ANALYTICAL GEOMETRY

Solutions

DBE's own marking guidelines, in booklet order

Every solution here is a clip of the official DBE marking guideline published with the paper the question came from. The working, the mark ticks and the accepted alternatives are DBE's. Nothing has been rewritten or re-derived.

The numbers match the question booklet: solution 34 answers question 34, and the chapters are in the same order.

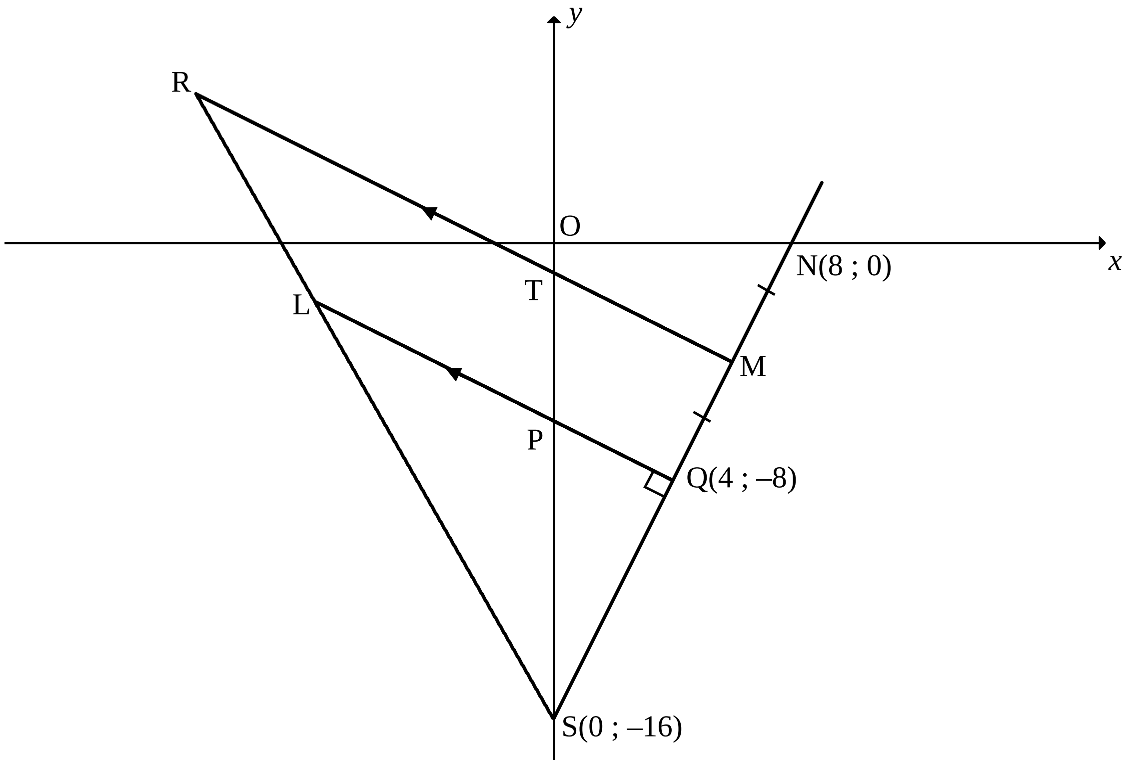
DBE writes one guideline for both languages, so these pages are bilingual and their marking notes are largely in English. That is DBE's own page, reproduced as printed.

A marking guideline is written for markers, not learners. It shows the shortest route to the marks, so where a step looks compressed, that is the guideline being terse rather than a step missing.

1. A right angle at Q, and a line drawn parallel to LQ

Official DBE marking guideline · May/June 2021 · Q3 · 19 marks

QUESTION/VRAAG 3



3.1	$M\left(\frac{4+8}{2};\frac{-8+0}{2}\right)$ $M(6;-4)$	✓ x_M ✓ y_M (2)
3.2	$m_{NS}=\frac{0-(-16)}{8-0}$ or $m_{NQ}=\frac{0-(-8)}{8-4}$ or $m_{QS}=\frac{-8-(-16)}{4-0}$ $=2$	✓ subst N and Q or N and Q or Q and S into gradient formula ✓ answer (2)
3.3	$m_{LQ}\times 2=-1$ [LQ $\perp$ NS] $\therefore m_{LQ}=-\frac{1}{2}$ $-8=-\frac{1}{2}(4)+c$ **OR** $y+8=-\frac{1}{2}(x-4)$ $c=-6$ $y+8=-\frac{1}{2}x+2$ $\therefore y=-\frac{1}{2}x-6$	✓ m_{LQ} ✓ substitution of Q ✓ calculation of *c* or simplification (3)
3.4	OS is the radius of circle passing through S $(x-0)^2+(y-0)^2=(16)^2$ $x^2+y^2=256$ Answer only: Full marks	✓ identifying radius = 16 ✓ Equation of circle (2)

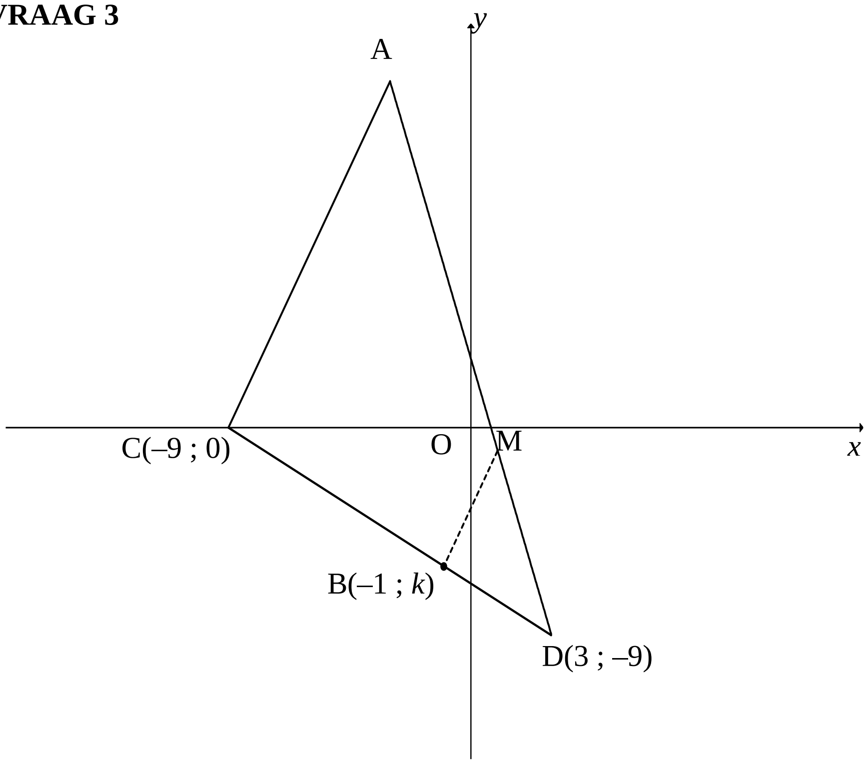
3.5	$m_{RM} = m_{LQ} = -\frac{1}{2}$ [RM ‖ LQ] $-4 = -\frac{1}{2}(6) + c$ **OR** $y + 4 = -\frac{1}{2}(x - 6)$ $c = -1$ $y + 4 = -\frac{1}{2}x + 3$ $\therefore y = -\frac{1}{2}x - 1$ $T(0;-1)$	✓ m_{RM} ✓ substitution of $M(6;-4)$ ✓ coordinates of T (3)
3.6	$T(0;-1)$, $P(0;-6)$ and $S(0;-16)$ $\therefore$ PS = 10 units and TS = 15 units $\frac{LS}{RS} = \frac{PS}{TS} = \frac{2}{3}$ [prop theorem; RM ‖ LP] **OR** [line ‖ one side of Δ/*lyn* ‖ *een sy v Δ*] Answer only: Full marks **OR** M(6 ; –4), Q(4 ; –8) and S(0 ; –16) MS = $\sqrt{180} = 6\sqrt{5}$ and QS = $\sqrt{80} = 4\sqrt{5}$ $\frac{LS}{RS} = \frac{QS}{MS} = \frac{2}{3}$ [prop theorem; RM ‖ LQ] **OR** [line ‖ one side of Δ/*lyn* ‖ *een sy v Δ*] Answer only: Full marks	✓ PS = 10 units ✓ TS = 15 units ✓ answer (3) ✓ MS $= 6\sqrt{5}$ units ✓ QS $= 4\sqrt{5}$ units ✓ answer (3)
3.7	area of PTMQ = area of ΔTSM – area of ΔPSQ $= \frac{1}{2}.ST.\perp h_M - \frac{1}{2}.PS.\perp h_Q$ $= \frac{1}{2}(15)(6) - \frac{1}{2}(10)(4)$ = 45 – 20 = 25 square units **OR** $TM = \sqrt{45} = 3\sqrt{5} = 6{,}71$ $MQ = \sqrt{20} = 2\sqrt{5} = 4{,}47$ $PQ = \sqrt{20} = 2\sqrt{5} = 4{,}47$ area of trapezium PTMQ = $\frac{1}{2}\left(3\sqrt{5} + 2\sqrt{5}\right)\left(2\sqrt{5}\right)$ $= \frac{1}{2}\left(5\sqrt{5}\right)\left(2\sqrt{5}\right)$ = 25 square units	✓ area of ΔTSM – area of ΔPSQ ✓ area ΔTSM = 45 ✓ area ΔPSQ = 20 ✓ answer (4) ✓ $TM = 3\sqrt{5}$ $MQ = 2\sqrt{5}$ $PQ = 2\sqrt{5}$ ✓ area of trapezium = $\frac{1}{2}$ (sum of ‖sides)(height) ✓ substitute into formula ✓ answer (4)

	OR $\text{MQ}=\sqrt{20}=2\sqrt{5}$ $\text{PQ}=\sqrt{20}=2\sqrt{5}$ $\text{TP}=5$ area of PTMQ= area of ΔMTP+ area of ΔPQM $\boxed{\text{area of PTMQ}=\frac{1}{2}\text{TP}\times\perp h_M+\frac{1}{2}\text{MQ}\times\text{PQ}}$ area of PTMQ$=\frac{1}{2}(5)\times 6+\frac{1}{2}\left(2\sqrt{5}\right)\left(2\sqrt{5}\right)$ area of PTMQ$=10+15=25$	✓ area of ΔMTP + area of ΔPQM ✓ area ΔMTP = 10 ✓ area ΔPQM = 15 ✓ answer (4)
		[19]

2. The gradient of DC, then a point that has to lie on it

Official DBE marking guideline · November 2024 · Q3 · 19 marks

QUESTION / VRAAG 3



3.1	$m_{DC}=\frac{-9-0}{3-(-9)}$ **OR/*OF*** $m_{DC}=\frac{0-(-9)}{-9-3}$ $m_{DC}=-\frac{3}{4}$ $\quad$ $m_{DC}=-\frac{3}{4}$	✓ correct substitution of $D(3;-9)$ & $C(-9;0)$ into gradient formula ✓ answer (2)
3.2	Equation of DC: $0=-\frac{3}{4}(-9)+c$ **OR/*OF*** $y-0=-\frac{3}{4}\left(x-(-9)\right)$ $c=\frac{-27}{4}$ or $-6\frac{3}{4}$ $\quad$ $y=-\frac{3}{4}(x+9)$ $y=-\frac{3}{4}x-\frac{27}{4}$ $\quad$ $y=-\frac{3}{4}x-\frac{27}{4}$	✓ correct substitution of $C(-9;0)$ or $D(3;-9)$ into equation of line ✓ answer (2)
3.3	$k=-\frac{3}{4}(-1)-\frac{27}{4}$ **OR/*OF*** $\frac{k-(-9)}{-1-3}=\frac{-3}{4}$ **OR/*OF*** $\frac{k-0}{-1-(-9)}=\frac{-3}{4}$ $k=\frac{3}{4}-\frac{27}{4}$ **OR/*OF*** $k+9=3$ **OR/*OF*** $k=-\frac{3}{4}(8)$ $k=-6$ $\quad$ $k=-6$ $\quad$ $k=-6$	✓ substitution of $B(-1;k)$ (1)
3.4	$DC=\sqrt{(3+9)^2+(-9-0)^2}$ DC = 15 units	✓ correct substitution of $D(3;-9)$ & $C(-9;0)$ into distance formula ✓ answer (2)

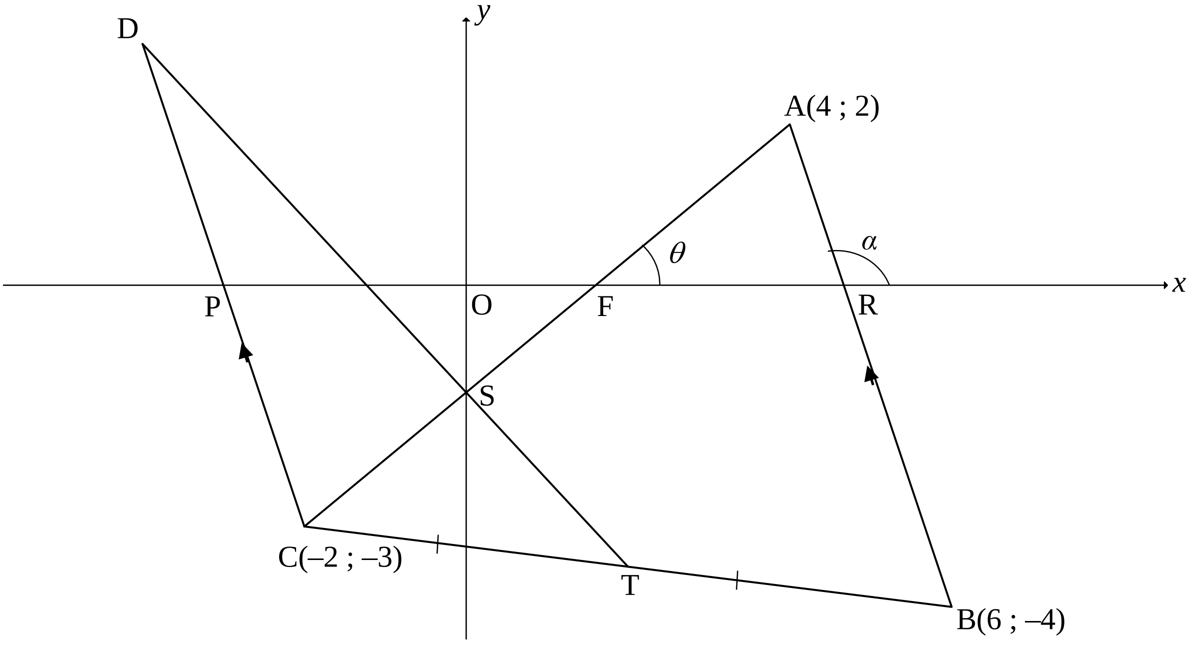
3.5	$DB = \sqrt{\left(3-(-1)\right)^2 + \left(-9-(-6)\right)^2}$ DB = 5 $\therefore \frac{DB}{DC} = \frac{5}{15} = \frac{1}{3}$	✓ DB = 5 ✓ answer (2)
3.6	$\frac{DM}{DA} = \frac{DB}{DC} = \frac{1}{3}$ $\frac{\text{Area } \Delta MBD}{\text{Area } \Delta ACD} = \frac{\frac{1}{2}(DM)(DB)\left(\sin \hat{D}\right)}{\frac{1}{2}(DA)(DC)\left(\sin \hat{D}\right)}$ $= \frac{1}{3} \times \frac{1}{3}$ $= \frac{1}{9}$	✓ $\frac{DM}{DA} = \frac{DB}{DC}$ ✓ correct use of area rule ✓ subst. for $\frac{BD}{DC}$ and $\frac{DM}{DA}$ into correct formula ✓ answer (4)
3.7	$y = -4x + c$ $m_{AD} = -4$ $-9 = -4(3) + c$ **OR/*OF*** $\frac{y+9}{x-3} = -4$ $c = 3$ $y + 9 = -4x + 12$ $y = -4x + 3$ $y = -4x + 3$ $(x-3)^2 + (y+9)^2 = 612$ $(x-3)^2 + (-4x+3+9)^2 = \left(\sqrt{612}\right)^2$ $(x-3)^2 + (-4x+12)^2 = 612$ $x^2 - 6x + 9 + 16x^2 - 96x + 144 = 612$ $17x^2 - 102x - 459 = 0$ $x^2 - 6x - 27 = 0$ $(x-9)(x+3) = 0$ $x = 9$ or $x = -3$ N/A $y = -4(-3) + 3$ $y = 15$ $A(-3;15)$	✓ correct substitution of $m_{AD} = -4$ and D(3 ; –9) ✓ $(x-3)^2 + (y+9)^2 = 612$ ✓ substitution of equation AD into distance formula ✓ standard form ✓ *x* values with rejection ✓ *y* coordinate (6)

	OR/*OF* $-9=-4(3)+c$ $c=3$ $y=-4x+3$ N(0 ; 3) $\text{ND} = \sqrt{(3-0)^2+(-9-3)^2}$ $=3\sqrt{17}$ $\text{AD} = 6\sqrt{17}$ $\text{ND} = \frac{1}{2}\text{AD}$ N is the midpoint of AD A(–3 ; 15)	**OR/*OF*** ✓ correct substitution of $m_{\text{AD}} = -4$ and D(3 ; –9) ✓ N(0 ; 3) ✓ substitution into distance formula to calculate ND ✓ $\text{ND} = \frac{1}{2}\text{AD}$ ✓ x – value ✓ y – value (6)
			[19]

3. A midpoint, an angle of inclination, and a parallel triangle

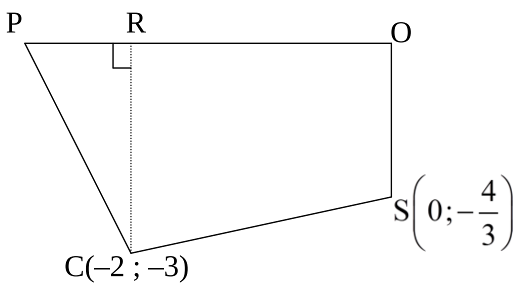
Official DBE marking guideline · November 2022 · Q3 · 20 marks

QUESTION/*VRAAG* 3



3.1.1	$m_{AB}=\dfrac{2-(-4)}{4-6}$ **OR** $m_{AB}=\dfrac{-4-2}{6-4}$ $m_{AB}=-3$ ANSWER ONLY: Full marks	✓ substitution ✓ answer (2)
3.1.2	$\tan\alpha=m_{AB}=-3$ $\alpha=108{,}43°$ ANSWER ONLY: Full marks	✓ $\tan\alpha=m_{AB}=-3$ ✓ answer (2)
3.1.3	$T\left(\dfrac{x_1+x_2}{2};\dfrac{y_1+y_2}{2}\right)$ $T\left(\dfrac{-2+6}{2};\dfrac{-3-4}{2}\right)$ $T\left(2\,;\dfrac{-7}{2}\right)$	✓ $x_T=2$ ✓ $y_T=\dfrac{-7}{2}$ (2)
3.1.4	$5(0)-6y=8$ $y=-\dfrac{4}{3}$ $S\left(0;\dfrac{-4}{3}\right)$	✓ $x_S=0$ ✓ $y_S=\dfrac{-4}{3}$ (2)
3.2	$m_{CD}=m_{AB}=-3$ $-3=-3(-2)+c$ OR $y-(-3)=-3(x-(-2))$ $c=-9$ $y=-3x-9$ $y=-3x-9$	✓ gradient ✓ substitution of C(–2 ; –3) ✓ equation (3)

3.3.1	$5x-6y=8$ $y=\frac{5}{6}x-\frac{8}{6}$ $\tan\theta=m_{AC}=\frac{5}{6}$ $\theta=39{,}81°$ $\hat{A}=108{,}43°-39{,}81°$ $=68{,}62°$ $D\hat{C}A=68{,}62°$ [alt $\angle$s ; DC‖AB]	✓ $\tan\theta=m_{AC}=\frac{5}{6}$ ✓ $\theta=39{,}81°$ ✓ $\hat{A}=68{,}62°$ ✓ answer (4)
3.3.2	$P(-3;0)$ and F(1,6 ; 0) Area POSC = Area ΔFPC – Area ΔOFS $=\frac{1}{2}(4{,}6)(3)-\frac{1}{2}(1{,}6)\left(\frac{4}{3}\right)$ = 6,9 – 1,07 = 5,83 units^2 **OR/*OF*** $P(-3;0)$ $FC=\sqrt{\left(-2-\frac{8}{5}\right)^2+(-3-0)^2}=\frac{3\sqrt{61}}{5}$ Area $\Delta PFC=\frac{1}{2}(PF)(FC)\sin O\hat{F}S$ $=\frac{1}{2}\left(\frac{23}{5}\right)\left(\frac{3\sqrt{61}}{5}\right)\sin 39{,}81°$ $=6{,}90$ Area $\Delta OFS=\frac{1}{2}\left(\frac{8}{5}\right)\left(\frac{4}{3}\right)$ = 1,07 Area POSC = 6,90 – 1,07 = 5,83 units^2 **OR/*OF***	✓ $P(-3;0)$ ✓ method ✓ $\frac{1}{2}(4{,}6)(3)$ ✓ $\frac{1}{2}(1{,}6)\left(\frac{4}{3}\right)$ ✓ answer (5) ✓ $P(-3;0)$ ✓ $\frac{1}{2}\left(\frac{23}{5}\right)\left(\frac{3\sqrt{61}}{5}\right)\sin 39{,}81°$ ✓ $\frac{1}{2}\left(\frac{8}{5}\right)\left(\frac{4}{3}\right)$ ✓ method ✓ answer (5)



$P(-3;0)$

Area of POSC = Area of OSCR + Area of ΔPRC

$$= \frac{1}{2}\left(\frac{4}{3}+3\right)\times 2+\frac{1}{2}(1\times 3)$$

$$= \frac{35}{6}$$

$$= 5{,}83\,\text{units}^2$$

✓ $P(-3;0)$

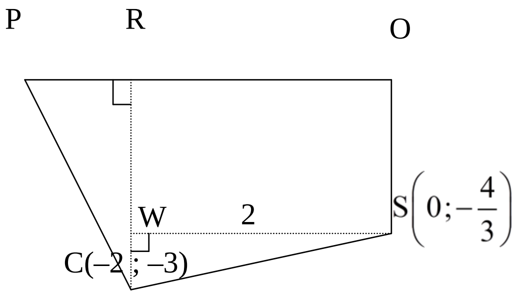
✓ method

✓ $\frac{1}{2}\left(\frac{4}{3}+3\right)\times 2$ ✓ $\frac{1}{2}(1\times 3)$

✓ answer

(5)

OR/ *OF*



$P(-3;0)$

Area POSC = Area ROSW + Area ΔPRC + Area ΔWSC

$$= \left(\frac{4}{3}\right)(2)+\frac{1}{2}(1)(3)+\frac{1}{2}(2)\left(\frac{5}{3}\right)$$

$$= \frac{35}{6}$$

$$= 5{,}83\text{units}^2$$

✓ $P(-3;0)$

✓ method

✓ $\frac{1}{2}(1)(3)$

✓ $\left(\frac{4}{3}\right)(2)+\frac{1}{2}(2)\left(\frac{5}{3}\right)$

✓ answer

(5)

OR/*OF*

	$P(-3;0)$ Area of $\Delta PSC = \frac{1}{2}(PC)(CS)\sin D\hat{C}A$ $= \frac{1}{2}\left(\sqrt{10}\right)\left(\frac{\sqrt{61}}{3}\right)\sin 68,62°$ $= 3,833..$ Area of $\Delta POS = \frac{1}{2}(PO)(OS)$ $= \frac{1}{2}(3)\left(\frac{4}{3}\right)$ $= 2$ Area POSC = 3,833… + 2 $= 5,83\text{units}^2$	✓ $P(-3;0)$ ✓ $\frac{1}{2}\left(\sqrt{10}\right)\left(\frac{\sqrt{61}}{3}\right)\sin 68,62°$ ✓ $\frac{1}{2}(3)\left(\frac{4}{3}\right)$ ✓ method ✓ answer (5)
		[20]

4. A right angle at L, and where LN crosses the x-axis

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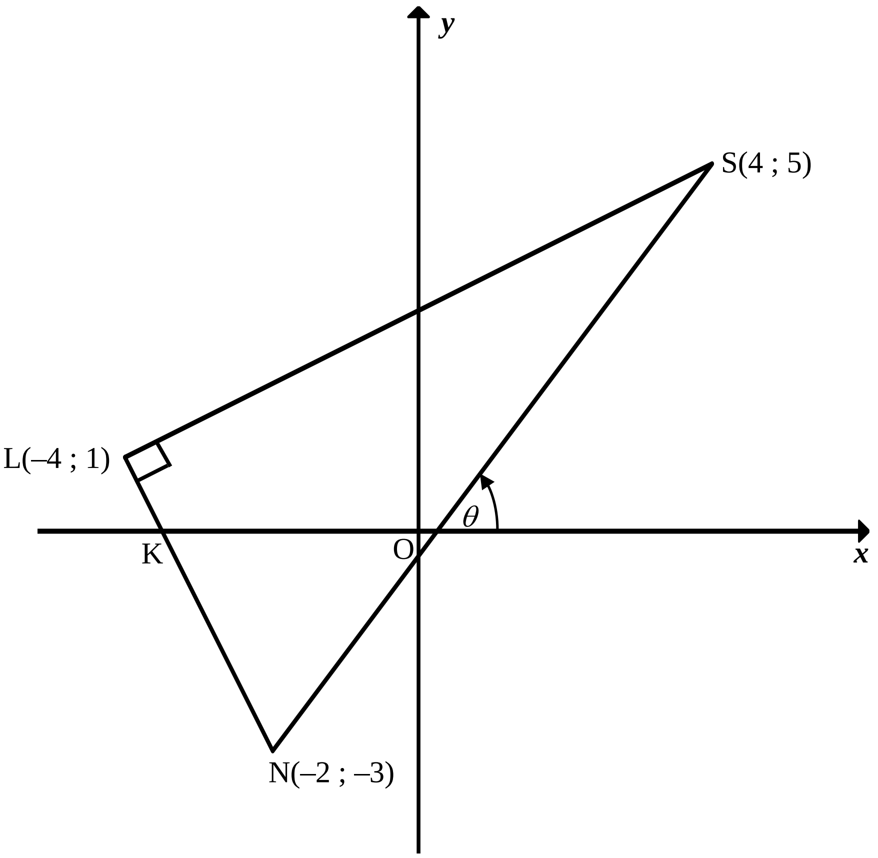
QUESTION/*VRAAG* 3



3.1	$SL = \sqrt{(x_S - x_L)^2 + (y_S - y_L)^2}$ $SL = \sqrt{(4-(-4))^2 + (5-1)^2}$ $SL = \sqrt{80} = 4\sqrt{5} = 8{,}94$ units	✓ substitution of S and L into correct formula ✓ answer (2)
3.2	$m_{SN} = \dfrac{5-(-3)}{4-(-2)}$ $m_{SN} = \dfrac{4}{3}$	✓ substitution of S and N into correct formula ✓ answer (2)
3.3	$m = \tan\theta = \dfrac{4}{3}$ $\theta = 53{,}13°$	✓ $\tan\theta = m_{SN}$ ✓ answer (2)
3.4	$m_{LN} = \dfrac{1-(-3)}{-4-(-2)}$ $m_{LN} = -2$ $L\hat{K}O = 116{,}565...°$ $L\hat{N}S = 116{,}565...° - 53{,}13°$ $L\hat{N}S = 63{,}44°$	✓ $m_{LN} = -2$ ✓ size of $L\hat{K}O$ ✓ answer (3)

	OR SN = 10 units $\sin L\hat{N}S = \frac{4\sqrt{5}}{10}$ $L\hat{N}S = 63{,}44°$	✓ SN = 10 units ✓ correct trig ratio ✓ answer (3)
	OR $LN = 2\sqrt{5}$ units $\tan L\hat{N}S = \frac{4\sqrt{5}}{2\sqrt{5}}$ $L\hat{N}S = 63{,}44°$	✓ $LN = 2\sqrt{5}$ units ✓ correct trig ratio ✓ answer (3)
	OR SN = 10 units $LN = 2\sqrt{5}$ units $\cos L\hat{N}S = \frac{2\sqrt{5}}{10}$ $L\hat{N}S = 63{,}44°$	✓ SN = 10 units and $LN = 2\sqrt{5}$ units ✓ correct trig ratio ✓ answer (3)
3.5	$m = \frac{4}{3}$ $1 = \frac{4}{3}(-4) + c$ **OR** $y - 1 = \frac{4}{3}(x - (-4))$ $c = \frac{19}{3}$ $y - 1 = \frac{4}{3}x + \frac{16}{3}$ $y = \frac{4}{3}x + \frac{19}{3}$ $y = \frac{4}{3}x + \frac{19}{3}$	✓ m_{SN} ✓ substitution of m_{SN} & L ✓ equation (3)
3.6	$SL = 4\sqrt{5}$ $LN = \sqrt{(-4-(-2))^2 + (1-(-3))^2}$ $LN = \sqrt{20} = 2\sqrt{5}$ Area $\Delta LSN = \frac{1}{2}(4\sqrt{5})(2\sqrt{5})$ $= 20$ units^2 **OR**	✓ $LN = \sqrt{20} = 2\sqrt{5}$ ✓ substitution into formula ✓ answer (3)

	SN = 10 units LN = $\sqrt{(-4-(-2))^2+(1-(-3))^2}$ LN = $\sqrt{20}$ = $2\sqrt{5}$ Area ΔLSN = $\frac{1}{2}(10)(2\sqrt{5})\sin 63,44°$ = 20 units^2	✓ LN = $\sqrt{20}$ = $2\sqrt{5}$ ✓ substitution into formula ✓ answer (3)
3.7	$\hat{L}$ = 90° SN is a diameter of circle S, L, N [chord subtends 90° **OR** converse ∠ in semi-circle] Centre of circle = $P\left(\frac{4+(-2)}{2};\frac{5+(-3)}{2}\right)$ = $P(1\,;1)$ **OR** Let the coordinates of P be (a ; b). Then, PL = PN: $(-4-a)^2+(1-b)^2=(-2-a)^2+(-3-b)^2$ $a-2b=-1$ …..equation 1 If PS = PN, then: $4a+2b=6$ ………….. equation 2 Solving simultaneously yields: $a=1$ and $b=1$ and $P(1\,;1)$ **OR** If PL = PN, then: $a-2b=-1$ …..equation 1 If PS = PL, then: $2a+b=3$.....equation 2 Solving simultaneously yields: $a=1$ and $b=1$ and $P(1\,;1)$	✓ SN is a diameter of circle S, L, N ✓ x-value ✓ y-value (3) ✓ 2 correct linear equations ✓ x-value ✓ y-value (3) ✓ 2 correct linear equations ✓ x-value ✓ y-value (3)
3.8	$L\hat{P}N=\theta=53{,}13°$ [alt ∠s; LP ‖ x –axis] ∴ $L\hat{P}S$ = 126,87° **OR** $L\hat{N}S$ = 63,44° ∴ $L\hat{P}S$=126,88° [∠ at centre = 2 ×∠ at circumference] **OR** $L\hat{S}N$ = 26,56° [sum of ∠s in Δ] $S\hat{L}P$ = 26,56° [∠s opp equal radii] ∴ $L\hat{P}S$ = 126,88° [sum of ∠s in Δ] **OR** $(4\sqrt{5})^2=5^2+5^2-2(5)(5)\cos L\hat{P}S$ $\cos L\hat{P}S=-\frac{3}{5}$ ∴ $L\hat{P}S$ = 126,87°	✓ $L\hat{P}N$ ✓ answer (2) ✓ $L\hat{N}S$ ✓ answer (2) ✓ $L\hat{S}N$ ✓ answer (2) ✓ correct substitution into cosine formula ✓ answer (2)
		[20]

5. A trapezium with one pair of sides parallel

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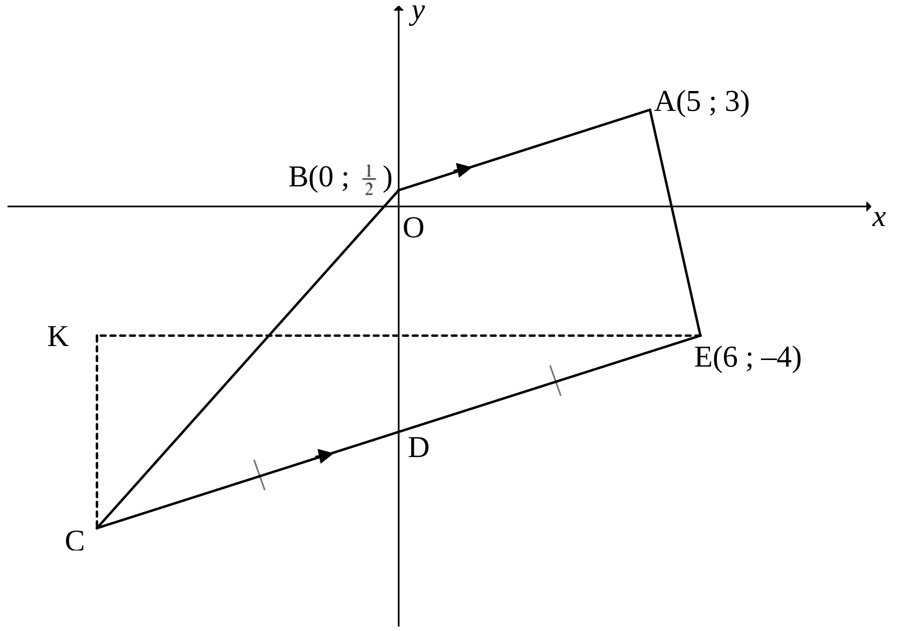
QUESTION/*VRAAG* 3



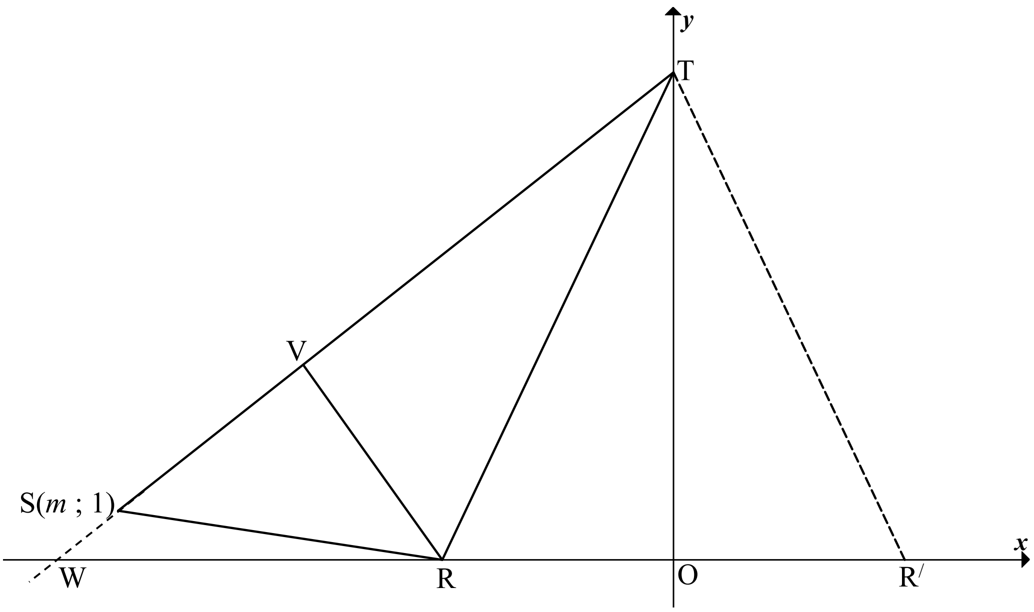
3.1	$m_{AB} = \dfrac{3-\frac{1}{2}}{5-0}$ $m_{AB} = \dfrac{1}{2}$ Answer only 2/2	✓ substitution ✓ answer (2)
3.2	$m_{CE} = m_{BA} = \dfrac{1}{2}$ $-4 = \dfrac{1}{2}(6) + c$ **OR/*OF*** $y-(-4) = \dfrac{1}{2}(x-6)$ $c = -7$ $y = \dfrac{1}{2}x - 7$	✓ gradient ✓ substitution of E ✓ answer (3)
3.3.1	D(0 ; –7) $\dfrac{x_C + 6}{2} = 0$ $\dfrac{y_C + (-4)}{2} = -7$ $x_C = -6$ $y_C = -10$ C(–6 ; –10) Answer only 3/3	✓ D(0 ; –7) ✓ $x_C = -6$ ✓ $y_C = -10$ (3)
3.3.2	Area Δ BCD $= \dfrac{1}{2}(7,5)(6)$ $= 22,5$ Area Δ ABD $= \dfrac{1}{2}(7,5)(5)$ $= 18,75$ Area ABCD = 22,5 + 18,75 = 41,25 units^2	✓ subst of correct base and height into the area formula ✓ area ΔBCD = 22,5 ✓ area ΔABD = 18,75 ✓ answer (4)

3.4.1	K(–6 ; –4)	✓ $x_K = -6$ ✓ $y_K = -4$ (2)
3.4.2a	KC = 6 units; KE = 12 units; $CE = \sqrt{(6)^2 + (12)^2}$ [Pythagoras] $CE = \sqrt{180} = 6\sqrt{5} = 13,42$ Perimeter ΔKEC = 6 + 12 + $\sqrt{180}$ = 31,42 units	✓ KC = 6 units ✓ KE = 12 units ✓ CE ✓ answer (4)
3.4.2b	$\tan K\hat{C}E = \frac{KE}{KC} = \frac{12}{6} = 2$ $K\hat{C}E = 63,43°$ **OR/*OF*** $\sin K\hat{C}E = \frac{KE}{CE} = \frac{12}{\sqrt{180}} = \frac{2\sqrt{5}}{5}$ $K\hat{C}E = 63,43°$ **OR/*OF*** $m_{CE} = \frac{1}{2}$ $\tan\theta = \frac{1}{2}$ $\theta = 26,57°$ $K\hat{C}E = 90° - 26,57°$ $K\hat{C}E = 63,43°$ **OR/*OF*** $KE^2 = KC^2 + CE^2 - 2(KC)(CE)\cos K\hat{C}E$ $(12)^2 = (6)^2 + \left(\sqrt{180}\right)^2 - 2(6)\left(\sqrt{180}\right)(\cos K\hat{C}E)$ $\cos K\hat{C}E = \frac{\sqrt{5}}{5}$ $K\hat{C}E = 63,43°$	✓ trig ratio ✓ $\tan K\hat{C}E = 2$ ✓ answer (3) ✓ trig ratio ✓ $\sin K\hat{C}E = \frac{12}{\sqrt{180}}$ ✓ answer (3) ✓ $\tan\theta = \frac{1}{2}$ ✓ $\theta = 26,57°$ ✓ answer (3) ✓ substitution into cosine rule ✓ trig ratio ✓ answer (3)
		[21]

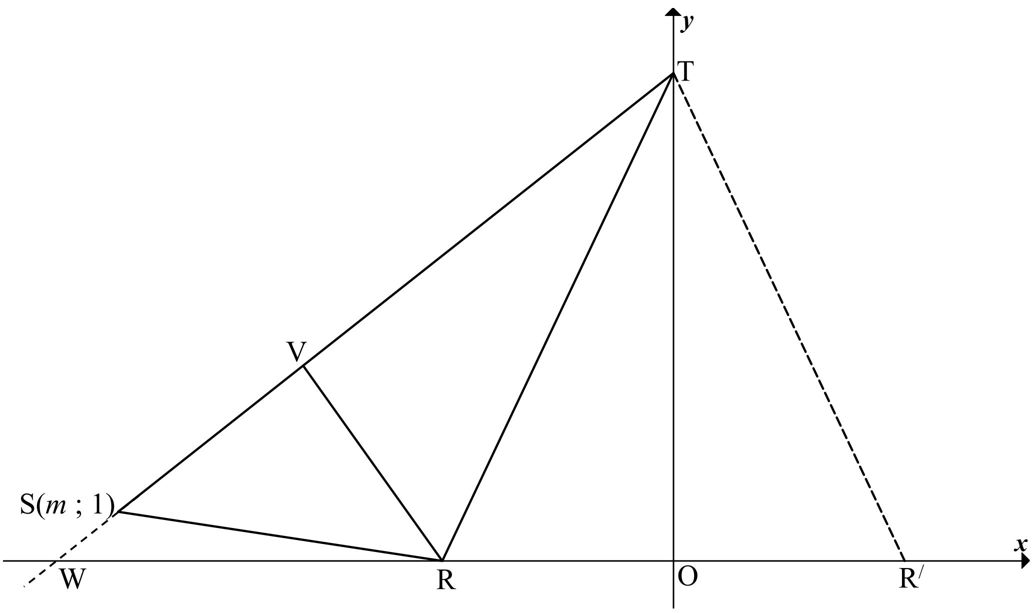
6. A whole triangle built from one line's equation

Official DBE marking guideline · May/June 2025 · Q3 · 21 marks

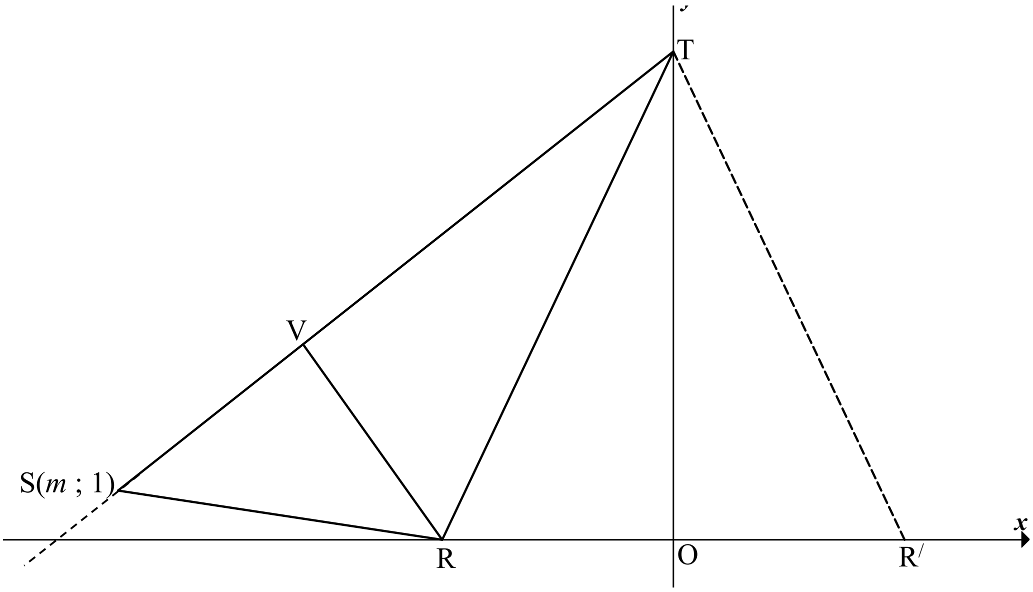
QUESTION/*VRAAG* 3



3.1	$2x-y+10=0$ $2x-0+10=0$ $x=-5$ R(–5 ; 0)	✓ $y=0$ ✓ $x=-5$ (2)
3.2	R(–5 ; 0) T(0 ; 10) $(\text{RT})^2=(-5-0)^2+(0-10)^2$ **OR** $(\text{RT})^2=5^2+10^2$ (Pythag) $\text{RT}=\sqrt{125}$ units **OR/*OF*** $\text{RT}=5\sqrt{5}$ units	✓ T(0 ; 10) ✓ subst of R & T into distance formula or Pythagoras ✓ answer (3)
3.3	$2\text{RT}^2=5\text{SR}^2$ $2(125)=5[(m-(-5))^2+(1-0)^2]$ $5[(m+5)^2+(1)^2]=250$ $(m+5)^2+1=50$ $m^2+10m-24=0$ **OR/*OF*** $(m+5)^2=49$ $(m-2)(m+12)=0$ $m+5=\pm7$ $m=-5\pm7$ $m=2$ or $m=-12$ $m=2$ or $m=-12$ N/A N/A $\therefore m=-12$ $\therefore m=-12$	✓ length of 2RT^2 ✓ length of 5SR^2 ✓ standard form or isolating square ✓ negative answer (4)



3.4	$m_{\text{ST}} = \dfrac{1-10}{-12-0}$ $m_{\text{ST}} = \dfrac{3}{4}$ $\therefore m_{\text{VR}} = -\dfrac{4}{3}$ $y = -\dfrac{4}{3}x + c$ **OR/*OF*** $y - y_1 = -\dfrac{4}{3}(x - x_1)$ $0 = -\dfrac{4}{3}(-5) + c$ $\quad$ $y - 0 = -\dfrac{4}{3}(x-(-5))$ $c = -\dfrac{20}{3}$ $\quad$ $y = -\dfrac{4}{3}(x+5)$ $y = -\dfrac{4}{3}x - \dfrac{20}{3}$ $\quad$ $y = -\dfrac{4}{3}x - \dfrac{20}{3}$	✓ substitution of S & T into gradient formula ✓ m_{ST} ✓ $m_{\text{VR}} = -\dfrac{1}{m_{\text{ST}}}$ ✓ substitution of R ✓ equation (5)
3.5	VR: $y = -\dfrac{4}{3}x - \dfrac{20}{3}$ ST: $y = \dfrac{3}{4}x + 10$ $\dfrac{3}{4}x + 10 = -\dfrac{4}{3}x - \dfrac{20}{3}$ $9x + 120 = -16x - 80$ $25x = -200$ $x = -8$ $y = 4$ $\therefore$ V$(-8 ; 4)$	✓ equating VR and ST ✓ simplification leading to $x = -8$ (2)



3.6	$R'(5;0)$ $VR=\sqrt{[-8-(-5)]^2+(4-0)^2}=5$ $VT=\sqrt{(-8-0)^2+(4-10)^2}=10$ Area of $RVTR'=\frac{1}{2}(VR)(VT)+\frac{1}{2}(RR')(OT)$ $=\frac{1}{2}(5)(10)+\frac{1}{2}(10)(10)$ $=25+50$ $=75\text{ units}^2$ **OR/*OF*** ST: $y=\frac{3}{4}x+10$ $0=\frac{3}{4}x+10$ $x=-\frac{40}{3}$ or $-13\frac{1}{3}$ $W\left(-\frac{40}{3}\ ;\ 0\right)$ $WR'=5-\left(-\frac{40}{3}\right)=\frac{55}{3}=18\frac{1}{3}$ Area of $RVTR'=$ Area of $\Delta TWR'-$ Area of ΔWVR $=\frac{1}{2}\left(5+\frac{40}{3}\right)(10)-\frac{1}{2}\left(-5+\frac{40}{3}\right)(4)$ $=75\text{ units}^2$	✓ length of VR ✓ length of VT ✓ area ΔVRT ✓ area Δ RTR′ ✓ answer (5) ✓ x-intercept of ST ✓ length of WR′ ✓ area Δ TWR′ ✓ area Δ WVR ✓ answer (5)
		[21]

7. A midpoint, an angle of inclination, and the point S beyond B

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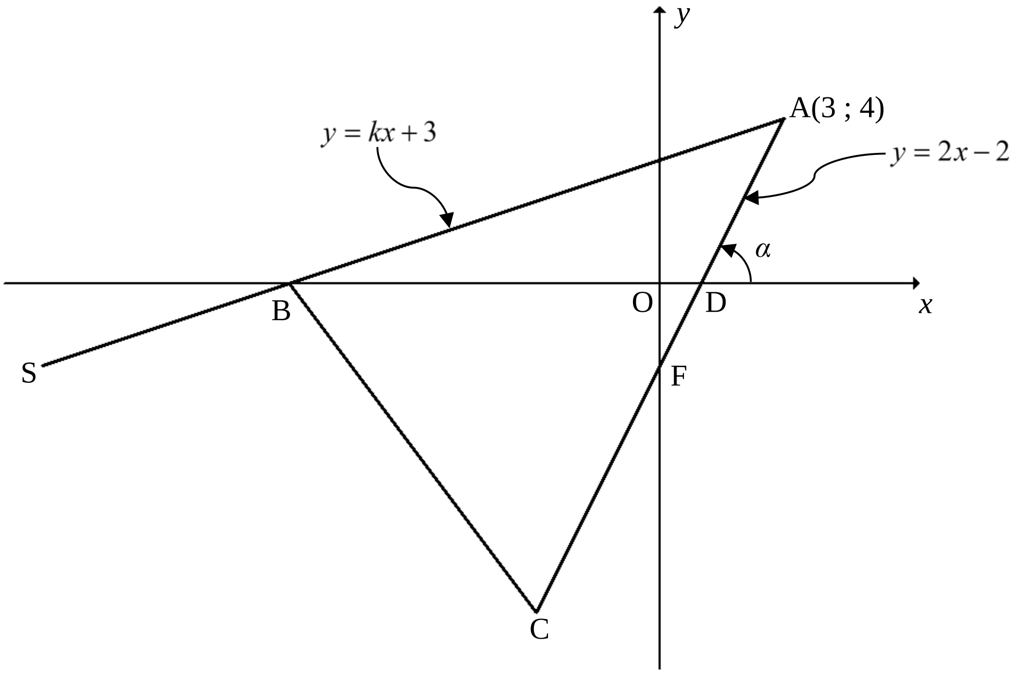
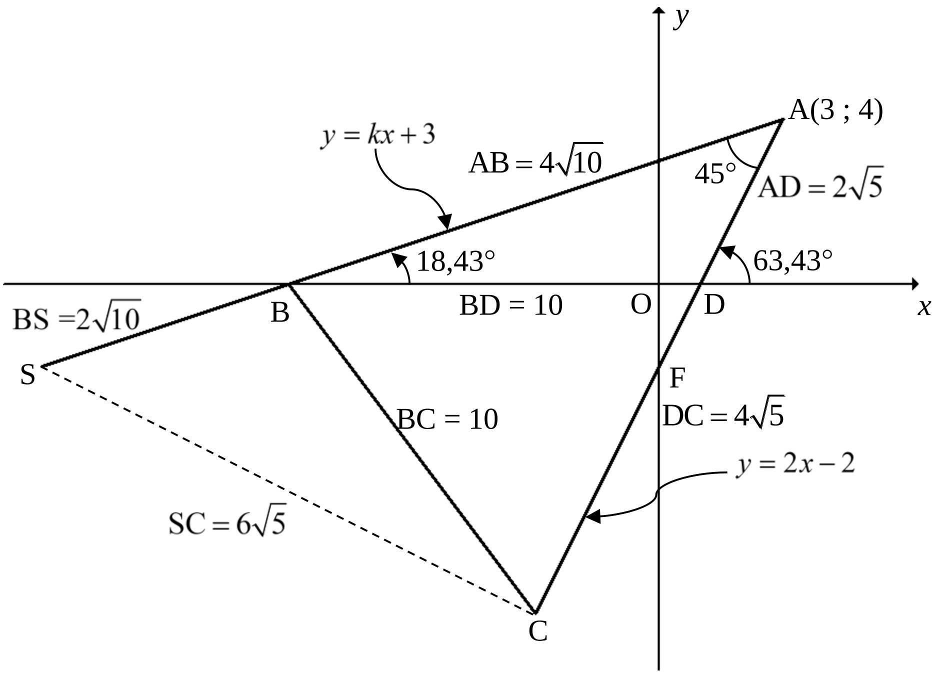
QUESTION/*VRAAG* 3



3.1	$y = kx + 3$ $4 = k(3) + 3$ $3k = 1$ $\therefore k = \frac{1}{3}$ **OR** y-intercept of AB: (0 ; 3) $m_{\text{AB}} = \frac{4-3}{3-0}$ $= \frac{1}{3}$ $\therefore k = \frac{1}{3}$	✓ substitution (3 ; 4) (1) ✓ substitution (0 ; 3) (1)
3.2	$0 = \frac{1}{3}x + 3$ $-3 = \frac{1}{3}x$ $x = -9$ $\text{B}(-9\ ;\ 0)$	✓ $y = 0$ ✓ answer (2)

3.3	$F(0\,;-2)$ $F\left(\frac{x+3}{2}\ ;\ \frac{y+4}{2}\right)$ $\frac{x+3}{2}=0 \qquad \frac{y+4}{2}=-2$ $x=-3 \qquad y=-8$ $C(-3\,;-8)$ **OR** by translation $F(0;-2)$ $A \to F\ (x;y) \to (x-3;y-6)$ $F \to C(0;-2) \to (0-3;-2-6) = (-3;-8)$	✓ $F(0\,;-2)$ ✓ $\frac{x+3}{2}=0\ ;\ \frac{y+4}{2}=-2$ ✓ x-value ✓ y-value (4) ✓ $F(0;-2)$ ✓ $(x-3\ ;y-6)$ ✓ x-value ✓ y-value (4)
3.4	$m_{BC}=\frac{0-(-8)}{-9-(-3)}$ OR $m_{BC}=\frac{-8-0}{-3-(-9)}$ $m_{BC}=-\frac{4}{3}$ $y=-\frac{4}{3}x+c$ $(-2)=-\frac{4}{3}(-15)+c$ $c=-22$ $y=-\frac{4}{3}x-22$ **OR** $m_{BC}=\frac{0-(-8)}{-9-(-3)}$ OR $m_{BC}=\frac{-8-0}{-3-(-9)}$ $m_{BC}=-\frac{4}{3}$ $y-y_1=-\frac{4}{3}(x-x_1)$ $y-(-2)=-\frac{4}{3}(x-(-15))$ $y+2=-\frac{4}{3}x-20$ $y=-\frac{4}{3}x-22$	✓ substitution of B and C into the gradient formula ✓ m_{BC} ✓ $m_{line}=m_{BC}$ ✓ substitution of $S(-15;-2)$ ✓ equation (5) ✓ substitution into the gradient formula ✓ m_{BC} ✓ $m_{line}=m_{BC}$ ✓ substitution of $S(-15;-2)$ ✓ equation (5)



3.5	$\tan\alpha = m_{AC} = 2$ $\alpha = 63,43°$ $\tan A\hat{B}D = m_{AS} = \frac{1}{3}$ $A\hat{B}D = 18,43°$ $B\hat{A}C = \alpha - A\hat{B}D$ $B\hat{A}C = 63,43° - 18,43°$ $B\hat{A}C = 45°$	✓ $\tan\alpha = m_{AC} = 2$ ✓ $\alpha = 63,43°$ ✓ $\tan A\hat{B}D = m_{AS} = \frac{1}{3}$ ✓ $A\hat{B}D = 18,43°$ ✓ answer (5)
	OR $AB = \sqrt{(-9-3)^2 + (0-4)^2}$ $AB = 4\sqrt{10}$ BD = 10 $AD = \sqrt{(3-1)^2 + (4-0)^2}$ $AD = 2\sqrt{5}$ $BD^2 = AB^2 + AD^2 - 2AB.AD\cos B\hat{A}C$ $(10)^2 = (4\sqrt{10})^2 + (2\sqrt{5})^2 - 2(4\sqrt{10})(2\sqrt{5})\cos B\hat{A}C$ $\cos B\hat{A}C = \frac{\sqrt{2}}{2}$ $B\hat{A}C = 45°$	✓ length of AB ✓ calculation of remaining 2 lengths ✓ substitution into cosine-rule ✓ rewriting in terms of $\cos B\hat{A}C$ ✓ answer (5)

3.6	A(3 ; 4) and S(–15 ; – 2) $AS=\sqrt{(x_A-x_S)^2+(y_A-y_S)^2}$ $AS=\sqrt{(3-(-15))^2+(4-(-2))^2}$ $AS=\sqrt{360}=6\sqrt{10}=18{,}97$ $\dfrac{\text{Area of }\Delta ABD}{\text{Area of }\Delta ASC}=\dfrac{\frac{1}{2}(BD)(\perp h)}{\frac{1}{2}(AS)(AC)\sin B\hat{A}C}$ $\dfrac{\text{Area of }\Delta ABD}{\text{Area of }\Delta ASC}=\dfrac{\frac{1}{2}(10)(4)}{\frac{1}{2}(6\sqrt{10})(6\sqrt{5})\sin 45^\circ}$ $\dfrac{\text{Area of }\Delta ABD}{\text{Area of }\Delta ASC}=\dfrac{2}{9}$	✓ $AS=\sqrt{(3-(-15))^2+(4-(-2))^2}$ ✓ length of AS ✓ Area ΔABD ✓ Area ΔASC ✓ answer (5)
	OR $AS=\sqrt{(3-(-15))^2+(4-(-2))^2}$ $AS=\sqrt{360}=6\sqrt{10}=18{,}97$ $AB=\sqrt{(-9-3)^2+(0-4)^2}=4\sqrt{10}$ $AD=\sqrt{(3-1)^2+(4-0)^2}=2\sqrt{5}$ $\dfrac{\text{Area of }\Delta ABD}{\text{Area of }\Delta ASC}=\dfrac{\frac{1}{2}(AB)(AD)\sin\hat{A}}{\frac{1}{2}(AS)(AC)\sin\hat{A}}$ $=\dfrac{\frac{1}{2}(4\sqrt{10})(2\sqrt{5})\sin\hat{A}}{\frac{1}{2}(6\sqrt{10})(6\sqrt{5})\sin\hat{A}}$ $=\dfrac{2}{9}$	✓ $AS=\sqrt{(3-(-15))^2+(4-(-2))^2}$ ✓ length of AS ✓ Area ΔABD ✓ Area ΔASC ✓ answer (5)
		[22]

8. Find k, then three lines all cutting the x-axis

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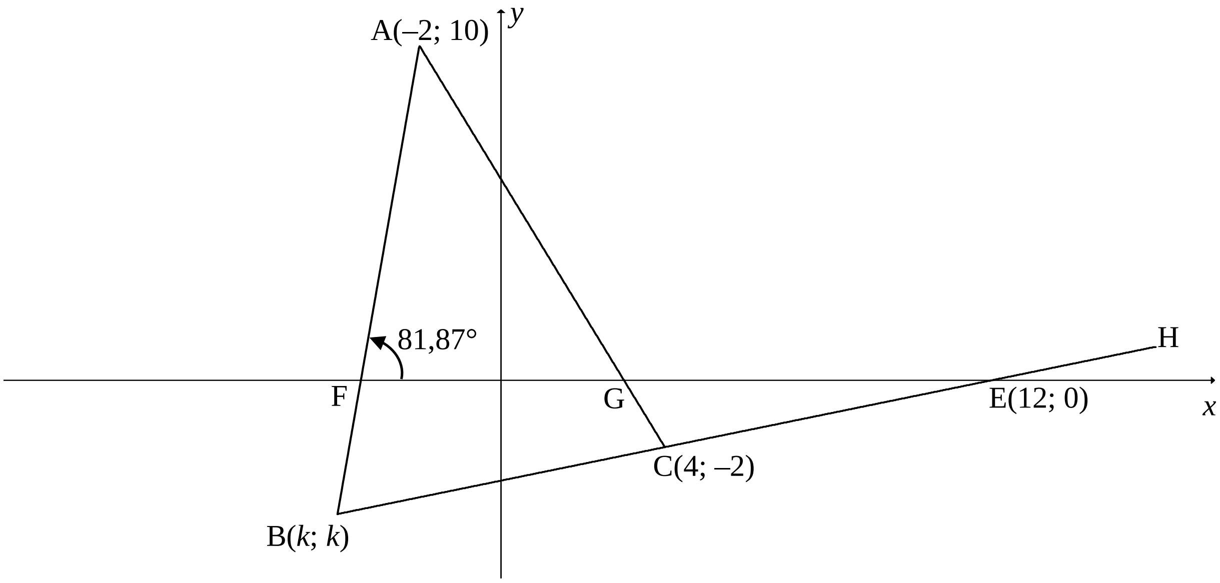
QUESTION/*VRAAG* 3



3.1.1	$m_{BE} = m_{CE} = \frac{0-(-2)}{12-4}$ **OR/*OF*** $m_{BE} = m_{CE} = \frac{-2-0}{4-12}$ $= \frac{1}{4}$ $= \frac{1}{4}$	✓ substitution C & E ✓ answer (2)
3.1.2	$m_{AB} = \tan 81{,}87°$ $m_{AB} = 7$ Answer only: Full marks *Slegs antw: Volpunte*	✓ substitution ✓ answer (2)
3.2	$y = mx + c$ $y - y_1 = m(x - x_1)$ $0 = \frac{1}{4}(12) + c$ or $y - 0 = \frac{1}{4}(x - 12)$ $c = -3$ $y = \frac{1}{4}x - 3$ $y = \frac{1}{4}x - 3$ **OR/*OF*** $y = mx + c$ $y - y_1 = m(x - x_1)$ $-2 = \frac{1}{4}(4) + c$ or $y - (-2) = \frac{1}{4}(x - 4)$ $c = -3$ $y = \frac{1}{4}x - 3$ $y = \frac{1}{4}x - 3$	✓ substitution of E ✓ answer (2) ✓ substitution of C ✓ answer (2)

3.3.1	$y=\frac{1}{4}x-3$ $k=\frac{1}{4}k-3$ $\frac{3}{4}k=-3$ $k=-4$ $\therefore B(-4;-4)$	✓ substitution ✓ answer (2)
	OR/*OF* $m_{BE}=\frac{1}{4}$ **OR/*OF*** $m_{BE}=\frac{1}{4}$ $\frac{0-k}{12-k}=\frac{1}{4}$ $\frac{k}{k-12}=\frac{1}{4}$ $-4k=12-k$ $4k=k-12$ $k=-4$ $k=-4$ $\therefore B(-4;-4)$	✓ substitution ✓ answer (2)
	OR/*OF* $m_{AB}=\tan 81{,}87°$ $m_{AB}=7$ $m_{AB}=\frac{10-k}{-2-k}$ $7(-2-k)=10-k$ $-14-7k=10-k$ $-6k=24$ $k=-4$ $\therefore B(-4;-4)$	✓ substitution ✓ answer (2)
	OR/*OF* EB: $y=\frac{1}{4}x-3$ and AB: $y=7x+24$ $\frac{1}{4}x-3=7x+24$ $\frac{27}{4}x=-27$ $x=k=-4$ $\therefore B(-4;-4)$	✓ equating EB & AB ✓ answer (2)

3.3.2	In ΔAFG: $m_{AC} = \frac{10-(-2)}{-2-4} = -2$ $\tan\theta = m_{AC} = -2$ $\theta = 180° - 63{,}43...°$ $\therefore \theta = 116{,}57°$ $\therefore \hat{A} = 116{,}57° - 81{,}87°$ [ext $\angle$ of Δ] $\therefore \hat{A} = 34{,}70°$ **OR/*OF*** In ΔABC: $a = BC = 2\sqrt{17}; b = AC = 6\sqrt{5}; c = AB = 10\sqrt{2}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\left(2\sqrt{17}\right)^2 = \left(6\sqrt{5}\right)^2 + \left(10\sqrt{2}\right)^2 - 2(6\sqrt{5})(10\sqrt{2}).\cos A$ $\cos A = \frac{\left(6\sqrt{5}\right)^2 + \left(10\sqrt{2}\right)^2 - \left(2\sqrt{17}\right)^2}{2\left(6\sqrt{5}\right)\left(10\sqrt{2}\right)}$ $= 0{,}822...$ $\therefore A = 34{,}7°$	✓ $m_{AC} = -2$ ✓ $\tan\theta = -2$ ✓ $\theta = 116{,}57°$ ✓ answer (4) ✓ all 3 lengths ✓ substitution into the correct cosine rule ✓ cos A subject ✓ answer (4)
3.3.3	$M\left(\frac{12+(-2)}{2}; \frac{10+(0)}{2}\right)$ Diagonals intersect at the point (5 ; 5)	✓ x-value ✓ y-value (2)
3.4.1	BE = ET $4\sqrt{17} = \sqrt{(12-p)^2 + (0-p)^2}$ $\left(4\sqrt{17}\right)^2 = \left(\sqrt{(12-p)^2 + (0-p)^2}\right)^2$ $272 = 144 - 24p + p^2 + p^2$ $p^2 - 12p - 64 = 0$ $(p-16)(p+4) = 0$ $\therefore p = 16$ or $p = -4$ (n.a.) $\therefore$T(16; 16)	✓ substitution of E & T ✓ equating ✓ standard form ✓ factors ✓ $p = 16$ (5)
3.4.2a	$(x-12)^2 + y^2 = \left(4\sqrt{17}\right)^2 = 272$	✓ LHS ✓ RHS (2)
3.4.2b	$m_{radius} = \frac{1}{4}$ $m_{tangent} = -4$ $y = -4x + c$ $-4 = -4(-4) + c$ $c = -20$ $y = -4x - 20$ **OR/*OF*** $y - y_1 = -4(x - x_1)$ $y - (-4) = -4(x - (-4))$ $y = -4x - 20$	✓ $m_{tangent}$ ✓ substitution of B ✓ equation (3)
		[24]

9. A circle inside a parallelogram, with CD as its tangent

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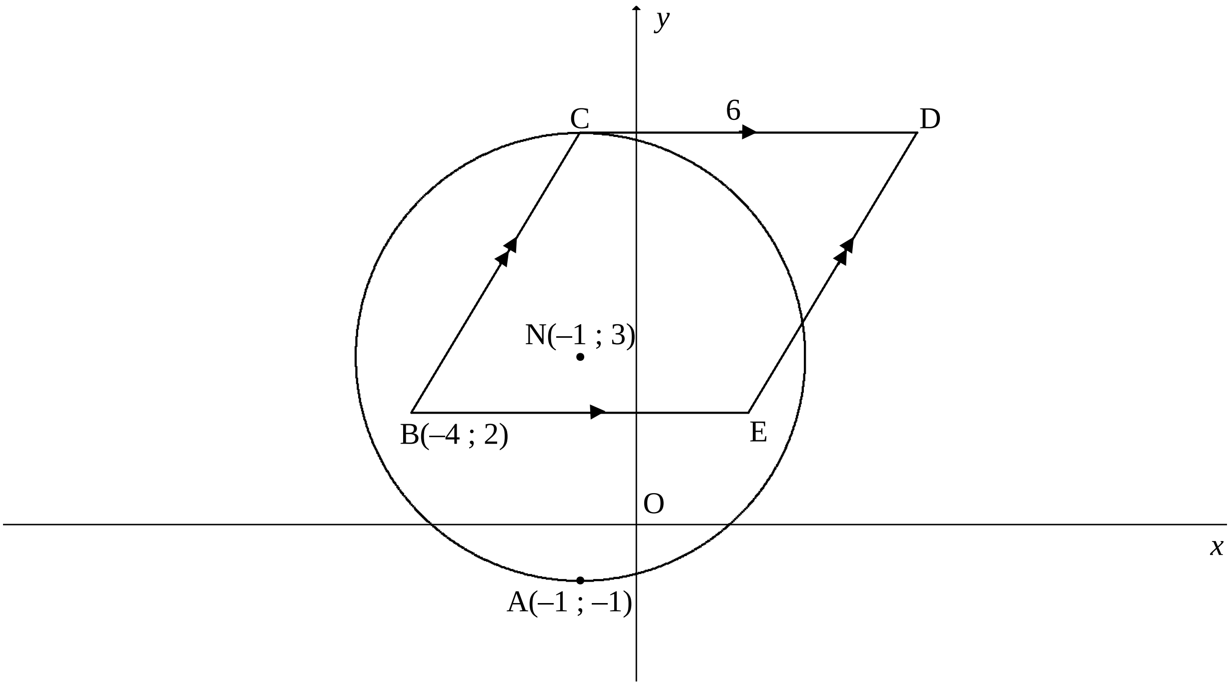
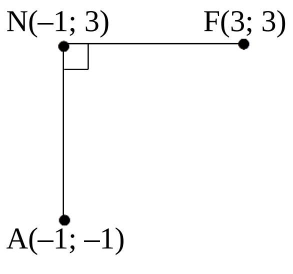
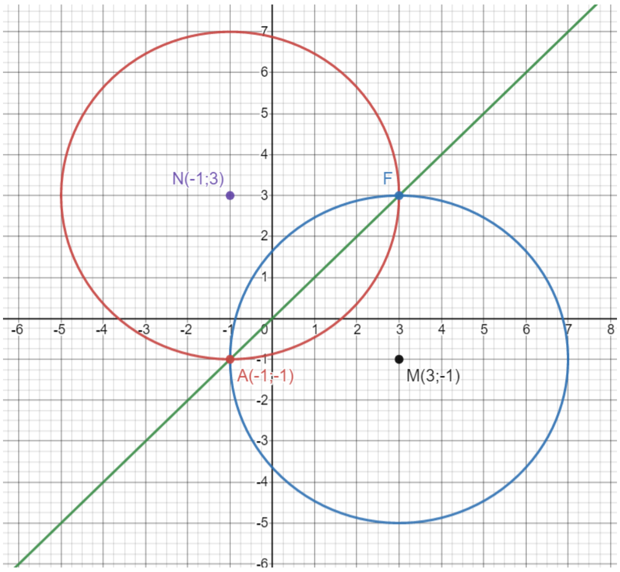
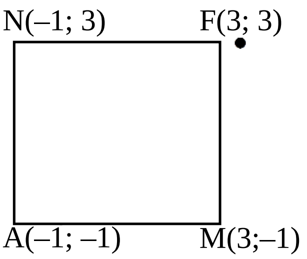
QUESTION/*VRAAG* 4



4.1	Radius = 4 units/*eenhede*	✓ answer (1)
4.2.1	$CD \perp CN$ $\therefore C(-1\ ;\ 7)$	✓x value ✓ y value (2)
4.2.2	CD = 6 units $\therefore D(5\ ;\ 7)$	✓x value ✓ y value (2)
4.2.3	$\perp h = 5$ units DC = 6 units $\text{Area } \Delta BCD = \frac{1}{2}(6)(5)$ $= 15 \text{ units}^2$ **OR/*OF*** $\perp h = 5$ units DC = 6 units $\text{Area } \Delta BCD = \frac{1}{2}[\text{Area of } \|^m]$ $= \frac{1}{2}[(5)(6)]$ $= 15 \text{ units}^2$	✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3) ✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3)

	OR/*OF* Let angle of inclination of BC = α $\tan\alpha = \frac{5}{3}$ $\alpha = 59{,}036...^\circ$ $\hat{BCD} = 180^\circ - \alpha$ $\hat{BCD} = 180^\circ - 59{,}036...^\circ$ $\hat{BCD} = 120{,}96^\circ$ Area $\Delta BCD = \frac{1}{2}\left(\sqrt{34}\right)(6)\sin 120{,}96^\circ$ $= 15$ units^2	✓ $\hat{BCD} = 120{,}96^\circ$ ✓ substitution into Area rule ✓ answer (3)
4.3.1	M(3 ; –1) [reflection of N(–1 ; 3) about the line $y = x$] $\therefore MN = \sqrt{(3-(-1))^2 + (-1-3)^2}$ $MN = \sqrt{32} = 4\sqrt{2} = 5{,}66$ units	✓ coordinates of M (A) ✓ substitution of M&N ✓ answer (3)
4.3.2	M(3 ; –1) $m_{MN} = \frac{3-(-1)}{-1-3} = -1$ MN: $-1 = -(3) + c$ or $y - 3 = -1(x+1)$ $c = 2$; $y - 3 = -x - 1$ $\therefore y = -x + 2$; $y = -x + 2$ $x = -x + 2$ $2x = 2$ $x = 1$ $\therefore y = 1$ midpoint $(1 ; 1)$ **OR/*OF***  N(–1 ; 3) $y_F = y_N = 3$ Reflected about $y = x$ $\therefore$ F(3 ; 3) midpoint $\left(\frac{-1+3}{2} ; \frac{-1+3}{2}\right) = (1 ; 1)$	 ✓ equation of MN ✓ equating AF & MN ✓ x value ✓ y value (4) ✓✓ coordinates of F ✓ x value ✓ y value (4)

	OR/*OF*   NAMF is a square (NA=NF=AM=MF and NA $\perp$ AM) Midpoint NM = $(1\ ;\ 1)$ = Midpoint of AF	✓ NAMF = square ✓x ✓ y of midpt NM ✓ midpt AF (4)
		[15]

10. p from two points on the circle, using the midpoint of AB

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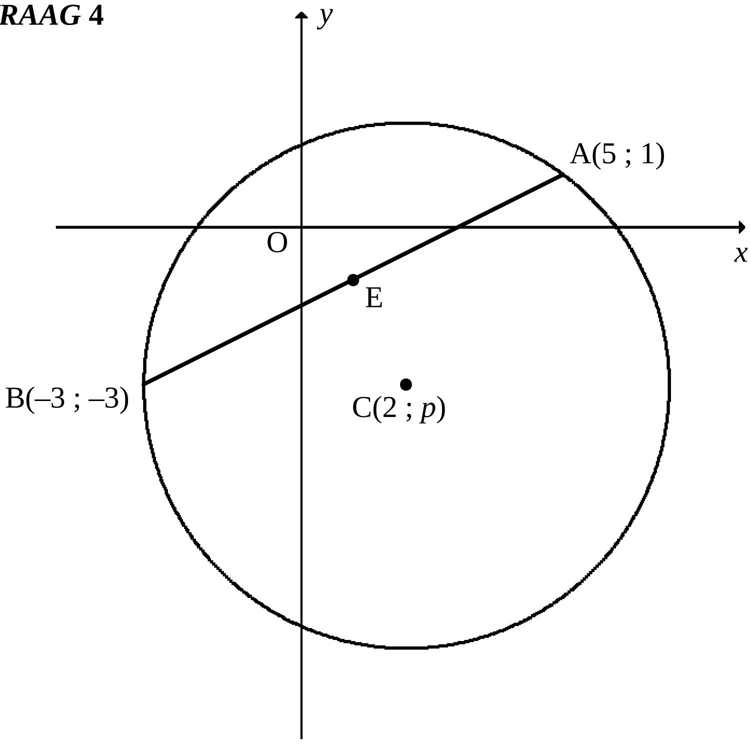
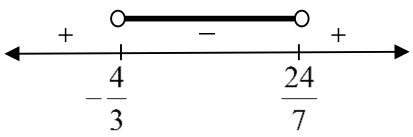
QUESTION/*VRAAG* 4



4.1	$E\left(\dfrac{5+(-3)}{2};\dfrac{1+(-3)}{2}\right)$ $\therefore E(1;-1)$	✓ $x=1$ ✓ $y=-1$ (2)
4.2	$AB=\sqrt{(x_A-x_B)^2+(y_A-y_B)^2}$ $AB=\sqrt{(5-(-3))^2+(1-(-3))^2}$ $AB=\sqrt{80}=4\sqrt{5}=8,94$ units	✓ $AB=\sqrt{80}=4\sqrt{5}=8,94$ (1)
4.3	$m_{AB}=\dfrac{1-(-3)}{5-(-3)}$ $m_{AB}=\dfrac{1}{2}$ $\therefore m_{CE}=-2$ [CE ⊥ AB] $E(1;-1)$ $y=-2x+c$ **OR** $y-y_1=-2(x-x_1)$ $(-1)=-2(1)+c$ $y-(-1)=-2(x-1)$ $c=1$ $y=-2x+1$ $y=-2x+1$	✓ $m_{AB}=\dfrac{1}{2}$ ✓ m_{CE} ✓ substitution of E ✓ equation (4)

4.4	$y=-2x+1$ $p=-2(2)+1$ $p=-3$ **OR** $m_{CE}=-2$ $\frac{p-(-1)}{2-1}=-2$ $p+1=-2$ $p=-3$	✓ substitution of C(2 ; p) into $\perp$ bisector of AB (1) ✓ substitution of C and E into the gradient formula (1)
4.5	BC = r = 5 units $\therefore (x-2)^2+(y+3)^2=25$ $x^2-4x+4+y^2+6y+9=25$ $x^2+y^2-4x+6y-12=0$	✓ BC = r = 5 units ✓ $(x-2)^2+(y+3)^2$ ✓ r^2 ✓ $x^2-4x+4+y^2+6y+9=25$ (4)

4.6	$(x-2)^2+(y+3)^2=25$ **OR** $x^2+y^2-4x+6y-12=0$ $y=tx+8$ $(x-2)^2+(tx+8+3)^2=25$ **OR** $x^2+(tx+8)^2-4x+6(tx+8)-12=0$ $x^2-4x+4+t^2x^2+22tx+121-25=0$ **OR** $x^2+t^2x^2+16tx+64-4x+6tx+48-12=0$ $x^2(t^2+1)+x(22t-4)+100=0$ $\Delta<0$ $(22t-4)^2-4(t^2+1)(100)<0$ $484t^2-176t+16-400t^2-400<0$ $84t^2-176t-384<0$ $21t^2-44t-96<0$ $(7t-24)(3t+4)<0$ CV: $\frac{24}{7}$; $-\frac{4}{3}$  $\therefore t\in\left(-\frac{4}{3};\frac{24}{7}\right)$ **OR** $-\frac{4}{3}<t<\frac{24}{7}$	✓ substitution of $y=tx+8$ ✓ standard form ✓ $\Delta<0$ ✓standard form of Δ ✓ critical values ✓ answer (6
		[18

11. A diameter, a cyclic quadrilateral, and where CV leads

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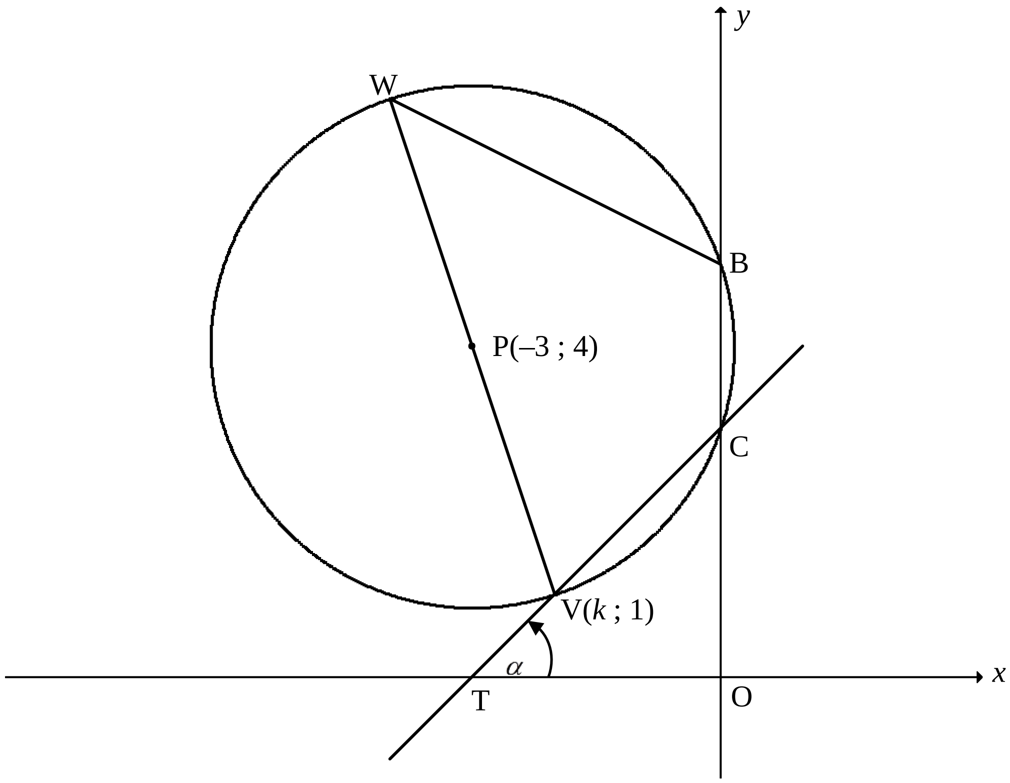
QUESTION 4



4.1	$\text{PV} = r = \sqrt{10}$ $\text{PV} = \sqrt{(k-(-3))^2 + (1-4)^2} = \sqrt{10}$ $(\text{PV})^2 = (k-(-3))^2 + (1-4)^2 = 10$ $k^2 + 6k + 9 + 9 = 10$ **OR** $(k+3)^2 + 9 = 10$ $k^2 + 6k + 8 = 0$ $\quad (k+3)^2 = 1$ $(k+4)(k+2) = 0$ $\quad k+3 = 1$ or $k+3 = -1$ $k = -4$ or $k = -2$ $\therefore k = -2$	✓ $\text{PV} = r = \sqrt{10}$ ✓ substitution into distance formula ✓ standard form ✓ factors ✓ answer (5)
4.2	$x^2 + 6x + y^2 - 8y + 15 = 0$ y-intercepts: $(0)^2 + 6(0) + y^2 - 8y + 15 = 0$ $(y-3)(y-5) = 0$ $y_C = 3$ or $y_B = 5$ $\therefore \text{BC} = 2$ units	✓ $x = 0$ ✓ factors ✓ both values ✓ answer (4)

4.3.1	$m_{TC} = \frac{3-1}{0-(-2)}$ $= 1$ $\tan\alpha = 1$ $\therefore \alpha = 45°$ **OR** $y = mx + 3$ $1 = m(-2) + 3$ $m_{TC} = 1$ $\tan\alpha = 1$ $\therefore \alpha = 45°$	✓ substitution into gradient formula ✓ $\tan\alpha = 1$ ✓ answer (3) ✓ substitution into equation of a line ✓ $\tan\alpha = 1$ ✓ answer (3)
4.3.2	$B\hat{C}V = 135°$ [ext $\angle$ of Δ/*buite* $\angle$ v Δ] $\therefore V\hat{W}B = 45°$ [opp $\angle$s of cyclic quad/*teenoorst. $\angle$e v kvh*] Answer only: Full marks **OR** $T\hat{C}O = 45°$ [$\angle$s of Δ/$\angle$*e v* Δ] $\therefore V\hat{W}B = 45°$ [ext $\angle$s of cyclic quad/*buite $\angle$ v kvh*] Answer only: Full marks	✓ $B\hat{C}V = 135°$ ✓ answer (2) ✓ $T\hat{C}O = 45°$ ✓ answer (2)
4.4.1	$Q(-3; -2)$	✓ x_Q ✓ y_Q (2)
4.4.2	$(x+3)^2 + (y+2)^2 = 10$	✓ LHS ✓ RHS (2)
4.4.3	$x = -2$ or $x = -4$	✓ $x = -2$ ✓ $x = -4$ (2)
		[20]

12. The centre recovered from a tangent and an intercept

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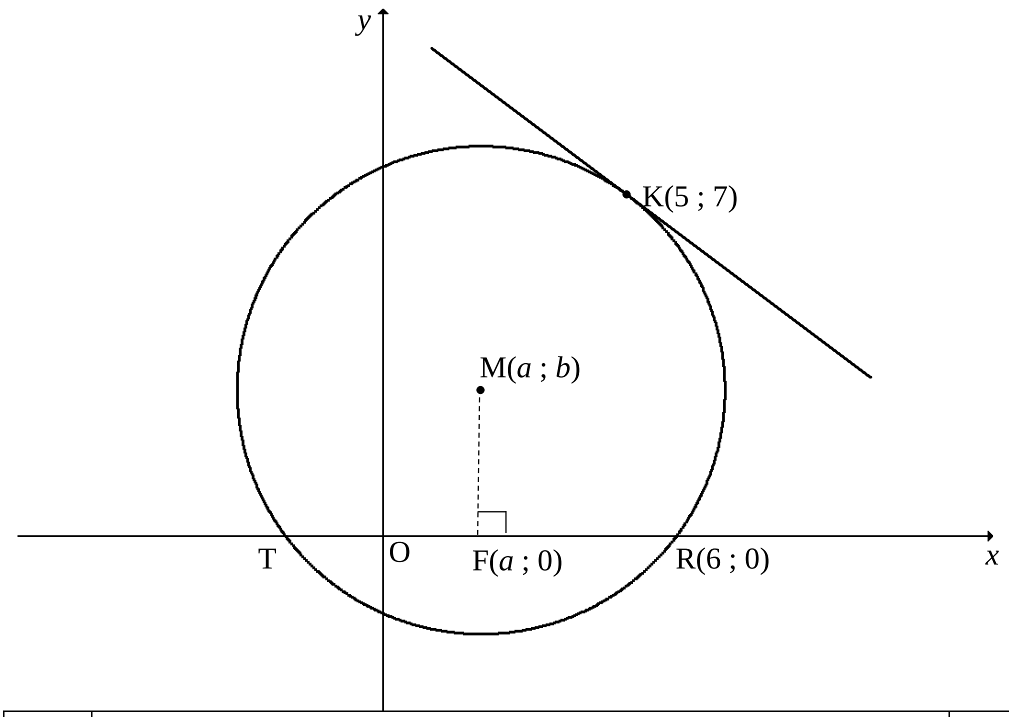
QUESTION/*VRAAG* 4



4.1.1	$y = x + 1$ $b = a + 1$	✓ $b = a + 1$ (1)
4.1.2	$MR^2 = MK^2$ $(a-6)^2 + (b-0)^2 = (a-5)^2 + (b-7)^2$ $(a-6)^2 + (a+1)^2 = (a-5)^2 + (a+1-7)^2$ $a^2 + 2a + 1 = a^2 - 10a + 25$ $12a = 24$ $a = 2$ $b = 3$ $\therefore M(2 ; 3)$	✓ equating radii / solving simultaneously ✓ substitution $b = a + 1$ ✓ $12a = 24$ ✓ $a = 2$ ✓ $b = 3$ (5)
4.2.1	$(6-2)^2 + (0-3)^2 = r^2$ $r = 5$ **OR/*OF*** $(2-5)^2 + (3-7)^2 = r^2$ $r = 5$ Answer only 2/2	✓ substitution R and M ✓ $r = 5$ (2) ✓ substitution K and M ✓ $r = 5$ (2)

4.2.2	T(–2 ; 0) TR = 8 units [line from centre ⊥ to chord] **OR/*OF*** M(2 ; 3) F(a ; 0) FR = 4 units TR = 8 units [line from centre ⊥ to chord] **OR/*OF*** $(x-2)^2+(0-3)^2=25$ $x^2-4x+4+9=25$ $x^2-4x-12=0$ $(x-6)(x+2)=0$ $x=6$ or $x=-2$ TR = 8 units Answer only 2/2	✓ T(–2 ; 0) ✓ answer (2) ✓ 4 units ✓ answer (2) ✓ x values ✓ answer (2)
4.3	$m_{\text{radius}}=\frac{7-3}{5-2}$ $m_{\text{radius}}=\frac{4}{3}$ $m_{\text{tangent}}=-\frac{3}{4}$ $7=-\frac{3}{4}(5)+c$ **OR/*OF*** $y-7=-\frac{3}{4}(x-5)$ $c=\frac{43}{4}$ $y=-\frac{3}{4}x+\frac{43}{4}$ $y=-\frac{3}{4}x+\frac{43}{4}$	✓ substitution ✓ $m_{\text{radius}}=\frac{4}{3}$ ✓ $m_{\text{tangent}}=-\frac{3}{4}$ ✓ substitution ✓ answer (5)
4.4.1	N(2 ; –2)	✓ $x_N=2$ ✓ $y_N=-2$ (2)
4.4.2	$(-2-2)^2+(0+2)^2=r^2$ $r^2=20$ $(x-2)^2+(y+2)^2=20$	✓ substitution ✓ $r^2=20$ ✓ answer (3)
		[20]

13. PN as a diameter, and the tangent at N

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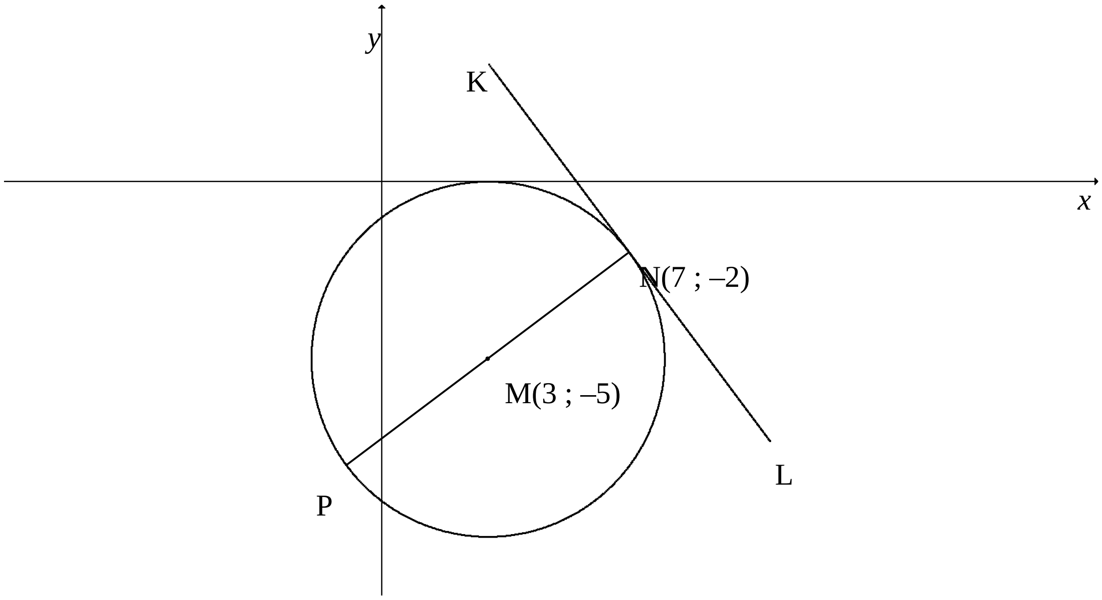
QUESTION/*VRAAG* 4



4.1	$P(x;y)$; $N(7;-2)$; $M(3;-5)$ $\frac{x+7}{2}=3$ $\quad\quad$ $\frac{y-2}{2}=-5$ $x=-1$ $\quad\quad$ $y=-8$ $P(-1;-8)$	✓ $x_P=-1$ ✓ $y_P=-8$ (2)
4.2.1	$r^2=(7-3)^2+(-2-(-5))^2$ **OR/*OF*** $r^2=(-1-3)^2+(-8-(-5))^2$ $r^2=25$ $(x-3)^2+(y+5)^2=25$	✓ substitution into distance formula ✓ $(x-3)^2+(y+5)^2$ ✓ $r^2=25$ (3)
4.2.2	$m_{\text{radius}}=\frac{-5-(-2)}{3-7}=\frac{3}{4}$ $m_{\text{tangent}}=-\frac{4}{3}$ [radius ⊥ tangent/*raaklyn* ⊥ *radius*] $-2=-\frac{4}{3}(7)+c$ **OR** $y-(-2)=-\frac{4}{3}(x-7)$ $c=\frac{22}{3}$ $\quad\quad$ $y=-\frac{4}{3}x+\frac{22}{3}$ $y=-\frac{4}{3}x+\frac{22}{3}$	✓ substitution ✓ $m_{\text{radius}}=\frac{-3}{-4}=\frac{3}{4}$ ✓ $m_{\text{tangent}}=-\frac{4}{3}$ ✓ substitution of *m* and N(7 ; –2) ✓ equation (5)
4.3	$-8=-\frac{4}{3}(-1)+c$ $\therefore\ c=-\frac{28}{3}$ $-\frac{28}{3}<k<\frac{22}{3}$	✓ subst *m* and P ✓ value of *c* ✓✓ answer (4)

4.4.1	$AB^2 = AM^2 - MB^2$ $AB^2 = \left[(t-3)^2 + (t+5)^2\right] - 5^2$ $= t^2 - 6t + 9 + t^2 + 10t + 25 - 25$ $AB = \sqrt{2t^2 + 4t + 9}$	✓ substitution into Pythagoras ✓ simplification (A) (2)
4.4.2	$t = \frac{-4}{2(2)}$ $= -1$ Minimum at $t = -1$ $AB = \sqrt{2(-1)^2 + 4(-1) + 9}$ $AB = \sqrt{7}$ **OR/*OF*** $4t + 4 = 0$ $t = -1$ Minimum at $t = -1$ $AB = \sqrt{2(-1)^2 + 4(-1) + 9}$ $AB = \sqrt{7}$ **OR/*OF*** Length of $AB = \sqrt{2t^2 + 4t + 9}$ $= \sqrt{2\left(t^2 + 2t + \frac{9}{2}\right)}$ $= \sqrt{2\left[(t+1)^2 + \frac{7}{2}\right]}$ $= \sqrt{2(t+1)^2 + 7}$ Minimum at $t = -1$ $AB = \sqrt{2(-1)^2 + 4(-1) + 9}$ $AB = \sqrt{7}$	✓ substitution into correct formula ✓ $t = -1$ ✓ substitution ✓ answer (4) ✓ derivative = 0 ✓ $t = -1$ ✓ substitution ✓ answer (4) ✓ completing of the square ✓ $t = -1$ ✓ substitution ✓ answer (4)
		[20]

14. Two circles, and the tangent common to both

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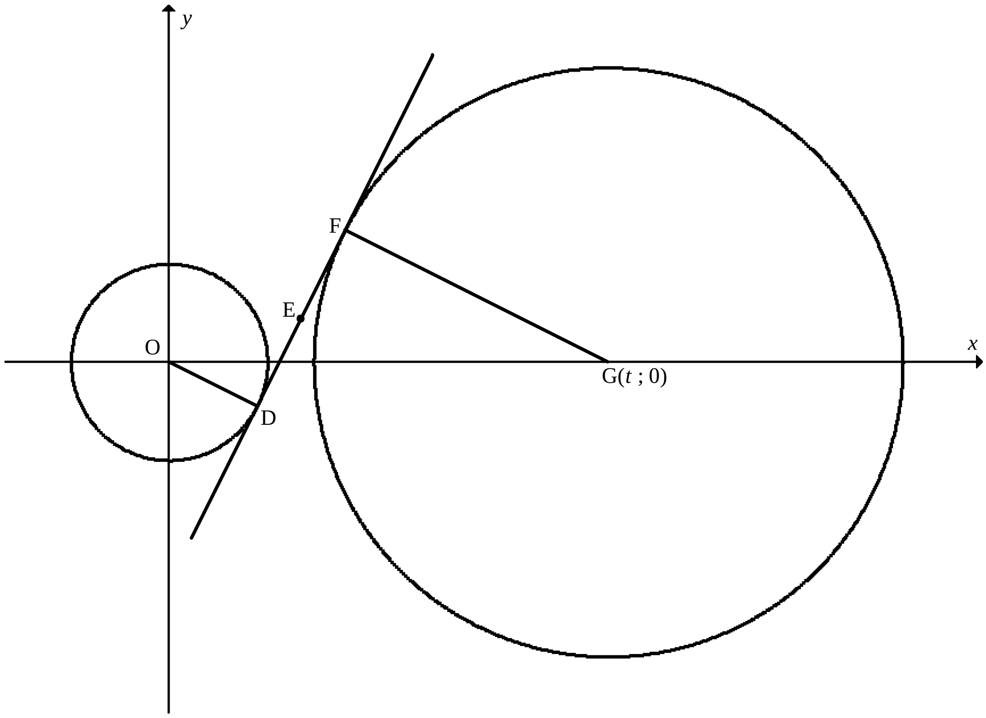
QUESTION/*VRAAG 4*



4.1	D$(p\ ;\ -2)$ $x^2+y^2=20$ $p^2+(-2)^2=20$ $p^2=16$ $p=\pm 4$ $p=4$	✓ substitution of point D$(p\ ;-2)$ ✓ $p^2=16$ (2)
4.2	$\frac{4+x_F}{2}=6$ $\quad$ $\frac{-2+y_F}{2}=2$ $x_F=8$ $\quad$ $y_F=6$ F$(8;6)$ **OR** $x_E-x_D=6-4$ $\quad$ $y_E-y_D=2-(-2)$ $=2$ $\quad$ $=4$ $x_F=6+2=8$ $\quad$ $y_F=2+4=6$ F$(8;6)$	✓ method ✓ x-value ✓ y-value (3) ✓ method ✓ x-value ✓ y-value (3)

4.3	$m_{DE}=\frac{-2-2}{4-6}$ $m_{DE}=2$ $-2=2(4)+c$ **OR** $y-(-2)=2(x-4)$ $c=-10$ $\quad y+2=2x-8$ $y=2x-10$ $\quad y=2x-10$ **OR** $m_{OD}=-\frac{2}{4}=-\frac{1}{2}$ $\therefore m_{DE}=2$ [tan ⊥ radius] $-2=2(4)+c$ **OR** $y-(-2)=2(x-4)$ $c=-10$ $\quad y+2=2x-8$ $y=2x-10$ $\quad y=2x-10$	✓ correct substitution ✓ gradient of DE, DF or EF ✓ substitution of point D(4 ; −2) or E(6 ; 2) or F(8 ; 6) ✓ answer (4) ✓ correct gradient of OD ✓ gradient of DE ✓ substitution of point D(4 ; −2) or E(6 ; 2) or F(8 ; 6) ✓ answer (4)
4.4	$m_{DE}=2$ $\therefore m_{GF}=-\frac{1}{2}$ [tan ⊥ radius] $\frac{0-6}{t-8}=-\frac{1}{2}$ $-(t-8)=2(-6)$ $t=20$ **OR** $y=2x-10$ $0=2x-10$ $x=5$ A(5 ; 0) In ΔAFG: FA ⊥ FG $FA^2=(6-0)^2+(8-5)^2=45$ $FG^2=(t-8)^2+(0-6)^2$ $=t^2-16t+100$ $GA^2=(t-5)^2$ $=t^2-10t+25$ $\therefore GA^2=GF^2+FA^2$ $t^2-10t+25=t^2-16t+100+45$ $6t=120$ $t=20$	✓ correct gradient of GF ✓ substitution of F ✓ answer (3) ✓ x-intercept of DF ✓ substitution into Pythagoras ✓ answer (3)

4.5	$F(8;6)$ G(20 ; 0) $(8-20)^2+(6-0)^2=r^2$ $r^2=180$ $(x-20)^2+y^2=180$ $x^2+y^2-40x+220=0$	✓ substitution of F and G ✓ value of r^2 ✓ equation of circle ✓ answer (4)
4.6	Smaller circle $r=2\sqrt{5}$ Larger circle $r=6\sqrt{5}$ G(20 ; 0) $k=20-(6\sqrt{5}-2\sqrt{5})$ or $k=20+(6\sqrt{5}-2\sqrt{5})$ $=20-4\sqrt{5}$ $=20+4\sqrt{5}$ = 11,06 units = 28,94 units **OR** Smaller circle $r=2\sqrt{5}$ $k=2\left(2\sqrt{5}\right)+20-8\sqrt{5}$ or $k=2\left(6\sqrt{5}\right)+20-8\sqrt{5}$ $=20-4\sqrt{5}$ $=20+4\sqrt{5}$ = 11,06 units = 28,94 units **OR** $x^2+y^2-40x+220=0$ $y=0$ $\therefore x^2-40x+220=0$ $\therefore x=20+6\sqrt{5}$ or $x=20-6\sqrt{5}$ $\therefore k=20+6\sqrt{5}-\sqrt{20}$ or $k=20-6\sqrt{5}+\sqrt{20}$ $\therefore k=20+4\sqrt{5}$ $\therefore k=20-4\sqrt{5}$ = 11,06 units = 28,94 units	✓ $r=2\sqrt{5}$ ✓ method ✓ answer ✓ answer (4) ✓ $r=2\sqrt{5}$ ✓ method ✓ answer ✓ answer (4) ✓ x-intercepts ✓ method ✓ answer ✓ answer (4)
		[20]

15. A tangent at P, then the other points that lie on the circle

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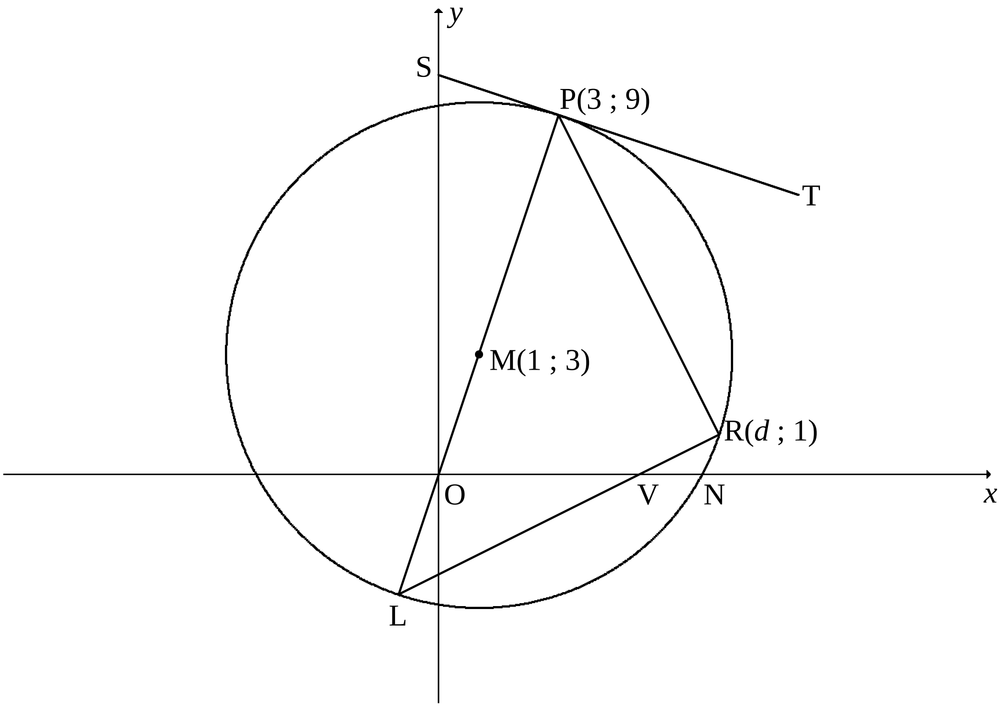
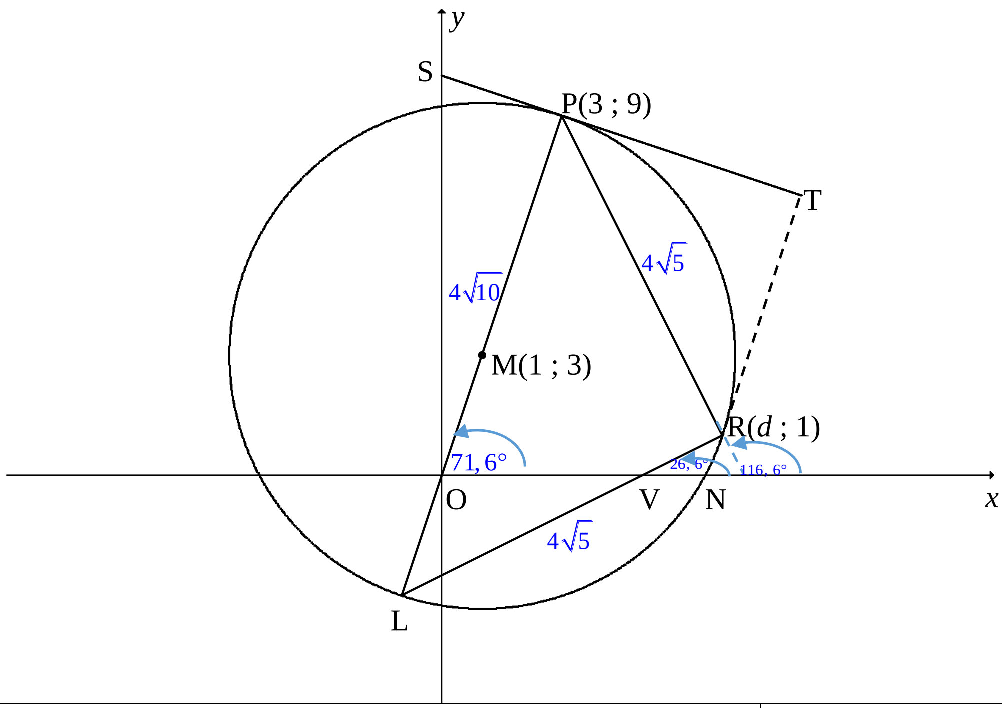
QUESTION/*VRAAG* 4



4.1	L(–1 ; –3)	✓ $x = -1$ ✓ $y = -3$ (2)
4.2	$m_{MP} = \frac{9-3}{3-1}$ $m_{MP} = 3$ $m_{ST} = -\frac{1}{3}$ $9 = -\frac{1}{3}(3) + c$ **OR/*OF*** $y - 9 = -\frac{1}{3}(x-3)$ $c = 10$ $\quad$ $y - 9 = -\frac{1}{3}x + 1$ $y = -\frac{1}{3}x + 10$ $\quad$ $y = -\frac{1}{3}x + 10$	✓ $m_{MP} = 3$ ✓ $m_{ST} = -\frac{1}{m_{MP}}$ ✓ substitution of m_{ST} & P(3; 9) into equation of a line ✓ equation of tangent ST (4)
4.3	$(x-1)^2 + (y-3)^2 = r^2$ $(3-1)^2 + (9-3)^2 = r^2$ $r^2 = 40$ $(x-1)^2 + (y-3)^2 = 40$ $x^2 - 2x + 1 + y^2 - 6y + 9 = 40$ $x^2 + y^2 - 2x - 6y - 30 = 0$	✓ $(3-1)^2 + (9-3)^2 = r^2$ ✓ value of r^2 ✓ LHS of equation of circle ✓ expanding LHS (4)

4.4	$d^2+(1)^2-2d-6(1)-30=0$ $d^2-2d-35=0$ $(d-7)(d+5)=0$ $d=7$ or $d=-5$ $\therefore d=7$ **OR/*OF*** $(x-1)^2+(y-3)^2=40$ $(d-1)^2+(1-3)^2=40$ $(d-1)^2=36$ $d-1=6$ or $d-1=-6$ $d=7$ or $d=-5$ $\therefore d=7$ **OR/*OF*** $\hat{PRL}=90°$ (∠ in semi-circle) $\frac{9-1}{3-d}\times\frac{1-(-3)}{d-(-1)}=-1$ $d^2-2d-35=0$ $(d-7)(d+5)=0$ $d=7$ or $d=-5$ $\therefore d=7$	✓ $d^2+(1)^2-2d-6(1)-30=0$ ✓ standard form (2) **OR/*OF*** ✓ $(d-1)^2+(1-3)^2=40$ ✓ standard form (2) **OR/*OF*** ✓ $m_{PR}\times m_{RL}=-1$ ✓ standard form (2)



4.5	$m_{PO} = 3$ $\therefore \tan \mathrm{P\hat{O}V} = 3$ $\mathrm{P\hat{O}V} = 71{,}565...°$ $m_{RL} = \dfrac{1-(-3)}{7-(-1)}$ $= \dfrac{1}{2}$ $\therefore \tan \mathrm{R\hat{V}N} = \dfrac{1}{2}$ $\mathrm{R\hat{V}N} = 26{,}565...°$ $\hat{L} = 71{,}565...° - 26.565...°$ [ext. ∠ of Δ / *buite ∠ van* Δ] $\hat{L} = 45°$ **OR/*OF*** $\hat{R} = 90°$ [∠ in semi-circle /∠ *in 'n halwe sirkel*] $PR^2 = (3-7)^2 + (9-1)^2$ $PR = \sqrt{80} = 4\sqrt{5}$ units $PL^2 = (3-(-1))^2 + (9-(-3))^2$ **OR** $RL^2 = (7+1)^2 + (1+3)^2$ $PL = \sqrt{160} = 4\sqrt{10}$ $RL = \sqrt{80} = 4\sqrt{5}$ $\sin \hat{L} = \dfrac{4\sqrt{5}}{4\sqrt{10}}$ **OR** $\cos \hat{L} = \dfrac{4\sqrt{5}}{4\sqrt{10}}$ **OR** $\tan \hat{L} = \dfrac{4\sqrt{5}}{4\sqrt{5}}$ $\hat{L} = 45°$	✓ $\tan \mathrm{P\hat{O}V} = m_{PO}$ ✓ $\mathrm{P\hat{O}V}$ ✓ m_{RL} using R(7 ; 1) & L ✓ $\mathrm{R\hat{V}N}$ ✓ answer (5) **OR/*OF*** ✓ $\hat{R} = 90°$ ✓ $PR = \sqrt{80} = 4\sqrt{5}$ ✓ length of PL **OR** RL ✓ trig ratio of $\hat{L}$ ✓ answer (5)

	OR/*OF* $PL = \sqrt{(3+1)^2 + (9+3)^2} = \sqrt{160} = 4\sqrt{10}$ $PR = \sqrt{(7-3)^2 + (1-9)^2} = \sqrt{80} = 4\sqrt{5}$ $LR = \sqrt{(7+1)^2 + (1+3)^2} = \sqrt{80} = 4\sqrt{5}$ $\cos L = \dfrac{80+160-80}{2\sqrt{80}\times\sqrt{160}}$ $\cos L = \dfrac{\sqrt{2}}{2}$ $\hat{L} = 45°$	**OR/*OF*** ✓ length of PL ✓ $PR = \sqrt{80} = 4\sqrt{5}$ ✓ length of LR ✓ substitution into the cos rule ✓ answer (5)
4.6	$m_{RM} = \dfrac{1-3}{7-1}$ $= -\dfrac{1}{3}$ $m_{RT} = 3$ (tan ⊥ rad) $m_{PT} = -\dfrac{1}{3}$ $m_{RT} \times m_{PT} = -1$ $PT \perp RT$ **OR/*OF*** $m_{MR} = \dfrac{3-1}{1-7}$ $= -\dfrac{1}{3}$ $m_{PT} = -\dfrac{1}{3}$ [proved in Q4.2] $m_{PT} = m_{MR}$ $\therefore PT \parallel MR$ $M\hat{R}T = 90°$ [radius ⊥ tangent / *raaklyn* ⊥ *radius*] $P\hat{T}R = 90°$ [co-int ∠s; PT ∥ MR/*ooreenkomst.* ∠*e*; PT ∥ MR] $PT \perp RT$ **OR/*OF*** $T\hat{P}R = \hat{L} = 45°$ [tan-chord theorem/ ∠ *tussen raaklyn en koord*] $TP = TR$ [tans from common pt] $\therefore T\hat{P}R = T\hat{R}P = 45°$ [∠s opp equal sides/ ∠*e teenoor gelyke sye*] $\therefore P\hat{T}R = 90°$ [sum of ∠s in Δ / *binne* ∠*e van* Δ] $PT \perp RT$	✓ m_{RM} ✓ m_{RT} ✓ $m_{RT} \times m_{PT} = -1$ (3) **OR/*OF*** ✓ $PT \parallel MR$ ✓ $M\hat{R}T = 90°$ ✓ $P\hat{T}R = 90°$ (3) **OR/*OF*** ✓ $T\hat{P}R = \hat{L}$ ✓ $T\hat{P}R = T\hat{R}P$ ✓ $P\hat{T}R = 90°$ (3)
		[20]

16. A tangent at E, and fifteen units along MT

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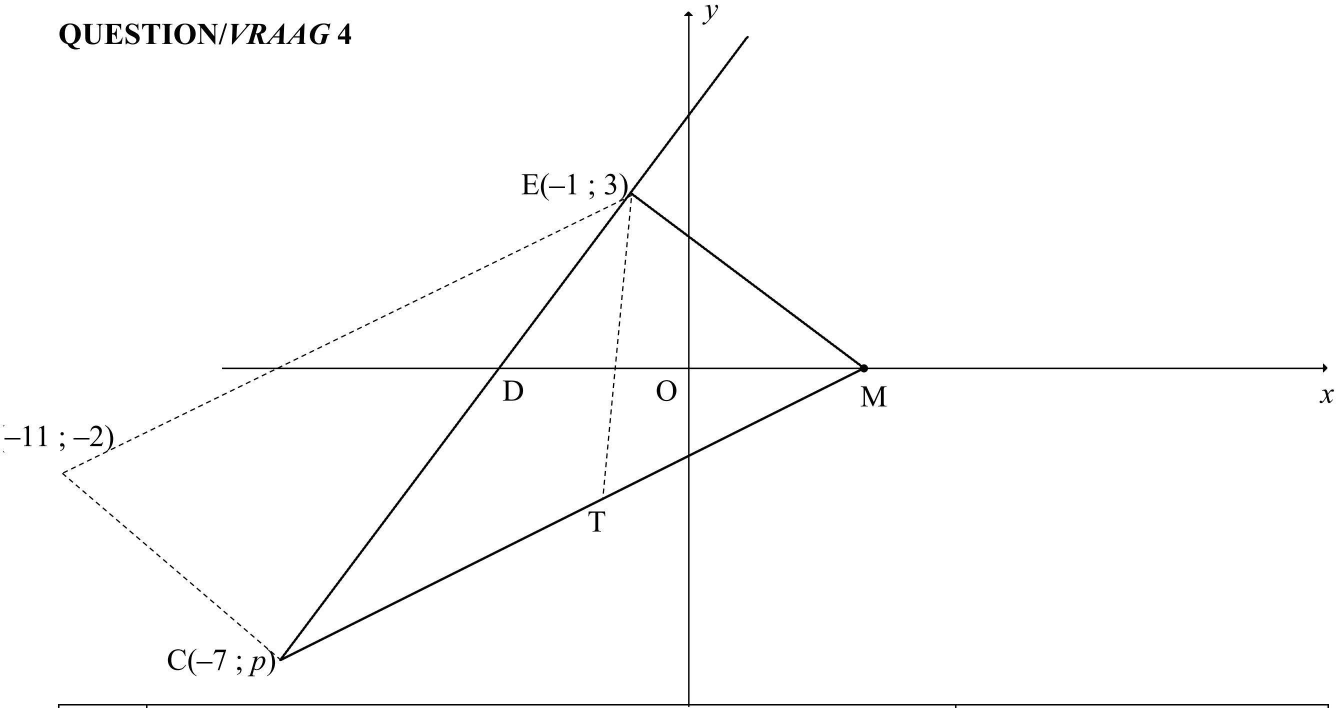
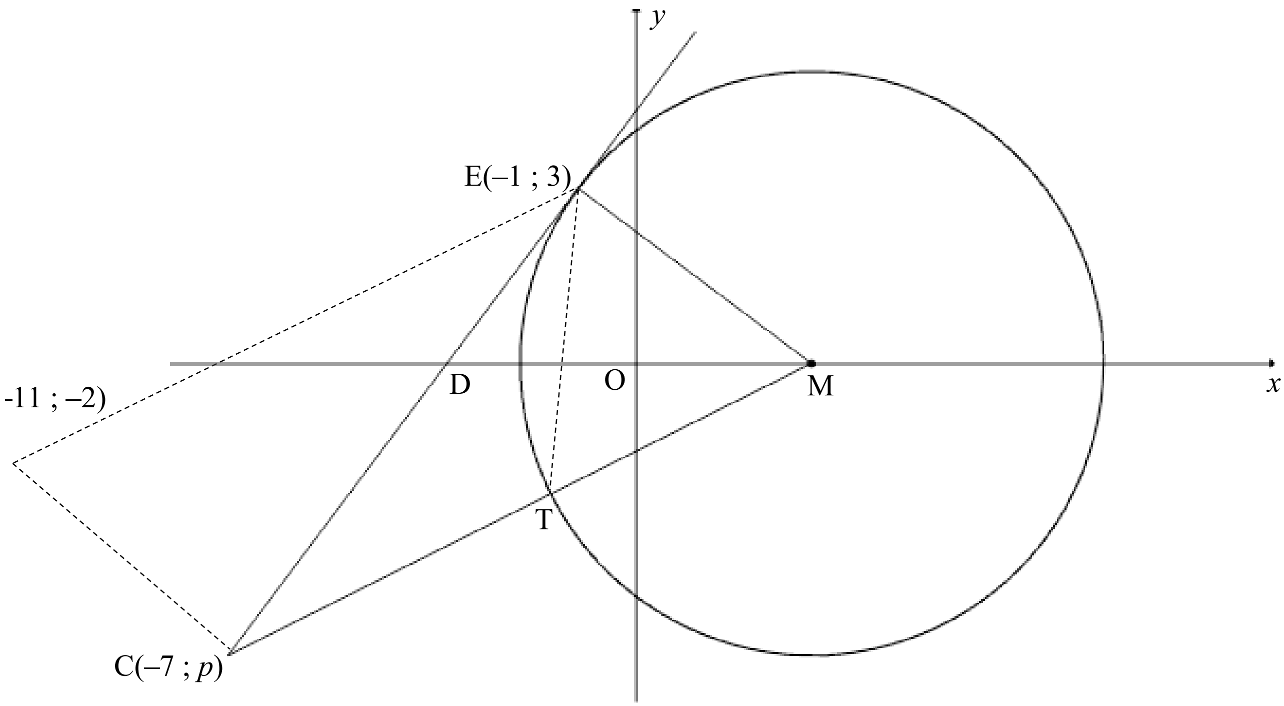
QUESTION/*VRAAG* 4



4.1	$\hat{CEM} = 90°$	✓ answer (1)
4.2	$m_{ME} = \dfrac{0-3}{3-(-1)}$ $m_{ME} = -\dfrac{3}{4}$ $\therefore m_{ED} = \dfrac{4}{3}$ $3 = \dfrac{4}{3}(-1)+c$ **OR/*OF*** $y-3 = \dfrac{4}{3}(x-(-1))$ $y = \dfrac{4}{3}x+\dfrac{13}{3}$ $\quad y = \dfrac{4}{3}x+\dfrac{13}{3}$ **OR/*OF*** $DM = \sqrt{(5)^2+\left(\dfrac{15}{4}\right)^2}$ [Pythagoras] $DM = \dfrac{25}{4}$ or 6,25 units $\therefore D\left(-\dfrac{13}{4};0\right)$ $m_{ED} = \dfrac{3-0}{-1-\left(-\dfrac{13}{4}\right)}$ $\therefore m_{ED} = \dfrac{4}{3}$ $3 = \dfrac{4}{3}(-1)+c$ **OR/*OF*** $y-3 = \dfrac{4}{3}(x-(-1))$ $y = \dfrac{4}{3}x+\dfrac{13}{3}$ $\quad y = \dfrac{4}{3}x+\dfrac{13}{3}$	✓ $m_{ME} = -\dfrac{3}{4}$ ✓ m_{ED} ✓ substitution of E(–1 ; 3) ✓ equation (4) ✓ coordinates of D ✓ m_{ED} ✓ substitution of E(–1 ; 3) ✓ equation

4.3	$y=\frac{4}{3}x+\frac{13}{3}$ $0=\frac{4}{3}x+\frac{13}{3}$ $x_D=-\frac{13}{4}$ $\therefore \text{DM}=3-\left(-\frac{13}{4}\right)$ $\text{DM}=\frac{25}{4}$ or $6{,}25$ units **OR/*OF*** EM = 5 units $\text{ED}=\frac{15}{4}$ units $\text{DM}=\sqrt{(5)^2+\left(\frac{15}{4}\right)^2}$ [Pythagoras] $\text{DM}=\frac{25}{4}$ or $6{,}25$ units	✓ x_D ✓ x_M-x_D ✓ answer (3) ✓ EM = 5 units ✓ substitution of EM & ED ✓ answer (3)
4.4	EC: $y=\frac{4}{3}x+\frac{13}{3}$ $p=\frac{4}{3}(-7)+\frac{13}{3}$ $p=-5$ **OR/*OF*** $m_{EC}=\frac{4}{3}$ $\frac{p-3}{-7-(-1)}=\frac{4}{3}$ $p-3=\frac{4}{3}(-6)$ $p=-5$	✓ substitution of C(–7 ; p) into equation of EC (1) ✓ substitution of C(–7 ; p) into gradient of EC (1)

4.5	M → E: $(x ; y) \rightarrow (x - 4 ; y + 3)$ [translation] C → S: $(-7 ; -5) \rightarrow (-7 - 4 ; -5 + 3)$ $\therefore$ S(–11 ; –2) **OR/*OF*** M → C: $(x ; y) \rightarrow (x - 10 ; y - 5)$ [translation] E → S: $(-1 ; 3) \rightarrow (-1 - 10 ; 3 - 5)$ $\therefore$ S(–11 ; –2) **OR/*OF*** E(–1 ; 3) and C(–7 ; –5) $\left(\frac{-1+(-7)}{2} ; \frac{3+(-5)}{2}\right)$ [Midpoint of EC] $=(-4;-1)$ S$(x ; y)$ and M(3 ; 0) $\frac{x_S+3}{2}=-4$ $\frac{y_S+0}{2}=-1$ $x_S=-11$ $y_S=-2$ $\therefore$ S(–11 ; –2)	✓ method: translation ✓ $x_S=-11$ ✓ $y_S=-2$ (3) ✓ method: translation ✓ $x_S=-11$ ✓ $y_S=-2$ (3) ✓ method: midpoint ✓ $x_S=-11$ ✓ $y_S=-2$ (3)
4.6	$r_{NEW}=5+7$ $r_{NEW}=12$ $MS=\sqrt{(3-(-11))^2+(0-(-2))^2}$ MS $=\sqrt{200}$ or $10\sqrt{2}$ or 14,14 units 14,14 > 12 $\therefore$ S$(-11;-2)$ lies outside the circle	✓ $r_{NEW}=12$ ✓ MS ✓ conclusion (3)



4.7	Inclination of EM: $\tan\hat{M} = m_{ME} = -\frac{3}{4}$ ref.$\angle = 36{,}87°$ Inclination of EM = 143,13° $\therefore E\hat{M}D = 36{,}87°$ Inclination of CM: $\tan\hat{M} = m_{CM} = \frac{5}{10}$ or $\frac{1}{2}$ $\therefore \hat{M} = 26{,}57°$ $\therefore E\hat{M}T = 26{,}57° + 36{,}87°$ $= 63{,}44°$ But EM = MT [radii] $\therefore E\hat{T}M = \frac{180° - 63{,}44°}{2}$ $\therefore E\hat{T}M = 58{,}28°$	✓ inclination of EM ✓ $E\hat{M}D$ ✓ inclination of CM ✓ $E\hat{M}T$ ✓ answer (5)
		[20]