

# FUNCTIONS AND GRAPHS

## Solutions

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### **DBE's own marking guidelines, in booklet order**

Every solution here is a clip of the official DBE marking guideline published with the paper the question came from. The working, the mark ticks and the accepted alternatives are DBE's. Nothing has been rewritten or re-derived.

The numbers match the question booklet: solution 34 answers question 34, and the chapters are in the same order.

DBE writes one guideline for both languages, so these pages are bilingual and their marking notes are largely in English. That is DBE's own page, reproduced as printed.

A marking guideline is written for markers, not learners. It shows the shortest route to the marks, so where a step looks compressed, that is the guideline being terse rather than a step missing.

**1. The inverse of 3 to the x, and where a translated point lands**

Official DBE marking guideline · May/June 2021 · Q6 · 8 marks

**QUESTION/VRAAG 6**

|       |                                                                                                                                    |                                                                                                      |
|-------|------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------|
| 6.1.1 | $y = 3^x$<br>$x = 3^y$<br>$y = \log_3 x$                                                                                           | ✓ swop $x$ and $y$<br>✓ equation<br>(2)                                                              |
| 6.1.2 | $h(x) = 3^{x-4} + 2$<br>Transformation: 4 units left, 2 units down<br>$P'(2;9)$                                                    | ✓ $x = 2$ (A)<br>✓ $y = 9$ (A)<br>(2)                                                                |
| 6.2   | $f(x) = 2^{x+p} + q$<br>$q = -16$<br>$16 = 2^{p+3} - 16$<br>$2^{p+3} = 32$<br>$2^{p+3} = 2^5$<br>$\therefore p + 3 = 5$<br>$p = 2$ | ✓ $q = -16$<br>✓ substitute (3 ; 16)<br>✓ $2^{p+3} = 2^5$ or $p + 3 = \log_2 32$<br>✓ $p = 2$<br>(4) |
|       |                                                                                                                                    | <b>[8]</b>                                                                                           |

**2. Log base 4: find k, then write down the inverse**

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**QUESTION/VRAAG 6**

|     |                                                                                                                                                     |                                                            |
|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------|
| 6.1 | $f(x) = \log_4 x$<br>$2 = \log_4 k$<br>$4^2 = k$<br>$\therefore k = 16$                                                                             | ✓ substitution of $(k ; 2)$<br><br>✓ answer<br>(2)         |
| 6.2 | $-1 = \log_4 x \quad \therefore x = \frac{1}{4}$<br><br>$\frac{1}{4} \leq x \leq 16 \quad \text{or/of} \quad x \in \left[ \frac{1}{4} ; 16 \right]$ | ✓ $x = \frac{1}{4}$<br><br>✓ answer<br>(2)                 |
| 6.3 | $f(x) = \log_4 x$<br>$y = \log_4 x$<br>$x = \log_4 y$<br>$y = 4^x$                                                                                  | ✓swopping x and y<br><br>✓answer<br>(2)                    |
| 6.4 | $x < 0$<br><br><b>OR/OF</b><br><br>$x \in (-\infty ; 0)$                                                                                            | ✓✓answer<br>(2)<br><br><b>OR/OF</b><br><br>✓✓answer<br>(2) |
|     |                                                                                                                                                     | <b>[8]</b>                                                 |

### 3. Find a, then describe the translation from g to h

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#### QUESTION 4

|       |                                                                                                                                                                                                                                                                                 |                                                      |
|-------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------|
| 4.1   | $10 = a\left(\frac{1}{3}\right)^{-2} + 7$ $3 = 9a$ $\therefore a = \frac{1}{3}$                                                                                                                                                                                                 | ✓ subs (-2 ; 10)<br>✓ simplification<br>✓ answer (3) |
| 4.2   | $y = g(0)$ $y = \frac{1}{3} \times \left(\frac{1}{3}\right)^0 + 7$ $y = \frac{22}{3} = 7,33$ $\therefore \left(0 ; \frac{22}{3}\right)$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;">ANSWER ONLY:<br/>FULL MARKS</div>          | ✓ substitution of $x = 0$<br>✓ answer (2)            |
| 4.3.1 | Translation by 1 unit to the right and 7 units downwards                                                                                                                                                                                                                        | ✓ 1 unit right<br>✓ 7 units downwards (2)            |
| 4.3.2 | $h(x) = \left(\frac{1}{3}\right)^x$ $h^{-1}: x = \left(\frac{1}{3}\right)^y$ $y = \log_{\frac{1}{3}}(x) \quad \text{OR/OR} \quad y = -\log_3(x)$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;">ANSWER ONLY:<br/>FULL MARKS</div> | ✓ swap $x$ and $y$<br>✓ answer (2)                   |
|       |                                                                                                                                                                                                                                                                                 | <b>[9]</b>                                           |

#### 4. A logarithm and its reflection in the line $y = x$

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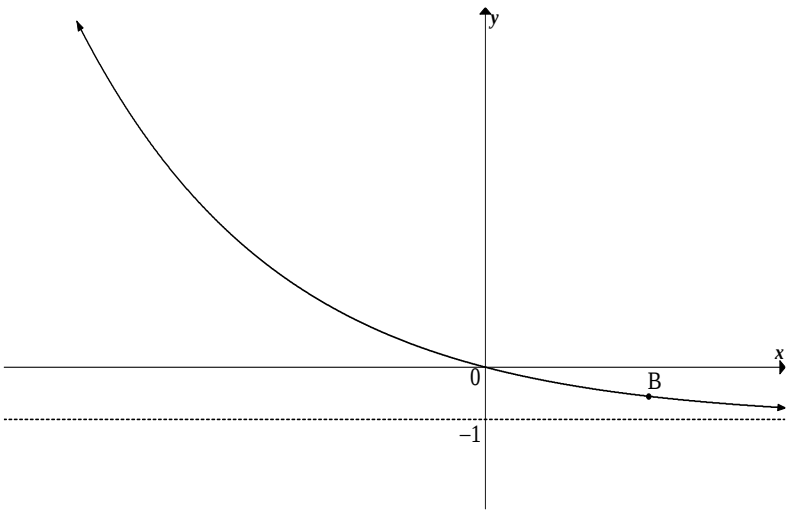
#### QUESTION5/VRAAG 5

|     |                                                                                                                                                                                                           |                                                                                                                     |
|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------|
| 5.1 | $P'(2;4)$                                                                                                                                                                                                 | $\checkmark x = 2$<br>$\checkmark y = 4$<br>(2)                                                                     |
| 5.2 | $f(x) = \log_a x$<br>$2 = \log_a 4$<br>$a^2 = 4$<br>$a = 2$                                                                                                                                               | $\checkmark$ substitute (4 ; 2)<br>$\checkmark a^2 = 4$<br>(2)                                                      |
| 5.3 | $y = 2^x$                                                                                                                                                                                                 | $\checkmark y = 2^x$<br>(1)                                                                                         |
| 5.4 | $1 = \log_2 x$<br>$\therefore x = 2$ T(2 ; 1)<br><br>RT = 2 units<br>P'T = 3 units<br><br>Area of $\Delta RTP' = \frac{1}{2} \cdot RT \cdot TP'$<br>$= \frac{1}{2} \times 2 \times 3 = 3 \text{ units}^2$ | $\checkmark x = 2$<br><br>$\checkmark$ RT = 2 units<br>$\checkmark$ P'T = 3 units<br><br>$\checkmark$ answer<br>(4) |
|     |                                                                                                                                                                                                           | <b>[9]</b>                                                                                                          |

## 5. An exponential shifted down one, and the range that gives it

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### QUESTION/VRAAG 4

|     |                                                                                                                                                                                                                                                                           |                                                   |
|-----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------|
| 4.1 | $f(x) = a^x - 1$<br>$-\frac{5}{9} = a^2 - 1$<br>$a^2 = \frac{4}{9}$<br>$a = \frac{2}{3}$                                                                                                                                                                                  | ✓ substitution<br><br>✓ answer (2)                |
| 4.2 | $y > -1$<br><b>OR/OF</b><br>$y \in (-1; \infty)$                                                                                                                                                                                                                          | ✓ answer (1)<br>✓ answer (1)                      |
| 4.3 |                                                                                                                                                                                        | ✓(0 ; 0)<br>✓ asymptote<br>✓decreasing shape (3)  |
| 4.4 | $\frac{19}{8} = \left(\frac{2}{3}\right)^x - 1$<br>$\frac{27}{8} = \left(\frac{2}{3}\right)^x$<br>$\left(\frac{3}{2}\right)^3 = \left(\frac{3}{2}\right)^{-x}$<br>$\therefore x = -3$<br>$\therefore C\left(-3; \frac{19}{8}\right)$<br>$C'\left(\frac{19}{8}; -3\right)$ | ✓substitution<br><br>✓ answer<br><br>✓ answer (3) |
|     |                                                                                                                                                                                                                                                                           | <b>[9]</b>                                        |

## 6. An exponential and a straight line meeting at (3 ; 4)

Official DBE marking guideline · May/June 2025 · Q6 · 11 marks

### QUESTION/VRAAG 6

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                                                                                                                                                                                           |
|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 6.1 | $(0 ; -3) \quad -3 = p^0 + q \quad \text{OR} \quad y = p^x - 4$<br>$q = -4$<br>$(3 ; 4) \quad 4 = p^3 - 4$<br>$p^3 = 8$<br>$p = 2$<br>$\therefore f(x) = 2^x - 4$                                                                                                                                                                                                                                                                                                                          | ✓ substitute (0 ; -3)<br>✓ answer for $q$<br>✓ substitute (3 ; 4)<br>✓ answer for $p$<br>(4)                                                                                              |
| 6.2 | $y > -4 \quad \text{OR/OF} \quad y \in (-4 ; \infty)$                                                                                                                                                                                                                                                                                                                                                                                                                                      | ✓ answer<br>(1)                                                                                                                                                                           |
| 6.3 | $g(x) = mx + c$<br>$E(-3 ; 0)$<br>$m = \frac{4-0}{3-(-3)} = \frac{2}{3}$<br>$y = \frac{2}{3}x + c$<br>$0 = \frac{2}{3}(-3) + c \quad \text{OR} \quad y-0 = \frac{2}{3}(x+3)$<br>$\therefore c = 2$<br>$y = \frac{2}{3}x + 2 \quad y = \frac{2}{3}x + 2$<br><b>OR/OF</b><br>For $g^{-1}$ :<br>$y = mx - 3$<br>$3 = m(4) - 3$<br>$m = \frac{3}{2}$<br>$\therefore g^{-1}(x) = \frac{3}{2}x - 3$<br>For $g$ :<br>$2x + 6 = 3y$<br>$\frac{2}{3}x + 2 = y$<br>$\therefore y = \frac{2}{3}x + 2$ | ✓ $E(-3 ; 0)$<br>✓ $m_{AE}$<br>✓ substitution<br>✓ equation<br>(4)<br><b>OR/OF</b><br>✓ substitution of (4 ; 3)<br>✓ $m$ of inverse<br>✓ equation of $g^{-1}$<br>✓ equation of $g$<br>(4) |
| 6.4 | $g(x) = \frac{2}{3}x + 2$<br>$x = \frac{2}{3}y + 2$<br>$g^{-1}(x) = \frac{3}{2}x - 3$                                                                                                                                                                                                                                                                                                                                                                                                      | ✓ swop $x$ and $y$<br>✓ equation<br>(2)                                                                                                                                                   |
|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | <b>[11]</b>                                                                                                                                                                               |

## 7. A straight line and its inverse, and where the two cross

Official DBE marking guideline · November 2022 · Q5 · 13 marks

### QUESTION 5/VRAAG 5

|     |                                                                                                                                                                                   |                                                                                                                      |
|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------|
| 5.1 | $g(x) = 2x + 6$<br>$y = 6$                                                                                                                                                        | ✓ $y = 6$ (1)                                                                                                        |
| 5.2 | $y = 2x + 6$<br>$x = 2y + 6$<br>$y = \frac{1}{2}x - 3$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;">Answer only: Full marks</div> | ✓ swop $x$ and $y$<br>✓ equation (2)                                                                                 |
| 5.3 | $\frac{1}{2}x - 3 = 2x + 6$<br>$x - 6 = 4x + 12$<br>$3x = -18$<br>$x = -6$<br>$A(-6; -6)$<br><b>OR/OF</b><br>$2x + 6 = x$<br>$x = -6$<br>$y = -6$                                 | ✓ equating<br><br>✓ $x = -6$<br>✓ $y = -6$ (3)<br><br><b>OR/OF</b><br><br>✓ equating<br>✓ $x = -6$<br>✓ $y = -6$ (3) |
| 5.4 | $AB = \sqrt{(6)^2 + (12)^2}$<br>$= \sqrt{180} = 6\sqrt{5} = 13,42$                                                                                                                | ✓ substitution<br>✓ answer (2)                                                                                       |
|     |                                                                                                                                                                                   |                                                                                                                      |

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|-----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5.5 | $BC = \sqrt{6^2 + 6^2} = \sqrt{72} = 6\sqrt{2}$ $AB = AC = \sqrt{180} \quad \text{symmetry of } g \text{ and } g^{-1}$ $\perp h = (\sqrt{180})^2 - \left(\frac{\sqrt{72}}{2}\right)^2$ $= \sqrt{162} = 9\sqrt{2}$ $\text{area of } \triangle ABC = \frac{1}{2} BC \times h$ $= \frac{1}{2} \times \sqrt{72} \times \sqrt{162} = 54 \text{ units}^2$ <p><b>OR/OF</b></p> $\tan \hat{BDC} = 2$ $\therefore \hat{BDC} = 63,43^\circ$ $\tan \hat{DCA} = \frac{1}{2}$ $\therefore \hat{DCA} = 26,57^\circ$ $\therefore \hat{DAC} = 36,86^\circ \quad (\text{ext angle triangle})$ $\text{Area of } \triangle ABC = \frac{1}{2} (\sqrt{180}) (\sqrt{180}) \sin 36,86^\circ$ $= 53,99 \text{ units}^2$ <p><b>OR/OF</b></p> $\text{Area of } \triangle ABC = \text{Area of } \triangle BDC + \text{Area of } \triangle ADC$ $= \frac{1}{2} DC \cdot BO + \frac{1}{2} DC \cdot \text{height}$ $= \frac{1}{2} (9)(6) + \frac{1}{2} (9)(6)$ $= 54 \text{ units}^2$ | <p>✓ BC</p> <p>✓ AB = AC /midpoint (3 ; 3)</p> <p>✓ <math>\perp h</math> (A)</p> <p>✓ substitution</p> <p>✓ answer (A) (5)</p> <p><b>OR/OF</b></p> <p>✓ <math>\hat{BDC} = 63,43^\circ</math></p> <p>✓ <math>\hat{DAC} = 36,86^\circ</math></p> <p>✓ <math>AC = \sqrt{180}</math></p> <p>✓ substitution into the correct formula</p> <p>✓ answer (A) (5)</p> <p><b>OR/OF</b></p> <p>✓ Areas (<math>\triangle BDC + \triangle ADC</math>)</p> <p>✓ <math>\frac{1}{2} DC \cdot BO</math></p> <p>✓ <math>\frac{1}{2} DC \cdot \text{height}</math></p> <p>✓ substitution</p> <p>✓ answer (A) (5)</p> |
|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | <b>[13]</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |

## 8. 2 to the x minus 4, on a closed interval

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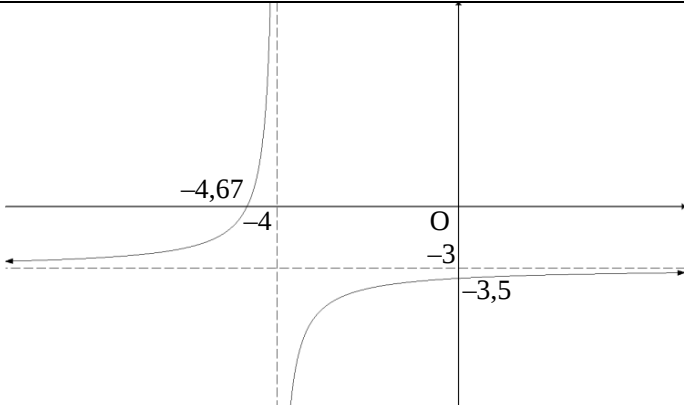
### QUESTION 4/VRAAG 4

|     |                                                                                                                                              |                                                    |
|-----|----------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------|
| 4.1 | $y = -4$                                                                                                                                     | ✓ $y = -4$ (1)                                     |
| 4.2 | $x$ – intercept: $0 = 2^x - 4$<br>$4 = 2^x$<br>$x = 2$<br>$\therefore B(2;0)$                                                                | ✓ $y = 0$<br>✓ $x = 2$ (2)                         |
| 4.3 | $y = 2^0 - 4 = -3$<br>$\therefore A(0; -3)$<br>$y = mx + c$<br>$m = \frac{3}{2}$<br>$k(x) = \frac{3}{2}x - 3$                                | ✓ $y = -3$<br><br>✓ gradient<br><br>✓ equation (3) |
| 4.4 | $k(1) = \frac{3}{2}(1) - 3 = \frac{-3}{2}$<br>$f(1) = 2^1 - 4 = -2$<br>Vertical distance $= -\frac{3}{2} - (-2) = \frac{1}{2}$ units         | ✓ $k(1)$<br>✓ $f(1) = -2$<br>✓ answer (3)          |
| 4.5 | $g(x) = f(x) + 4$<br>$g(x) = 2^x ; x \in [-2; 4)$                                                                                            | ✓ $g(x) = 2^x$ (1)                                 |
| 4.6 | Range of $g : y \in \left[\frac{1}{4}; 16\right)$<br>Domain of $g^{-1} : x \in \left[\frac{1}{4}; 16\right)$ or/of $\frac{1}{4} \leq x < 16$ | <br>✓ ✓ $x \in \left[\frac{1}{4}; 16\right)$ (2)   |
| 4.7 | $g : y = 2^x$<br>$g^{-1} : x = 2^y$<br>$g^{-1}(x) = \log_2 x, x \in \left[\frac{1}{4}; 16\right)$                                            | ✓ swop $x$ and $y$<br>✓ equation (2)               |
|     |                                                                                                                                              | [14]                                               |

## 9. Show that $p = 4$ and $q = -3$ , then sketch the hyperbola

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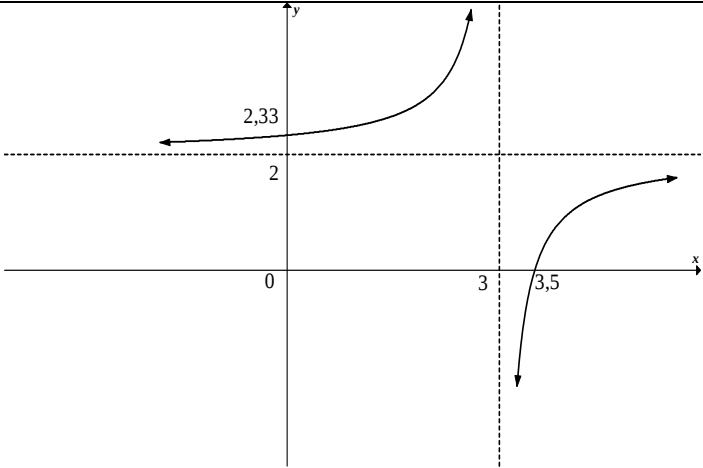
### QUESTION/VRAAG 4

|     |                                                                                                                                                                                                                                                                    |                                                                                                                                                                                                                                  |
|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 4.1 | $x+1=-x-7$<br>$2x=-8$<br>$x=-4$<br>$\therefore y=-3$<br>$\therefore f(x)=\frac{-2}{x+4}-3$<br>$\therefore p=4$ and $q=-3$<br><br><b>OR/OF</b><br>$p+q=1$ .....(1)<br>$-p+q=-7$<br>$q=p-7$ .....(2)<br>subs. (2) into (1)<br>$p+p-7=1$<br>$2p=8$<br>$p=4$<br>$q=-3$ | $\checkmark x+1=-x-7$<br>$\checkmark 2x=-8$<br>$\checkmark x=-4$<br>$\checkmark y=-3$<br><br><b>OR/OF</b><br>$\checkmark p+q=1$<br>$\checkmark q=p-7$<br><br>$\checkmark$ substitution<br>$\checkmark$ simplification<br><br>(4) |
| 4.2 | $y=\frac{-2}{x+4}-3$<br>$0=\frac{-2}{x+4}-3$<br><br>$-2-3(x+4)=0$<br>$-3x-14=0$<br>$\therefore x=-\frac{14}{3}$                                                                                                                                                    | $\checkmark y=0$<br><br><br>$\checkmark x=-\frac{14}{3}$ (2)                                                                                                                                                                     |
| 4.3 |                                                                                                                                                                                 | $\checkmark$ horizontal asymptote<br>$\checkmark$ vertical asymptote<br>$\checkmark$ y intercept<br>$\checkmark$ shape<br><br>(4)                                                                                                |
|     |                                                                                                                                                                                                                                                                    | <b>[10]</b>                                                                                                                                                                                                                      |

## 10. Asymptotes, domain, and both intercepts

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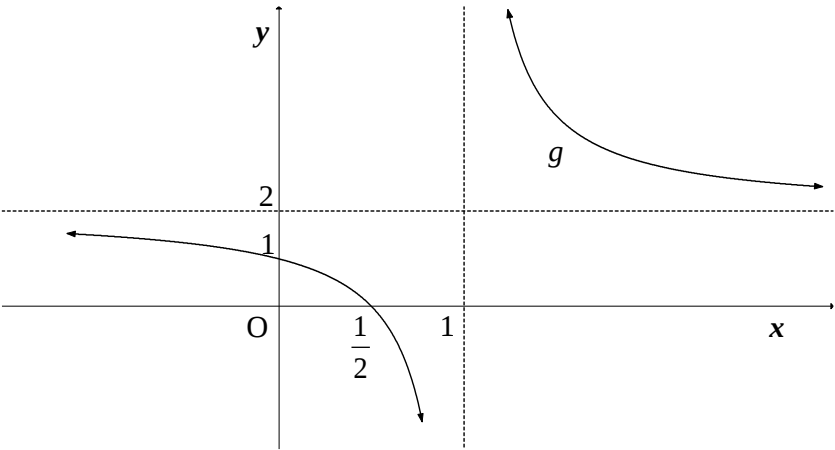
### QUESTION/VRAAG 5

|     |                                                                                                                                               |                                                                                                                               |
|-----|-----------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|
| 5.1 | $x = 3$<br>$y = 2$                                                                                                                            | $\checkmark x = 3$<br>$\checkmark y = 2$<br>(2)                                                                               |
| 5.2 | $x \in \mathbb{R}, x \neq 3$<br><br><b>OR/OF</b><br><br>$x \in (-\infty ; 3) \cup (3 ; \infty)$<br><br><b>OR/OF</b><br><br>$x < 3$ or $x > 3$ | $\checkmark$ answer (1)<br><br><b>OR/OF</b><br><br>$\checkmark$ answer (1)<br><br><b>OR/OF</b><br><br>$\checkmark$ answer (1) |
| 5.3 | $0 = \frac{-1}{x-3} + 2$ $-2x + 6 = -1$ $x = \frac{7}{2}$ x-int: $\left(\frac{7}{2} ; 0\right)$                                               | $\checkmark y = 0$<br><br>$\checkmark$ answer<br>(2)                                                                          |
| 5.4 | y-int: $\left(0 ; \frac{7}{3}\right)$                                                                                                         | $\checkmark x = 0$<br>$\checkmark \frac{7}{3}$<br>(2)                                                                         |
| 5.5 |                                                             | $\checkmark$ asymptotes<br>$\checkmark$ intercepts with the axes<br>$\checkmark$ shape<br>(3)                                 |
|     |                                                                                                                                               | <b>[10]</b>                                                                                                                   |

## 11. Asymptotes, a sketch, and where the graph is positive

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### QUESTION 4/VRAAG 4

|     |                                                                                                                                                                                        |                                                                                                                                                                                      |
|-----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 4.1 | $x = 1$<br>$y = 2$                                                                                                                                                                     | ✓ $x = 1$<br>✓ $y = 2$<br>(2)                                                                                                                                                        |
| 4.2 |                                                                                                      | ✓ x-intercept<br>✓ y-intercept<br>✓ asymptotes<br>✓ shape<br>(4)                                                                                                                     |
| 4.3 | $x < \frac{1}{2}$ or $x > 1$                                                                                                                                                           | ✓ $x < \frac{1}{2}$<br>✓ $x > 1$<br>(2)                                                                                                                                              |
| 4.4 | $y = -(x-1) + 2$<br>$y = -x + 3$<br><br><b>OR/OF</b><br>$y - 2 = -(x - 1)$<br>$y = -x + 3$<br><br><b>OR/OF</b><br>$y = -x + c$<br>$2 = -(1) + c$<br>$c = 3$<br>$\therefore y = -x + 3$ | ✓ substitution of (1 ; 2)<br>✓ answer<br>(2)<br><br><b>OR/OF</b><br>✓ substitution of (1 ; 2)<br>✓ answer<br>(2)<br><br><b>OR/OF</b><br>✓ substitution of (1 ; 2)<br>✓ answer<br>(2) |
|     |                                                                                                                                                                                        | [10]                                                                                                                                                                                 |

## 12. A line of symmetry is given - find p and q from it

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### QUESTION/VRAAG 5

|     |                                                                                                                                                                                                                                                                                                                                                                                                        |                                                                                                                                                                                                                                                                          |
|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5.1 | $p = -1$                                                                                                                                                                                                                                                                                                                                                                                               | ✓ $p = -1$ (1)                                                                                                                                                                                                                                                           |
| 5.2 | $y = x - 3$<br>$y = 1 - 3$<br>$y = -2$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-left: 20px;">           Answer only: <math>\frac{2}{2}</math> </div>                                                                                                                                                                                                           | ✓ substitution<br>✓ answer (2)                                                                                                                                                                                                                                           |
| 5.3 | $1 = \frac{a}{0-1} - 2$<br>$3 = \frac{a}{-1}$<br>$a = -3$                                                                                                                                                                                                                                                                                                                                              | ✓ substitute (0 ; 1)<br>✓ answer (2)                                                                                                                                                                                                                                     |
| 5.4 | $\frac{-3}{x-1} - 2 = 0$<br>$-3 = 2x - 2$<br>$2x = -1$<br>$\therefore x = -\frac{1}{2}$<br>$x \in \left[-\frac{1}{2}; 1\right)$ <b>OR/OF</b> $-\frac{1}{2} \leq x < 1$                                                                                                                                                                                                                                 | ✓ x-intercept<br>✓✓ answer (3)                                                                                                                                                                                                                                           |
| 5.5 | <p><math>f</math> is reflected in the <math>y</math>-axis, then translated 2 units to the right</p> <p><b>OR/OF</b></p> <p><math>f</math> is reflected in the <math>x</math>-axis, then translated 4 units down</p> <p><b>OR/OF</b></p> <p><math>f</math> is reflected about the line <math>x = 1</math></p> <p><b>OR/OF</b></p> <p><math>f</math> is reflected about the line <math>y = -2</math></p> | ✓ reflection in $y$ -axis<br>✓ translation 2 units right (2)<br><br><b>OR/OF</b><br>✓ reflection in $x$ -axis<br>✓ translation 4 units down (2)<br><br><b>OR/OF</b><br>✓✓ reflection about line $x = 1$ (2)<br><br><b>OR/OF</b><br>✓✓ reflection about line $y = -2$ (2) |
|     |                                                                                                                                                                                                                                                                                                                                                                                                        | [10]                                                                                                                                                                                                                                                                     |

### 13. Build the equation back from its domain and one intercept

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#### QUESTION 5

|     |                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                 |
|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5.1 | $g(x) = \frac{a}{x+2} + q$<br>Subs (1 ; 0):<br>$0 = \frac{a}{1+2} + q$<br>$0 = a + 3q$<br>Subs $\left(0 ; -\frac{1}{2}\right)$<br>$-\frac{1}{2} = \frac{a}{0+2} + q$<br>$-1 = a + 2q$<br>Solving simultaneously:<br>$q = 1$<br>$a = -3$<br>$\therefore g(x) = \frac{-3}{x+2} + 1$ | $\checkmark g(x) = \frac{a}{x+2} + q$<br><br>$\checkmark 0 = a + 3q$<br><br>$\checkmark -1 = a + 2q$<br>$\checkmark$ solving simultaneously<br>$\checkmark q = 1$<br>$\checkmark a = -3$<br>(6) |
| 5.2 | $y \in \mathbb{R}; y \neq 1$<br><b>OR/OF</b><br>$(-\infty; 1)$ or $(1; \infty)$<br><b>OR/OF</b><br>$y < 1$ or $y > 1$                                                                                                                                                             | $\checkmark$ answer<br>(1)                                                                                                                                                                      |
| 5.3 | $y - 1 = 1(x + 2)$ <b>OR/OF</b> $1 = 1(-2) + c$<br><div style="border: 1px solid black; padding: 5px; display: inline-block; margin: 5px;"> ANSWER ONLY:<br/> FULL MARKS </div> $c = 3$<br>$y = x + 3$ $y = x + 3$                                                                | $\checkmark m = 1$<br>$\checkmark$ subs point $(-2; 1)$<br>$\checkmark$ answer<br>(3)                                                                                                           |
| 5.4 | $K'(-3; 4)$                                                                                                                                                                                                                                                                       | $\checkmark$ x-value<br>$\checkmark$ y-value<br>(2)                                                                                                                                             |
|     |                                                                                                                                                                                                                                                                                   | <b>[12]</b>                                                                                                                                                                                     |

# 14. Where the asymptotes meet, and the point on f closest to it

Official DBE marking guideline · May/June 2025 · Q4 · 15 marks

## QUESTION/VRAAG 4

|     |                                                                                                                                                                                                                                                                                                                                    |                                                                                                                              |
|-----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------|
| 4.1 | M(3 ; 4)                                                                                                                                                                                                                                                                                                                           | ✓ $x = 3$<br>✓ $y = 4$<br>(2)                                                                                                |
| 4.2 | $f(x) = \frac{4}{x-3} + 4$ $y = \frac{4}{0-3} + 4 = \frac{8}{3}$ $\therefore D\left(0; \frac{8}{3}\right)$                                                                                                                                                                                                                         | ✓ $x = 0$<br>✓ y-value<br>(2)                                                                                                |
| 4.3 | M(3 ; 4)<br>$y = x + t$ <b>OR/OF</b> $y = (x + p) + q$<br>$4 = 3 + t$ $y = x - 3 + 4$<br>$t = 1$ $y = x + 1$<br>$\therefore t = 1$                                                                                                                                                                                                 | ✓ substituting M<br>✓ value of $t$<br>(2)                                                                                    |
| 4.4 | $\frac{4}{x-3} + 4 = 0$ $-4(x-3) = 4$ $x-3 = -1$ $x = 2$ C(2 ; 0)<br>$\therefore 2 \leq x < 3$                                                                                                                                                                                                                                     | ✓ $y = 0$<br><br>✓ $x = 2$<br><br>✓✓ answer<br>(4)                                                                           |
| 4.5 | $\frac{4}{x-3} + 4 = x + 1$ $\frac{4}{x-3} = x - 3$ $4 = (x-3)^2$ $\pm 2 = x - 3$ $\therefore x = 5 \quad \text{or} \quad x \neq 1$ $\therefore A(5 ; 6)$<br><b>OR/OF</b><br>Point closest to the origin in $y = \frac{a}{x}$ is $(\sqrt{a}; \sqrt{a})$<br>By translation:<br><br>A( $\sqrt{a} + 3$ ; $\sqrt{a} + 4$ )<br>A(5 ; 6) | ✓ equating<br><br><br><br><br>✓ $x_A$ ✓ $y_A$<br><br><b>OR/OF</b><br><br><br><br><br>✓ translation<br>✓ $x_A$ ✓ $y_A$<br>(3) |
| 4.6 | $h(x) = \frac{-4}{x+3} + 4$ $= \frac{4}{-x-3} + 4$ $\therefore \text{Reflection in } y - \text{axis.}$ A'(-5; 6)<br>AA' = 10                                                                                                                                                                                                       | <br><br><br><br><br><br>✓ coordinates<br>✓ distance<br>(2)                                                                   |

# 15. Asymptotes at (1 ; 2), and everything that follows from them

Official DBE marking guideline · November 2022 · Q4 · 20 marks

## QUESTION 4/VRAAG 4

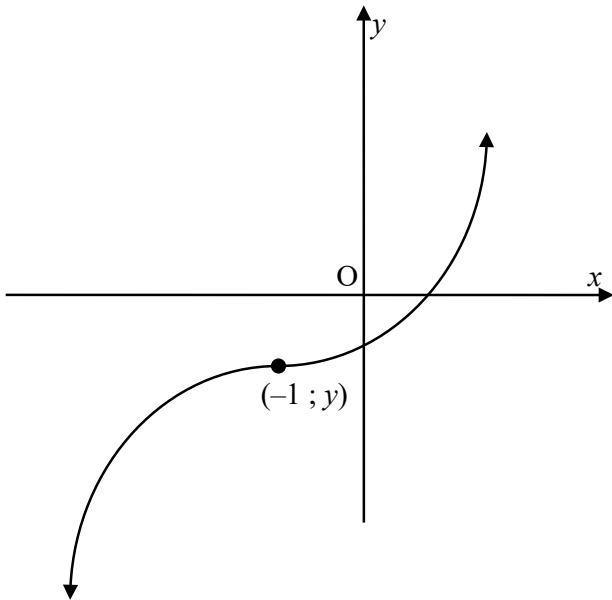
|       |                                                                                                                                                                                                                        |                                                                                    |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------|
| 4.1.1 | $p = -1$ and $q = 2$                                                                                                                                                                                                   | ✓ $p = -1$ ✓ $q = 2$ (2)                                                           |
| 4.1.2 | $\frac{1}{x-1} + 2 = 0$ $-2x + 2 = 1$ $x = \frac{1}{2}$ $\left(\frac{1}{2}; 0\right)$                                                                                                                                  | ✓ = 0<br><br>✓ answer (2)                                                          |
| 4.1.3 | $x = \frac{1}{2} - 3$ $= \frac{-5}{2}$ <div>Answer only: full marks</div>                                                                                                                                              | ✓ -3<br>✓ $x = \frac{-5}{2}$ (2)                                                   |
| 4.1.4 | $y = x + t$ $2 = 1 + t$ $t = 1$                                                                                                                                                                                        | ✓ subst (1 ; 2)<br>✓ $t = 1$ (2)                                                   |
| 4.1.5 | $-2 \leq \frac{1}{x-1}$ <div>Answer only: full marks</div> $\frac{1}{x-1} + 2 \geq 0$ $\therefore x \leq \frac{1}{2} \text{ or } x > 1$ <b>OR/OF</b> $x \in \left(-\infty; \frac{1}{2}\right] \text{ or } (1; \infty)$ | ✓ $\frac{1}{x-1} + 2 \geq 0$<br>✓ $x \leq \frac{1}{2}$<br>✓ $x > 1$ (3)            |
| 4.2.1 | $y = -5$                                                                                                                                                                                                               | ✓ answer (1)                                                                       |
| 4.2.2 | $x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = 2$ $f(2) = 2^2 - 4(2) - 5 = -9$ $\therefore D(2; -9)$ <b>OR/OF</b> $f'(x) = 2x - 4$ $2x - 4 = 0$ $x = 2$ $f(2) = 2^2 - 4(2) - 5 = -9$ $\therefore D(2; -9)$                  | ✓ $x = 2$<br>✓ $y = -9$ (2)<br><br><b>OR/OF</b><br><br>✓ $x = 2$<br>✓ $y = -9$ (2) |

|       |                                                                                        |                                                                                                      |
|-------|----------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------|
| 4.2.3 | $q = -5$<br>$-9 = a(2)^2 - 5$<br>$-4 = 4a$<br>$a = -1$<br>$\therefore g(x) = -2^x - 5$ | $\checkmark q = -5$<br>$\checkmark$ substitution of $(2 ; -9)$<br><br>$\checkmark a = -1$<br><br>(3) |
| 4.2.4 | $y \in (-\infty ; -5)$ OR $y < -5; y \in R$                                            | $\checkmark$ answer (1)                                                                              |
| 4.2.5 | $k < -9$                                                                               | $\checkmark -9$<br>$\checkmark k < -9$ (2)                                                           |
|       |                                                                                        | <b>[20]</b>                                                                                          |

## 16. From a turning point and one other point to the equation

Official DBE marking guideline · May/June 2025 · Q5 · 9 marks

### QUESTION/VRAAG 5

|     |                                                                                                                    |                                                                                                                                                                             |
|-----|--------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5.1 | $y = a(x+1)^2 + 4$<br>$-4 = a(-3+1)^2 + 4$<br>$-8 = 4a$<br>$-2 = a$<br>$y = -2(x+1)^2 + 4$<br>$y = -2x^2 - 4x + 2$ | $\checkmark$ substitute $(-1 ; 4)$<br>$\checkmark$ substitute $(-3 ; -4)$<br>$\checkmark -8 = 4a$<br><br>(3)                                                                |
| 5.2 | $k < -4$                                                                                                           | $\checkmark \checkmark k < -4$<br>(2)                                                                                                                                       |
| 5.3 |                                 | $\checkmark$ point of inflection<br>$\checkmark$ change of concavity at<br>$x = -1$<br>$\checkmark$ y-intercept below<br>x-axis<br>$\checkmark$ increasing curve<br><br>(4) |
|     |                                                                                                                    | <b>[9]</b>                                                                                                                                                                  |

# 17. The turning point, then a vertical distance on the parabola

Official DBE marking guideline · May/June 2022 · Q6 · 14 marks

## QUESTION 6

|     |                                                                                                                                                                                                                                                                                                         |                                                                                      |
|-----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|
| 6.1 | $f(x) = -x^2 - 6x + 7$<br>$f'(x) = -2x - 6$<br>$-2x - 6 = 0$ <b>OR/OF</b> $x = -\frac{(-6)}{2(-1)}$<br>$x = -3$<br>$E(-3; 16)$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;">ANSWER ONLY:<br/>FULL MARKS</div>                                           | ✓ method<br>✓ x-value<br>✓ y-value<br><br>(3)                                        |
| 6.2 | $k = f(-5)$<br>$k = -(-5)^2 - 6(-5) + 7$<br>$\therefore k = 12$                                                                                                                                                                                                                                         | ✓ answer (A)      (1)                                                                |
| 6.3 | $C(0; 7)$<br>$D(-5; 12)$<br>$m_{CD} = \frac{12-7}{-5-0}$<br>$m_{CD} = -1$<br>Equation of CD:<br>$y = -x + 7$                                                                                                                                                                                            | ✓ coordinates of C<br><br>✓ substitution<br>✓ m<br><br>✓ answer<br><br>(4)           |
| 6.4 | $-2x - 6 = -1$<br>$-2x = 5$<br>$x = -\frac{5}{2}$<br>$y = f\left(-\frac{5}{2}\right) = -\left(-\frac{5}{2}\right)^2 - 6\left(-\frac{5}{2}\right) + 7 = \frac{63}{4} = 15,75$<br>$\therefore P\left(-\frac{5}{2}; \frac{63}{4}\right)$                                                                   | ✓ $f'(x) = -2x - 6$<br>✓ equating to -1<br>✓ x-value<br><br>✓ y-value (A)<br><br>(4) |
| 6.5 | Point by symmetry: $(-1; 12)$<br>$-5 < x < -1$<br><b>OR/OF</b><br>$-x^2 - 6x + 7 > 12$<br>$-x^2 - 6x - 5 > 0$<br>$x^2 + 6x + 5 < 0$<br>$(x+1)(x+5) < 0$<br>$-5 < x < -1$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;">ANSWER ONLY:<br/>FULL MARKS</div> | ✓ -1<br>✓ answer<br><br>(2)<br><br><br><br>✓ -1<br>✓ answer<br><br>(2)               |
|     |                                                                                                                                                                                                                                                                                                         | <b>[14]</b>                                                                          |

## 18. A parabola and a line, and the gap EH between them

Official DBE marking guideline · November 2024 · Q6 · 15 marks

### QUESTION MARKS

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                            |
|-----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------|
| 6.1 | $x = \frac{-4}{2(-1)} = 2 \quad \text{OR/OF} \quad f'(x) = -2x + 4 = 0$ $x = 2$ $y = -(2)^2 + 4(2) + 5$ $= 9$ $\therefore B(2; 9)$                                                                                                                                                                                                                                                                                                                      | ✓ method for $x$<br>✓ value of $x$<br>✓ value for $y$<br>(3)                                               |
| 6.2 | $x^2 - 4x - 5 = 0$ $(x - 5)(x + 1) = 0$ $x = 5 \quad \text{or} \quad x = -1$ $\therefore A(-1; 0)$ <p>Equation of AC:</p> $\frac{8-0}{3+1} = \frac{y-0}{x+1} \quad \text{OR/OF} \quad m = \frac{8-0}{3+1} = 2$ $8x + 8 = 4y \quad y - 0 = 2(x + 1)$ $2x + 2 = y \quad y = 2x + 2$                                                                                                                                                                       | ✓ $A(-1; 0)$<br>✓ method for gradient<br>✓ substitute point A or C into equation of a straight line<br>(3) |
| 6.3 | $EH = -x^2 + 4x + 5 - (2x + 2)$ $EH = -x^2 + 2x + 3$ <p>Max EH at:</p> $-2x + 2 = 0 \quad \text{OR/OF} \quad x = \frac{-b}{2a} = \frac{-(-2)}{2(-1)}$ $x = 1 \quad x = 1$ <p>Max length of EH <math>= -(1)^2 + 2(1) + 3</math></p> <div style="border: 1px solid black; padding: 2px; display: inline-block;">Answer only: <math>\frac{1}{4}</math></div>                                                                                               | ✓ difference<br>✓ $EH = -x^2 + 2x + 3$<br>✓ $x$ - value<br>✓ length<br>(4)                                 |
| 6.4 | $k(x) = f(x + m)$ $= -x^2 - 2mx - m^2 + 4x + 4m + 5$ $k'(x) = -2x - 2m + 4$ $k'(x) = 2$ $-2x - 2m + 4 = 2$ $-2x = 2m - 2$ $x = 1 - m$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">Answer only: <math>\frac{0}{5}</math></div> $k(1 - m) = g(1 - m)$ $-(1 - m)^2 - 2m(1 - m) - m^2 + 4(1 - m) + 4m + 5 = 2(1 - m) + 2$ $-1 + 2m - m^2 - 2m + 2m^2 - m^2 + 4 - 4m + 4m + 5 = 2 - 2m + 2$ $-2m - 4 = 0$ $\therefore m = -2$ | ✓ $k'(x)$<br>✓ $k'(x) = 2$<br>✓ $x = 1 - m$<br>✓ substitution<br>✓ answer                                  |

|  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                                                                   |
|--|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  | <p><b>OR/OF</b></p> $k(x) = f(x+m) = g(x)$ $-x^2 - 2mx - m^2 + 4x + 4m + 5 = 2x + 2$ $0 = x^2 + (2m-2)x + m^2 - 4m - 3$<br>$\Delta = (2m-2)^2 - 4(1)(m^2 - 4m - 3)$ $\Delta = 4m^2 - 8m + 4 - 4m^2 + 16m + 12$ $\Delta = 8m + 16 = 0$ $\therefore m = -2$<br><p><b>OR/OF</b></p> $f(x) = -x^2 + 4x + 5$ $f'(x) = -2x + 4 = 2$ $-2x = -2$ $x = 1$<br>$f(1) = -(1)^2 + 4(1) + 5$ $\therefore y = 8$ $\therefore \text{translate 2 units to the right}$ $\therefore m = -2$ | <p><b>OR/OF</b></p><br>✓ equating<br>✓ simplification<br><br>✓ substitution into $\Delta$<br><br>✓ $\Delta = 0$<br>✓ answer<br>(5)<br><br><p><b>OR/OF</b></p><br>✓ $f'(x) = 2$<br><br>✓ x-value<br><br><br>✓ y-value<br>✓ 2 units to the right<br>✓ answer<br>(5) |
|  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | <b>[15]</b>                                                                                                                                                                                                                                                       |

### 19. A parabola, and a line whose angle of inclination is given

Official DBE marking guideline · November 2021 · Q7 · 17 marks

#### QUESTION 7

|       |                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                 |
|-------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 7.1   | B(-4 ; 0)<br>D(6 ; 0)                                                                                                                                                                                                                                                                                                                                                                                                                   | ✓ B(-4 ; 0)<br>✓ D(6 ; 0) (2)                                                                                                                                                                   |
| 7.2   | $f(x) = x^2 - 2x - 24$<br>$x_{tp} = \frac{-b}{2a}$ <b>OR/OF</b> $2x - 2 = 0$ <b>OR/OF</b> $x = \frac{-4+6}{2}$<br>$x = \frac{-(-2)}{2(1)}$<br>$\therefore x_{tp} = 1$<br>$y_{tp} = f(1)$<br>$= 1^2 - 2(1) - 24$<br>$= -25$<br>C(1 ; -25)                                                                                                                                                                                                | ✓ $x_{tp} = 1$<br><br><br><br><br><br><br><br>✓ $y_{tp} = -25$ (2)                                                                                                                              |
| 7.3   | $y \geq -25$<br><br><b>OR/OF</b><br><br>$y \in [-25 ; \infty)$                                                                                                                                                                                                                                                                                                                                                                          | ✓ answer (1)<br><br><b>OR/OF</b><br><br>✓ answer (1)                                                                                                                                            |
| 7.4.1 | $m_{AE} = \tan 14,04^\circ = 0,25 = \frac{1}{4}$                                                                                                                                                                                                                                                                                                                                                                                        | ✓ answer (1)                                                                                                                                                                                    |
| 7.4.2 | $m_{\text{tang}} = -4$<br>$f'(x) = 2x - 2$<br><br>$2x - 2 = -4$<br>$x_T = -1$<br>$y_T = -21$                                                                                                                                                                                                                                                                                                                                            | ✓ $m_{\text{tang}} = -4$<br>✓ $f'(x) = 2x - 2$<br><br>✓ equating<br>✓ $x_T = -1$<br>✓ $y_T = -21$ (5)                                                                                           |
| 7.5   | $m_{\text{line}} = \frac{1}{4}$<br>$y + 9 = \frac{1}{4}(x + 3)$ <b>OR/OF</b> $-9 = \frac{1}{4}(-3) + c$<br>$y + 9 = \frac{1}{4}x + \frac{3}{4}$ $c = -\frac{33}{4} = -8,25$<br>$y = \frac{1}{4}x - \frac{33}{4}$ <b>OR/OF</b> $y = 0,25x - 8,25$<br>$x^2 - 2x - 24 = \frac{1}{4}x - \frac{33}{4}$<br>$4x^2 - 8x - 96 = x - 33$<br>$4x^2 - 9x - 63 = 0$<br>$(4x - 21)(x + 3) = 0$<br>$\therefore x = \frac{21}{4} = 5,25$ or $x \neq -3$ | ✓ $m_{\text{line}} = \frac{1}{4}$<br><br>✓ substitution $m$ and $K(-3 ; -9)$<br><br>✓ $y = \frac{1}{4}x - \frac{33}{4}$<br><br>✓ equating<br>✓ standard form<br><br>✓ answer with selection (6) |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                         | [17]                                                                                                                                                                                            |

## 20. A parabola meeting a straight line, and the distance between them

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# QUESTION/VRAAG 5

|     |                                                                                                                                                                                                                                                                                                |                                                                                                       |
|-----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------|
| 5.1 | $-2x^2 + 4x + 16 = 0$<br>$x^2 - 2x - 8 = 0$<br>$(x-4)(x+2) = 0$<br>$x = 4$ or $x = -2$<br>$\therefore A(-2;0)$ and $B(4;0)$                                                                                                                                                                    | $\checkmark$ factors<br>$\checkmark x = -2$ $\checkmark x = 4$<br>(3)                                 |
| 5.2 | $f(x) = -2x^2 + 4x + 16$<br>$-\frac{b}{2a} = -\frac{-4}{-2(2)} = 1$<br>$f(1) = -2(1)^2 + 4(1) + 16 = 18$<br>$\therefore C(1;18)$<br><br><b>OR/OF</b><br>$f(x) = -2x^2 + 4x + 16$<br>$f'(x) = -4x + 4$<br>$-4x + 4 = 0$<br>$x = 1$<br>$f(1) = -2(1)^2 + 4(1) + 16 = 18$<br>$\therefore C(1;18)$ | $\checkmark 1$<br><br>$\checkmark 18$<br><b>OR/OF</b><br><br>$\checkmark 1$<br>$\checkmark 18$<br>(2) |
| 5.3 | $y \leq 18$<br><br><b>OR/OF</b><br>$y \in (-\infty; 18]$                                                                                                                                                                                                                                       | $\checkmark y \leq 18$<br>(1)<br><br><b>OR/OF</b><br>$\checkmark y \in (-\infty; 18]$<br>(1)          |
| 5.4 | TP (1 ; 18) for $f$<br>TP (2 ; 15) for $h$<br>$\therefore p = -1$ $q = -3$                                                                                                                                                                                                                     | $\checkmark$ TP for $h$ at (2 ; 15)<br>$\checkmark p = -1$<br>$\checkmark q = -3$<br>(3)              |
| 5.5 | $y = 2x + 4$<br>$x = 2y + 4$<br>$\therefore y = \frac{1}{2}x - 2$                                                                                                                                                                                                                              | $\checkmark$ swop $x$ and $y$<br>$\checkmark y = \frac{1}{2}x - 2$<br>(2)                             |
| 5.6 | $g(x) = 0$ or $g^{-1}(x) = 0$<br>$x = 4$ or $x = -2$ (product 0 at x-intercepts)                                                                                                                                                                                                               | $\checkmark x = 4$<br>$\checkmark x = -2$<br>(2)                                                      |

|     |                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                    |
|-----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5.7 | $-2x^2 + 4x + 16 + k = 2x + 4$<br>$-2x^2 + 2x + 12 + k = 0$<br>$b^2 - 4ac < 0$<br>$(2)^2 - 4(-2)(12 + k) < 0$<br>$4 + 8(12 + k) < 0$<br>$100 + 8k < 0$<br>$k < -12,5$<br><br><b>OR/OF</b><br><br>$g'(x) = 2$<br>$f'(x) = -4x + 4 = 2$<br><br>$x = \frac{1}{2}$<br>$f\left(\frac{1}{2}\right) = 17,5$<br>$g\left(\frac{1}{2}\right) = 5$<br>$\therefore k < -12,5$ | ✓equating<br>✓ standard form<br>✓ $b^2 - 4ac < 0$<br>✓ substitution<br><br><br>✓ answer<br>(5)<br><br><b>OR/OF</b><br>✓ $g'(x) = 2$<br>✓ $f'(x) = -4x + 4$<br><br><br>✓ $f\left(\frac{1}{2}\right) = 17,5$<br>✓ $g\left(\frac{1}{2}\right) = 5$<br>✓ answer<br>(5) |
|     |                                                                                                                                                                                                                                                                                                                                                                   | <b>[18]</b>                                                                                                                                                                                                                                                        |

## 21. A turning point that is also a point of intersection

Official DBE marking guideline · November 2023 · Q5 · 18 marks

### QUESTION 5/VRAAG 5

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                           |
|-----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5.1 | $(1; 8)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $\checkmark x = 1 \checkmark y = 8$ (2)                                                                                                                                                                                                                                                                                                                   |
| 5.2 | $y = -\frac{1}{2}(0-1)^2 + 8$ $= 7\frac{1}{2}$ $C\left(0; \frac{15}{2}\right)$                                                                                                                                                                                                                                                                                                                                                                                                                                                               | $\checkmark x = 0$<br><br>$\checkmark$ answer (2)                                                                                                                                                                                                                                                                                                         |
| 5.3 | $8 = \frac{d}{1}$ $\therefore d = 8$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | $\checkmark$ substitution $(1; 8)$ (1)                                                                                                                                                                                                                                                                                                                    |
| 5.4 | $y \in R; y \neq 0$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | $\checkmark y \neq 0$ (1)                                                                                                                                                                                                                                                                                                                                 |
| 5.5 | $-3 \leq x < 0$ or $x \geq 5$<br><b>OR/OF</b><br>$x \in [-3; 0) \cup [5; \infty)$                                                                                                                                                                                                                                                                                                                                                                                                                                                            | $\checkmark \checkmark -3 \leq x < 0$<br>$\checkmark x \geq 5$<br>(3)                                                                                                                                                                                                                                                                                     |
| 5.6 | $-2x + k = \frac{8}{x}$ $-2x^2 + kx - 8 = 0$ $\Delta = (k)^2 - 4(-2)(-8)$ $k^2 - 64 < 0$ $CV: k = 8; k = -8$ $\therefore -8 < k < 8 \quad \text{or/of} \quad k \in (-8; 8)$<br><b>OR/OF</b><br>$g'(x) = h'(x)$ $-\frac{8}{x^2} = -2$ $-8 = -2x^2$ $x = \pm 2$ $y = \pm 4 \quad \therefore B(2; 4) \text{ and } A(-2; -4)$ For tangents:<br>$h(x) = -2x + k \quad \text{or} \quad h(x) = -2x + k$ $4 = -2(2) + k \quad \quad -4 = -2(-2) + k$ $k = 8 \quad \quad \quad k = -8$ $\therefore -8 < k < 8 \quad \text{or/of} \quad k \in (-8; 8)$ | $\checkmark -2x + k = \frac{8}{x}$<br>$\checkmark$ standard form<br>$\checkmark$ substitution into $\Delta$<br>$\checkmark \Delta < 0$ or $\Delta = 0$<br><br>$\checkmark$ inequality (5)<br><br><b>OR/OF</b><br>$\checkmark -\frac{8}{x^2} \checkmark = -2$<br><br>$\checkmark$ x-values<br>$\checkmark$ y-values<br><br><br>$\checkmark$ inequality (5) |

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|-----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5.7 | $\begin{aligned} h(x) &= -2x + 8 \\ -2x + 8 &= \frac{8}{x} \\ -2x^2 + 8x &= 8 \\ -2x^2 + 8x - 8 &= 0 \\ x^2 - 4x + 4 &= 0 \\ (x-2)^2 &= 0 \\ \therefore x &= 2 \end{aligned}$<br>$\begin{aligned} f(2) &= \frac{15}{2} \\ h(2) &= 4 \\ 4 &= \frac{15}{2} + t \\ \therefore t &= -\frac{7}{2} \end{aligned}$ <p><b>OR/OF</b></p> $f(2) = \frac{15}{2}$ <p>Tangent point of contact (2 ; 4)</p> $\begin{aligned} \therefore 4 &= -\frac{1}{2}(2-1)^2 + 8 + t \\ 4 &= \frac{15}{2} + t \\ \therefore t &= -\frac{7}{2} \end{aligned}$ <p><b>OR/OF</b></p> $\begin{aligned} g(x) &= 8x^{-1} \\ g'(x) &= -8x^{-2} \\ -2 &= -8x^{-2} \\ \frac{1}{4} &= \frac{1}{x^2} \\ x &= 2 \\ y &= \frac{8}{2} = 4 \\ R(2; 4) \\ y &= -\frac{1}{2}(x-1)^2 + 8 + t \\ 4 &= -\frac{1}{2}(2-1)^2 + 8 + t \\ t &= -\frac{7}{2} \end{aligned}$ | <div style="text-align: right;">✓ <math>x = 2</math></div><br><br><div style="text-align: right;">✓ <math>f(2)</math></div> <div style="text-align: right;">✓ <math>h(2)</math></div><br><br><div style="text-align: right;">✓ answer</div><br><br><p>(4)</p><br><br><p><b>OR/OF</b></p> <div style="text-align: right;">✓ <math>x = 2</math></div> <div style="text-align: right;">✓ <math>f(2)</math></div><br><br><div style="text-align: right;">✓ <math>h(2)</math></div><br><br><div style="text-align: right;">✓ answer</div><br><br><p>(4)</p><br><br><p><b>OR/OF</b></p><br><br><div style="text-align: right;">✓ <math>x = 2</math></div><br><br><div style="text-align: right;">✓ <math>h(2)</math></div> <div style="text-align: right;">✓ <math>f(2)</math></div><br><br><div style="text-align: right;">✓ answer</div> <div style="text-align: right;">(4)</div> |
|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | [18]                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |

## 22. Range, both intersections, and the line drawn through them

Official DBE marking guideline · May/June 2024 · Q6 · 18 marks

### QUESTION 6/VRAAG 6

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                           |
|-----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 6.1 | $y \geq -4$ or $y \in [-4; \infty)$                                                                                                                                                                                                                                                                                                                                                                                                       | ✓ answer (1)                                                                                                                                                                                                                                                              |
| 6.2 | $x^2 - 2x - 3 = 0$<br>$(x - 3)(x + 1) = 0$<br>$x = 3$ or $x = -1$<br>$\therefore E(3; 0)$ and $D(-1; 0)$                                                                                                                                                                                                                                                                                                                                  | ✓ = 0<br><br>✓ both x-values<br>✓ correct identification of coordinates (3)                                                                                                                                                                                               |
| 6.3 | $P(0; -3)$<br>$\therefore m_{PE} = 1$<br>$\therefore g(x) = x - 3$                                                                                                                                                                                                                                                                                                                                                                        | ✓ $m_{PE} = 1$<br>✓ $g(x) = x - 3$ (2)                                                                                                                                                                                                                                    |
| 6.4 | $f(x) > g(x)$<br>$x < 0$ or $x > 3$                                                                                                                                                                                                                                                                                                                                                                                                       | ✓ $x < 0$<br>✓ $x > 3$ (2)                                                                                                                                                                                                                                                |
| 6.5 | Distance $= -x^2 + 2x + 3 - x + 3 = -x^2 + x + 6$<br>$D' = -2x + 1 = 0$ or/of $x = -\frac{b}{2a}$<br>$\therefore x = \frac{1}{2}$ $\therefore x = \frac{1}{2}$<br>$D\left(\frac{1}{2}\right) = -\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 6$<br>$= \frac{25}{4} = 6,25$                                                                                                                                                                  | ✓ $D = -x^2 + x + 6$<br><br>✓ method<br>✓ $x = \frac{1}{2}$<br><br>✓ substitution<br><br>✓ answer (5)                                                                                                                                                                     |
| 6.6 | $f'(x) = m$<br>$1 = 2x - 2$<br>$x = \frac{3}{2}$<br>Point on $f: \left(\frac{3}{2}; -\frac{15}{4}\right)$<br>$-\frac{15}{4} = \left(\frac{3}{2} - 3\right) - n$<br>$\therefore n = 2\frac{1}{4} = \frac{9}{4} = 2,25$<br><br><b>OR/OF</b><br>$f(x) = k(x)$<br>$x^2 - 2x - 3 = x - 3 - n$<br>$x^2 - 3x + n = 0$<br>$\Delta = b^2 - 4ac$<br>$= (-3)^2 - 4(1)(n)$<br>To touch: $\Delta = 0$<br>$0 = 9 - 4n$<br>$4n = 9$<br>$n = \frac{9}{4}$ | ✓ $1 = 2x - 2$<br>✓ $x = \frac{3}{2}$<br><br>✓ $-\frac{15}{4}$<br>✓ $-\frac{15}{4} = \left(\frac{3}{2} - 3\right) - n$<br><br>✓ answer (5)<br><br><b>OR/OF</b><br>✓ equating<br>✓ standard form<br><br>✓ substitution into $\Delta$<br>✓ $\Delta = 0$<br><br>✓ answer (5) |
|     |                                                                                                                                                                                                                                                                                                                                                                                                                                           | [18]                                                                                                                                                                                                                                                                      |