

PATTERNS, SEQUENCES AND SERIES

Solutions

DBE's own marking guidelines, in booklet order

Every solution here is a clip of the official DBE marking guideline published with the paper the question came from. The working, the mark ticks and the accepted alternatives are DBE's. Nothing has been rewritten or re-derived.

The numbers match the question booklet: solution 34 answers question 34, and the chapters are in the same order.

DBE writes one guideline for both languages, so these pages are bilingual and their marking notes are largely in English. That is DBE's own page, reproduced as printed.

A marking guideline is written for markers, not learners. It shows the shortest route to the marks, so where a step looks compressed, that is the guideline being terse rather than a step missing.

1. A quadratic pattern, a difference of 33, then adding m to every term

Official DBE marking guideline · May/June 2025 · Q3 · 9 marks

QUESTION/VRAAG 3

3.1	$ \begin{array}{ccccccc} 14 & ; & 9 & ; & 6 & ; & 5 & ; & \dots \\ & \swarrow & & \searrow & \swarrow & & \searrow & & \\ & -5 & & -3 & & -1 & & & \\ & \swarrow & & \searrow & \swarrow & & \searrow & & \\ & 2 & & 2 & & & & & \end{array} $ $ \begin{array}{lcl} 2a = 2 & 3(1) + b = -5 & 1 - 8 + c = 14 \\ a = 1 & b = -8 & c = 21 \\ \therefore T_n = n^2 - 8n + 21 \end{array} $	$ \begin{array}{l} \checkmark 2a = 2 \\ \checkmark 3(1) + b = -5 \\ \checkmark 1 - 8 + c = 14 \end{array} $ (3)
3.2	$ \begin{array}{l} T_n = -5 + (n-1)(2) \\ T_n = 2n - 7 \\ 2n - 7 = 33 \\ \therefore n = 20 \\ \therefore T_{21} = (21)^2 - 8(21) + 21 \\ T_{21} = 294 \end{array} $ OR/OF $ \begin{array}{l} \therefore T_{n+1} - T_n = (n+1)^2 - 8(n+1) + 21 - n^2 + 8n - 21 \\ n^2 + 2n + 1 - 8n - 8 + 21 - n^2 + 8n - 21 = 33 \\ 2n - 7 = 33 \\ \therefore n = 20 \\ \therefore T_{21} = (21)^2 - 8(21) + 21 \\ T_{21} = 294 \end{array} $	$ \begin{array}{l} \checkmark \text{general term} \\ \checkmark \text{equating to 33} \\ \\ \checkmark \text{answer} \end{array} $ (3) OR/OF $ \begin{array}{l} \checkmark T_{n+1} - T_n \\ \checkmark \text{equating to 33} \\ \\ \checkmark \text{answer} \end{array} $ (3)
3.3	$ \begin{array}{l} T_7 = T_1 = 14 \\ \therefore 14 + m \geq 0 \\ m \geq -14 \\ \text{And } T_6 = T_2 \\ \therefore 9 + m < 0 \\ m < -9 \\ \therefore -14 \leq m < -9 \end{array} $	$ \begin{array}{l} \checkmark -14 \\ \\ \checkmark -9 \\ \checkmark -14 \leq m < -9 \end{array} $ (3)
		[9]

2. Find b from the first difference, then the 60th term

Official DBE marking guideline · November 2022 · Q3 · 10 marks

QUESTION 3/VRAAG 3

3.1	$3a + b = 7$ $3 + b = 7$ $b = 4$ OR/OF $T_2 - T_1 = 7$ $4 + 2b + 9 - (1 + b + 9) = 7$ $b = 4$	$\checkmark 3a + b = 7$ $\checkmark 3 + b = 7$ (2) OR/OF $\checkmark T_2 - T_1 = 7$ $\checkmark$ substitution (2)
3.2	$T_n = n^2 + 4n + 9$ $T_{60} = (60)^2 + 4(60) + 9$ $= 3849$ Answer only: full marks	$\checkmark$ substitution $\checkmark$ answer (2)
3.3	14 ; 21 ; 30 ; 41; First difference: 7 ; 9 ; 11 ; ... Common 2 nd difference: 2 $T_p = 2p + 5$ Answer only: full marks OR/OF First difference: 7 ; 9 ; 11 ; ... $T_n = a + (n-1)d$ $T_p = 7 + (p-1)(2)$ $T_p = 2p + 5$	$\checkmark$ first difference $\checkmark 2$ $\checkmark 2p + 5$ (3) OR/OF $\checkmark$ first difference $\checkmark 2$ $\checkmark 2p + 5$ (3)
3.4	$157 = 2p + 5$ $p = 76$ $\therefore$ Between T_{76} and T_{77} OR/OF $T_{n+1} - T_n = 157$ $(n+1)^2 + 4(n+1) + 9 - (n^2 + 4n + 9) = 157$ $n^2 + 2n + 1 + 4n + 4 + 9 - n^2 - 4n - 9 = 157$ $2n = 152$ $n = 76$ $\therefore$ Between T_{76} and T_{77}	$\checkmark 157 = 2p + 5$ $\checkmark p = 76$ $\checkmark T_{76}$ and T_{77} (3) OR/OF $\checkmark T_{n+1} - T_n = 157$ $\checkmark n = 76$ $\checkmark T_{76}$ and T_{77} (3)
		[10]

3. A pattern that turns, and where its terms start to fall

Official DBE marking guideline · November 2021 · Q3 · 11 marks

QUESTION 3

3.1	-82	✓ answer (1)
3.2	-145 ; -122 ; -101 ; ... $2a = -2 \quad \therefore a = -1$ $3a + b = 23 \quad \therefore 3(-1) + b = 23 \quad \therefore b = 26$ $a + b + c = -145 \quad \therefore -1 + 26 + c = -145 \quad \therefore c = -170$ $\therefore T_n = -n^2 + 26n - 170$ OR/OF $2a = -2 \quad \therefore a = -1$ $c = -145 + (-2) - 23 = -170$ $\therefore T_n = -n^2 + bn - 170$ $-145 = -1 + b - 170$ $b = 26$ $\therefore T_n = -n^2 + 26n - 170$	$\checkmark 2a = -2$ $\checkmark 3(-1) + b = 23$ $\checkmark -1 + 26 + c = -145$ (3) OR/OF $\checkmark 2a = -2$ $\checkmark c = -145 + (-2) - 23$ $\checkmark -145 = -1 + b - 170$ (3)
3.3	$T_n = bn + c$ $T_n = -2n + 25$ $-2n + 25 = -121$ $-2n = -146$ $n = 73$ Between T_{73} and T_{74} OR/OF $T_{n+1} - T_n = -(n+1)^2 + 26(n+1) - 170 - (-n^2 + 26n - 170)$ $-121 = -2n + 25$ $n = 73$ Between T_{73} and T_{74} $T_n = a + (n-1)d$ $= 23 + (n-1)(-2)$ $= 25 - 2n$	$\checkmark T_n = -2n + 25$ $\checkmark T_n = -121$ $\checkmark n = 73$ $\checkmark$ answer (4) OR/OF $\checkmark T_n = -2n + 25$ $\checkmark T_n = -121$ $\checkmark n = 73$ $\checkmark$ answer (4)
3.4	$n = \frac{-b}{2a} = \frac{-26}{2(1)} = 13$ $T_{13} = -1$ $\therefore$ add 2 OR/OF $T'_n = -2n + 26 = 0$ $n = 13$ $T_{13} = -(13)^2 + 26(13) - 170 = -1$ $\therefore$ add 2	$\checkmark 13$ $\checkmark T_{13} = -1$ $\checkmark$ add 2 (3) OR/OF $\checkmark 13$ $\checkmark T_{13} = -1$ $\checkmark$ add 2 (3)

4. 3 ; 7 ; 12 ..., then what must be added to reach 13 527

Official DBE marking guideline · May/June 2024 · Q3 · 15 marks

QUESTION 3/VRAAG 3

3.1.1	$\begin{array}{c} 3 ; 7 ; 12 ; 18 \\ \quad \vee \quad \vee \quad \vee \\ \text{First diff: } 4 ; 5 ; 6 \\ \quad \quad \vee \quad \vee \\ \text{Second diff: } 1 ; 1 \\ 2a = 1 \\ a = \frac{1}{2} \\ 3a + b = 4 \\ 3\left(\frac{1}{2}\right) + b = 4 \\ b = \frac{5}{2} \\ a + b + c = 3 \\ \frac{1}{2} + \frac{5}{2} + c = 3 \\ c = 0 \\ T_n = \frac{1}{2}n^2 + \frac{5}{2}n \end{array}$	$\begin{array}{l} \checkmark 2a = 1 \\ \\ \checkmark 3\left(\frac{1}{2}\right) + b = 4 \\ \\ \checkmark \frac{1}{2} + \frac{5}{2} + c = 3 \end{array}$ (3)
3.1.2	$\begin{array}{l} 13\,527 = \frac{1}{2}n^2 + \frac{5}{2}n \\ n^2 + 5n - 27\,054 = 0 \\ (n - 162)(n + 167) = 0 \\ n = 162 \text{ or } n = -167 \\ T_{161} = 13\,363 \\ \therefore T_{161} + 164 = 13\,527 \\ 164 \text{ must be added.} \\ \\ \textbf{OR/OF} \\ \\ T_n = 3 + \text{sum of 1}^{\text{st}} \text{ differences} \\ 13\,527 = 3 + 4 + 5 + \dots + n \\ \frac{n - 3 + 1}{2}[3 + n] = 13\,527 \\ n^2 + n - 27\,060 = 0 \\ (n + 165)(n - 167) = 0 \\ n = 164 \end{array}$	$\begin{array}{l} \checkmark 13\,527 = \frac{1}{2}n^2 + \frac{5}{2}n \\ \checkmark \text{standard form} \\ \\ \checkmark \text{answers for } n \\ \\ \checkmark 164 \end{array}$ (4) OR/OF $\begin{array}{l} \checkmark 13\,527 = 3 + 4 + 5 + \dots + n \\ \checkmark n^2 + n - 27\,060 = 0 \\ \\ \checkmark \text{answers for } n \\ \checkmark 164 \end{array}$ (4)

3.2.1	$T_n = 8 + (n-1)(3)$ $T_n = 3n + 5$ $41 = 3n + 5$ $36 = 3n$ $n = 12$	✓ $T_n = 3n + 5$ ✓ $T_n = 41$ ✓ answer (3)
3.2.2a	$P_{41} = 12$	✓ answer (1)
3.2.2b	$P_8 = a + 7d = 1$ $P_{11} = a + 10d = 2$ $3d = 1$ $d = \frac{1}{3}$ $a + 7\left(\frac{1}{3}\right) = 1$ $a = -\frac{4}{3}$	✓ $a + 7d = 1$ ✓ $a + 10d = 2$ ✓ value of d ✓ value of a (4)
		[15]

5. When a term of the pattern equals a term of its own differences

Official DBE marking guideline · May/June 2021 · Q2 · 17 marks

QUESTION/VRAAG 2

2.1.1	$2a = -8$ $a = -4$ $3a + b = 28$ $b = 40$ $a + b + c = 72$ $c = 36$ $T_n = -4n^2 + 40n + 36$	✓ second differences = -8 ✓ $a = -4$ ✓ $b = 40$ ✓ $c = 36$ (4)
2.1.2	$T_{12} = 36 - 8n = 36 - 8(12) = -60$ $-4n^2 + 40n + 36 = -60$ $n^2 - 10n - 24 = 0$ $(n - 12)(n + 2) = 0$ $\therefore n = 12$	✓ $36 - 8n$ ✓ -60 ✓ standard form ✓ factors ✓ $n = 12$ (5)
2.1.3	$T_n' = -8n + 40 = 0$ $n = 5$ $T_n = -4(5)^2 + 40(5) + 36 = 136$ OR/OF $\frac{-b}{2a} = \frac{40}{8} = 5$ $T_n = -4(5)^2 + 40(5) + 36$ $= 136$	✓ $-8n + 40$ ✓ $n = 5$ ✓ 136 (3) OR/OF ✓ substitution ✓ $n = 5$ ✓ 136 (3)
2.1.4	Maximum value = 41	✓ value (1)
2.2	$2\sin 3x - (-11) = 15 - 2\sin 3x$ $4\sin 3x = 4$ $\sin 3x = 1$ $3x = 90^\circ$ $\therefore x = 30^\circ$	✓ equating ✓ $4\sin 3x = 4$ ✓ $\sin 3x = 1$ ✓ answer (4)
		[17]

6. A linear pattern, its sum, and a sigma expansion

Official DBE marking guideline · November 2021 · Q4 · 10 marks

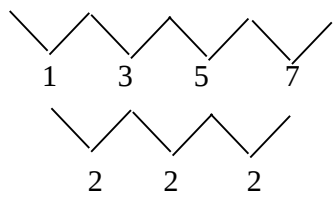
QUESTION/VRAAG 4

4.1	$a = 5$ and/en $d = 2$ $T_{51} = 5 + (51-1)(2)$ $= 105$	✓ a and d ✓ substitution into correct formula ✓ answer (3)
4.2	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{51} = \frac{51}{2}[2(5) + (51-1)2]$ or/of $S_{51} = \frac{51}{2}[5 + 105]$ $= 2\,805$	✓ substitution into correct formula ✓ answer (2)
4.3	$\sum_{n=1}^{5\,000} (2n+3) = 5 + 7 + 9 + \dots + 10\,003$	✓ expansion (1)
4.4	$T_1 = -3$ $T_{4\,999} = -2(4\,999) - 1 = -9\,999$ $\therefore \sum_{n=1}^{5\,000} (2n+3) + \sum_{n=1}^{4\,999} (-2n-1)$ $= (5 + 7 + 9 + \dots + 9\,999 + 10\,001 + 10\,003) +$ $\quad \quad \quad (-3 - 5 - 7 - 9 - \dots - 9\,999)$ $= 10\,001 + 10\,003 - 3$ $= 20\,001$ OR/OF $S_{4\,999} = \frac{4\,999}{2}[2(-3) + (4\,999-1)(-2)] = -24\,999\,999$ $S_{5\,000} = \frac{5\,000}{2}((2)(5) + (5\,000-1)(2)) = 25\,020\,000$ $\sum_{n=1}^{5\,000} (2n+3) + \sum_{n=1}^{4\,999} (-2n-1) = 25\,020\,000 - 24\,999\,999$ $= 20\,001$	✓ $T_1 = -3$ ✓ $T_{4\,999} = -9\,999$ ✓ both expansions ✓ answer (A) (4) OR/OF ✓ $T_1 = -3$ ✓ $S_{4\,999} = -24\,999\,999$ ✓ $S_{5\,000} = 25\,020\,000$ ✓ answer (A) (4)
		[10]

7. Twenty terms, then seventy-five, and the term that had to change

Official DBE marking guideline · November 2024 · Q2 · 14 marks

QUESTION/VRAAG 2

2.1.1	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{20} = \frac{20}{2}[2(7) + (20-1)(5)]$ $S_{20} = 1\,090$	✓ substitution ✓ answer (2)
2.1.2	$T_n = 7 + (n-1)(5)$ $T_n = 5n + 2$ $S_{75} - S_{20} = 14\,400 - 1\,090 = 13\,310$ $\sum_{n=1}^{75} (5n+2) - \sum_{n=1}^{20} (5n+2) = 13\,310$ $\therefore \sum_{n=21}^{75} (5n+2) = 13\,310$ OR/OF $T_{21} + T_{22} + T_{23} + \dots + T_{75} = 14\,400 - 1\,090$ $107 + 112 + 117 + \dots = 13\,310$ $T_n = 102 + 5n$ $\therefore \sum_1^{55} (5n+102) = 13\,310$	✓ general term ✓ $S_{75} - S_{20}$ ✓ $\sum_{n=21}^{75} \checkmark = 13\,310$ (4) OR/OF ✓ $S_{75} - S_{20}$ ✓ general term ✓ $\sum_{n=1}^{55} \checkmark = 13\,310$ (4)
2.2.1	$T_n = 2n - 1$ 98 th first difference = $2(98) - 1$ = 195 Quadratic: $T_{98} = 9\,632 - 195$ = 9 437	✓ answer ✓ method ✓ answer (3)
2.2.2	28 ; 29 ; 32  $\begin{array}{lll} 3a + b = 1 & 9a + 3b + c = 32 & \\ 2a = 2 & 3(1) + b = 1 & T_3 = 1(3)^2 - 2(3) + c = 32 \\ a = 1 & b = -2 & c = 29 \end{array}$ OR/OF $1 - 2 + c = 28$ $c = 29$ $\therefore T_n = n^2 - 2n + 29$	✓ second difference ✓ $a = 1$ ✓ $b = -2$ ✓ method to calculate c ✓ $c = 29$ (5)
		[14]

8. From the 7th term to the number of terms, then the first forty

Official DBE marking guideline · May/June 2022 · Q2 · 16 marks

QUESTION/VRAAG 2

2.1.1	$a + 6d = 35$ $-1 + 6d = 35$ $6d = 36$ $d = 6$ OR/OF $\frac{35 - (-1)}{7 - 1} = 6$ ANSWER ONLY: FULL MARKS	✓ substitution ✓ answer (2) OR/OF ✓ substitution ✓ answer (2)
2.1.2	$T_n = a + (n-1)d$ $473 = -1 + (n-1)(6)$ $79 = n - 1$ $\therefore n = 80$ ANSWER ONLY: FULL MARKS	✓ substitution into the correct formula ✓ equating to 473 ✓ answer (3)
2.1.3	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{40} = \frac{40}{2}[2(-1) + (40-1)(6)]$ $\therefore S_{40} = 4640$ OR/OF $T_{40} = 6(40) - 7$ $= 233$ $S_n = \frac{n}{2}(a + l)$ $= \frac{40}{2}(-1 + 233)$ $= 4640$	✓ substitution ✓ answer (2) OR/OF ✓ substitution ✓ answer (2)
2.2.1	$75 \quad \quad 53 \quad \quad 35 \quad \quad 21$ $\quad \backslash \quad / \quad \backslash \quad / \quad \backslash \quad /$ $\quad -22 \quad \quad -18 \quad \quad -14$ $\quad \quad \backslash \quad / \quad \backslash \quad /$ $\quad \quad \quad 4 \quad \quad \quad 4$ $T_5 = 11$	✓ answer (A) (1)
2.2.2	$T_n = an^2 + bn + c$ $2a = 4$ $a = 2$ $3a + b = -22$ $6 + b = -22$ $b = -28$ $a + b + c = 75$ $2 - 28 + c = 75$ $c = 101$ $\therefore T_n = 2n^2 - 28n + 101$	✓ $T_n = an^2 + bn + c$ ✓ $a = 2$ ✓ $b = -28$ ✓ $c = 101$ (4)

2.2.3

Minimum value of T_n

$$n = -\frac{b}{2a} = -\frac{(-28)}{2(2)}$$

$$n = 7$$

$$\text{Minimum value of } T_n = 2(7)^2 - 28(7) + 101 = 3$$

Each term in the new pattern is $-\frac{1}{5}$ the value of the terms in the old pattern.

$$\text{Maximum value of new pattern} = -\frac{3}{5}$$

OR/OF

$$T'_n = 4n - 28$$

$$4n - 28 = 0$$

$$4n = 28$$

$$n = 7$$

$$\text{Minimum value of } T_n = 2(7)^2 - 28(7) + 101 = 3$$

Each term in the new pattern is $-\frac{1}{5}$ the value of the terms in the old pattern.

$$\text{Maximum value of new pattern} = -\frac{3}{5}$$

OR/OF

$$T_n = -\frac{2}{5}n^2 + \frac{28}{5}n - \frac{101}{5}$$

$$n = -\frac{b}{2a} = \frac{-\frac{28}{5}}{2\left(\frac{-2}{5}\right)}$$

$$= 7$$

$$T_7 = -\frac{3}{5}$$

OR/OF

$$\checkmark n = 7$$

$$\checkmark \text{ min value} = 3$$

$$\checkmark -\frac{1}{5} \text{ value of term of old pattern}$$

$$\checkmark \text{ max value} = -\frac{3}{5} \quad (4)$$

OR/OF

$$\checkmark n = 7$$

$$\checkmark \text{ min value} = 3$$

$$\checkmark -\frac{1}{5} \text{ value of term of old pattern}$$

$$\checkmark \text{ max value} = -\frac{3}{5} \quad (4)$$

OR/OF

$$\checkmark \checkmark T_n \div (-5)$$

$$\checkmark n = 7$$

$$\checkmark \text{ max value} = -\frac{3}{5} \quad (4)$$

OR/OF

$$T_n = -\frac{2}{5}n^2 + \frac{28}{5}n - \frac{101}{5}$$

$$T'_n = -\frac{4}{5}n + \frac{28}{5}$$

$$\checkmark\checkmark T_n \div (-5)$$

$$-\frac{4}{5}n + \frac{28}{5} = 0$$

$$-4n = -28$$

$$n = 7$$

$$\text{Minimum value of } T_n = 2(7)^2 - 28(7) + 101 = 3$$

Each term in the new pattern is $-\frac{1}{5}$ the value of the terms in the old pattern.

$$\checkmark n = 7$$

$$\text{Maximum value of new pattern} = -\frac{3}{5}$$

$$\checkmark \text{ max value} = -\frac{3}{5}$$

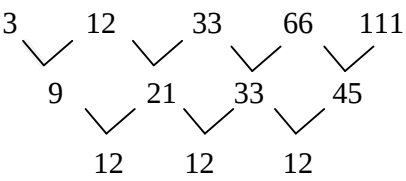
(4)

[16]

9. T91 and S91, then a quadratic pattern from three differences

Official DBE marking guideline · November 2023 · Q2 · 16 marks

QUESTION 2/VRAAG 2

2.1.1	$7 + 12 + 17 + \dots$ $T_n = a + (n-1)d$ $T_{91} = 7 + (91-1)(5)$ $T_{91} = 457$ OR/OF $d = 5$ $T_n = 5n + 2$ $T_{91} = 5(91) + 2$ $T_{91} = 457$	$\checkmark d = 5$ $\checkmark$ substitution into correct formula $\checkmark$ answer (3) OR/OF $\checkmark d = 5$ $\checkmark$ substitution $n = 91$ $\checkmark$ answer (3)
2.1.2	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{91} = \frac{91}{2}[2 \times 7 + (91-1)(5)]$ $S_9 = 21\ 112$ OR/OF $S_n = \frac{n}{2}(a + l)$ $S_{91} = \frac{91}{2}(7 + 457)$ $S_{91} = 21\ 112$	$\checkmark$ substitution into correct formula $\checkmark$ answer (2) OR/OF $\checkmark$ substitution into correct formula $\checkmark$ answer (2)
2.1.3	$T_n = 7 + (n-1)(5)$ $5n + 2 = 517$ $5n = 515$ $n = 103$	$\checkmark$ substitution into correct formula $\checkmark$ equate $\checkmark$ answer (3)
2.2.1	$T_1 = 3; T_2 - T_1 = 9$ and $T_3 - T_2 = 21$  $\therefore T_5 = 3 + 9 + 21 + 33 + 45 = 111$ OR/OF $2a = 12$ $a = 6$ $3(6) + b = 9$ $b = -9$ $6 - 9 + c = 3$ $T_5 = 6(5)^2 - 9(5) + 6 = 111$	$\checkmark$ constant second diff = 12 $\checkmark$ first differences : 33 and 45 (2) OR/OF $\checkmark$ constant second diff = 12 $\checkmark$ substitute 5 (2)

2.2.2	$2a = 12$ $a = 6$ $3(6) + b = 9 \quad \text{or} \quad 5 \times 6 + b = 21$ $b = -9$ $6 - 9 + c = 3$ $c = 6$ $T_n = 6n^2 - 9n + 6$	$\checkmark 2a = 12$ $\checkmark 3(6) + b = 9 / 5 \times 6 + b = 21$ $\checkmark 6 - 9 + c = 3 \quad (3)$
2.2.3	$T_n' = 12n - 9 > 0$ $n > \frac{3}{4}$ $\therefore T_n$ is increasing for $n \in N$ OR/OF $n = -\frac{b}{2a} = -\frac{-9}{2(6)}$ $n = \frac{3}{4}$ $\therefore \text{min at } n = 1 \text{ for } n \in N$ $\therefore T_n$ is increasing for $n \in N$	$\checkmark T_n' = 12n - 9$ $\checkmark n > \frac{3}{4}$ $\checkmark \text{increasing for } n \in N \quad (3)$ OR/OF $\checkmark n = -\frac{b}{2a} = \frac{9}{2(6)}$ $\checkmark n = \frac{3}{4}$ $\checkmark \text{increasing for } n \in N \quad (3)$
		[16]

10. An athlete's weekly kilometres as an arithmetic series

Official DBE marking guideline · May/June 2025 · Q2 · 18 marks

QUESTION/VRAAG 2

2.1.1	$T_n = 2n + 3$	✓ $2n$ ✓ 3 (2)
2.1.2	$93 = 2n + 3$ $90 = 2n$ $45 = n$	✓equating ✓answer (2)
2.1.3	$50 + 70 + 90 + \dots + 930$ $S_{45} = \frac{45}{2}[2(50) + (45 - 1)(20)]$ $S_{45} = R22\ 050$ Total raised = R22 050 OR/OF $50 + 70 + 90 + \dots + 930$ $S_{45} = \frac{45}{2}[50 + 930]$ $S_{45} = R22\ 050$ Total raised = R22 050 OR/OF $5 + 7 + 9 + \dots + 93$ $S_{45} = \frac{45}{2}[2(5) + (45 - 1)(2)]$ $S_{45} = 2\ 205\text{km}$ $S_{45} = R22\ 050$ Total raised = R22 050 OR/OF $5 + 7 + 9 + \dots + 93$ $S_{45} = \frac{45}{2}[5 + 93]$ $S_{45} = 2\ 205\text{km}$ $S_{45} = R22\ 050$ Total raised = R22 050	✓convert to money ✓substitution ✓answer (3) OR/OF ✓convert to money ✓substitution ✓answer (3) OR/OF ✓substitution ✓answer ✓convert to money (3) OR/OF ✓substitution ✓answer ✓convert to money (3)
2.2.1 a)	$T_1 = (2)^{1+2}$ $T_1 = 8$	✓ $a = 8$ (1)
2.2.1 b)	$r = 2$	✓ $r = 2$ (1)
2.2.2	$T_{20} = (2)^{20+2}$ $T_{20} = 2^{22} = (2^2)^{11}$ $= 4^{11}$	✓substitution ✓answer (2)

2.2.3	$\frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots$ $S_{\infty} = \frac{a}{1-r}$ $= \frac{\frac{1}{8}}{1 - \frac{1}{2}}$ $\therefore S_{\infty} = \frac{1}{4}$	✓ series ✓ substitution ✓ answer (3)
2.2.4	$8 + 4^2 + 32 + 4^3 + \dots + 4^{11} + \dots$ $S_{21} - S_{10} = \frac{8(2^{21} - 1)}{2 - 1} - \frac{16(4^{10} - 1)}{4 - 1}$ $= 16\,777\,208 - 5\,592\,400$ $= 11\,184\,808$ OR/OF $8 + 32 + 128 + \dots$ $S_{11} = \frac{8(4^{11} - 1)}{4 - 1}$ $\therefore S_{11} = 11\,184\,808$	✓ $\frac{8(2^{21} - 1)}{2 - 1}$ ✓ $n = 10$ ✓ $\frac{16(4^{10} - 1)}{4 - 1}$ ✓ $11\,184\,808$ OR/OF ✓ $n = 11$ ✓ $r = 4$ ✓ $\frac{8(4^{11} - 1)}{4 - 1}$ ✓ $11\,184\,808$ (4)
		[18]

11. Find x, show the sum formula, then the sum to infinity

Official DBE marking guideline · November 2021 · Q2 · 6 marks

QUESTION/VRAAG 2

2.1	$\frac{90}{x} = \frac{81}{90}$ $81x = 8100$ $x = 100$ OR/OF $x = 90 \times \frac{10}{9}$ $x = 100$	$\checkmark \frac{90}{x} = \frac{81}{90}$ $\checkmark \text{ answer} \quad (2)$ OR/OF $\checkmark \frac{10}{9}$ $\checkmark \text{ answer} \quad (2)$
2.2	$S_n = \frac{a(1-r^n)}{1-r}$ $S_n = \frac{100(1-(0,9)^n)}{1-0,9}$ $S_n = \frac{100(1-(0,9)^n)}{0,1}$ $\therefore S_n = 1\,000(1-(0,9)^n)$	$\checkmark r = 0,9$ $\checkmark \text{substitution into correct formula} \quad (2)$
2.3	$S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{100}{1-\frac{9}{10}}$ $S_\infty = 1000$ OR/OF $S_\infty = \lim_{n \rightarrow \infty} [1\,000(1-(0,9)^n)]$ $S_\infty = 1000$	$\checkmark \text{ substitution}$ $\checkmark \text{ answer} \quad (2)$ OR/OF $\checkmark S_\infty = \lim_{n \rightarrow \infty} [1\,000(1-(0,9)^n)]$ $\checkmark \text{ answer} \quad (2)$
		[6]

12. The general term, then which term of the series equals 15 625

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QUESTION MARKS

3.1.1	$T_n = ar^{n-1} = 2000\left(\frac{1}{5}\right)^{n-1}$	✓ $2000\left(\frac{1}{5}\right)^{n-1}$ (1)
3.1.2	$T_7 = 2000\left(\frac{1}{5}\right)^{7-1} = \frac{16}{125}$	✓ $\frac{16}{125}$ (1)
3.1.3	$\frac{16}{15625} = 2000\left(\frac{1}{5}\right)^{n-1}$ $\frac{1}{1953125} = \left(\frac{1}{5}\right)^{n-1}$ $\left(\frac{1}{5}\right)^9 = \left(\frac{1}{5}\right)^{n-1}$ OR $n-1 = \log_{\frac{1}{5}} \frac{1}{1953125}$ $n-1 = 9$ $n = 10$	✓ equating ✓ same base / use of log ✓ answer (3)
3.2	$S_\infty = 27 = \frac{a}{1-r}$ $S_3 = \frac{a(1-r^3)}{1-r} = 26$ $27(1-r^3) = 26$ $1-r^3 = \frac{26}{27}$ $r^3 = \frac{1}{27}$ $\therefore r = \frac{1}{3}$ OR/OF $S_\infty = 27 = \frac{a}{1-r}$ $a = 27(1-r)$ But $a + ar + ar^2 = 26$ $a(1+r+r^2) = 26$ $27(1-r)(1+r+r^2) = 26$ $(1-r)(1+r+r^2) = \frac{26}{27}$ $r^2 + r + 1 - r^3 - r^2 - r = \frac{26}{27}$ $-r^3 + 1 = \frac{26}{27}$ $r^3 = \frac{1}{27}$ $\therefore r = \frac{1}{3}$	✓ $S_\infty = 27 = \frac{a}{1-r}$ ✓ $S_3 = \frac{a(1-r^3)}{1-r} = 26$ ✓ substitution ✓ $r = \frac{1}{3}$ (4) OR/OF ✓ $a = 27(1-r)$ ✓ $a + ar + ar^2 = 26$ ✓ substitution ✓ $r = \frac{1}{3}$ (4)
		[9]

13. Does it converge? Then k from a sigma that comes to 29 520

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QUESTION 2/VRAAG 2

2.1.1	$r = \frac{1}{2}$ Yes, because $-1 < \frac{1}{2} < 1$	$\checkmark r = \frac{1}{2}$ $\checkmark$ answer with reason (2)
2.1.2	$S_{\infty} = \frac{a}{1-r}$ $S_{\infty} = \frac{4}{1-\frac{1}{2}}$ $\therefore S_{\infty} = 8$	$\checkmark$ substitution $\checkmark$ answer (2)
2.2	$\sum_{p=k}^{10} 3^{p-1} = 3^{k-1} + 3^{k+1-1} + 3^{k+2-1} + \dots + 3^9$ $= 3^{k-1} + 3^k + 3^{k+1} + \dots + 3^9$ $S_n = \frac{a(r^n - 1)}{r - 1}$ $29\,520 = \frac{3^{k-1}(3^{11-k} - 1)}{3 - 1}$ $3^{10} - 3^{k-1} = 59\,040$ $3^{k-1} = 9$ $k - 1 = 2$ $\therefore k = 3$	$\checkmark a = 3^{k-1}$ $\checkmark r = 3$ $\checkmark n = 11 - k$ $\checkmark$ substitution $\checkmark$ answer (5)
		[9]

14. k from a sum of 98 301, then a geometric and an arithmetic together

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QUESTION 3/VRAAG 3

3.1.1	$T_n = ar^{n-1}$ $T_n = 3(2)^{n-1}$	$\checkmark T_n = 3(2)^{n-1}$ (1)
3.1.2	$\sum_{p=1}^k \frac{3}{2} \cdot 2^p = 98\,301$ $\sum_{p=1}^k \frac{3}{2} \cdot 2^p = 3 + 6 + 12 + \dots$ $n = k$ $\frac{3[(2)^k - 1]}{2 - 1} = 98\,301$ $(2)^k = 32\,768$ $2^k = 2^{15}$ OR/OF $k = \log_2 32\,768$ $\therefore k = 15$	$\checkmark$ expansion $\checkmark n = k$ $\checkmark$ substitution into correct formula $\checkmark k = 15$ (4)
3.2	$S_{22} = \frac{22}{2}[2a + 21(3)]$ $S_{22} = 22a + 693$ $S_{\infty} = \frac{a}{1 - \frac{1}{3}}$ $= \frac{3a}{2}$ $\therefore 22a + 693 = \frac{3a}{2} + 734$ $44a + 1386 = 3a + 1468$ $41a = 82$ $a = 2$	$\checkmark$ substitution into S_n $\checkmark S_{22} = 22a + 693$ $\checkmark$ substitution into S_{∞} $\checkmark S_{22} = S_{\infty} + 734$ $\checkmark$ answer (5)
		[10]

15. Circles inside circles, and the series their radii make

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QUESTION/VRAAG 3

3.1	2nd circle: $\frac{6}{2} = 3 \text{ cm}$ 3rd circle: $\frac{3}{2} = 1,5 \text{ cm}$ OR/OF 3rd circle = $6\left(\frac{1}{2}\right)^2 = 1,5$	✓✓ radius 3rd circle (2) OR/OF ✓✓ radius 3rd circle (2)
3.2	$36\pi ; 9\pi ; \frac{9}{4}\pi ; \dots$ $r = \frac{1}{4} ; a = 36\pi$ $S_{10} = \frac{36\pi \left(\left(\frac{1}{4} \right)^{10} - 1 \right)}{\frac{1}{4} - 1}$ $S_{10} = 150,80$	✓ a ✓ r ✓ substitution ✓ answer (4)
3.3	$6 ; 3 ; \frac{3}{2} \dots$ radius = $\frac{1}{2} \left(\frac{3}{128} \right) = \frac{3}{256}$ $\frac{3}{256} = 6 \left(\frac{1}{2} \right)^{n-1}$ $\frac{1}{512} = \left(\frac{1}{2} \right)^{n-1}$ $\left(\frac{1}{2} \right)^9 = \left(\frac{1}{2} \right)^{n-1} \quad \text{or/of} \quad n-1 = \log_{\left(\frac{1}{2} \right)} \left(\frac{1}{512} \right)$ $\therefore n-1=9$ $\therefore n=10$ $\therefore n-1=9$ $\therefore n=10$	✓ $\frac{3}{256}$ ✓ equating to radius general term ✓ simplification ✓ answer (4)

	OR/OF 12 ; 6 ; 3 ; ... diameter = $\frac{3}{128}$ $\frac{3}{128} = 12 \left(\frac{1}{2}\right)^{n-1}$ $\frac{1}{512} = \left(\frac{1}{2}\right)^{n-1}$ $\left(\frac{1}{2}\right)^9 = \left(\frac{1}{2}\right)^{n-1}$ or/of $n-1 = \log_{\left(\frac{1}{2}\right)}\left(\frac{1}{512}\right)$ $\therefore n-1=9$ $\therefore n-1=9$ $\therefore n=10$ $\therefore n=10$	OR/OF ✓ $12 \left(\frac{1}{2}\right)^{n-1}$ ✓ equating general term of diameter to $\frac{3}{128}$ ✓ simplification ✓ answer (4)
		[10]

16. The 10th term, then the values of t that let it converge

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QUESTION/VRAAG 3

3.1.1	$T_n = ar^{n-1}$ $T_{10} = 1024 \left(\frac{1}{4}\right)^{10-1}$ $\therefore T_{10} = \frac{1}{256}$ ANSWER ONLY: FULL MARKS	✓ substitution into the correct formula ✓ answer (2)
3.1.2	$\sum_{p=0}^8 256(4^{1-p}) = 1024 + 256 + 64 + \dots$ $S_n = \frac{a[1-r^n]}{1-r}$ $S_9 = \frac{1024 \left[1 - \left(\frac{1}{4}\right)^9\right]}{1 - \frac{1}{4}}$ $S_9 = \frac{87381}{64}$ $= 1365,33$ OR/OF $\sum_{p=0}^8 256(4^{1-p})$ $= 1024 + 256 + 64 + 16 + 4 + 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64}$ $S_9 = \frac{87381}{64}$ $= 1365,33$	✓ 1024 ✓ $n = 9$ ✓ substitution into the correct formula ✓ answer (4) OR/OF ✓ 1024 ✓ rest of expansion ✓ $n = 9$ terms ✓ answer (4)
3.2	$-t^2 - 6t - 9; \frac{t^3 + 9t^2 + 27t + 27}{2}$ $-(t^2 + 6t + 9); \frac{1}{2}(t+3)(t^2 + 6t + 9)$ $-(t+3)^2; \frac{1}{2}(t+3)^3$ $r = \frac{-(t+3)}{2}$ $-1 < \frac{-t-3}{2} < 1$ $-2 < -t-3 < 2$ $1 < -t < 5$ $-5 < t < -1$	$\frac{t^3 + 9t^2 + 27t + 27}{-t^2 - 6t - 9}$ ✓ $r = \frac{2}{-t^2 - 6t - 9}$ ✓ $-(t^2 + 6t + 9)$ ✓ $\frac{1}{2}(t+3)(t^2 + 6t + 9)$ ✓ $-1 < \frac{-t-3}{2} < 1$ ✓ answer (5)
		[11]

17. r from the 6th term, then how many terms reach 114 674

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QUESTION 2/VRAAG 2

2.1.1	$a = 14$ $T_6 = 14r^5 = 448$ $r^5 = 32$ $\therefore r = 2$ Answer only: full marks	$\checkmark T_6 = 14r^5 = 448$ $\checkmark r = 2$ (2)
2.1.2	$T_n = 14(2)^{n-1}$ $S_n = \frac{14(2^6 - 1)}{2 - 1}$ $S_6 = 882$ $114\,674 - 882 = 113\,792$ $113\,792 = 896(2^n - 1)$ $128 = 2^n$ $n = 7$ OR/OF $S_n = \frac{a(r^n - 1)}{r - 1}$ $114\,674 = \frac{14(2^n - 1)}{2 - 1}$ $8\,191 = 2^n - 1$ $2^n = 8\,192$ $n = \log_2 8\,192$ $n = 13$ $\therefore 7$ more terms must be added to the first 6 terms.	$\checkmark$ substitution into correct formula $\checkmark S_6 = 882$ $\checkmark 128 = 2^n$ $\checkmark 7$ (4) OR/OF $\checkmark$ substitution into correct formula $\checkmark 2^n = 8\,192$ $\checkmark n = 13$ $\checkmark 7$ (4)
2.1.3	$r = \frac{1}{2}$ OR $448r^5 = 14$ $\therefore r = \frac{1}{2}$ $S_\infty = \frac{a}{1 - r}$ $S_\infty = \frac{448}{1 - \frac{1}{2}}$ $S_\infty = 896$	$\checkmark r = \frac{1}{2}$ $\checkmark$ substitution $\checkmark$ answer (3)

