^2-2xh-h^2-(-x^2)=-2xh-h^2$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{-2xh-h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(-2x-h)}{h}$ $=\lim\limits_{h\to 0}(-2x-h)$ $\therefore f'(x)=-2x$	**OR/*OF*** ✓ $-x^2-2xh-h^2$ ✓ $-2xh-h^2$ ✓substitution into the formula ✓ $-2x-h$ ✓answer (5)
8.2.1	$f(x)=4x^3-5x^2$ $f'(x)=12x^2-10x$	✓ $12x^2$ (A) ✓ $-10x$ (A) (2)
8.2.2	$D_x\left[\dfrac{-6\sqrt[3]{x}+2}{x^4}\right]$ $=D_x\left[\dfrac{-6(x)^{\frac{1}{3}}}{x^4}+\dfrac{2}{x^4}\right]$ $=D_x\left[-6x^{-\frac{11}{3}}+2x^{-4}\right]$ $=22x^{-\frac{14}{3}}-8x^{-5}$	✓ $x^{\frac{1}{3}}$ ✓ $-6x^{-\frac{11}{3}}+2x^{-4}$ ✓ $22x^{-\frac{14}{3}}$ ✓ $-8x^{-5}$ (4)
		[11]

2. First principles, then differentiating a product of surds

Official DBE marking guideline · May/June 2021 · Q8 · 12 marks

QUESTION/*VRAAG* 8

8.1	$f(x)=3x^2$ $f^{/}(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f^{/}(x)=\lim_{h\to 0}\dfrac{3(x+h)^2-3x^2}{h}$ $f^{/}(x)=\lim_{h\to 0}\dfrac{3x^2+6xh+3h^2-3x^2}{h}$ $=\lim_{h\to 0}\dfrac{6xh+3h^2}{h}$ $=\lim_{h\to 0}\dfrac{h(6x+3h)}{h}$ $=6x$	✓ substitution ✓ expansion ✓ simplification ✓ $\lim_{h\to 0}\dfrac{h(6x+3h)}{h}$ ✓$6x$ (5)
8.2.1	$f(x)=x^2-3+9x^{-2}$ $f^{/}(x)=2x-18x^{-3}$	✓$9x^{-2}$ ✓$2x$ ✓$-18x^{-3}$ (3)
8.2.2	$g(x)=(\sqrt{x}+3)(\sqrt{x}-1)$ $g(x)=x+2x^{\frac{1}{2}}-3$ $g^{/}(x)=1+x^{-\frac{1}{2}}$	✓ x ✓ $2x^{\frac{1}{2}}$ ✓1 ✓ $x^{-\frac{1}{2}}$ (4)
		[12]

3. First principles, then dy/dx of a quartic and of a surd

Official DBE marking guideline · November 2021 · Q9 · 12 marks

QUESTION/VRAAG 9

9.1	$f^{/}(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f^{/}(x)=\lim_{h\to 0}\frac{2(x+h)^2-3(x+h)-(2x^2-3x)}{h}$ $f^{/}(x)=\lim_{h\to 0}\frac{2x^2+4xh+2h^2-3x-3h-2x^2+3x}{h}$ $=\lim_{h\to 0}\frac{4xh+2h^2-3h}{h}$ $=\lim_{h\to 0}\frac{h(4x+2h-3)}{h}$ $=\lim_{h\to 0}(4x+2h-3)$ $\therefore f^{/}(x)=4x-3$	✓substitution ✓ $2x^2+4xh+2h^2-3x-3h$ ✓ $4xh+2h^2-3h$ ✓factorisation ✓answer (5)
	OR/OF	**OR/OF**
	$f(x)=2x^2-3x$ $f(x+h)=2(x+h)^2-3(x+h)$ $f(x+h)=2x^2+4xh+2h^2-3x-3h$ $f(x+h)-f(x)=4xh+2h^2-3h$ $f^{/}(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{4xh+2h^2-3h}{h}$ $=\lim_{h\to 0}\frac{h(4x+2h-3)}{h}$ $=\lim_{h\to 0}(4x+2h-3)$ $\therefore f^{/}(x)=4x-3$	✓substitution ✓ $2x^2+4xh+2h^2-3x-3h$ ✓ $4xh+2h^2-3h$ ✓factorisation ✓answer (5)
9.2.1	$y=4x^5-6x^4+3x$ $\frac{dy}{dx}=20x^4-24x^3+3$	✓ $20x^4$ ✓ $-24x^3$ ✓3 (3)

9.2.2	$D_x\left[\dfrac{-\sqrt[3]{x}}{2}+\left(\dfrac{1}{3x}\right)^2\right]$ $D_x\left[\dfrac{-x^{\frac{1}{3}}}{2}+\dfrac{x^{-2}}{9}\right]$ $D_x\left[-\dfrac{1}{2}x^{\frac{1}{3}}+\dfrac{1}{9}x^{-2}\right]$ $=-\dfrac{1}{6}x^{-\frac{2}{3}}-\dfrac{2x^{-3}}{9}$ $=-\dfrac{1}{6x^{\frac{2}{3}}}-\dfrac{2}{9x^3}$	✓ $\dfrac{-x^{\frac{1}{3}}}{2}$ ✓ $\dfrac{x^{-2}}{9}$ ✓ $-\dfrac{1}{6}x^{-\frac{2}{3}}$ ✓ $-\dfrac{2x^{-3}}{9}$ (4)
		[12]

4. First principles, then the tangent with the least gradient

Official DBE marking guideline · November 2022 · Q7 · 12 marks

QUESTION 7/*VRAAG* 7

7.1	$f(x)=x^2+x$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{(x+h)^2+(x+h)-(x^2+x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{x^2+2xh+h^2+x+h-x^2-x}{h}$ $=\lim_{h\to 0}\frac{2xh+h^2+h}{h}$ $=\lim_{h\to 0}\frac{h(2x+h+1)}{h}$ $\therefore f'(x)=2x+1$	✓substitution into the formula ✓ $x^2+2xh+h^2+x+h$ ✓ $2xh+h^2+h$ ✓common factor ✓answer (5)
	OR/*OF* $f(x)=x^2+x$ $f(x+h)=(x+h)^2+(x+h)=x^2+2xh+h^2+x+h$ $f(x+h)-f(x)=x^2+2xh+h^2+x+h-x^2-x$ $=2xh+h^2+h$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{2xh+h^2+h}{h}$ $=\lim_{h\to 0}\frac{h(2x+h+1)}{h}$ $\therefore f'(x)=2x+1$	**OR/*OF*** ✓ $x^2+2xh+h^2+x+h$ ✓ $2xh+h^2+h$ ✓substitution into the formula ✓common factor ✓answer (5)
7.2	$f(x)=2x^5-3x^4+8x$ $f'(x)=10x^4-12x^3+8$	✓ $10x^4$ ✓ $-12x^3$ ✓ 8 (3)
7.3	$g(x)=ax^3+3x^2+bx+c$ $g'(x)=3ax^2+6x+b$ $g''(x)=6ax+6$ $g''(-1)=6a(-1)+6=0$ $\therefore a=1$ For concave up $g''(x)>0$ $6x+6>0$ $x>-1$	✓ $g'(x)=3ax^2+6x+b$ ✓ $g''(-1)=6a(-1)+6=0$ ✓ $a=1$ ✓ $x>-1$ (4)

OR/OF

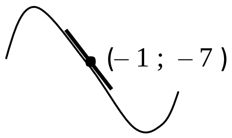
Min gradient at (– 1 ; – 7) implies:

at $x = -1$ - point of inflection
and g will be positive cubic
hence $a > 0$

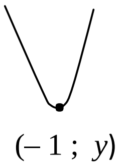
Since g is concave up

$x > -1$

OR/OF



Since g is concave up
$x > -1$



Since g is concave up
$x > -1$

Answer only: $\frac{1}{4}$

OR/OF

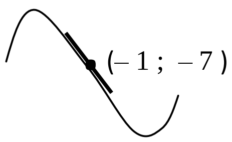
Min gradient at (– 1 ; – 7) implies:

at $x = -1$ - point of inflection
and g will be positive cubic
hence $a > 0$

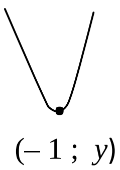
Since g is concave up

$x > -1$

OR/OF

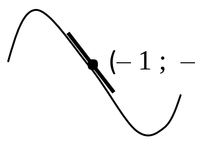


Since g is concave up
$x > -1$



Since g is concave up
$x > -1$

Answer only: $\frac{1}{4}$

	OR/OF Min gradient at (– 1 ; – 7) implies: at $x = -1$ - point of inflection and g will be positive cubic hence $a > 0$ Since g is concave up $x > -1$ **OR/OF**  Since g is concave up $x > -1$  Since g is concave up $x > -1$ Answer only: $\frac{1}{4}$	**OR/OF** ✓ point of inflection ✓✓ $a > 0$ ✓ $x > -1$ (4) **OR/OF** ✓✓pos graph ✓point of inflection ✓ $x > -1$ (4)
		[12]

5. First principles, then where a tangent's gradient stays positive

Official DBE marking guideline · November 2023 · Q7 · 13 marks

QUESTION 7/*VRAAG* 7

7.1	$f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\dfrac{-4(x+h)^2-(-4x^2)}{h}$ $f'(x)=\lim_{h\to 0}\dfrac{-4x^2-8xh-4h^2+4x^2}{h}$ $f'(x)=\lim_{h\to 0}\dfrac{-8xh-4h^2}{h}$ $f'(x)=\lim_{h\to 0}\dfrac{h(-8x-4h)}{h}$ $f'(x)=\lim_{h\to 0}(-8x-4h)$ $f'(x)=-8x$ **OR/*OF*** $f(x+h)=-4(x+h)^2=-4x^2-8xh-4h^2$ $f(x+h)-f(x)=-4x^2-8xh-4h^2-(-4x^2)$ $=-8xh-4h^2$ $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\dfrac{-8xh-4h^2}{h}$ $f'(x)=\lim_{h\to 0}\dfrac{h(-8x-4h)}{h}$ $f'(x)=\lim_{h\to 0}(-8x-4h)$ $f'(x)=-8x$	✓substitution into correct formula ✓ $f(x+h)=-4x^2-8xh-4h^2$ ✓simplification ✓common factor ✓answer (5) **OR/*OF*** ✓ $f(x+h)=-4x^2-8xh-4h^2$ ✓simplification ✓substitution into correct formula ✓common factor ✓answer (5)
7.2.1	$f(x)=2x^3-3x$ $f'(x)=6x^2-3$	✓ $6x^2$ ✓ -3 (2)
7.2.2	$D_x\left[7.\sqrt[3]{x^2}+2x^{-5}\right]$ $D_x\left[7.x^{\frac{2}{3}}+2x^{-5}\right]$ $=\dfrac{14}{3}x^{-\frac{1}{3}}-10x^{-6}$	✓ $x^{\frac{2}{3}}$ ✓derivative with rational exp ✓ $-10x^{-6}$ (3)
7.3	$-6x^2+8>0$ $x^2<\dfrac{8}{6}$ CV's: $x=-\dfrac{2}{\sqrt{3}}$ or $x=\dfrac{2}{\sqrt{3}}$ Positive for : $-\dfrac{2}{\sqrt{3}}<x<\dfrac{2}{\sqrt{3}}$	✓CV's: $x=\pm\dfrac{2}{\sqrt{3}}$ ✓✓answer (3)
		[13]

6. First principles, then b and c recovered from a given tangent

Official DBE marking guideline · May/June 2024 · Q8 · 15 marks

QUESTION8/*VRAAG* 8

8.1	$f(x)=\frac{1}{x}$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{\frac{1}{(x+h)}-\frac{1}{x}}{h}$ $f'(x)=\lim_{h\to 0}\frac{x-(x+h)}{x(x+h)}\times\frac{1}{h}$ $f'(x)=\lim_{h\to 0}\frac{-h}{x(x+h)}\times\frac{1}{h}$ $f'(x)=\lim_{h\to 0}\frac{-1}{x(x+h)}$ $f'(x)=-\frac{1}{x^2}$	✓ $f(x+h)=\frac{1}{x+h}$ ✓ $\frac{x-(x+h)}{x(x+h)}\times\frac{1}{h}$ ✓ $\frac{-h}{x(x+h)}\times\frac{1}{h}$ ✓ $\frac{-1}{x(x+h)}$ ✓answer (5)
	OR/*OF*	**OR/*OF***
	$f(x)=\frac{1}{x}$ $f(x+h)=\frac{1}{x+h}$ $f(x+h)-f(x)=-\frac{h}{x(x+h)}$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{-h}{x(x+h)}\times\frac{1}{h}$ $f'(x)=\lim_{h\to 0}\frac{-1}{x(x+h)}$ $f'(x)=-\frac{1}{x^2}$	✓ $f(x+h)=\frac{1}{x+h}$ ✓ $f(x+h)-f(x)=-\frac{h}{x(x+h)}$ ✓ $\frac{-h}{x(x+h)}\times\frac{1}{h}$ ✓ $\frac{-1}{x(x+h)}$ ✓answer (5)

8.2.1	$\frac{d}{dx}\left(\sqrt{4x^6}+\sqrt{2}.x^2\right)$ $=\frac{d}{dx}\left(2x^3+\sqrt{2}.x^2\right)$ $=6x^2+2\sqrt{2}x$	✓ $2x^3$ ✓ $6x^2$ ✓ $2\sqrt{2}x$ (3)
8.2.2	$g(x)=\frac{3x^4-4x^2+6}{x^2}$ $g(x)=3x^2-4+6x^{-2}$ $g^/(x)=6x-12x^{-3}$	✓ $3x^2-4+6x^{-2}$ ✓ $6x$ ✓ $-12x^{-3}$ (3)
8.3	$f(x)=3x^2+bx+c$ $f^/(x)=6x+b$ $f^/(1)=6+b=9$ $\therefore b=3$ $f(1)=3+3+c=0$ $c=-6$ $\therefore\ f(x)=3x^2+3x-6$	✓ $f^/(1)=6+b=9$ ✓ $b=3$ ✓ $f(1)=3+3+c=0$ ✓ $c=-6$ (4)
		[15]

7. First principles, then a tangent common to two curves

Official DBE marking guideline · May/June 2025 · Q8 · 17 marks

QUESTION/*VRAAG* 8

8.1	$f(x)=x^2-2$ $f^{/}(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f^{/}(x)=\lim_{h\to 0}\frac{x^2+2xh+h^2-2-(x^2-2)}{h}$ $f^{/}(x)=\lim_{h\to 0}\frac{2xh+h^2}{h}$ $f^{/}(x)=\lim_{h\to 0}\frac{h(2x+h)}{h}$ $f^{/}(x)=\lim_{h\to 0}(2x+h)$ $\therefore f^{/}(x)=2x$ **OR/*OF*** $f(x)=x^2-2$ $f(x+h)=(x+h)^2-2=x^2+2xh+h^2-2$ $f(x+h)-f(x)=2xh+h^2$ $f^{/}(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f^{/}(x)=\lim_{h\to 0}\frac{2xh+h^2}{h}$ $f^{/}(x)=\lim_{h\to 0}\frac{h(2x+h)}{h}$ $f^{/}(x)=\lim_{h\to 0}(2x+h)$ $\therefore f^{/}(x)=2x$	✓ $f(x+h)$ ✓ substitution ✓ simplification ✓ factorisation ✓ answer (5) **OR/*OF*** ✓ $f(x+h)$ ✓ simplification ✓ substitution ✓ factorisation ✓ answer (5)
8.2.1	$\frac{d}{dx}\left[3x^2-4x\right]$ $=6x-4$	✓$6x$ ✓ -4 (2)
8.2.2	$g(x)=-2\sqrt{x}(x-1)^2$ $g(x)=-2x^{\frac{1}{2}}(x^2-2x+1)$ $g(x)=-2x^{\frac{5}{2}}+4x^{\frac{3}{2}}-2x^{\frac{1}{2}}$ $g^{/}(x)=-5x^{\frac{3}{2}}+6x^{\frac{1}{2}}-x^{\frac{-1}{2}}$	✓ $x^{\frac{1}{2}}$ ✓ expansion ✓ $-5x^{\frac{3}{2}}+6x^{\frac{1}{2}}$ ✓ $-x^{\frac{-1}{2}}$ (4)

8.3	$y = 4x - 14$ tangent $\therefore 4x - 4 = 4$ $4x = 8$ $x = 2$ $\therefore y = 4(2) - 14$ at point $(2\ ;-6)$ $g(2) = a(2)^2 + b(2) - 18 = -6$ $4a + 2b = 12 \quad ...(1)$ $g^{/}(2) = 2a(2) + b = 4$ $4a + b = 4 \quad ...(2)$ $(1) - (2)$ $b = 8$ $4a + b = 4$ $4a + 8 = 4$ $4a = -4$ $a = -1$ **OR/OF** $2x^2 - 4x - 6 = 4x - 14$ $2x^2 - 8x + 8 = 0$ $x^2 - 4x + 4 = 0$ $(x-2)^2 = 0$ $\therefore x = 2$ $\therefore y = 4(2) - 14$ at point $(2\ ;-6)$ $g(2) = a(2)^2 + b(2) - 18 = -6$ $4a + 2b = 12 \quad ...(1)$ $g^{/}(2) = 2a(2) + b = 4$ $4a + b = 4 \quad ...(2)$ $(1) - (2) \quad b = 8$ $4a + b = 4$ $4a + 8 = 4$ $4a = -4$ $a = -1$	✓ $4x - 4$ ✓ $= 4$ ✓ y-value ✓ $g(2) = y -$ value ✓ $g^{/}(2) = 4$ ✓ a and b (6) **OR/OF** ✓equating ✓ y-value ✓ $g(2) = y -$ value ✓ $g^{/}(x)$ ✓ $g^{/}(2) = 4$ ✓ a and b (6)
		[17]

8. The rules, a tangent at x = 2, then first principles

Official DBE marking guideline · November 2024 · Q8 · 18 marks

QUESTION/*VRAAG* 8

8.1.1	$\frac{d}{dx}\left[3x-5x^2\right]$ $=3-10x$	✓3 ✓$-10x$ (2)
8.1.2	$g(x)=\frac{2}{x^2}-\sqrt[3]{x^7}$ $g(x)=2x^{-2}-x^{\frac{7}{3}}$ $g^{/}(x)=-4x^{-3}-\frac{7}{3}x^{\frac{4}{3}}$	✓ $2x^{-2}$ ✓ $-x^{\frac{7}{3}}$ ✓ $-4x^{-3}$ ✓ $-\frac{7}{3}x^{\frac{4}{3}}$ (4)
8.2	$f(x)=x^3-4x^2+2x+3$ $f^{/}(x)=3x^2-8x+2$ $m=f^{/}(2)=3(2)^2-8(2)+2$ $m=-2$ $y=f(2)=(2)^3-4(2)^2+2(2)+3$ $y=-1$ $y+1=-2(x-2)$ $y=-2x+3$	✓ $m=-2$ ✓ $y=-1$ ✓ answer (3)
8.3.1	$f(x)=-6x^2$ $f^{/}(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{-6(x+h)^2+6x^2}{h}$ $=\lim_{h\to 0}\frac{-6x^2-12xh-6h^2+6x^2}{h}$ $=\lim_{h\to 0}\frac{-12xh-6h^2}{h}$ $=\lim_{h\to 0}\frac{h(-12x-6h)}{h}$ $=\lim_{h\to 0}(-12x-6h)$ $=-12x$	✓substitution ✓ $f(x+h)=-6x^2-12xh-6h^2$ ✓simplification ✓common factor ✓answer (5)

	OR/*OF* $f(x)=-6x^2$ $f(x+h)=-6x^2-12xh-6h^2$ $f(x+h)-f(x)=-6x^2-12xh-6h^2+6x^2=-12xh-6h^2$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{-12xh-6h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(-12x-6h)}{h}$ $=\lim\limits_{h\to 0}(-12x-6h)$ $=-12x$	**OR/*OF*** ✓ $f(x+h)=-6x^2-12xh-6h^2$ ✓simplification ✓substitution ✓common factor ✓answer (5)
8.3.2	$x\geq 0$ **OR/*OF*** $x\leq 0$	✓answer (1) ✓answer (1)
8.3.3	$y=-6x^2$ $x=-6y^2$ $y^2=\dfrac{-1}{6}x$ $y=\pm\sqrt{-\dfrac{1}{6}x}$ $\therefore y=-\sqrt{-\dfrac{1}{6}x}\quad ;\quad x\leq 0$	✓ swopping x and y ✓ $y=\pm\sqrt{-\dfrac{1}{6}x}$ ✓ answer (3)
		[18]

9. Reading f' off the turning points of f

Official DBE marking guideline · November 2024 · Q9 · 8 marks

QUESTION/*VRAAG* 9

9.1	$1 < x < \frac{5}{2}$	✓✓answer (2)
9.2	$(1\,;\,0)$ and $\left(\frac{5}{2}\,;0\right)$	✓ $x = 1$ ✓ $x = \frac{5}{2}$ (2)
9.3	$\dfrac{\frac{5}{2}+1}{2}$ $= \frac{7}{4}$ Concave up for $x > \frac{7}{4}$	✓method ✓answer (2)
9.4	$-9 < k < -8$	✓✓answer (2)
		[8]

10. Turning points, then where the cubic is concave up

Official DBE marking guideline · May/June 2021 · Q9 · 11 marks

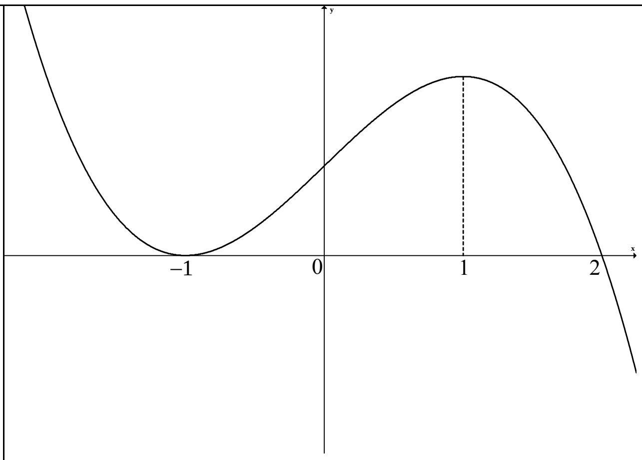
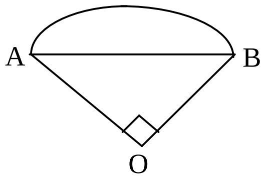
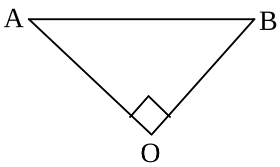
QUESTION/*VRAAG* 9

9.1	$f^{/}(x)=6x^2+6x-12$ $6x^2+6x-12=0$ $x^2+x-2=0$ $(x+2)(x-1)=0$ $x=-2$ or $x=1$ $y=20$ or $y=-7$ $\therefore$ A(–2 ; 20) and B(1 ; –7)	✓ $6x^2+6x-12$ ✓ = 0 ✓factors ✓ x -values ✓ y -values (5)
9.2	$f^{//}(x)=12x+6$ $12x+6>0$ $12x>-6$ $x>-\frac{1}{2}$ **OR/OF** $x=\frac{-2+1}{2}=-\frac{1}{2}$ $\therefore x>-\frac{1}{2}$	✓$12x+6$ ✓ $f^{//}(x)>0$ ✓ $x>-\frac{1}{2}$ (3) **OR/OF** ✓ $x=-\frac{1}{2}$ ✓✓ $x>-\frac{1}{2}$ (3)
9.3	$f^{/}(2)=24$ Equation of the tangent: $y-4=24(x-2)$ $y=24x-44$	✓ $f^{/}(2)$ ✓ 24 ✓ equation (3)
		[11]

11. Sketch a cubic from four facts, without ever finding its equation

Official DBE marking guideline · May/June 2021 · Q10 · 13 marks

QUESTION/*VRAAG* 10

10.1		✓ $x = -1$ and $x = 2$ ✓ TP at $x = -1$ ✓ TP at $x = 1$ ✓ shape (4)
10.2.1	 Area of segment = $\frac{1}{4}$ Area of big circle $= \frac{1}{4}\pi\ (x - x^2)^2$  Area triangle ABO counted $=$ Area $\Delta = \frac{1}{2}\left(x - x^2\right)^2$ Area of shaded region $= \frac{1}{4}\pi(x - x^2)^2 - \frac{1}{2}\left(x - x^2\right)^2$ $= \frac{\pi - 2}{4}\left(x - x^2\right)^2$ $= \left(\frac{\pi - 2}{4}\right)\left(x^2 - 2x^3 + x^4\right)$	✓✓ $\frac{1}{4}\pi(x - x^2)^2$ ✓ Area $\Delta = \frac{1}{2}\left(x - x^2\right)^2$ ✓ subtract areas ✓ common factor (5)

10.2.2	Area of shaded region $=\frac{(\pi-2)}{4}\left(x^4-2x^3+x^2\right)$ $\frac{dA}{dx}=\left(\frac{\pi-2}{4}\right)\left(4x^3-6x^2+2x\right)$ $4x^3-6x^2+2x=0$ $x(2x^2-3x+1)=0$ $x(2x-1)(x-1)=0$ $x\neq 0$ or $x=\frac{1}{2}$ or $x\neq 1$	✓ $\left(\frac{\pi-2}{4}\right)\left(4x^3-6x^2+2x\right)$ ✓ factors ✓ $x=0;x=1;x=\frac{1}{2}$ ✓ $x=\frac{1}{2}$ (4)
		[13]

12. a, b and c from two intercepts, then the local minimum

Official DBE marking guideline · May/June 2024 · Q9 · 13 marks

QUESTION9/*VRAAG* 9

9.1	$f(x) = ax^3 + bx^2 + cx - 5$ $-5 = a(0+1)^2(0-5)$ $-5 = -5a$ $a = 1$ $f(x) = (x+1)(x+1)(x-5)$ $f(x) = (x^2 + 2x + 1)(x-5)$ $f(x) = x^3 - 3x^2 - 9x - 5$ $\therefore\ b = -3$ and $c = -9$	✓ substitution of x-intercepts ✓ simplification ✓ simplification (3)
9.2	$f(x) = x^3 - 3x^2 - 9x - 5$ $f'(x) = 3x^2 - 6x - 9$ $x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$ $x = 3$ or $x = -1$ Minimum value at $x = 3$	✓ $f'(x) = 3x^2 - 6x - 9$ ✓ $f'(x) = 0$ ✓factors ✓ $x = 3$ (4)
9.3	$f''(x).f(x) > 0$ Point of inflection: $x = 1$ $x < 1$; $x \neq -1$ or $x > 5$	✓ $x = 1$ ✓ $x < 1$; $x \neq -1$ ✓ $x > 5$ (3)
9.4	$-32 < -t < -5$ $5 < t < 32$ **OR/*OF*** Shift up more than 5 units and less than 32 units $\therefore 5 < t < 32$	✓-32 ✓$-32 < -t < -5$ ✓ $5 < t < 32$ (3) **OR/*OF*** ✓more than 5 units ✓less than 32 units ✓ $5 < t < 32$ (3)
		[13]

13. a and b from two turning points, then the x-intercept

Official DBE marking guideline · November 2021 · Q10 · 15 marks

QUESTION/VRAAG 10

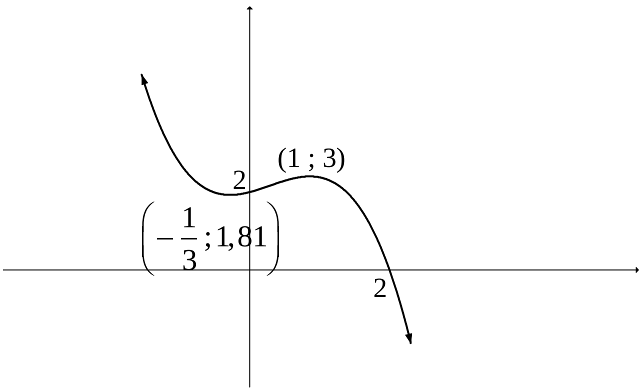
10.1	$h(x)=ax^3+bx^2$ $h^{/}(x)=3ax^2+2bx$ $h^{/}(4)=0$ $48a+8b=0$ $6a+b=0$ ...(1) $h(4)=32$ $64a+16b=32$ $4a+b=2$ ...(2) (1)– (2): $6a+b=0$ $4a+b=2$ $2a=-2$ $a=-1$ $4(-1)+b=2$ $b=6$	✓ $h^{/}(x)$ ✓ $h^{/}(4)=0$ ✓ $48a+8b=0$ or $6a+b=0$ ✓ $h(4)=32$ ✓ $64a+16b=32$ or $4a+b=2$ (5)
10.2	$h(x)=-x^3+6x^2$ $-x^3+6x^2=0$ $x^2(-x+6)=0$ $x=0$ or $x=6$ $\therefore$ A$(6\,;0)$	✓ $-x^3+6x^2=0$ ✓ factors ✓ A$(6\,;0)$ (3)
10.3.1	$0<x<4$ or $0\le x\le 4$ **OR/OF** $x\in(0\,;4)$ or $x\in[0\,;4]$	✓critical values ✓notation (2) **OR/OF** ✓critical values ✓notation (2)
10.3.2	$x>2$ **OR/OF** $x\in(2\,;\infty)$	✓ 2 ✓ notation (2) **OR/OF** ✓ 2 ✓ notation (2)
10.4	$f(x)=h(x-1)=-(x-1)^3+6(x-1)^2$ $f(0)=7$ $7<k<32$ or $k\in(7\,;32)$	✓ $k<32$ ✓ new y-intercept = 7 ✓ $7<k<32$ (3)
		[15]

14. Given the graph of f', work back to f

Official DBE marking guideline · November 2022 · Q8 · 17 marks

QUESTION 8/*VRAAG* 8

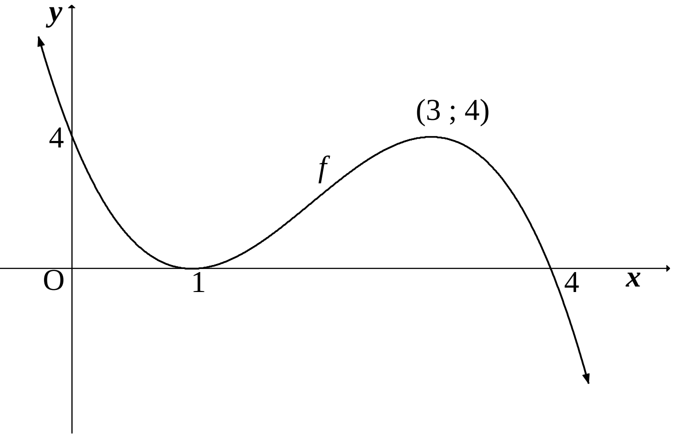
8.1	$f^{/}(x) = mx^2 + nx + k$ $f^{/}(x) = m\left(x + \frac{1}{3}\right)(x-1)$ $1 = m\left(0 + \frac{1}{3}\right)(0-1)$ $1 = -\frac{1}{3}m$ $\therefore m = -3$ $f^{/}(x) = -3\left(x + \frac{1}{3}\right)(x-1)$ $f^{/}(x) = -3\left(x^2 - \frac{2}{3}x - \frac{1}{3}\right)$ $f^{/}(x) = -3x^2 + 2x + 1$ $\therefore n = 2$ $\therefore k = 1$ **OR/OF** $k = 1$ $0 = m + n + 1$ and $\frac{1}{9}m - \frac{1}{3}n + 1 = 0$ $m + n = -1$ (1) $m - 3n = -9$ (2) (1) – (2) $4n = 8$ $\therefore n = 2$ $m + 2 = -1$ $\therefore m = -3$	✓ substitution of $\left(-\frac{1}{3}\ ;\ 0\right)$ and (1 ; 0) ✓ substitution of (0 ; 1) ✓ $m = -3$ ✓ $f^{/}(x) = -3\left(x^2 - \frac{2}{3}x - \frac{1}{3}\right)$ ✓ $n = 2$ ✓ $k = 1$ (6) **OR/OF** ✓ $k = 1$ ✓ $m + n = -1$ ✓ $m - 3n = -9$ ✓ $4n = 8$ ✓ $n = 2$ ✓ $m = -3$ (6)
8.2.1	$f(x) = -x^3 + x^2 + x + 2$ $f\left(-\frac{1}{3}\right) = \frac{49}{27} = 1{,}81$ T.P$\left(-\frac{1}{3}\ ;\ \frac{49}{27}\right)$ $f(1) = 3$ T.P$(1\ ;\ 3)$	✓ x-coordinates of the TP ✓ T.P$\left(-\frac{1}{3}\ ;\ \frac{49}{27}\right)$ ✓ T.P$(1\ ;\ 3)$ (3)

8.2.2	$f(x) = -x^3 + x^2 + x + 2$ $-x^3 + x^2 + x + 2 = 0$ $(x-2)(-x^2 - x - 1) = 0$ $x = 2$ or no solution 	✓ $x = 2$ ✓one x-intercept ✓ two turning points ✓y-intercept ✓ shape: neg cubic (5)
8.3.1	$a = \dfrac{-\frac{1}{3}+1}{2}$ $= \dfrac{1}{3}$ **OR/OF** $f'(x) = -3x^2 + 2x + 1$ $f''(x) = -6x + 2$ $f''(a) = -6a + 2 = 0$ $-6a = -2$ $a = \dfrac{1}{3}$	✓ answer (1) **OR/OF** ✓ answer (1)
8.3.2	$b < \dfrac{4}{3}$ units	✓ $\dfrac{4}{3}$ ✓ $b < \dfrac{4}{3}$ (2)
		[17]

15. Turning points, a sketch, then reading answers off it

Official DBE marking guideline · November 2023 · Q8 · 18 marks

QUESTION 8/*VRAAG* 8

8.1	$f'(x) = -3x^2 + 12x - 9$ $-3x^2 + 12x - 9 = 0$ $x^2 - 4x + 3 = 0$ $(x-3)(x-1) = 0$ $\therefore x = 3$ or $x = 1$ $f(3) = -(3)^3 + 6(3)^2 - 9(3) + 4 = 4$ $f(1) = -(1)^3 + 6(1)^2 - 9(1) + 4 = 0$ $\therefore$ turning points are: (3 ; 4) and (1 ; 0)	✓ $f'(x) = -3x^2 + 12x - 9$ ✓ $f'(x) = 0$ ✓ both x-values ✓ both y-values (4)
8.2		✓ y-intercept ✓ both x-intercepts ✓ both turning points ✓ shape (4)
8.3	$0 < k < 4$ or/*of* $k \in (0 ; 4)$	✓✓k between y-values of turning points (2)
8.4	$f''(x) = -6x + 12 = 0$ $x = 2$ Max at (2 ; 2) $f'(2) = 3$ $\therefore y - 2 = 3(x-2)$ or $2 = 3(2) + c$ $g(x) = 3x - 4$ $g(x) = 3x - 4$ **OR/*OF*** Point of inflection: $x = \frac{3+1}{2}$ $x = 2$ Max at (2 ; 2) $f'(2) = 3$ $\therefore y - 2 = 3(x-2)$ or $2 = 3(2) + c$ $g(x) = 3x - 4$ $g(x) = 3x - 4$	✓ $f''(x) = -6x + 12$ ✓ $f''(x) = 0$ ✓ x-value ✓ y-value ✓gradient at x-value ✓equation of tangent (6) **OR/*OF*** ✓✓ $\frac{3+1}{2}$ ✓ x-value ✓ y-value ✓gradient at x-value ✓equation of tangent (6)
8.5	$\tan\theta = 3$ $\therefore \theta = 71{,}57°$	✓ gradient of g ✓ answer (2)
		[18]

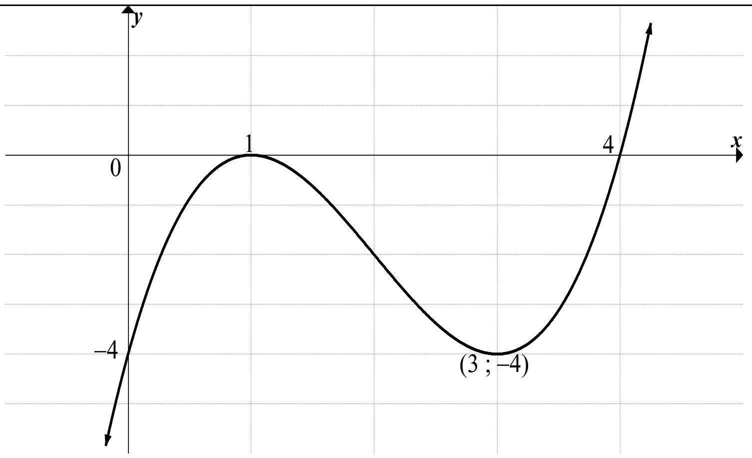
16. Show k = 1, then the turning points, the concavity and the sketch

Official DBE marking guideline · May/June 2025 · Q9 · 18 marks

QUESTION/*VRAAG* 9

9.1	$f(x)=(x-4)(x^2-2x+1)$ $f(x)=(x-4)(x-1)^2$ $\therefore k=1$ **OR/*OF*** $f(1)=1-6+9-4=0$ $(x-1)$ is a factor $\therefore k=1$ **OR/*OF*** $-4k^2=-4$ $k^2=1$ $\therefore k=1$	✓ (x^2-2x+1) ✓ $(x-1)^2$ (2) **OR/*OF*** ✓ $f(1)$ ✓ $f(1)=0$ (2) **OR/*OF*** ✓ $-4k^2=-4$ ✓ $k^2=1$ (2)
9.2	$3x^2-12x+9=0$ $x^2-4x+3=0$ $(x-3)(x-1)=0$ $x=3$ or $x=1$ TP's: $(3\,;-4)$ and $(1\,;0)$	✓ $f^{/}(x)$ ✓ values of x ✓✓turning points (4)

9.3	$f^{//}(x)=6x-12$
	$f^{//}(-3)=-30$
	$f^{//}(x)<0$, therefore concave down
		✓substitution $x=-3$ into $f^{//}(x)$
		✓ concave down (2)

9.4		✓ turning points ✓ x-intercepts ✓ y-intercept ✓ shape (4)
9.5	$\text{Distance}=(-6x^2+24x-18)-(x^3-6x^2+9x-4)$ $=-6x^2+24x-18-x^3+6x^2-9x+4$ $=-x^3+15x-14$ Max distance: $-3x^2+15=0$ $3x^2=15$ $x^2=5$ $x=\pm\sqrt{5}$, but $1<x<3$ $\therefore x=\sqrt{5}$ $d(\sqrt{5})=8{,}36$ max distance = 8,36	✓ $h(x)=-2f^{/}(x)$ ✓ simplification ✓ first derivative ✓= 0 ✓ x-value ✓ answer (6)
		[18]

17. A cubic from its intercepts, and everything that then follows

Official DBE marking guideline · May/June 2022 · Q9 · 23 marks

QUESTION/*VRAAG* 9

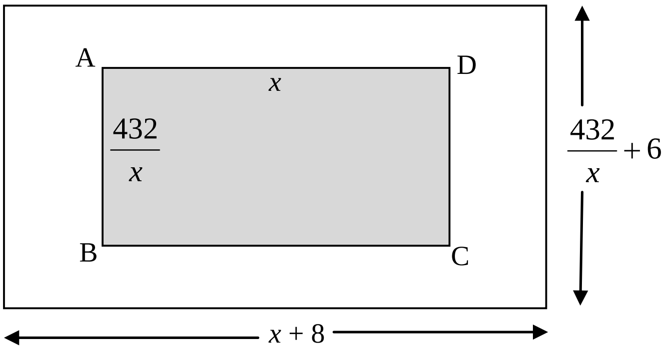
9.1	$f(x)=(x+t)^2(x-3)$ $-3=(0+t)^2(0-3)$ $1=t^2$ $t=\pm 1$ $\therefore t=1$ $f(x)=(x+1)^2(x-3)$ $f(x)=(x^2+2x+1)(x-3)$ $f(x)=x^3-x^2-5x-3$	✓ $f(x)=(x+t)^2(x-3)$ ✓ subs (0 ; –3) ✓ t ✓ $f(x)=(x+1)^2(x-3)$ ✓ expansion (5)
9.2	$f'(x)=3x^2-2x-5$ $0=3x^2-2x-5$ $0=(x+1)(3x-5)$ $x=-1$ or $x=\frac{5}{3}$ $N\left(\frac{5}{3}\,;-\frac{256}{27}\right)=(1,67\,;-9,48)$	✓ $f'(x)=3x^2-2x-5$ ✓ $=0$ ✓ factors ✓ x-value ($x>0$) ✓ y-value (A) (5)
9.3.1	$x<3$; $x\neq -1$ **OR/*OF*** $x<-1$ or $-1<x<3$ **OR/*OF*** $(-\infty\,;-1)$ or $(-1\,;3)$	✓ $x<3$ ✓ $x\neq -1$ (2) **OR/*OF*** ✓ $x<-1$ ✓ $-1<x<3$ (2) **OR/*OF*** ✓ $(-\infty\,;-1)$ ✓ $(-1\,;3)$ (2)
9.3.2	$x<-1$ or $x>\frac{5}{3}$ OR/OF $x\leq -1$ or $x\geq \frac{5}{3}$ **OR/*OF*** $(-\infty\,;-1)$ or $\left(\frac{5}{3}\,;\infty\right)$ OR/OF $(-\infty\,;-1]$ or $\left[\frac{5}{3}\,;\infty\right)$	✓ $x<-1$ ✓ $x>\frac{5}{3}$ (2) **OR/*OF*** ✓ $(-\infty\,;-1)$ ✓ $\left(\frac{5}{3}\,;\infty\right)$ (2)
9.3.3	$f''(x)>0$ $6x-2>0$ $x>\frac{1}{3}$ or $\left(\frac{1}{3}\,;\infty\right)$ ANSWER ONLY: FULL MARKS **OR/OF** $\dfrac{\frac{5}{3}+(-1)}{2}=\frac{1}{3}$ $x>\frac{1}{3}$ or $\left(\frac{1}{3}\,;\infty\right)$	✓ $6x-2$ ✓ $\frac{1}{3}$ ✓ $x>\frac{1}{3}$ (3) **OR/OF** ✓ substitution ✓ $\frac{1}{3}$ ✓ $x>\frac{1}{3}$ (3)

9.4	Distance $= x^3 - x^2 - 5x - 3 - (3x^2 - 2x - 5)$ $= x^3 - 4x^2 - 3x + 2$ $\frac{d\text{Distance}}{dx} = 3x^2 - 8x - 3$ $0 = 3x^2 - 8x - 3$ $0 = (3x+1)(x-3)$ $x = 3$ or $x = -\frac{1}{3}$ Max distance $= \left(-\frac{1}{3}\right)^3 - 4\left(-\frac{1}{3}\right)^2 - 3\left(-\frac{1}{3}\right) + 2$ $= \frac{68}{27} = 2{,}52$	✓ $x^3 - 4x^2 - 3x + 2$ ✓ $\frac{d\text{Distance}}{dx} = 3x^2 - 8x - 3$ ✓ factors ✓ x-values ✓ $x = -\frac{1}{3}$ ✓ answer (6)
		[23]

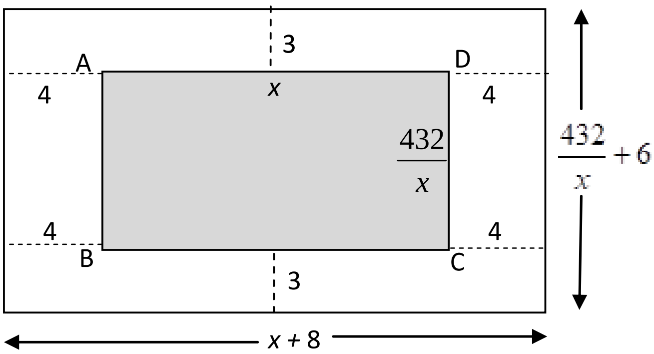
18. A printed poster, and the size that wastes the least page

Official DBE marking guideline · November 2023 · Q9 · 6 marks

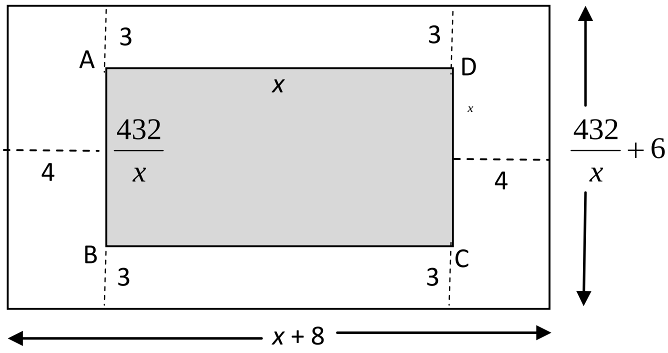
QUESTION 9/*VRAAG* 9



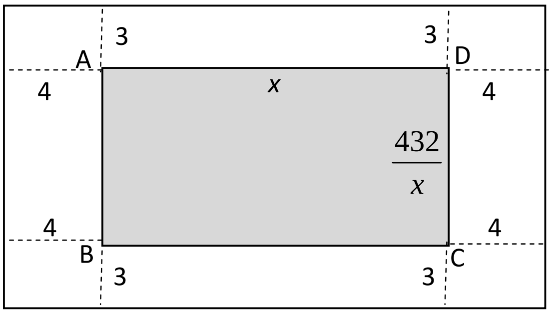
9.1	$432 = xb$ $\therefore b = \frac{432}{x}$ $A(x) = (x+8)\left(\frac{432}{x}+6\right)$ $A(x) = 432 + 6x + \frac{3456}{x} + 480$ $A(x) = \frac{3456}{x} + 6x + 480$	✓ $b = \frac{432}{x}$ ✓ $(x+8)$ ✓ $\left(\frac{432}{x}+6\right)$ (3)
9.2	$A(x) = 3456x^{-1} + 6x + 480$ $A'(x) = -\frac{3456}{x^2} + 6$ $-\frac{3456}{x^2} + 6 = 0$ $3456 = 6x^2$ $\therefore x = \sqrt{576} = 24\text{ cm}$	✓ $3456x^{-1} + 6x + 480$ ✓ $A'(x) = -\frac{3456}{x^2} + 6$ ✓answer (3)
		[6]



$$\text{total area} = 2(x+8)(3) + 2\left(\frac{432}{x}\right)(4) + \left(\frac{432}{x}\right)(x$$



$$\text{total area} = 2(4)\left(\frac{432}{x}+6\right)+(x)\left(\frac{432}{x}+6\right)$$



$$\text{total area} = 4(4)(3)+2(x)(3)+\left(\frac{432}{x}\right)(x)+2\left(\frac{432}{x}\right)(4$$

19. The speed that keeps a journey's cost as low as possible

Official DBE marking guideline · November 2021 · Q11 · 7 marks

QUESTION/VRAAG 11

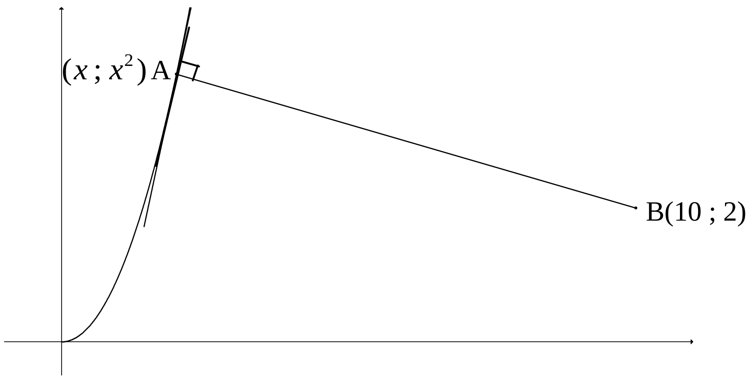
11	Time = $\frac{20}{x}$ Cost = (water cost per hour × time) + (kms × R/km) $C(x)=1{,}6\times\left(\frac{20}{x}\right)+20\left(1{,}2+\frac{x}{4000}\right)$ $C(x)=\frac{32}{x}+24+\frac{x}{200}$ $C'(x)=-\frac{32}{x^2}+\frac{1}{200}=0$ $x^2=6\,400$ $x=80\,km/h$	✓ $\frac{20}{x}$ ✓ $1{,}6\times\left(\frac{20}{x}\right)$ ✓ $20\left(1{,}2+\frac{x}{4000}\right)$ ✓ $C(x)=\frac{32}{x}+24+\frac{x}{200}$ ✓ $C'(x)=-\frac{32}{x^2}+\frac{1}{200}$ ✓ $C'(x)=0$ ✓ answer (A) (7)
		[7]

20. The shortest distance from a point to a parabola

Official DBE marking guideline · November 2022 · Q9 · 8 marks

QUESTION9/*VRAAG* 9

9.1	Any point on f : $(x\,;\,x^2)$ $\text{distance} = \sqrt{(x-10)^2 + (x^2-2)^2}$ $= \sqrt{x^2 - 20x + 100 + x^4 - 4x^2 + 4}$ $= \sqrt{x^4 - 3x^2 - 20x + 104}$ For min distance $\frac{d}{dx}\left(x^4 - 3x^2 - 20x + 104\right) = 0$ $4x^3 - 6x - 20 = 0$ $(x-2)(4x^2 + 8x + 10) = 0$ $\Delta = 8^2 - 4(4)(10) = -96 \quad \therefore$ no roots $\therefore x = 2$ $d = \sqrt{2^4 - 3(2)^2 - 20(2) + 104} = 2\sqrt{17} = 8{,}25$	✓ $(x\,;\,x^2)$ ✓ substitution ✓ simplification ✓ answer ✓ $4x^3 - 6x - 20$ ✓ derivative = 0 ✓ $x = 2$ ✓ answer (A) (8)

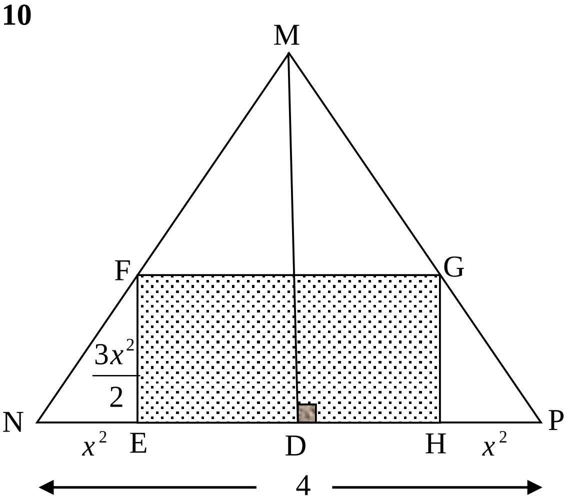


9.2	$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{x^2 - 2}{x - 10}$ $\therefore m_{\tan g} = -\frac{x-10}{x^2-2}$ $f'(x) = 2x$ $\therefore 2x = -\frac{x-10}{x^2-2}$ $-2x^3 + 4x = x - 10$ $2x^3 - 3x - 10 = 0$ $x = 2$ $y = (2)^2 = 4$ $\therefore AB = \sqrt{(2-10)^2 + (4-2)^2}$ $= 2\sqrt{17} = 8{,}25$	✓ m_{AB} ✓ $m_{\tan g} = -\frac{x-10}{x^2-2}$ ✓ $f'(x) = 2x$ ✓ equating ✓ standard form ✓ $x = 2$ ✓ substitute into distance ✓ answer (A) (8)
		[8]

21. Show the area is 6x squared minus 3x to the fourth, then maximise it

Official DBE marking guideline · May/June 2024 · Q10 · 8 marks

QUESTION 10/*VRAAG* 10



10.1	$\frac{NE}{EF} = \frac{2}{3} = \frac{x^2}{b}$ $3x^2 = 2b$ $\therefore b = \frac{3x^2}{2}$ $EH = 4 - 2x^2$ Area EFGH = $(4-2x^2)\left(\frac{3x^2}{2}\right)$ $A(x) = 6x^2 - 3x^4$	✓ $\frac{NE}{EF} = \frac{2}{3} = \frac{x^2}{b}$ ✓ $\therefore b = \frac{3x^2}{2}$ ✓ $EH = 4 - 2x^2$ ✓ $(4-2x^2)\left(\frac{3x^2}{2}\right)$ (4)
	OR/OF In ΔDMP: $\tan P = \frac{3}{2}$ In ΔHGP: $\tan P = \frac{GH}{x^2}$ $\frac{GH}{x^2} = \frac{3}{2}$ $\therefore b = \frac{3x^2}{2}$ $EH = 4 - 2x^2$ Area EFGH = $(4-2x^2)\left(\frac{3x^2}{2}\right)$ $A(x) = 6x^2 - 3x^4$	**OR/OF** ✓ $\frac{GH}{x^2} = \frac{2}{3}$ ✓ $\therefore b = \frac{3x^2}{2}$ ✓ $EH = 4 - 2x^2$ ✓ $(4-2x^2)\left(\frac{3x^2}{2}\right)$ (4)
10.2	$A(x) = 6x^2 - 3x^4$ $A'(x) = 12x - 12x^3 = 0$ $12x(1-x^2) = 0$ $\therefore x \neq 0$ or $x = -1$ or $x = 1$ $\therefore$ max area: $A(1) = 6(1)^2 - 3(1)^4 = 3 \text{ cm}^2$	✓ $12x - 12x^3 = 0$ ✓ values of x ✓ correct substitution ✓ answer (4)
		[8]

22. A cyclist's speed, given as a rate of change

Official DBE marking guideline · November 2024 · Q10 · 8 marks

QUESTION/*VRAAG* 10

10.1	$-6t+18=0$ $18=6t$ $3=t$ $s^{/}(3)=-3(3)^2+18(3)=27$	✓ $=0$ ✓ value of t ✓ answer (3)
10.2	$-3t^2+18t=0$ $-3t(t-6)=0$ $t=0$ or $t=6$ $s(t)=at^3+bt^2$ $s^{/}(t)=3at^2+2bt$ $3a=-3$ and $2b=18$ $a=-1$ $b=9$ $s(t)=-1t^3+9t^2$ $s(6)=-(6)^3+9(6)^2$ $s(6)=108$	 ✓ factors ✓ values ✓ a and b ✓ substitution ✓ answer (5)
		[8]