EUCLIDEAN GEOMETRY

Solutions

DBE's own marking guidelines, in booklet order

Every solution here is a clip of the official DBE marking guideline published with the paper the question came from. The working, the mark ticks and the accepted alternatives are DBE's. Nothing has been rewritten or re-derived.

The numbers match the question booklet: solution 34 answers question 34, and the chapters are in the same order.

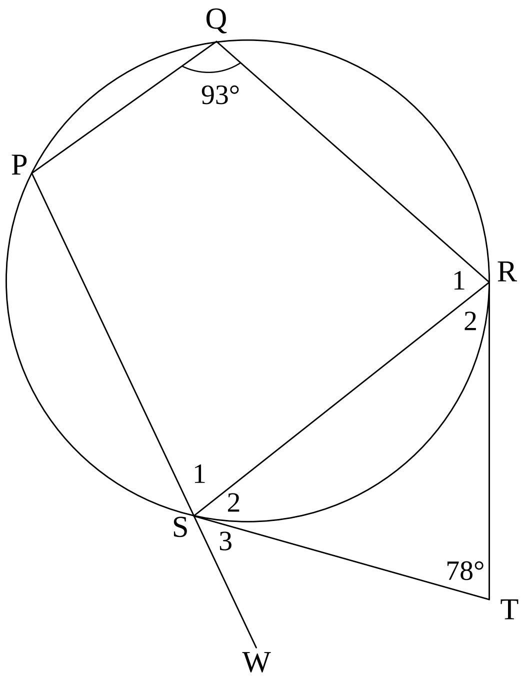
DBE writes one guideline for both languages, so these pages are bilingual and their marking notes are largely in English. That is DBE's own page, reproduced as printed.

A marking guideline is written for markers, not learners. It shows the shortest route to the marks, so where a step looks compressed, that is the guideline being terse rather than a step missing.

1. Why ST = TR, then two angles in a cyclic quadrilateral

Official DBE marking guideline · November 2021 · Q9 · 5 marks

QUESTION/VRAAG 9

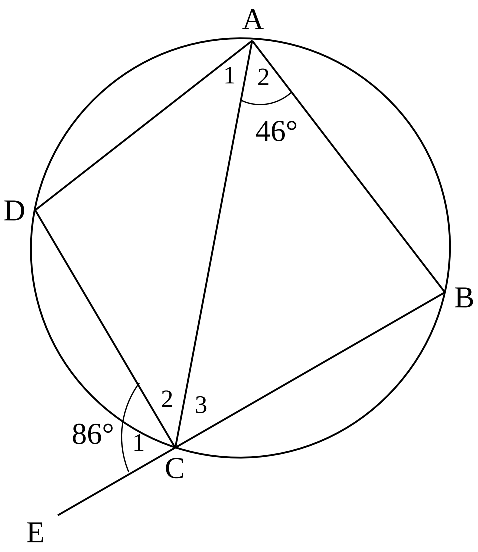


9.1	tangents from same(common) point/*raaklyne vanaf dieselfde punt*	✓ R (1)
9.2.1	$\hat{S}_2 = S\hat{R}T$ [∠s opp equal sides/*∠e teenoor gelyke sye*] $\therefore \hat{S}_2 = 51°$ [sum of ∠s in Δ/*som van ∠e in Δ*]	✓ R ✓ S (2)
9.2.2	$\hat{S}_2 + \hat{S}_3 = 93°$ [ext ∠ of cyclic quad/*buite∠ van koordevh*] $\hat{S}_3 = 42°$ **OR/*OF*** $\hat{S}_1 = 87°$ [opp ∠s of cyclic quad/*teenoorst ∠e v kdvh*] $\hat{S}_3 = 180° - (87° + 51°)$ $\hat{S}_3 = 42°$ [∠s on a str line/*∠e op reguitlyn*]	✓ R ✓ answer (2) ✓ R ✓ answer (2)
		[5]

2. An exterior angle of a cyclic quadrilateral, then AD = DC

Official DBE marking guideline · November 2024 · Q9 · 6 marks

QUESTION/VRAAG 9

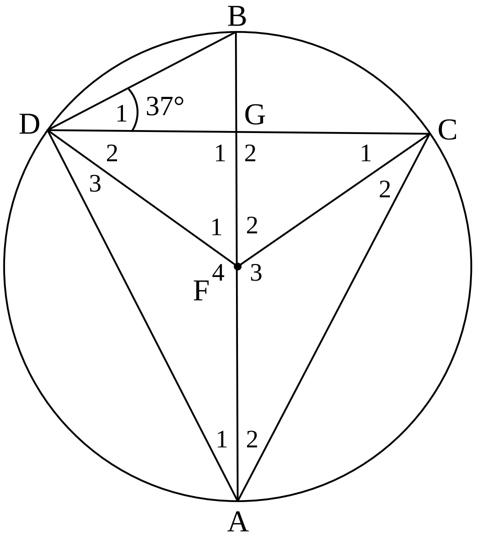


9.1	$\hat{A}_1 = 40°$ [ext. ∠ of a cyclic quad / *buite ∠ van kvh*]	✓S ✓R (2)
9.2	$\hat{B} = 80°$ $\left[\hat{A}_1 = \frac{1}{2}\hat{B}\right]$ $\hat{D} = 100°$ [opp ∠s of cyclic quad / *teenoorst. ∠e van kvh*] $\therefore \hat{C}_2 = 40°$ [sum of ∠s in Δ / *binne ∠e van* Δ] $\therefore \hat{C}_2 = \hat{A}_1 = 40°$ $\therefore$ AD = DC [sides opp = ∠s / *sye teenoor gelyke ∠*] **OR/*OF*** $\hat{B} = 80°$ $\left[\hat{A}_1 = \frac{1}{2}\hat{B}\right]$ $A\hat{C}E = \hat{A}_2 + \hat{B}$ [ext ∠ of Δ / *buite ∠ van* Δ] $\therefore \hat{C}_2 = 40°$ $\therefore \hat{C}_2 = \hat{A}_1 = 40°$ $\therefore$ AD = DC [sides opp = ∠s / *sye teenoor gelyke ∠*] **OR/*OF*** $\hat{B} = 80°$ $\left[\hat{A}_1 = \frac{1}{2}\hat{B}\right]$ $\therefore \hat{C}_3 = 180° - 46° - 80°$ [sum of ∠s in Δ / *binne ∠e van* Δ] $\therefore \hat{C}_3 = 54°$ $\therefore \hat{C}_2 = 180° - 86° - 54°$ [∠s on a str. line / *∠e op 'n reguitlyn*] $\therefore \hat{C}_2 = 40°$ $\therefore \hat{C}_2 = \hat{A}_1 = 40°$ $\therefore$ AD = DC [sides opp = ∠s / *sye teenoor gelyke ∠*]	✓S ✓S/R ✓S ✓R (4) ✓S ✓S/R ✓S ✓R (4) ✓S ✓S/R ✓S ✓R (4)
		[6]

3. A diameter and a bisected angle: find three more

Official DBE marking guideline · May/June 2024 · Q9 · 12 marks

QUESTION/*VRAAG* 9



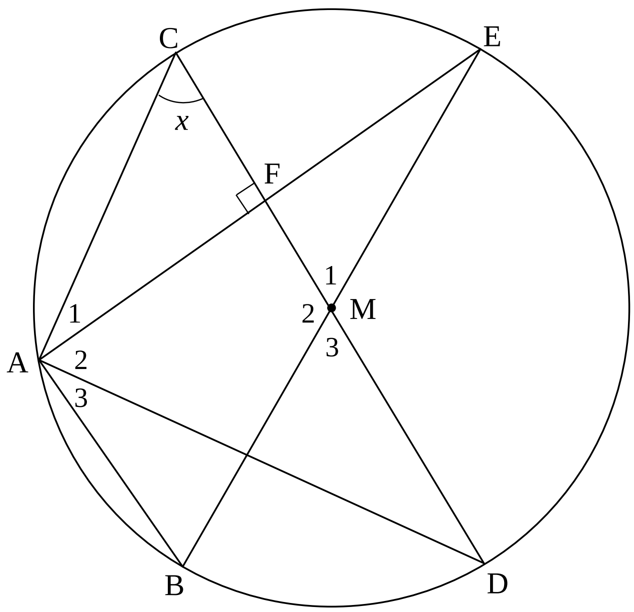
9.1	$\hat{A}_2 = \hat{D}_1 = 37°$ $\hat{A}_1 = \hat{A}_2 = 37°$ $\hat{D}_3 = \hat{A}_1 = 37°$ $\hat{C}_2 = \hat{A}_2 = 37°$	[∠s in the same seg/*∠e in dies segment*] [BA bisects $C\hat{A}D$/BA *halveer* $C\hat{A}D$] [∠s opp equal sides/*∠e teenoor gelyke sye*] [∠s opp equal sides/*∠e teenoor gelyke sye*]	✓ S ✓ R ✓✓ any other two statements (4)
9.2	$A\hat{D}G = 53°$ $\hat{A}_1 = 37°$ $\therefore \hat{G}_1 = 90°$ $\therefore CG = DG$ **OR** $\hat{F}_2 = 2\hat{D}_1 = 74°$ $\hat{D}_3 = 37°$ $\therefore \hat{D}_2 = 16°$ $\hat{C}_1 = \hat{D}_2 = 16°$ $\therefore \hat{G}_2 = 90°$ $\therefore CG = DG$	[∠ in semi circle/*∠ in halwe sirkel*] [proved in 9.1/*reeds bewys in 9.1*] [sum of ∠s in Δ/*binne ∠e van* Δ] [line from centre ⊥ to chord/ *lyn uit midpt.* ⊥ *op koord*] [∠ at centre = 2 ×∠ at circumference/ *midpt. ∠s = 2×omtreks∠*] [proved in 9.1/*reeds bewys in 9.1*] [∠ in semi circle/*∠ in halwe sirkel*] [∠s opp equal sides/*∠e teenoor gelyke sye*] [sum of ∠s in Δ/*binne ∠e van* Δ] [line from centre ⊥ to chord/ *lyn uit midpt.* ⊥ op *koord*]	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ R ✓ S ✓ R (4)

9.3	$\hat{F}_2 = 2\hat{D}_1 = 74°$ **OR** $\hat{F}_2 = 2\hat{A}_2 = 74°$ [$\angle$ at centre = 2 ×$\angle$ at circum./ *midpt.* $\angle$*s = 2×omtreks*$\angle$] $\frac{FG}{20} = \cos 74°$ $FG = 5,51$ $\therefore BG = 14,49$ units	✓ S ✓ trig ratio ✓ FG ✓ answer (4)
	OR	
	$\hat{F}_2 = 2\hat{D}_1 = 74°$ [$\angle$ at centre = 2 ×$\angle$ at circumference *midpt.* $\angle$*= 2×omtreks*$\angle$] $\frac{FG}{20} = \sin 16°$ $FG = 5,51$ $\therefore BG = 14,49$ units	✓ S ✓ trig ratio ✓ FG ✓ answer (4)
	OR	
	$\frac{DG}{20} = \cos 16°$ DG = 19,23 $\frac{BG}{19,23} = \tan 37°$ BG = 14,49 units	✓ trig ratio ✓ length of DG ✓ trig ratio ✓ answer (4)
	OR	
	$\frac{DG}{20} = \cos 16°$ DG = 19,23 $FG^2 = FD^2 - DG^2$ [Pythagoras] $FG^2 = 20^2 - (19,23)^2$ FG = 5,51 $BG = 20 - 5,51$ = 14,49 units	✓ trig ratio ✓ length of DG ✓ correct use of Pythagoras ✓ answer (4)
		[12]

4. Two diameters, a perpendicular chord, everything in terms of x

Official DBE marking guideline · November 2021 · Q10 · 13 marks

QUESTION/*VRAAG* 10



10.1	line from centre ⊥ to chord/*lyn vanaf middelpunt ⊥ op koord*	✓ R (1)
10.2	$\therefore \hat{A}_1 = 90° - x$ [sum of ∠s in Δ/*som van ∠e in Δ*] $\therefore \hat{M}_1 = 180° - 2x$ [∠at centre=2× at circumf/*midpts*∠= 2×omtreks∠]	✓ S ✓ S ✓ R (3)
10.3	$C\hat{A}D = 90°$ [∠ in semi circle/∠ *in halfsirkel*] $\hat{A}_2 = 90° - (90° - x)$ $\hat{A}_2 = x$ $\therefore \hat{A}_2 = \hat{C} = x$ ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF*** $E\hat{M}D = 2x$ [adj suppl ∠s/*aanligg suppl ∠e*] $\therefore\ \hat{A}_2 = x$ [∠at centre= 2×∠ at circumf/*midpts*∠= 2 ×omtreks∠] $\therefore\ \hat{A}_2 = C = x$ ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF*** $\hat{M}_3 = 180^{\circ} - 2x$ [vert. opp/ *regoorstaande ∠e*] $\therefore \hat{A}_3 = 90^{\circ} - x$ [∠at centre= 2×∠ at circumf/*midpts*∠= 2 ×omtreks∠] $B\hat{A}E = 90^{\circ}$ [∠ in semi-circle /∠ *in halfsirkel*] $\therefore\ \hat{A}_2 = C = x$ ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF***	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ S ✓ R ✓ R (4) ✓ S ✓ R ✓ S ✓ R (4)

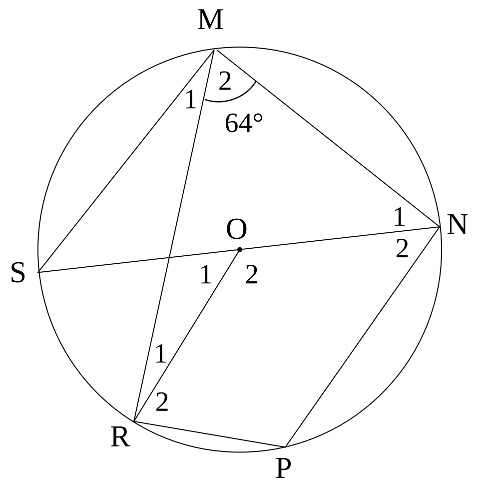
	CD ∥ AB [midpt. Thm/ *middelpuntst.*] $B\hat{A}E = 90^\circ$ [∠ in semi-circle/∠ *in halfsirkel*] $\therefore \hat{A}_3 = \hat{D} = 90^\circ - x$ [alt.∠s; CD∥AB/*verwiss ∠e*] $\therefore \hat{A}_2 = x = \hat{C}$ ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF*** $C\hat{A}D = 90°$ [∠ in semi circle/∠ *in halfsirkel*] AC = diameter [converse ∠ in semi circle/*omgek ∠ in halfsirkel*] ∴ AD is a tangent [converse radius ⊥ tangent/*omgek radius⊥rkl*]	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ R ✓ S ✓ R (4)
10.4	AF = FE and BM = ME [given & radii] $\therefore$ FM = $\frac{1}{2}$AB = 12 units [Midpt Theorem/*middelptstelling*] EM = MB = CM = 18 units [radii] ∴ EB = 36 units [diameter = 2 radius] $\therefore AE^2 = (36)^2 - (24)^2$ [Pythagoras] AE = $12\sqrt{5}$ or 26,83 units **OR/*OF*** AF = FE and BM = ME [given & radii] $\therefore$ FM = $\frac{1}{2}$AB = 12 units [Midpt Theorem/*middelpuntstelling*] EM = MB = CM = 18 units [radii] $\therefore FE^2 = (18)^2 - (12)^2$ [Pythagoras] FE = $6\sqrt{5}$ AE = $12\sqrt{5}$ or 26,83 units	✓ FM = 12 ✓ R ✓ EB = 36 ✓ using Pyth correctly ✓ answer (5) ✓ FM = 12 ✓ R ✓ EM = 18 ✓ using Pyth correctly ✓ answer (5)
		[13]

5. A diameter drawn through a cyclic quadrilateral

Official DBE marking guideline · November 2022 · Q8 · 13 marks

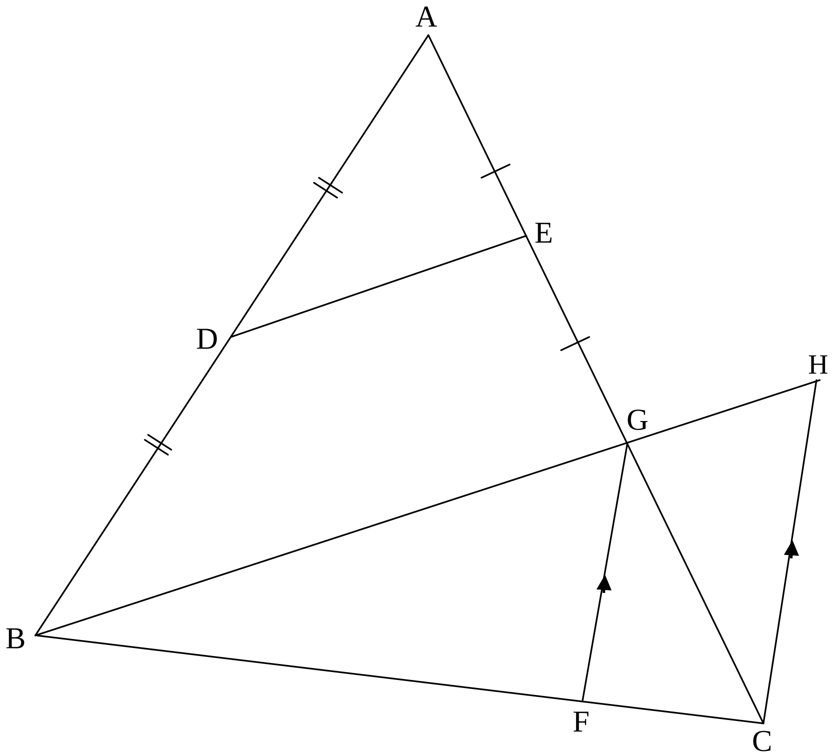
QUESTION/*VRAAG* 8

8.1



8.1.1	$\hat{P} = 116°$ [opp ∠s of cyclic quad/*teenoorst. ∠e van kvh*]	✓ S ✓ R (2)
8.1.2	$\hat{M}_1 + 64° = 90°$ [∠ in semi-circle/∠ *in halwe sirkel*] $\hat{M}_1 = 26°$	✓ R ✓ S (2)
8.1.3	$\hat{O}_1 = 52°$ [∠ at centre = 2 x ∠ at circumference/*midpts.* ∠ = 2 x *omtreks.* ∠]	✓ S ✓R (2)

8.2

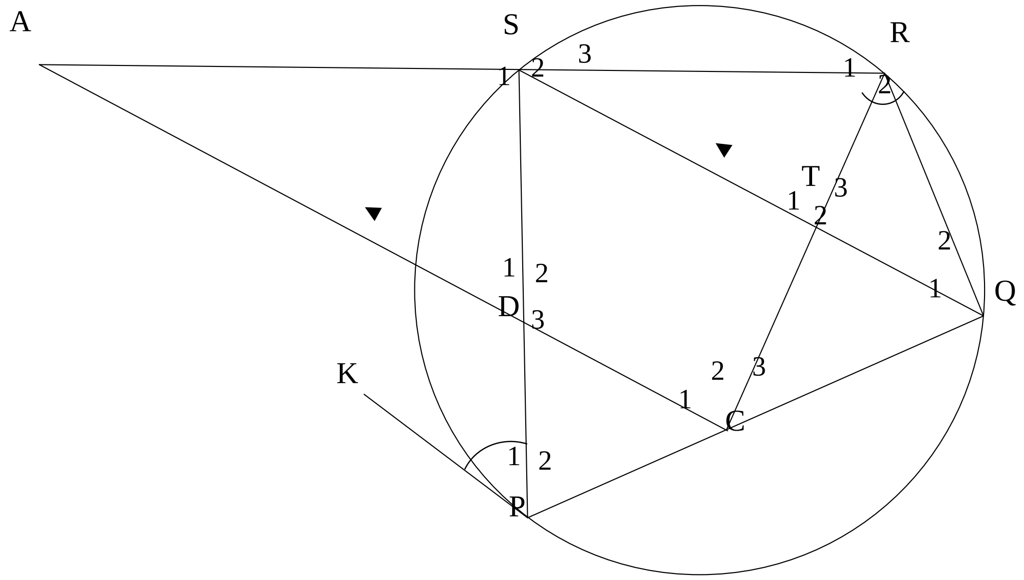


8.2.1	Midpt theorem/*Midpt. Stelling* **OR/*OF*** Converse prop intercept theorem	✓ R (1) ✓ R (1)
8.2.2	BG = 2DE or $6x - 2$ [Midpt theorem/*Midpt. stelling*] BG = $6x - 2$ $\frac{GH}{BG} = \frac{FC}{BF}$ [line ∥ one side of Δ **OR** prop theorem; FG ∥ CH / *lyn ∥ een sy v. Δ*] $\frac{x+1}{6x-2} = \frac{1}{4}$ $4x + 4 = 6x - 2$ $2x = 6$ $x = 3$ **OR/*OF***	✓ S ✓ R ✓ S ✓R ✓ equation into x ✓ answer (6)
	$\frac{BF}{FC} = \frac{BG}{GH}$ [line ∥ one side of Δ **OR** prop theorem; FG ∥ CH / *lyn ∥ een sy v. Δ*] $\frac{AE}{AG} = \frac{DE}{BG}$ [ΔADE ||| ΔABG] BG = $4x + 4$ $\frac{1}{2} = \frac{3x-1}{4x+4}$ $\therefore 4x + 4 = 6x - 2$ $\therefore x = 3$	✓ S ✓ R ✓ S ✓R ✓ equation into x ✓ answer (6)
		[13]

6. A tangent, a parallel line, and a cyclic quadrilateral to prove

Official DBE marking guideline · November 2022 · Q10 · 13 marks

QUESTION/*VRAAG* 10



10.1	$\hat{P}_1 = \hat{Q}_1$ [tan-chord theorem/$\angle$ *tussen raaklyn en koord*] $\hat{S}_1 = \hat{Q}_1 + \hat{Q}_2$ [ext $\angle$ of cyclic quad/*buite* $\angle$ *v. kvh*] $\therefore \hat{S}_1 = \hat{P}_1 + \hat{Q}_2$ $\hat{T}_2 = \hat{R}_2 + \hat{Q}_2$ [ext $\angle$ of Δ/*buite* $\angle$ *v.* Δ] but $\hat{P}_1 = \hat{R}_2$ [given/*gegee*] $\hat{T}_2 = \hat{P}_1 + \hat{Q}_2$ $\therefore \hat{S}_1 = \hat{T}_2 = \hat{P}_1 + \hat{Q}_2$	✓ S ✓ S / R ✓ S ✓ S (4)
10.2	In Δ ASD and ΔACR $\hat{A} = \hat{A}$ [common $\angle$/*gemeenskaplike* $\angle$] $\hat{S}_1 = \hat{T}_2$ [proven/*reeds bewys*] $\hat{T}_2 = \hat{C}_2$ [alt $\angle$s; QS || CA/verw. $\angle$*e*; QS || CA] $\therefore \hat{S}_1 = \hat{C}_2$ $\hat{D}_1 = \hat{R}_1$ [sum of $\angle$s in Δ/$\angle$*e v.* Δ] ΔASD ||| ΔACR $\therefore \frac{AD}{AR} = \frac{AS}{AC}$ [corresponding sides in proportion/ *ooreenstemmende sy in dies. verhouding*] **OR/*OF***	✓ identifying Δ's ✓ S ✓ S / R ✓ S ✓ S (5)

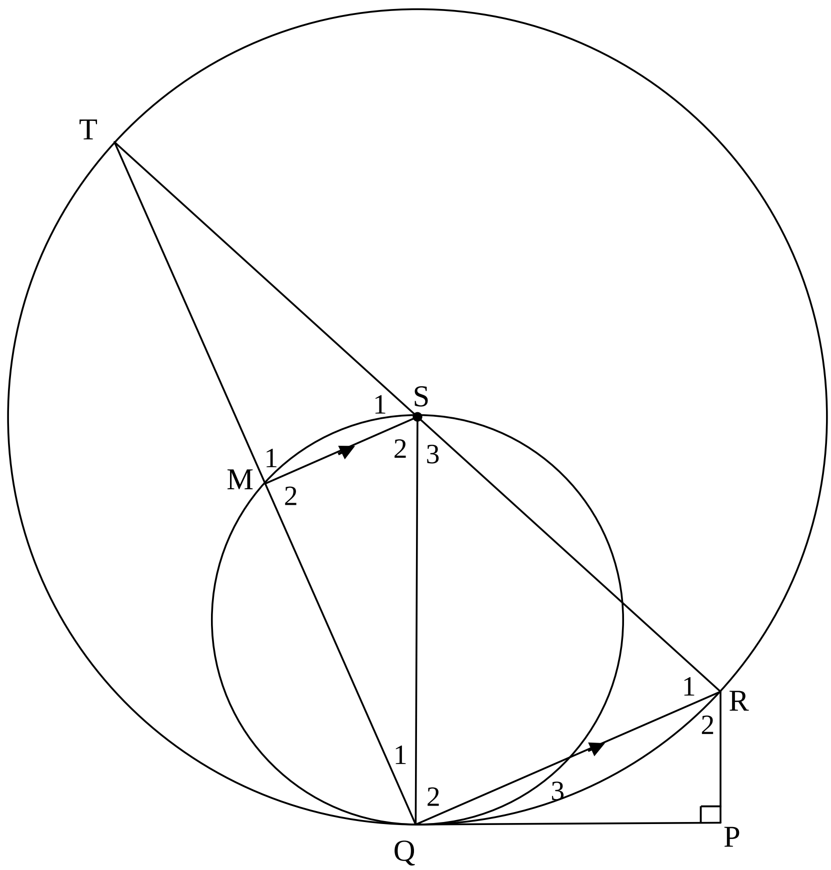
	In Δ ASD and ΔACR $\hat{A} = \hat{A}$ [common $\angle$/*gemeenskaplike* $\angle$] $\hat{S}_1 = \hat{T}_2$ [proven/*gegee*] $\hat{T}_2 = \hat{C}_2$ [alt $\angle$s; QS || CA/verw. $\angle$*e*; QS || CA] $\therefore \hat{S}_1 = \hat{C}_2$ $\Delta ASD \parallel\!\mid \Delta ACR$ [$\angle;\angle;\angle$] $\therefore \frac{AD}{AR} = \frac{AS}{AC}$ [corresponding sides in proportion/ *ooreenstemmende sy in dies. verhouding*]	✓ identifying Δ's ✓ S ✓ S/R ✓ S ✓ R (5)
10.3	$\frac{AS}{AC} = \frac{SD}{CR}$ [$\Delta ASD \parallel\!\mid \Delta ACR$] $\therefore AS = \frac{AC \times SD}{CR}$ $\frac{AS}{AR} = \frac{CT}{CR}$ [line || one side of Δ OR prop theorem; TS || CA/*lyn* || *een sy v.* Δ] $\therefore AS = \frac{AR \times CT}{CR}$ $\therefore \frac{AC \times SD}{CR} = \frac{AR \times CT}{CR}$ $\therefore AC \times SD = AR \times CT$	✓ S ✓ S ✓ R ✓ equating (4)
		[13]

TOTAL/*TOTAAL*: 150

7. Two circles with a tangent common to both at Q

Official DBE marking guideline · May/June 2021 · Q10 · 15 marks

QUESTION/VRAAG 10



10.1.1	$\hat{Q}_1 + \hat{Q}_2 = 90°$	[$\angle$ in semi circle/$\angle$ *in halwe sirkel*]	✓ S/R
	$\therefore \hat{M}_2 = 90°$	[co-interior $\angle$, MS ‖ QR/*ko-binne* $\angle$*e*, MS ‖ QR]	✓ S/R
	$\therefore$ SQ is a diameter	[converse: $\angle$ in semi circle/ *Omgekeerde:* $\angle$ *in halwe sirkel*]	✓ R (3)
	OR		
	MS ‖ QR		
	$\frac{TS}{SR} = \frac{TM}{MQ} = \frac{1}{1}$	[prop theorem; SM ‖ QR] **OR** [line ‖ one side of Δ]/*lyn* ‖ *een sy v*Δ	✓ S/R
	$\therefore TM = MQ$		
	$\therefore \hat{M}_2 = 90°$	[Line from centre bisects chord/*midpt. sirkel; midpt koord*]	✓ S/R
	$\therefore$ SQ is a diameter	[converse: $\angle$ in semi circle/ *Omgekeerde:* $\angle$ *in halwe sirkel*]	✓ R (3)
	OR		
	SQ $\perp$ QP	[tan $\perp$ rad/*raaklyn* $\perp$ *radius*]	✓ S ✓ R
	$\therefore$ SQ is a diameter	[converse: tan $\perp$ rad/*Omgekeerde: raaklyn* $\perp$ *radius*]	✓ R (3)

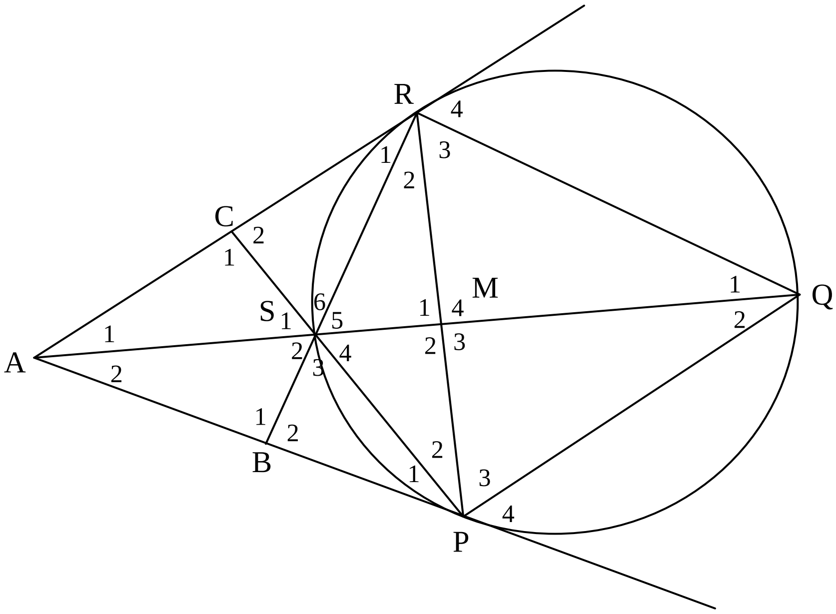
10.1.2	In ΔRTQ and ΔRQP $\hat{T} = \hat{Q}_3$ [tan-chord theorem/∠ *tussen raaklyn en koord*] $\hat{Q}_1 + \hat{Q}_2 = 90°$ [co-interior ∠s, MS ∥ QR/*ko-binne ∠e*, MS ∥ QR] **or** [∠ in semi circle/∠ *in halwe sirkel*] $\therefore \hat{Q}_1 + \hat{Q}_2 = \hat{P} = 90°$ $\hat{R}_1 = \hat{R}_2$ [∠s of Δ/∠*e van Δ*] $\Delta RTQ \parallel\!\mid \Delta RQP$ $\frac{RT}{RQ} = \frac{RQ}{RP}$ $RT = \frac{RQ^2}{RP}$ **OR** In ΔRTQ and ΔRQP $\hat{T} = \hat{Q}_3$ [tan-chord theorem ∠ *tussen raaklyn en koord*] $\hat{Q}_1 + \hat{Q}_2 = 90°$ [co-interior ∠s, MS ∥ QR/*ko-binne ∠e*, MS ∥ QR] **or** [∠ in semi circle/∠ *in halwe sirkel*] $\therefore \hat{Q}_1 + \hat{Q}_2 = \hat{P} = 90°$ $\Delta RTQ \parallel\!\mid \Delta RQP$ [∠,∠,∠] $\frac{RT}{RQ} = \frac{RQ}{RP}$ $RT = \frac{RQ^2}{RP}$	✓ S ✓ R ✓ S ✓ S ✓ S ✓ ratio (6) ✓ S ✓ R ✓ S ✓ S ✓ R ✓ ratio (6)
10.2	QR = 28 units [midpoint theorem/*midpt. stelling*] $RP^2 = 28^2 - \left(\sqrt{640}\right)^2$ [Pythagoras/*Pythagoras*] RP = 12 units $RT = \frac{RQ^2}{RP}$ $RT = \frac{28^2}{12}$ $RT = \frac{196}{3}$ Radius = $\frac{98}{3}$ units	✓ S ✓ R ✓ S ✓ RP = 12 ✓ RT ✓ answer (6)
		[15]

TOTAL/TOTAAL: 150

8. Two tangents meeting a chord that has been produced

Official DBE marking guideline · November 2023 · Q10 · 15 marks

QUESTION/*VRAAG 10*



10.1	$\hat{S}_3 = P\hat{Q}R$ [ext ∠ of cyclic quad] $\hat{R}_3 = P\hat{Q}R$ [∠s opp equal sides] $\therefore \hat{S}_3 = \hat{R}_3$ But $\hat{S}_4 = \hat{R}_3$ [∠s in the same seg] $\therefore \hat{S}_3 = \hat{S}_4$	✓ S ✓ R ✓ S / R ✓ S ✓ R (5)
10.2	$\hat{R}_1 + \hat{R}_2 = P\hat{Q}R$ [tan chord theorem] $\hat{S}_4 = P\hat{Q}R$ [proved in 10.1] $\therefore \hat{S}_4 = \hat{R}_1 + \hat{R}_2$ SMRC is a cyclic quad [converse ext ∠ of cyclic quad]	✓ S ✓ R ✓ S ✓ R (4)
10.3	$\hat{S}_3 = \hat{R}_2 + \hat{P}_2$ [ext ∠ of Δ] $\hat{S}_4 = \hat{P}_1 + \hat{A}_2$ [ext ∠ of Δ] $\therefore \hat{R}_2 + \hat{P}_2 = \hat{A}_2 + \hat{P}_1$ But $\hat{P}_1 = \hat{R}_2$ [tan chord theorem] $\therefore \hat{P}_2 = \hat{A}_2$ RP is a tangent to the circle [converse tan chord theorem] **OR** [∠ between line and chord] **OR** [converse alt seg theorem] **OR**	✓ S ✓ R ✓ S ✓ S ✓ R ✓ R (6)

	In ΔMSP and ΔMPA $\hat{M}_2$ is common $AR = AP$ [tans from same point] $\hat{R}_1 + \hat{R}_2 = \hat{P}_1 + \hat{P}_2$ [$\angle$s opp equal sides] $\hat{S}_4 = \hat{R}_1 + \hat{R}_2$ [proved in 10.2] $\therefore \hat{S}_4 = \hat{P}_1 + \hat{P}_2$ $\therefore \hat{P}_2 = \hat{A}_2$ [sum of $\angle$s in Δ] RP is a tangent to the circle [converse tan chord theorem]	✓ S ✓ S / R ✓ S ✓ S ✓ S ✓ R (6)
		[15]

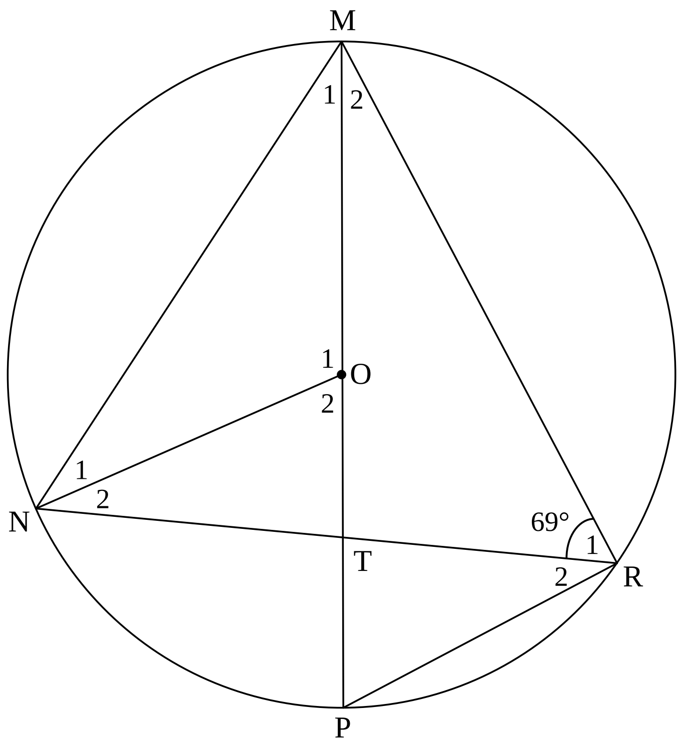
TOTAL/TOTAAL: 150

9. A diameter cutting a chord, and the angles around it

Official DBE marking guideline · May/June 2021 · Q8 · 16 marks

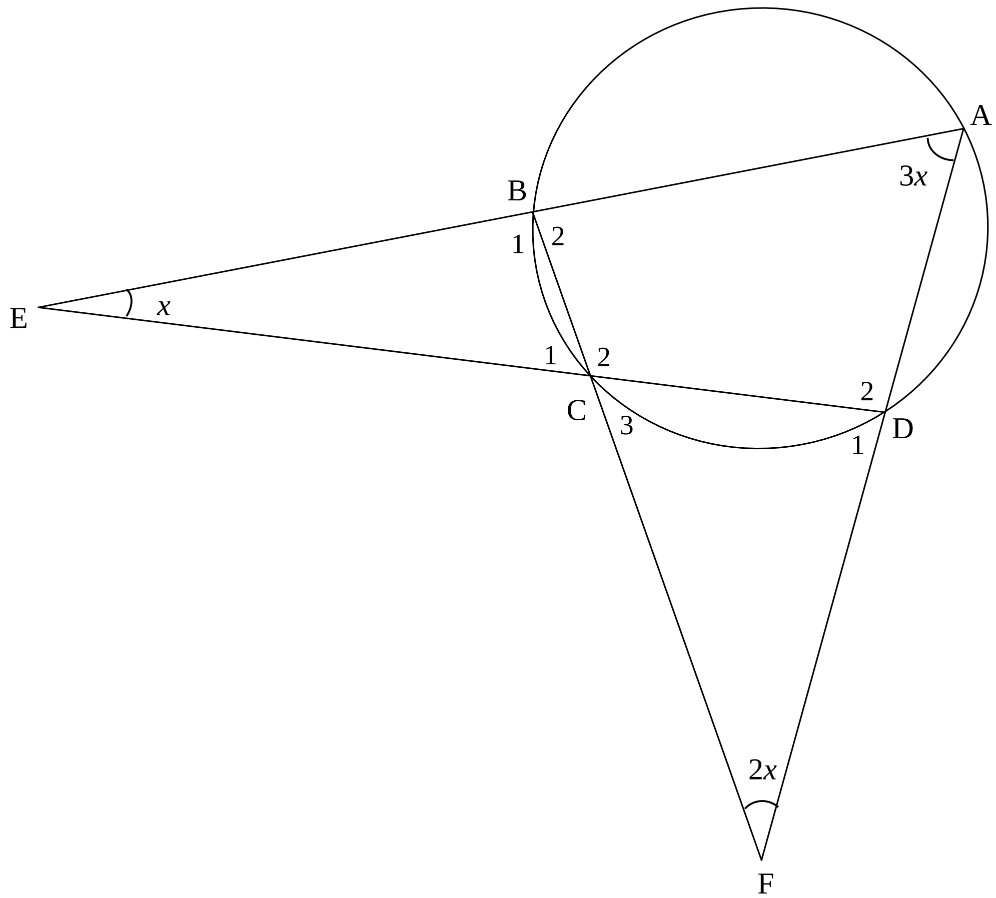
QUESTION/VRAAG 8

8.1



8.1.1	$\text{M}\hat{\text{R}}\text{P} = 90°$ [$\angle$ in semi circle/$\angle$ *in halwe sirkel*] $\hat{R}_2 = 21°$	✓ R ✓ S (2)
8.1.2	$\hat{O}_1 = 138°$ [$\angle$ at centre = 2 ×$\angle$ at circumference/ *midpts.* $\angle$ = 2× *omtreks* $\angle$]	✓ S ✓R (2)
8.1.3	$\hat{M}_1 = 21°$ [$\angle$s in the same segment/$\angle$*e in dieselfde sirkel segment*] **OR** $\hat{M}_1 + N_1 = 180° - 138°$ [sum of $\angle$s in Δ/$\angle$*e v* Δ] $\therefore \hat{M}_1 = 21°$ [$\angle$s opp equal sides/$\angle$*e teenoor gelyke sye*]	✓ S ✓R (2) ✓ S ✓R (2)
8.1.4	$\hat{O}_2 = 42°$ [$\angle$s on a str line/$\angle$*e op 'n reguitlyn*] $\hat{P} = 42°$ [alt $\angle$s; NO || PR/*Verw.* $\angle$*e*, NO || PR] $\hat{M}_2 = 48°$ [sum of $\angle$s in Δ/$\angle$*e v* Δ] **OR** $\hat{N}_2 = \hat{R}_2 = 21°$ [alt $\angle$s; NO || PR/*Verw.* $\angle$*e*, NO || PR] $\hat{N}_1 = \hat{M}_1 = 21°$ [$\angle$s opposite equal sides/$\angle$*e teenoor gelyke sye*] $\hat{M}_2 = 48°$ [sum of $\angle$s of Δ NMR//$\angle$*e v* ΔNMR]	✓ S ✓ S ✓R ✓ S (4) ✓ S ✓R ✓ S ✓ S (4)

8.2



8.2	$\hat{D}_1 = 4x$	[ext $\angle$ of Δ/*buite* $\angle$ *v* Δ]	✓ S/R
	$\hat{D}_2 = 180° - 4x$	[$\angle$s on a str line/$\angle$*e op ´n reguitlyn*]	✓ S
	$\hat{B}_1 = 5x$	[ext $\angle$ of Δ/*buite* $\angle$ *v* Δ]	✓ S
	$\hat{B}_1 = \hat{D}_2$	[ext $\angle$ of cyclic quad/*buite* $\angle$ *v kvh*]	✓S ✓R
	$180° - 4x = 5x$ $9x = 180°$ $x = 20°$		✓ answer (6)
	OR		
	$\hat{C}_1 = 3x$	[ext $\angle$ of cyclic quad/*buite* $\angle$ *v kvh*]	✓ S ✓ R
	$\hat{B}_2 = 4x$	[ext $\angle$ of Δ/*buite* $\angle$ *v* Δ]	✓ S
	$\hat{C}_1 = \hat{C}_3 = 3x$	[vert opp $\angle$s]	✓ S
	$\hat{D}_2 = 5x$	[ext $\angle$ of Δ/*buite* $\angle$ *v* Δ]	
	$4x + 5x = 180°$	[opp $\angle$ of cyclic quad/*teenoorst.* $\angle$*e v kvh*]	✓S/R
	$x = 20°$		✓ answer (6)

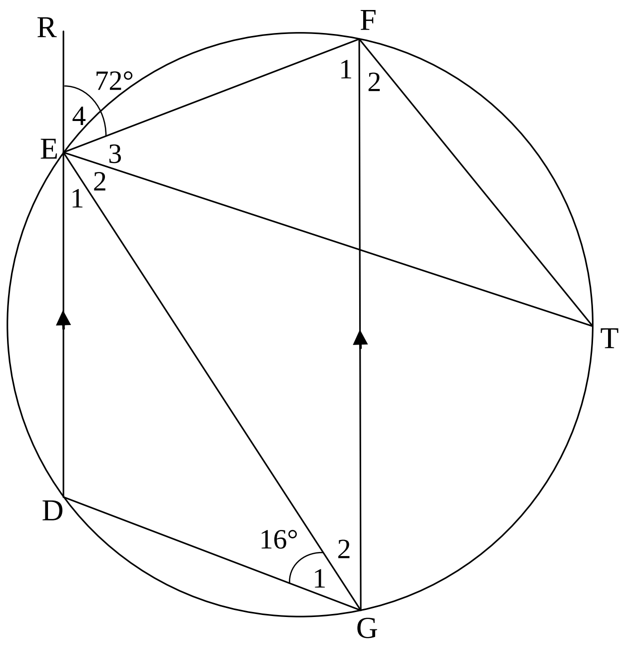
	OR		
	$\hat{C}_3 = 3x$	[ext $\angle$ of cyclic quad/*buite* $\angle$ *v kvh*]	
	$\hat{D}_1 = 4x$	[ext $\angle$ of Δ/*buite* $\angle$ *v* Δ]	✓ S ✓R
	$2x + 3x + 4x = 180°$	[sum of $\angle$s in Δ/$\angle$*e v* Δ]	✓ S
	$9x = 180°$		✓ S ✓R
	$x = 20°$		✓ answer (6)
			[16]

10. A cyclic quadrilateral with one pair of sides parallel

Official DBE marking guideline · May/June 2022 · Q9 · 16 marks

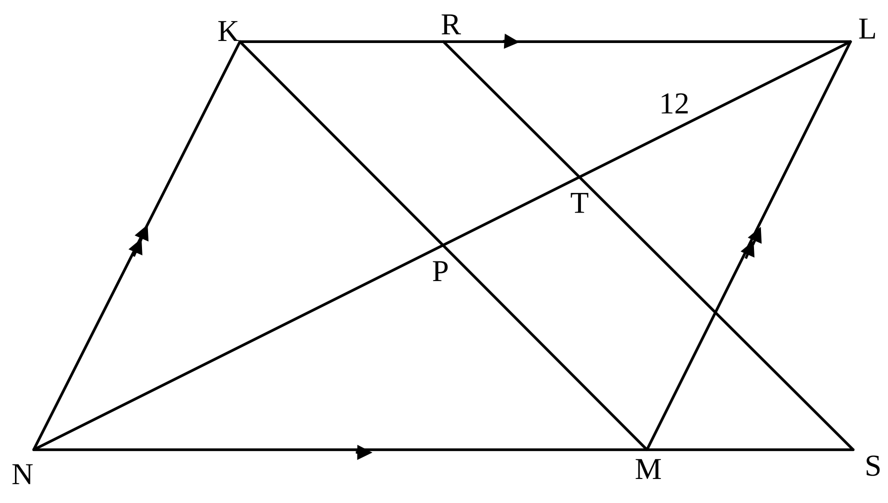
QUESTION/*VRAAG* 9

9.1



9.1.1	$D\hat{G}F = \hat{E}_4 = 72°$ [ext ∠ of cyclic quad/ *buite ∠ v kvh*]	✓ S ✓ R (2)
9.1.2	$\hat{G}_2 = 72° - 16° = 56°$ $\hat{T} = \hat{G}_2 = 56°$ [∠s in the same seg/ ∠*e in dies.* ⊙ *segment*]	✓ S ✓ S / R (2)
9.1.3	$\hat{F}_1 = \hat{E}_4 = 72°$ [alt ∠s; DE ‖ GF / *verw.* ∠*e*; DE ‖ GF] $\therefore G\hat{E}F = 52°$ [sum of ∠s in Δ / ∠*e van* Δ] **OR/*OF*** $\hat{E}_1 = 56°$ [alt ∠s; DE ‖ GF / *verw.* ∠*e*; DE ‖ GF] $\therefore G\hat{E}F = 52°$ [∠s on a str. line/ ∠*e op 'n reguitlyn*]	✓ S / R ✓ S (2) ✓ S / R ✓ S (2)

9.2



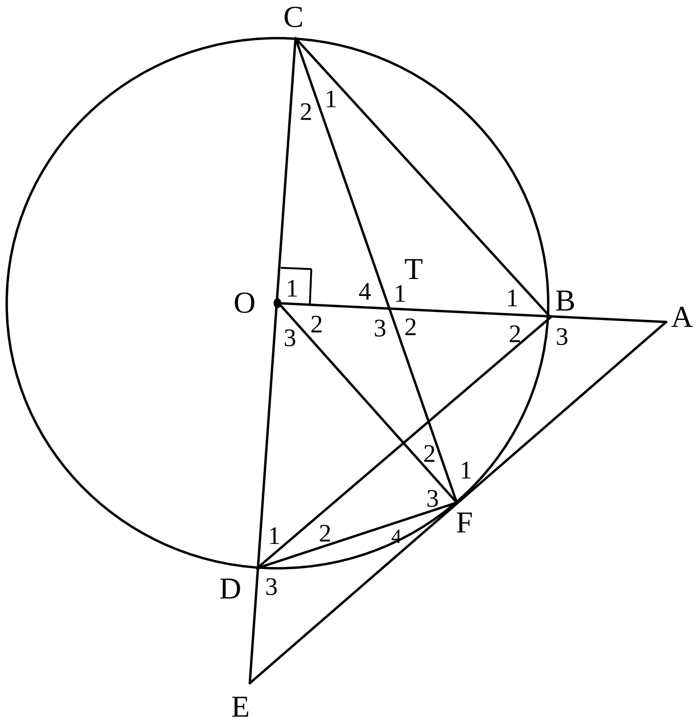
9.2.1	NP = PL = 16 [diag of ||m / *hoeklyne van ||m*] PT = 4 NP : PT = 16 : 4 = 4 : 1	✓ S ✓ R ✓ S ✓ answer (4)
9.2.2	NM : MS = 4 : 1 NP : PT = NM : MS KM || RS [line divides two sides of Δ in prop / *Lyn verdeel 2 sye v Δ eweredig*] **OR/*OF*** [converse prop theorem / *omgekeerde lyn || een sy v Δ*]	✓ S ✓ R (2)
9.2.3	$\frac{RL}{KL}=\frac{TL}{LP}$ [prop theorem; KM || RS **OR** line || one side of Δ / *Lyn || een sy v Δ*] $RL=\frac{12\times 21}{16}$ = 15,75	✓ S ✓ R ✓ S ✓ answer (4)

	OR / *OF* NM : MS = 4 : 1 KR = MS = 5,25 [opp side of $\|\|^m$ / *teenoorst. sye van* $\|\|^m$] KL = NM = 21 RL + 5,25 = 21 RL = 15,75	✓ S ✓ R ✓ S ✓ answer (4)
		[16]

11. A diameter produced until it meets a tangent

Official DBE marking guideline · May/June 2024 · Q10 · 19 marks

QUESTION/*VRAAG* 10



10.1	$\hat{O}_1 = 90°$ $\hat{F}_2 + \hat{F}_3 = 90°$ $\hat{O}_1 = \hat{F}_2 + \hat{F}_3 = 90°$ $\therefore$ TODF is a cyclic quad	[given/*gegee*] [∠ in semi circle/∠ *in halwe sirkel*] [ext ∠ = int opp ∠/ *buite ∠ = teenoorst. binne ∠*] **OR** [converse ext ∠ of cyclic quad/ *omgekeerde buite ∠ v kvh*]	✓ S ✓ R ✓ S ✓ R (4)
10.2	$\hat{T}_1 = \hat{T}_3$ But $\hat{D}_3 = \hat{T}_3$ $\therefore \hat{T}_1 = \hat{D}_3$	[vert opp ∠s =/ *regoorstaande ∠e*] [ext ∠ of cyclic quad/ *buite ∠ v kvh*]	✓ S / R ✓ S ✓ R (3)
10.3	In ΔDFE and ΔTFO 1) $\hat{D}_3 = \hat{T}_3$ 2) $\hat{F}_4 = \hat{C}_2$ but $\hat{C}_2 = \hat{F}_2$ $\therefore \hat{F}_4 = \hat{F}_2$ 3) $\hat{E} = \hat{O}_2$ ΔTFO ||| ΔDFE	[ext ∠ of cyclic quad/ *buite ∠ v kvh*] [tan-chord theorem/ *∠ tussen rkl en koord*] [∠s opp equal sides/ *∠e teenoor gelyke sye*] [3^{rd} ∠ of Δ/ *∠e van Δ*] [∠∠∠]	✓ S ✓ S / R ✓ S ✓ S ✓ S **OR** R (5)

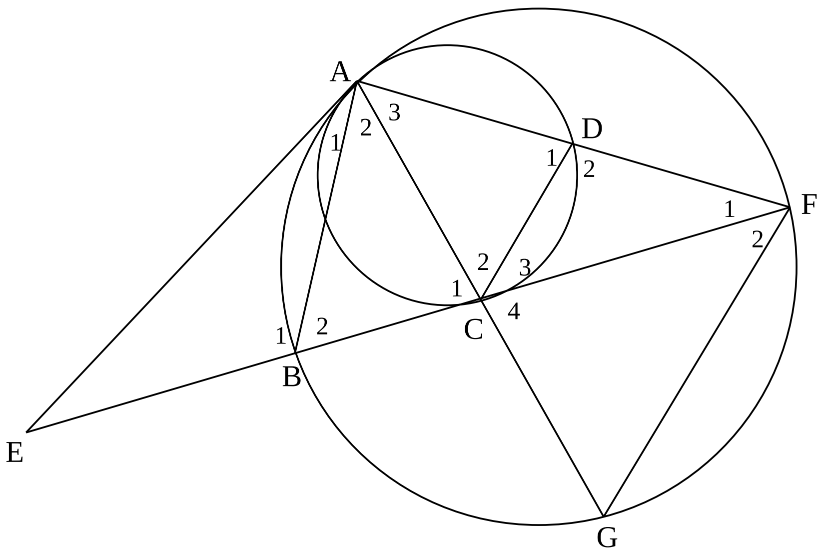
	OR In ΔDFE and ΔTFO 1) $\hat{D}_3 = \hat{T}_3$ [ext $\angle$ of cyclic quad/*buite $\angle$ van Δ*] 2) $\hat{F}_4 = \hat{C}_2$ [tan-chord theorem/*$\angle$ tussen rkl en koord*] $\hat{F}_2 + \hat{F}_3 = 90°$ [$\angle$ in semi circle/*$\angle$ in halwe sirkel*] $\hat{D}_1 + \hat{D}_2 = 90° - \hat{C}_2$ [sum of $\angle$s in Δ/ *binne $\angle$e van Δ*] $\hat{E} = 90° - 2\hat{F}_4$ [ext $\angle$ of Δ/ *buite $\angle$van Δ*] $\hat{O}_3 = 2\hat{C}_2$ [$\angle$ at centre = 2 $\times\angle$ at circumference/ *midpt. $\angle$s = 2$\times$omtreks$\angle$*] $\hat{O}_2 = 90° - 2\hat{F}_4$ [$\angle$s on a str line/*$\angle$e op 'n reguitlyn*] $\hat{O}_2 = \hat{E}$ 3) $\therefore \hat{F}_4 = \hat{F}_2$ [3rd $\angle$ of Δ/*$\angle$e van Δ*] ΔTFO ||| ΔDFE [$\angle\angle\angle$]	✓ S ✓ S / R ✓ S ✓ S ✓ S **OR** R (5)
10.4	$\hat{B}_2 = \hat{D}_1$ [$\angle$s opp equal sides/*$\angle$e teenoor gelyke sye*] $\hat{B}_2 = \hat{E}$ [given/*gegee*] $\therefore \hat{D}_1 = \hat{E}$ $\therefore$DB || EA [corresp $\angle$s =/*ooreenkomstige $\angle$e gelyk*]	✓ S / R ✓ R (2)
10.5	In ΔOEA DB || EA [proven/*reeds bewys*] $\frac{OD}{DE} = \frac{OB}{BA}$ [line || one side of Δ/*lyn || een sy van Δ*] **OR** [prop theorem; DB || EA/ *eweredigheid stelling; DB || EA*] $\therefore DE = \frac{DO.AB}{OB}$ $\frac{FO}{FE} = \frac{TO}{DE}$ [ΔTFO ||| ΔDFE] $DE = \frac{TO.FE}{FO}$ $\therefore \frac{DO.AB}{OB} = \frac{TO.FE}{FO}$ $\therefore \frac{DO.AB}{DO} = \frac{TO.FE}{DO}$ [DO = OB = FO] $\therefore DO = \frac{TO.FE}{AB}$	 ✓ R ✓ S ✓ S / R ✓ S ✓ S (5)
		[19]

TOTAL/*TOTAAL*: 150

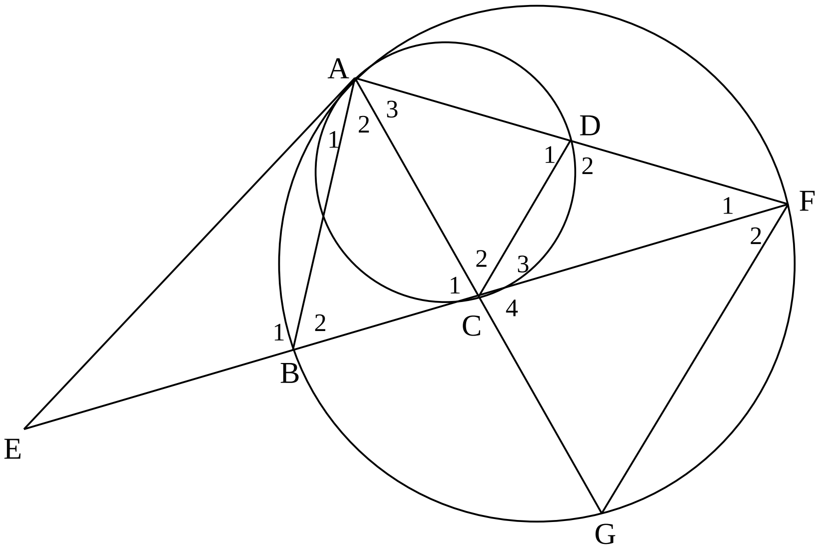
12. Two circles touching internally, and a tangent common to both

Official DBE marking guideline · November 2024 · Q11 · 20 marks

QUESTION/*VRAAG* 11



11.1	$\hat{D}_1 = E\hat{A}G = x$	[tan-chord theorem/ $\angle$ *tussen raaklyn en koord*]	✓ S ✓ R
	$\hat{C}_1 = \hat{D}_1 = x$	[tan-chord theorem/ $\angle$ *tussen raaklyn en koord*]	✓ S ✓ R
	$\hat{C}_4 = \hat{C}_1 = x$	[vert opp $\angle$s = / *regoorst.* $\angle$*e*]	✓ S/R
	$A\hat{F}G = E\hat{A}G = x$	[tan-chord theorem/ $\angle$ *tussen raaklyn en koord*]	✓ S (6)
	OR/*OF*		
	$EA = EC$	[tans from common pt/ *raaklyne vanuit dies. punt*]	✓ S/R
	$\hat{C}_1 = E\hat{A}G = x$	[$\angle$s opp equal sides/ $\angle$*e teenoor gelyke sye*]	✓ S
	$\hat{C}_4 = \hat{C}_1 = x$	[vert opp $\angle$s = / *regoorst.* $\angle$*e*]	✓ S/R
	$\hat{D}_1 = E\hat{A}G = x$	[tan-chord theorem/ $\angle$ *tussen raaklyn en koord*]	✓ S ✓ R
	$A\hat{F}G = E\hat{A}G = x$	[tan-chord theorem $\angle$ *tussen raaklyn en koord*]	✓ S (6)

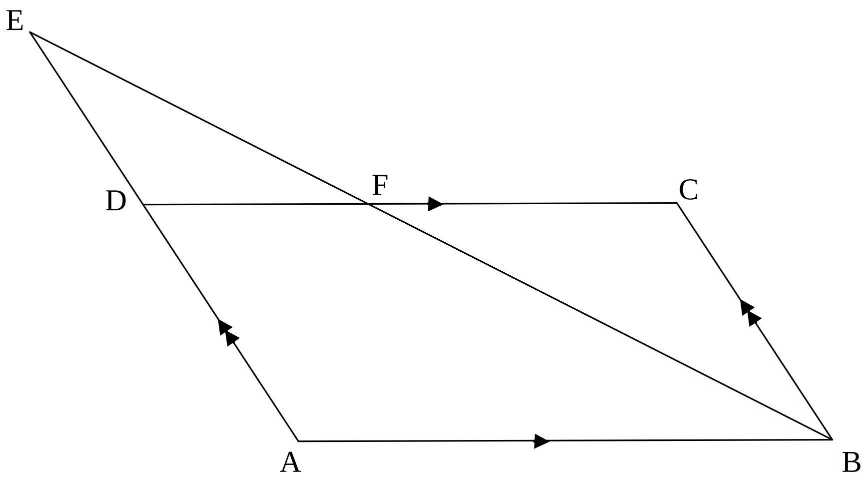


11.2	$\hat{D}_1 = A\hat{F}G = x$ $\therefore$ DC || FG [corresp ∠s = / *ooreenk ∠e* =] $\frac{AG}{AC} = \frac{AF}{AD}$ [prop theorem; DC || FG / *lyn || een sy van* Δ] $\therefore$ AG.AD = AC.AF **OR/OF** In ΔACD and ΔAGF $\hat{A}_3$ is common $A\hat{F}G = \hat{D}_1 = x$ [proved in 11.1 / *reeds bewys*] $\hat{C}_2 = A\hat{G}F = x$ [sum ∠ Δs/*binne ∠e Δ*] ΔACD ||| ΔAGF [∠∠∠] $\frac{AC}{AG} = \frac{AD}{AF}$ [||| Δs $\therefore$ sides in proportion / ||| Δ*e* $\therefore$ *sye in dieselfde verhouding*] $\therefore$ AG.AD = AC.AF	✓ S ✓ S/R ✓ S ✓ R (4) ✓ S ✓ S ✓ S/R ✓ S (4)
11.3	In Δ AGF and ΔABC $\hat{G} = \hat{B}_2$ [∠s in the same seg / *∠e in dies. segment*] $A\hat{F}G = \hat{C}_1 = x$ [proved in 11.1 / *reeds bewys*] $\hat{A}_3 = \hat{A}_2$ [sum of ∠s in Δ/*binne ∠e van* Δ] Δ AGF ||| ΔABC [∠∠∠]	✓ S ✓ R ✓ S ✓ S **OR/*OF*** R (4)

11.4	$\frac{GF}{BC}=\frac{AF}{AC}$ [$\Delta AGF \mid\mid\mid \Delta ABC$]	✓ S / R
	$\therefore GF=\frac{BC.AF}{AC}$	✓ S
	$\Delta\, ACD \mid\mid\mid \Delta FGC$ [$\angle\angle\angle$]	✓ S
	$\therefore \frac{AC}{GF}=\frac{AD}{FC}$	✓ S
	$\therefore AC=\frac{AD.FG}{FC}$	
	$\therefore GF=BC.AF\div\frac{AD.FG}{FC}$	✓ S
	$GF=BC.AF\times\frac{FC}{AD.FG}$	✓ S
	$\therefore GF^2=\frac{BC.FC.AF}{AD}$	
		(6)
	OR/*OF*	**OR/*OF***
	$\Delta\, AGF \mid\mid\mid \Delta ABC$ [$\angle\angle\angle$]	
	$\frac{GF}{BC}=\frac{AF}{AC}$	✓ S
	$GF=\frac{AF.BC}{AC}$	✓ S
	$\Delta\, ACD \mid\mid\mid \Delta AGF$ [$\angle\angle\angle$]	
	$\frac{AD}{AF}=\frac{CD}{GF}$	
	$GF=\frac{AF.CD}{AD}$	✓ S
	$GF\times GF=\frac{AF.BC}{AC}.\frac{AF.CD}{AD}$	✓ S
	$\Delta FCD \mid\mid\mid \Delta FAC$ [$\angle\angle\angle$]	✓ S
	$\frac{FC}{FA}=\frac{CD}{AC}$ from $\mid\mid\mid$ Δ\`s	
	$FC=\frac{CD.AF}{AC}$	✓ S
	$GF^2=\frac{AF.FC.BC}{AD}$	(6)
		[20]

TOTAL/*TOTAAL*: 150

QUESTION/VRAAG 9



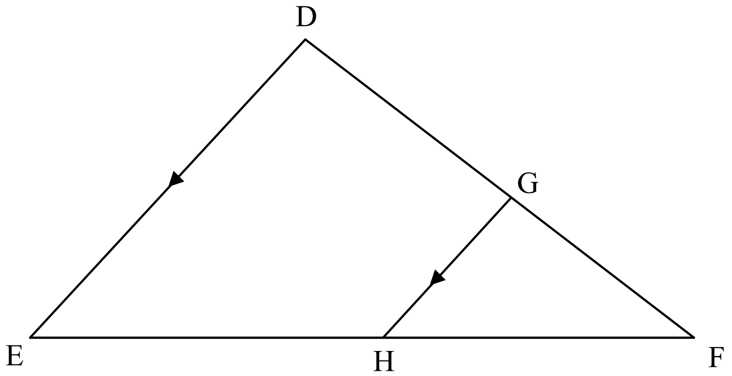
9.1	$\frac{FB}{EB}=\frac{DA}{EA}$ [prop theorem; DC ‖ AB] **OR** [line ‖ one side of Δ] $FB=\frac{4p\times 21}{7p}$ FB = 12 units	✓ S ✓ R ✓ answer (3)
9.2	In ΔEDF and ΔEAB: $\hat{E}$ is common $E\hat{D}F=\hat{A}$ [corresp ∠s; EA ‖ CB] $E\hat{F}D=E\hat{B}A$ [corresp ∠s; DC ‖ AB] ΔEDF ‖‖‖ ΔEAB [∠;∠;∠]	✓ S ✓ S/R ✓ S **OR** R (3)
9.3	$\frac{DF}{AB}=\frac{ED}{EA}$ [‖‖‖ Δs] DF = $\frac{3p\times 14}{7p}$ DF = 6 units FC = 8 units [DC = AB = 14 units; opp sides of ‖^{m}] **OR** ΔEDF ‖‖‖ ΔBCF [∠;∠;∠] $\frac{ED}{BC}=\frac{DF}{CF}$ [‖‖‖ Δs] $\frac{3}{4}=\frac{14-FC}{FC}$ [BC = AD; opp sides of ‖^{m}] $3FC=56-4FC$ $FC=8$	✓ S ✓ DF = 6 ✓ FC = 14 − DF (3) ✓ ΔEDF ‖‖‖ ΔBCF ✓ $\frac{3}{4}=\frac{14-FC}{FC}$ ✓ answer (3)
		[9]

14. A line drawn across a triangle, and the ratios it creates

Official DBE marking guideline · May/June 2025 · Q9 · 20 marks

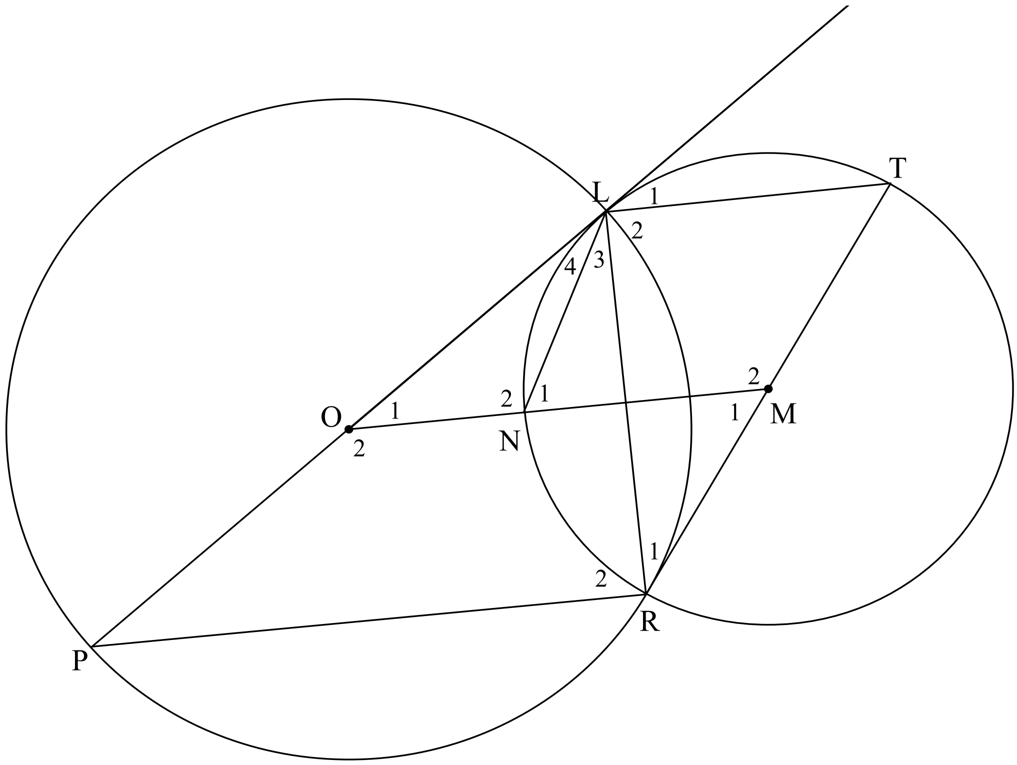
QUESTION/*VRAAG* 9

9.1



9.1.1	$\frac{HF}{EH}=\frac{GF}{GD}=\frac{2}{5}$ [line || to one side of Δ/*lyn || een sy van* Δ] **OR** [prop theorem; GH||DE/*eweredigheidst.; GH || DE*]	✓ S ✓ R (2)
9.1.2	$\frac{EH}{EF}=\frac{DG}{DF}=\frac{5}{7}$ [line || to one side of Δ/*lyn || een sy van* Δ] **OR** [prop theorem; GH||DE/*eweredigheidst.; GH || DE*] $\frac{EH}{21}=\frac{5}{7}$ $EH = 15cm$ **OR/*OF*** $\frac{HF}{EF}=\frac{2}{7}$ [line || to one side of Δ/*lyn || een sy van* Δ] **OR** [prop theorem; GH||DE/*eweredigheidst.; GH || DE*] $\frac{HF}{21}=\frac{2}{7}$ $HF = 6cm$ $EH = 21-6$ $EH = 15cm$	✓ S ✓ answer (2) ✓ S ✓ answer (2)
9.1.3	ΔFGH ||| **ΔFDE** [∠∠∠]	✓ S (1)
9.1.4	$\frac{GH}{DE}=\frac{FH}{FE}$ [|||Δ's] **OR/*OF*** $\frac{GH}{DE}=\frac{FG}{FD}$ [|||Δ's] $\frac{GH}{DE}=\frac{2}{7}$	✓ S ✓ S (2)

9.2



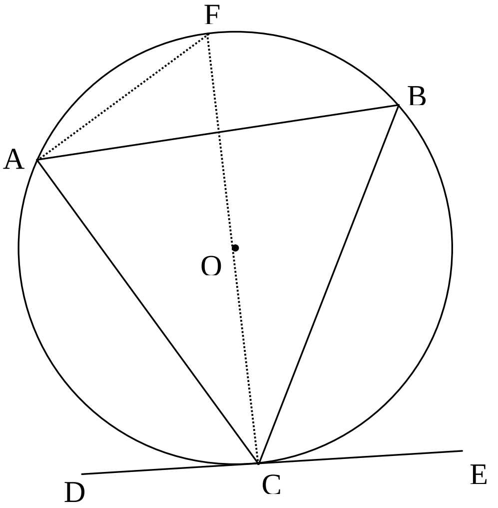
9.2.1	$\hat{L}_2 = 90°$ [$\angle$ in semi-circle/ *$\angle$ in halwe sirkel*] **OR** $\hat{L}_1 = \hat{R}_1$ [tan chord theorem/ *raaklyn-koordst.*] $\hat{R}_2 = 90°$ [$\angle$ in semi-circle] *$\angle$ in halwe sirkel*] $\hat{R}_1 = \hat{P}$ [tan chord theorem] *raaklyn-koordst.*] $\therefore\ \hat{L}_2 = \hat{R}_2$ $\therefore \hat{L}_1 = \hat{P}$ $\therefore$ LT || PR [alt $\angle$s =/*verw.* $\angle e$ =] $\therefore$ LT || PR [corresp. $\angle$s =/ *ooreenk.* $\angle e$ =]	✓ S ✓ R ✓ S/R ✓ R (4)
9.2.2	$\hat{L}_1 = \hat{R}_1$ [tan chord theorem/*raaklyn-koordst.*] $\hat{L}_1 = \hat{O}_1$ [corresp. $\angle$s; LT || OM/*ooreenk.* $\angle e$*; LT ||OM*] $\therefore \hat{R}_1 = \hat{O}_1$ $\therefore$ L, O, R and M are concyclic. $\therefore$ LORM is a cyclic quadrilateral [converse $\angle$s in the same seg/ *omgekeerde* $\angle e$ *in dies. sirkel segm*]	✓ S ✓ R ✓ S/R ✓ S ✓ R (5)
9.2.3	$O\hat{L}R = \hat{M}_1$ [$\angle$s in the same seg/$\angle e$ *in dieselfde segment*] $2\hat{L}_3 = \hat{M}_1$ [$\angle$ at centre = 2 ×$\angle$ at circumference/ *midpt.* $\angle$ = 2× *omtreks* $\angle$] $\therefore O\hat{L}R = 2\hat{L}_3$ $\therefore \hat{L}_4 = \hat{L}_3$ $\therefore$ LN bisects $O\hat{L}R$	✓ S/R ✓ S ✓ R ✓ S (4)
		[20]

15. The tangent-chord theorem, proved and then used

Official DBE marking guideline · May/June 2024 · Q8 · 11 marks

QUESTION/*VRAAG* 8

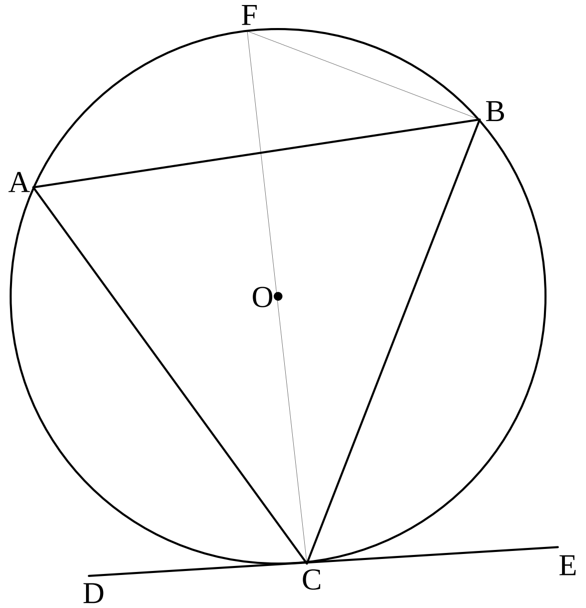
8.1



	Construction: Draw diameter CF and draw AF *Konstruksie: Trek middellyn* CF *en verbind* AF $F\hat{C}E = 90°$ [tan ⊥ radius/*raaklyn ⊥ radius*] $F\hat{A}C = 90°$ [∠ in semi circle/∠ *in halwe sirkel*] $F\hat{A}B = F\hat{C}B$ [∠s same segment/∠*e dieselfde segm*] $\therefore B\hat{A}C = B\hat{C}E$ $\therefore B\hat{C}E = \hat{A}$	✓Constr ✓ S ✓ R ✓ S/R ✓ S/R (5)

OR

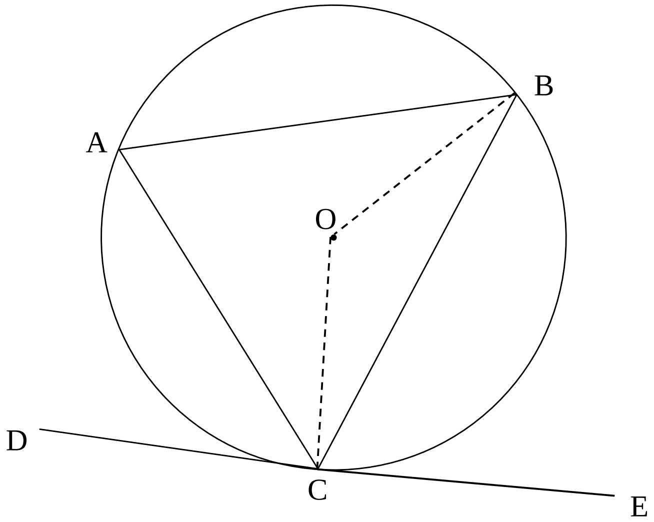
8.1



	Construction: Draw diameter CF and draw FB *Konstruksie: Trek middellyn* CF *en verbind* FB $\hat{FBC} = 90°$ [∠ in semi circle/∠ *in halwe sirkel*] $\hat{BFC} + \hat{FCB} = 90°$ [sum of ∠s in Δ/*binne ∠e v Δ*] $\hat{OCE} = 90°$ [tan ⊥ radius/ *raaklyn ⊥ radius*] $\therefore \hat{BCE} = \hat{F}$ but $\hat{A} = \hat{F}$ [∠s in same seg/∠ *in dies. segment*] $\therefore \hat{BCE} = \hat{A}$	✓ construction ✓ S / R ✓ S ✓ R ✓ S / R (5)

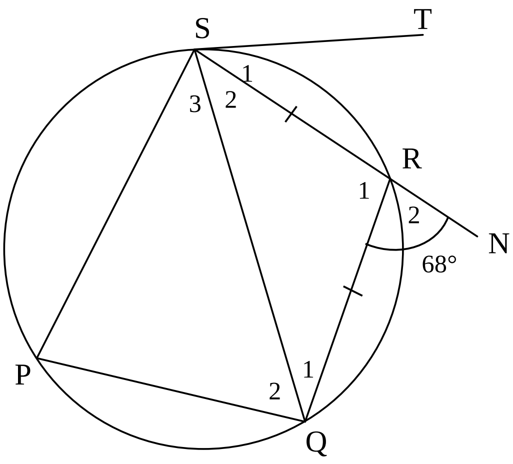
OR

8.1



	Construction: Draw radii BO and OC *Konstruksie: Trek radiusse BO en OC* $\hat{OCE} = 90°$ or $\hat{BCE} = 90° - \hat{OCB}$ [tan ⊥ radius / *raaklyn ⊥ radius*] $\hat{OCB} = \hat{OBC}$ [∠s opp equal sides/ *∠e teenoor gelyke sye*] $\therefore \hat{COB} = 180° - 2\hat{OCB}$ [∠s of Δ/*∠e van Δ*] $\hat{CAB} = 90° - \hat{OCB}$ [∠ at centre = 2×∠ circumf/ *midpts∠ = 2× omtreks∠*] $\therefore \hat{BCE} = \hat{CAB}$	✓ construction ✓ S ✓R ✓ S ✓ S/R (5)

8.2



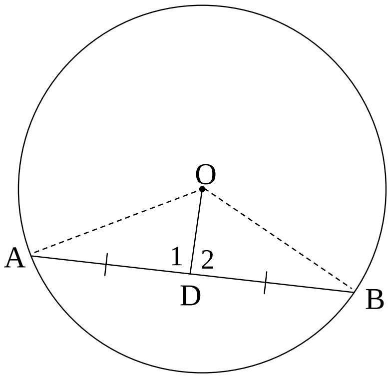
8.2.1	$\hat{P} = \hat{R}_2 = 68°$	[ext ∠ of cyclic quad /*buite ∠ van kvh*]	✓ S ✓ R (2)
8.2.2	$\hat{Q}_1 = \hat{S}_2$ $\hat{Q}_1 + \hat{S}_2 = 68°$ $\therefore \hat{Q}_1 = 34°$	[∠s opp equal sides /∠*e teenoor gelyke sye*] [ext ∠ of Δ/ *buite ∠ van* Δ]	✓ S ✓ S (2)
8.2.3	$\hat{S}_1 = \hat{Q}_1 = 34°$	[tan-chord theorem/∠ *tussen rkl en koord*]	✓ S ✓ R (2)
			[11]

16. The line from the centre that bisects a chord

Official DBE marking guideline · November 2022 · Q9 · 14 marks

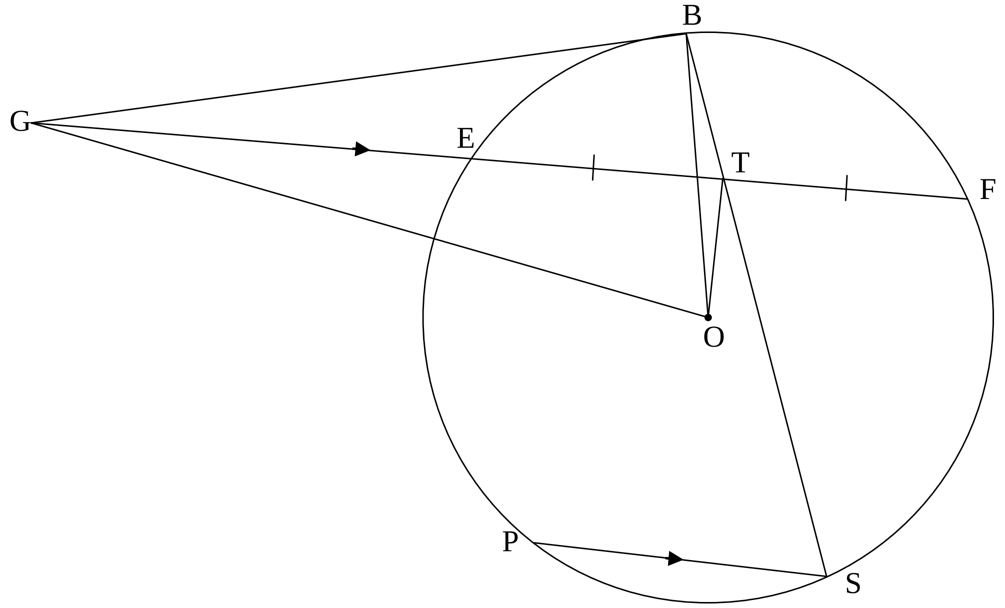
QUESTION/*VRAAG* 9

9.1



9.1.1	Construction: Draw OA and OB In ΔADO and ΔBDO OA = OB [radii/*radiusse*] OD = OD [common side/*gemeenskaplike sy*] AD = DB [given/*gegee*] $\therefore \Delta ADO \equiv \Delta BDO$ [S;S;S] ADB is a straight line $\therefore \hat{D}_1 = \hat{D}_2$ $\Delta ADO \equiv \Delta BDO$ $\therefore OD \perp AB$ [$\angle$s on a str line/$\angle$*e op 'n reguitlyn*]	✓ construction ✓ first pair of sides ✓ other 2 pairs ✓ R ✓ R (5)
	OR/*OF* Construction: Draw OA and OB In ΔADO and ΔBDO AD = DB [given/*gegee*] $\hat{A} = \hat{B}$ [$\angle$s opp; $\angle$s sides /$\angle$*e teenoor gelyke sye*] OA = OB [radii/*radiusse*] $\therefore \Delta ADO \equiv \Delta BDO$ [S;$\angle$;S] ADB is a straight line $\therefore \hat{D}_1 = \hat{D}_2$ $\Delta ADO \equiv \Delta BDO$ $\therefore OD \perp AB$ [$\angle$s on a str line/$\angle$*e op 'n reguitlyn*]	✓ construction ✓ first pair of sides ✓ other 2 pairs ✓ R ✓ R (5)

9.2



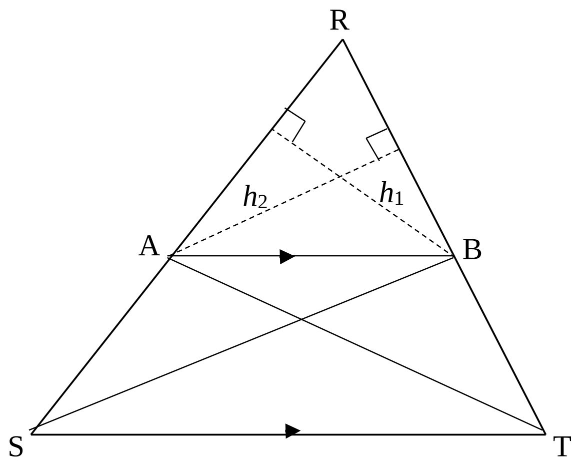
9.2.1	$\text{O}\hat{\text{T}}\text{G} = 90°$ [line from centre to midpt of chord/ *midpt. sirkel; midpt. koord*] $\text{O}\hat{\text{B}}\text{G} = 90°$ [tan ⊥ radius/*raaklyn* ⊥ *radius*] $\therefore \text{O}\hat{\text{T}}\text{G} = \text{O}\hat{\text{B}}\text{G} = 90°$ $\therefore$ OTBG is a cyclic quadrilateral [line subtends equal ∠s **OR** converse ∠s in the same segment/ *lyn onderspan gelyke ∠e*]	✓S ✓R ✓S ✓R ✓R (5)
9.2.2	$\hat{\text{S}} = \text{B}\hat{\text{T}}\text{G}$ [corresp ∠s; GF ∥ PS / *ooreenk.* ∠s; GF ∥ PS] But $\text{B}\hat{\text{T}}\text{G} = \text{G}\hat{\text{O}}\text{B}$ [∠s in the same segment/∠*e in dies. sirkelsegment*] $\text{G}\hat{\text{O}}\text{B} = \hat{\text{S}}$	✓S ✓R ✓S ✓R (4)
		[14]

17. A line parallel to one side of a triangle, proved

Official DBE marking guideline · November 2024 · Q10 · 15 marks

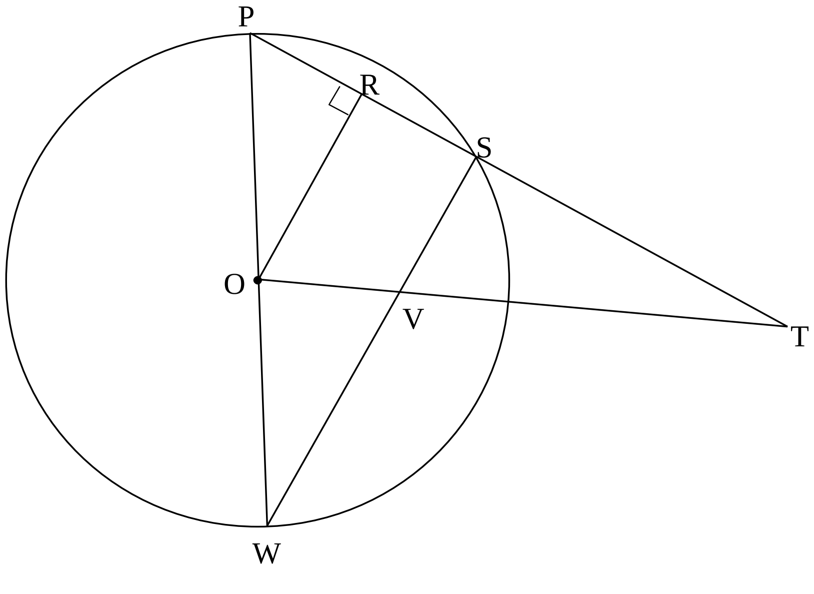
QUESTION/VRAAG 10

10.1



10.1	Construction: Join SB and TA and draw h_1 from B $\perp$ AR and h_2 from A $\perp$ RB *Konstruksie: Verbind SB en TA en trek* h_1 *vanaf B* $\perp$ *AR en* h_2 *vanaf A* $\perp$ *RB* Proof/*Bewys*: $\frac{\text{area } \Delta\text{RAB}}{\text{area } \Delta\text{ASB}} = \frac{\frac{1}{2}\text{RA} \times h_1}{\frac{1}{2}\text{AS} \times h_1} = \frac{\text{RA}}{\text{AS}}$ $\frac{\text{area } \Delta\text{RAB}}{\text{area } \Delta\text{ABT}} = \frac{\frac{1}{2}\text{RB} \times h_2}{\frac{1}{2}\text{BT} \times h_2} = \frac{\text{RB}}{\text{BT}}$ area ΔRAB = area ΔRAB [common/*gemeenskaplik*] But area ΔASB = area ΔABT [same base & height; AB || ST/ *dies. basis & hoogte; AB ||ST*] $\therefore \frac{\text{area } \Delta\text{RAB}}{\text{area } \Delta\text{ASB}} = \frac{\text{area } \Delta\text{RAB}}{\text{area } \Delta\text{ABT}}$ $\therefore \frac{\text{RA}}{\text{AS}} = \frac{\text{RB}}{\text{BT}}$	✓ construction ✓ $\frac{\text{area } \Delta\text{RAB}}{\text{area } \Delta\text{ASB}} = \frac{\frac{1}{2}\text{RA} \times h_1}{\frac{1}{2}\text{AS} \times h_1}$ ✓ $\frac{\text{RA}}{\text{AS}}$ ✓ $\frac{\text{area } \Delta\text{RAB}}{\text{area } \Delta\text{ABT}} = \frac{\text{RB}}{\text{BT}}$ ✓ S ✓ R (6)

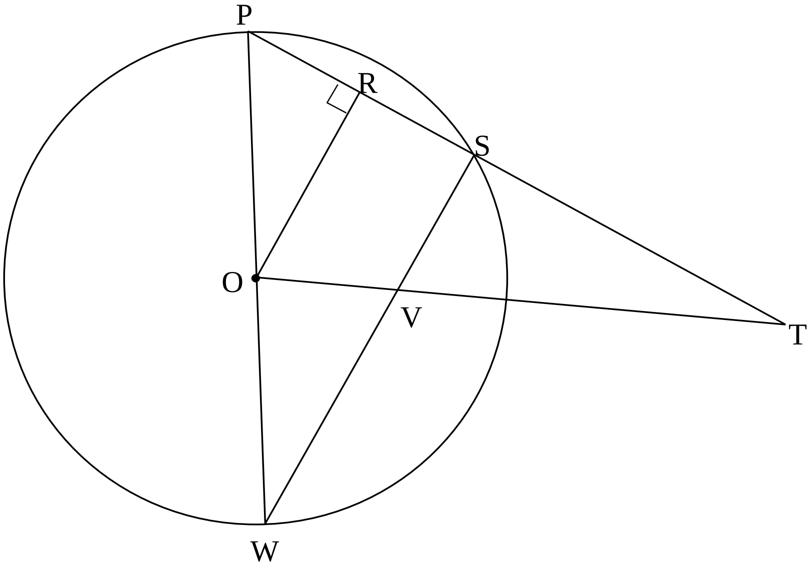
10.2



10.2.1	PR = RS PO = OW $\therefore$ OR = $\frac{1}{2}$WS $\therefore$ OR : WS = 1 : 2 **OR/*OF*** $P\hat{S}W = 90°$ $P\hat{R}O = 90°$ $\therefore P\hat{R}O = P\hat{S}W$ $\therefore$ RO ǁ SW $\frac{PO}{OW} = \frac{PR}{RS}$ PO = OW $\therefore$ PR = RS $\therefore$ OR : WS = 1 : 2	[line from centre ⊥ to chord/ *lyn vanuit midpt. sirkel ⊥ op koord*] [radii / *radiusse*] [midpt theorem/*midpt. stelling*] [∠ in semi circle/∠ *in halwe sirkel*] [given] [corresp ∠s = / *ooreenk. ∠e* =] **OR/*OF*** [co-int. ∠s suppl / *ko-binne ∠e suppl*] [prop theorem; RO ǁ SW/ *lyn* ǁ *een sy van* Δ] [radii / *radiusse*] [midpt theorem/ *midpt. stelling*]	✓ S ✓ R ✓ S ✓ S ✓ R (5) ✓ S ✓ S ✓ S ✓ R ✓ R (5)

	OR/*OF* ΔPRO and ΔPSW $P\hat{S}W = 90°$ [$\angle$ in semi circle/$\angle$ *in halwe sirkel*] $P\hat{R}O = 90°$ [given] $\therefore P\hat{R}O = P\hat{S}W$ $\hat{P}$ is common $P\hat{O}R = P\hat{W}S$ [sum of $\angle$s in Δ/ *som van* $\angle$*e in* Δ] $\therefore \Delta PRO \parallel\!\vert\, \Delta PSW$ [$\angle\angle\angle$] $\therefore \frac{PO}{PW} = \frac{RO}{SW}$ [$\parallel\!\vert\, \Delta$s / $\parallel\!\vert\, \Delta e$] but PW = 2 PO [diameter = 2 radius/*middellyn = 2 radius*] $\therefore \frac{RO}{SW} = \frac{PO}{2PO}$ $= \frac{1}{2}$ $\therefore$ OR : WS = 1 : 2	✓ S ✓ R ✓ S ✓ S ✓ S (5)
10.2.2	$\frac{OV}{VT} = \frac{RS}{ST} = \frac{1}{3}$ [prop theorem; RO ∥ SW/ *lyn* ∥ *een sy van* Δ] $\frac{RS}{15} = \frac{1}{3}$ RS = 5 units PR = RS = 5 units [line from centre ⊥ to chord / *lyn vanuit midpt. sirkel* ⊥ *op koord*] $\therefore$ PT = 25 units	✓ S /R ✓ S ✓ S ✓ answer (4)
		[15]

10.2

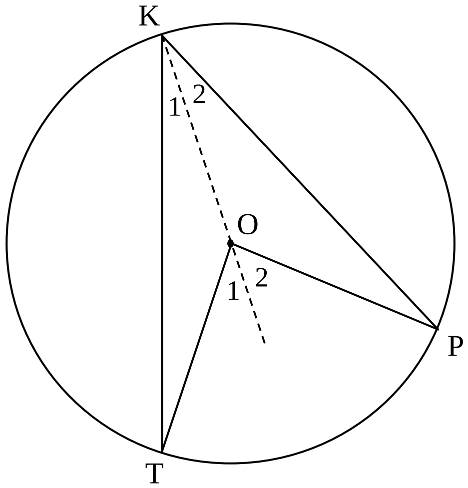


18. The angle at the centre is twice the angle at the circumference

Official DBE marking guideline · November 2023 · Q8 · 16 marks

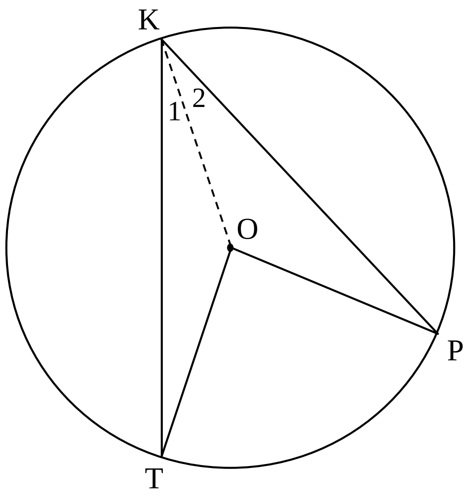
QUESTION/*VRAAG 8*

8.1



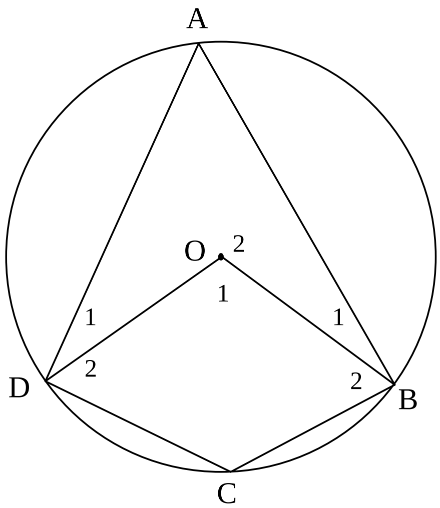
8.1	Construction: Draw KO produced $\hat{O}_1 = \hat{K}_1 + \hat{T}$ [ext $\angle$ of Δ] But $\hat{K}_1 = \hat{T}$ [$\angle$s opp equal sides] $\therefore \hat{O}_1 = 2\hat{K}_1$ $\hat{O}_2 = \hat{K}_2 + P$ [ext $\angle$ of Δ] But $\hat{K}_2 = P$ [$\angle$s opp equal sides] $\therefore \hat{O}_2 = 2\hat{K}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{K}_1 + 2\hat{K}_2$ $= 2\left(\hat{K}_1 + \hat{K}_2\right)$ $\therefore T\hat{O}P = 2T\hat{K}P$ **OR**	✓ construction ✓ S / R ✓ S ✓ S ✓ S (5)

8.1



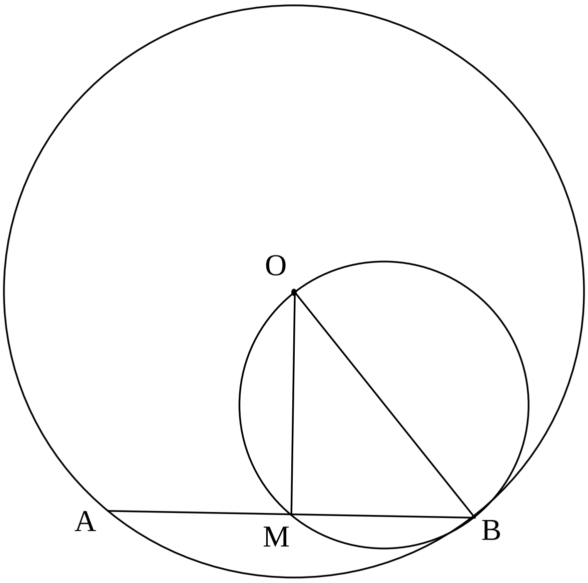
8.1	Construction: Draw KO		✓ construction
	$\hat{T} = \hat{K}_1$	[$\angle$s opp. equal sides]	✓ S / R
	$\therefore K\hat{O}T = 180° - 2\hat{K}_1$	[sum of $\angle$s of ΔKOT]	✓ S
	$\hat{P} = \hat{K}_2$	[$\angle$s opp. equal sides]	
	$\therefore K\hat{O}P = 180° - 2\hat{K}_2$	[sum of $\angle$s of ΔKOP]	✓ S
	$T\hat{O}P = 360° - \left(K\hat{O}T + K\hat{O}P\right)$	[$\angle$s around a point]	
	$= 360° - \left(180° - 2\hat{K}_1 + 180° - 2\hat{K}_2\right)$		✓ S
	$= 2\hat{K}_1 + 2\hat{K}_2$		
	$= 2\left(\hat{K}_1 + \hat{K}_2\right)$		
	$\therefore T\hat{O}P = 2T\hat{K}P$		
			(5)

8.2



8.2	$\hat{O}_1 = 4x + 100°$ [given] $\therefore \hat{A} = 2x + 50°$ [$\angle$ at centre = 2 ×$\angle$ at circumference] $x + 34° + 2x + 50° = 180°$ [opp $\angle$s of cyclic quad] $3x = 96°$ $x = 32°$ **OR** $\hat{O}_2 = 2x + 68°$ [$\angle$ at centre = 2 ×$\angle$ at circumference] $4x + 100° + 2x + 68° = 360°$ [$\angle$s round a pt] $6x = 192°$ $x = 32°$ **OR** $\hat{O}_2 = -4x + 260°$ [$\angle$s round a pt] $2\hat{C} = -4x + 260°$ [$\angle$ at centre = 2 ×$\angle$ at circumference] $\hat{C} = -2x + 130°$ $x + 34° = -2x + 130°$ $3x = 96°$ $x = 32°$	✓ S ✓ R ✓ S ✓ R ✓ answer (5) ✓ S ✓ R ✓ S ✓ R ✓ answer (5) ✓ S ✓ R ✓ S ✓ R ✓ answer (5)

8.3



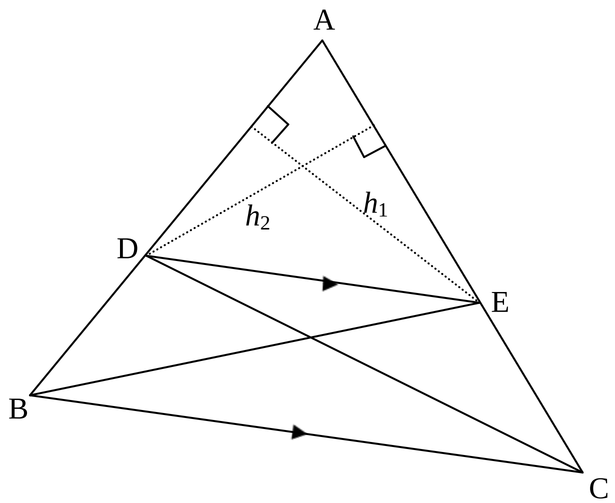
8.3.1	$O\hat{M}B = 90°$ [$\angle$ in semi circle]	✓ S ✓ R (2)
8.3.2	$AB = \sqrt{300} = 10\sqrt{3}$ $\therefore MB = 5\sqrt{3}$ [line from centre $\perp$ to chord] $OB^2 = OM^2 + MB^2$ [Pythagoras] $OB^2 = 5^2 + \left(5\sqrt{3}\right)^2$ OB = 10 units	 ✓ S ✓ R ✓ S ✓ answer (4)
		[16]

19. The proportion theorem, then the rider that follows it

Official DBE marking guideline · May/June 2021 · Q9 · 17 marks

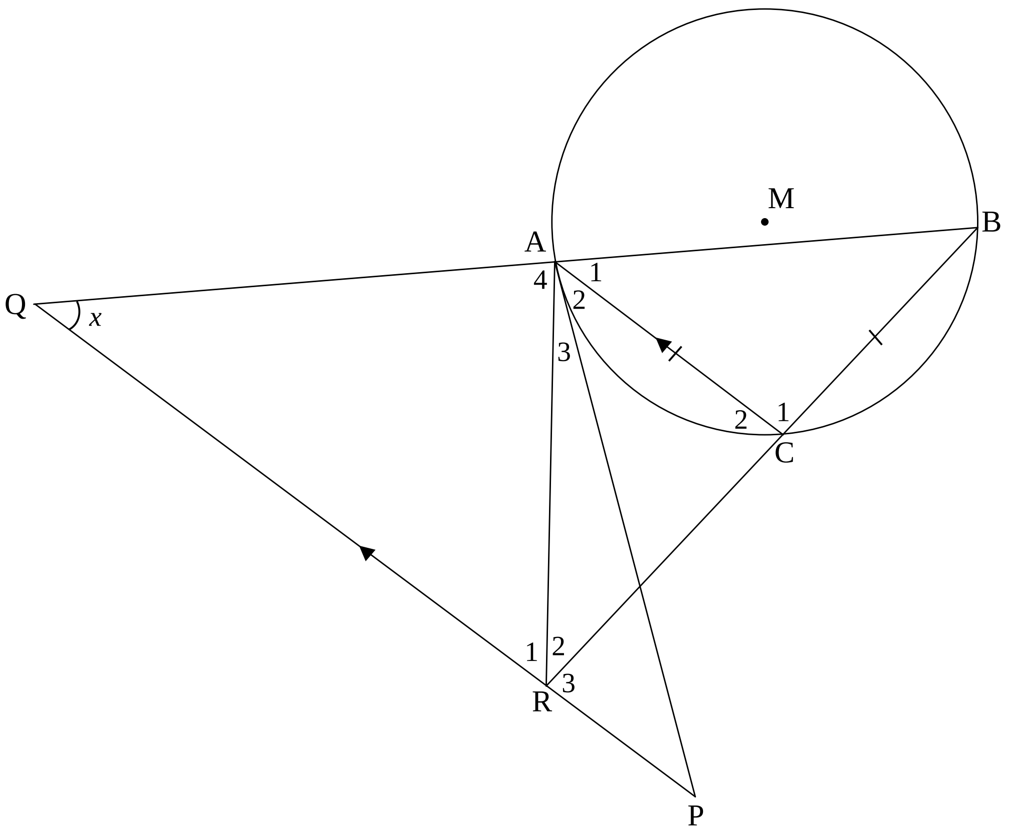
QUESTION/VRAAG 9

9.1



9.1	Constr: Join BE and CD and draw h_1 from E $\perp$ AD and h_2 from D $\perp$ AE *Konstr: Verbind BE en CD en trek* h_1 *vanaf* E $\perp$ AD *en* h_2 *vanaf* D $\perp$ AE Proof/*Bewys*: $\dfrac{\text{area } \Delta ADE}{\text{area } \Delta BDE} = \dfrac{\frac{1}{2} AD \times h_1}{\frac{1}{2} BD \times h_1} = \dfrac{AD}{BD}$ $\dfrac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \dfrac{\frac{1}{2} AE \times h_2}{\frac{1}{2} EC \times h_2} = \dfrac{AE}{EC}$ $\text{area } \Delta ADE = \text{area } \Delta ADE$ [common/*gemeenskaplik*] But $\text{area } \Delta BDE = \text{area } \Delta DEC$ [same base & height ; DE || BC/ *dies basis & hoogte ;* DE || BC] $\therefore \dfrac{\text{area } \Delta ADE}{\text{area } \Delta BDE} = \dfrac{\text{area } \Delta ADE}{\text{area } \Delta DEC}$ $\therefore \dfrac{AD}{BD} = \dfrac{AE}{EC}$	✓ constr/*konstr* ✓ $\dfrac{\text{area } \Delta ADE}{\text{area } \Delta BDE}$ ✓ $\dfrac{\frac{1}{2} AD \times h_1}{\frac{1}{2} BD \times h_1}$ **or R** ✓ $\dfrac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \dfrac{AE}{EC}$ ✓ S ✓R (6)

9.2



9.2.1	$\hat{A}_1 = x$ [corresp ∠s; PQ ‖ CA/*ooreenkomstige ∠e*, PQ ‖ CA] $\hat{B} = x$ [∠s opp equal sides/∠*e teenoor gelyke sye*] $\hat{A}_2 = x$ [tan-chord theorem/∠ *tussen raaklyn en koord*] $\hat{P} = x$ [alt ∠s; PQ ‖ CA/*verw. ∠e*, PQ ‖ CA]	✓ S ✓ R ✓ S/R ✓ S ✓ R ✓ S/R (6)
9.2.2	$\hat{B} = \hat{P}$ [proved in 9.2.1/*bewys in 9.2.1*] ∴ A, B, P and R are concyclic ∴ ABPR is a cyclic quadrilateral [conv ∠s in the same segment/ *koord onderspan gelyke omtreks ∠e*]	✓ S ✓ R (2)
9.2.3	$\frac{BA}{BQ} = \frac{BC}{BR}$ [prop th; AC ‖ QP] **OR** [line ‖ one side Δ/*lyn* ‖ *een syn v Δ*] But QR = BR [sides opp = ∠s/*sye teenoor* = ∠*e*] $\therefore \frac{BA}{BQ} = \frac{BC}{QR}$	✓ S ✓ R ✓ S (3)

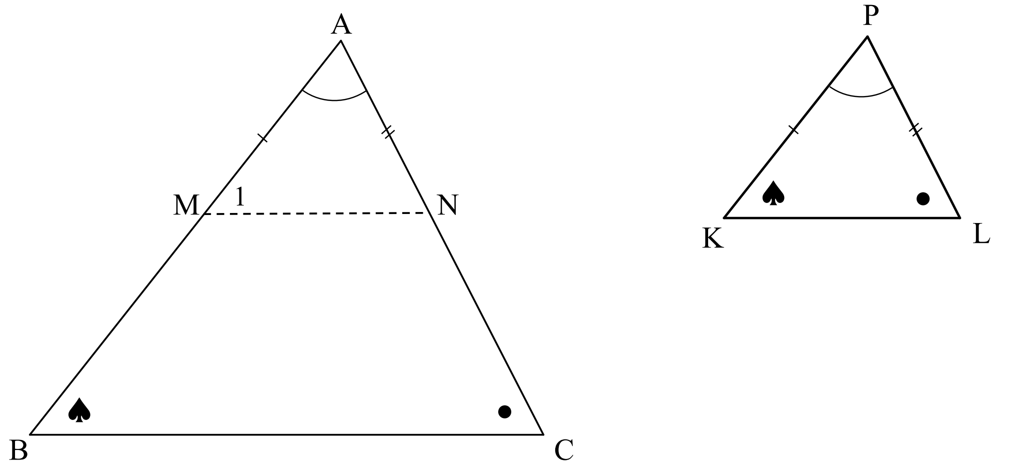
	OR In Δ ABC and Δ BQR: $\hat{A}_1 = \hat{B} = x$ [proved in 9.2.1] $\hat{B} = \hat{Q} = x$ [proved in 9.2.1] $\hat{C}_1 = B\hat{R}Q = 180° - 2x$ [sum of $\angle$s of Δ] $\therefore \Delta ABC \,\|\|\|\, \Delta BQR$ $\therefore \frac{BA}{BQ} = \frac{BC}{QR}$	✓ S ✓ S ✓ S (3)
	OR In Δ ABC and Δ BQR: $\hat{A}_1 = \hat{B} = x$ [proved in 9.2.1] $\hat{B} = \hat{Q} = x$ [proved in 9.2.1] $\hat{C}_1 = B\hat{R}Q = 180° - 2x$ [sum of $\angle$s of Δ] $\therefore \Delta ABC \,\|\|\|\, \Delta BQR$ [$\angle\angle\angle$] $\therefore \frac{BA}{BQ} = \frac{BC}{QR}$	✓ S ✓ S ✓ R (3)
	OR In Δ ABC and Δ QBR: $\hat{B}$ is common $\hat{A}_1 = \hat{Q} = x$ [corres $\angle$s; PQ || CA] $\hat{C}_1 = B\hat{R}Q = 180° - 2x$ [sum of $\angle$s of Δ] $\therefore \Delta ABC \,\|\|\|\, \Delta QBR$ [$\angle\angle\angle$] But QR = BR [sides opp = $\angle$s/*sye teenoor* = $\angle$*e*] $\therefore \frac{BA}{BQ} = \frac{BC}{QR}$	✓ S ✓ S ✓ S (3)
		[17]

20. Equiangular triangles have their sides in proportion

Official DBE marking guideline · May/June 2025 · Q10 · 20 marks

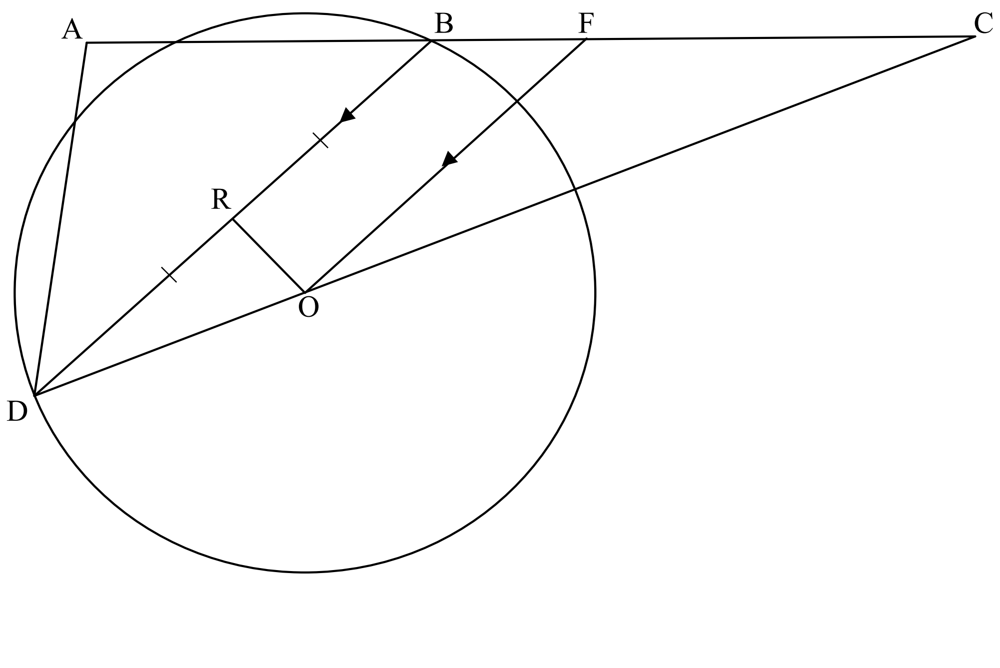
QUESTION/*VRAAG* 10

10.1



10.1	Construction: Draw line MN, where M and N are points on AB and AC respectively such that AM = PK and AN = PL. In Δ AMN and ΔPKL $\hat{A} = \hat{P}$ [given] AM = PK [construction] AN = PL [construction] ΔAMN ≡ ΔPKL [s∠s] $\therefore \hat{M}_1 = \hat{K}$ But $\hat{B} = \hat{K}$ [given] $\therefore \hat{M}_1 = \hat{B}$ ∴ MN || BC [corresp ∠s =/*ooreenk. ∠e* =] $\therefore \frac{AB}{AM} = \frac{AC}{AN}$ [line || one side of Δ/*lyn* || *een sy v* Δ] But AM = PK and AN = PL $\therefore \frac{AB}{PK} = \frac{AC}{PL}$	✓ construction ✓ S/R ✓ S ✓ S / R ✓ S ✓ R (6)

10.2



10.2.1	In ΔCFO and ΔCBD $\hat{C} = \hat{C}$ [common] $C\hat{F}O = C\hat{B}D$ [corresp $\angle$s; BD||FO/*ooreenk. $\angle$e; BD||FO*] $C\hat{O}F = C\hat{D}B$ [sum of $\angle$s in Δ/*binne $\angle$e v* Δ] **OR** [corresp $\angle$s; BD||FO/ *ooreenk. $\angle$e; BD||FO*] ΔCFO ||| ΔCBD [$\angle\angle\angle$]	✓ S ✓ S/R ✓ S/R **OR** R (3)
10.2.2	$\frac{FO}{BD} = \frac{CO}{CD}$ [||| Δs] FO.CD = CO.BD But $R\hat{D}O = F\hat{C}O$ [given] $\therefore$ BD = BC [sides opp equal $\angle$s/*sye teenoor gelyke $\angle$e*] $\therefore$ FO.CD = CO.BC	✓ S/R ✓ S/R (2)

10.2.3	RD = 6 units [DR = RB] $\frac{RO}{RD}=\frac{3}{4}$ $\therefore RO=\frac{3}{4}(6)$ RO = 4,5 units OR $\perp$ BD [line from centre to midpt of chord/ *midpt. sirkel; midpt koord*] $\therefore DO=\sqrt{6^2+4{,}5^2}$ [Pythagoras] DO = 7,5 units $\frac{BF}{BC}=\frac{DO}{DC}$ [line || to one side of Δ/*lyn || een sy v* Δ] **OR/*OF*** [prop theorem; BD||FO/*eweredigheidst.;*BD||FO] BC = BD = 12 units [sides opp equal ∠s/*sye teenoor gelyke ∠e*] $\therefore \frac{BF}{12}=\frac{7{,}5}{19{,}2}$ $BF=\frac{7{,}5\times 12}{19{,}2}$ $BF=\frac{75}{16}$ units	 ✓ S ✓ S/R ✓ S ✓ S/R ✓ S ✓ S (6)
10.2.4	$\tan R\hat{D}O=\frac{RO}{RD}=\frac{4{,}5}{6}=\frac{3}{4}$ $R\hat{D}O=36{,}87°$ $F\hat{C}O=R\hat{D}O=36{,}87°$ [given] $\therefore A\hat{B}D=73{,}74°$ [ext ∠ of Δ/*buite ∠ v.* Δ] **OR/*OF*** $CD^2=BC^2+BD^2-2(BC)(BD)\cos D\hat{B}C$ $\cos D\hat{B}C=\frac{12^2+12^2-19{,}2^2}{2(12)(12)}$ $\cos D\hat{B}C=-\frac{7}{25}$ $D\hat{B}C=106{,}26°$ $\therefore A\hat{B}D=73{,}74°$ [∠s on a straight line/*∠e op 'n reguitlyn*]	✓ ratio ✓ $R\hat{D}O$ ✓ answer (3) ✓ substitution into cosine-rule ✓ $D\hat{B}C$ ✓ answer (3)
		[20]

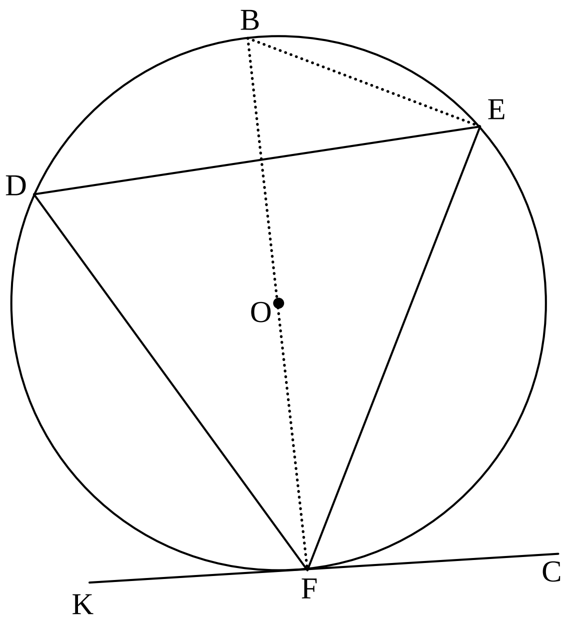
TOTAL/*TOTAAL*: 150

21. The tangent-chord theorem again, and a long rider after it

Official DBE marking guideline · November 2021 · Q11 · 23 marks

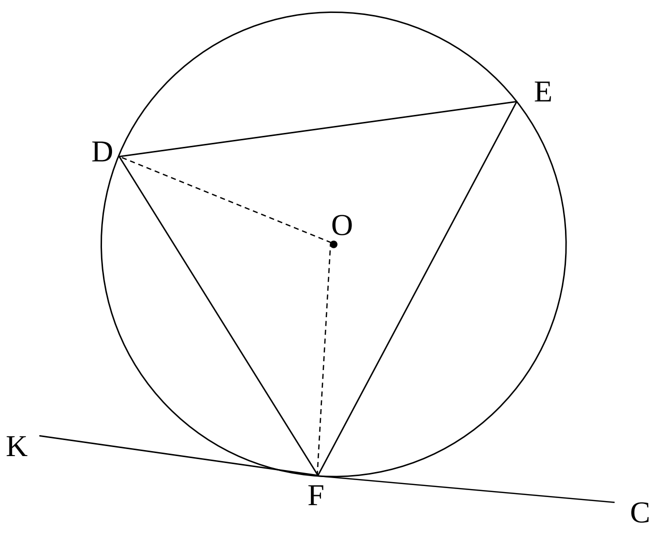
QUESTION/*VRAAG* 11

11.1



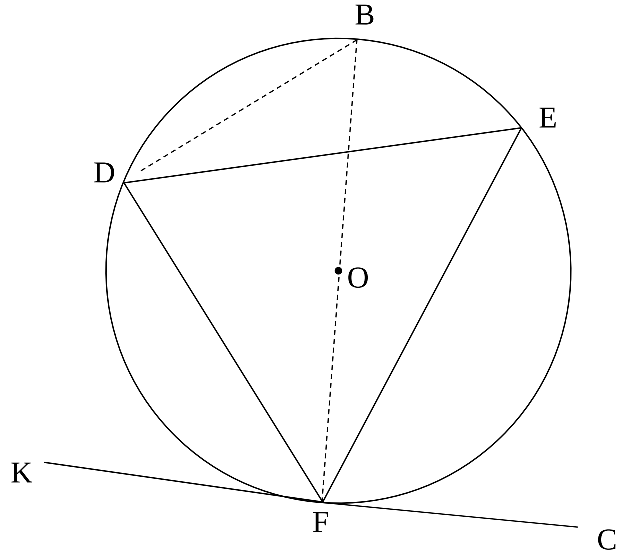
	Construction: Draw diameter BF and draw BE *Konstruksie: Trek middellyn* BF *en verbind* BE	✓Constr
	$B\hat{F}K = 90°$ or $D\hat{F}K = 90° - B\hat{F}D$ [radius ⊥ tangent/*raaklyn*]	✓ S ✓ R
	$B\hat{E}F = 90°$ [∠ in semi-circle/*semi-sirkel*]	✓ S
	$\therefore D\hat{E}F = 90° - B\hat{E}D$	
	$= 90° - B\hat{F}D$ [∠s same segment/∠*e dieselfde segment*]	✓ S/R
	$\therefore D\hat{F}K = D\hat{E}F$	(5)

OR/*OF*



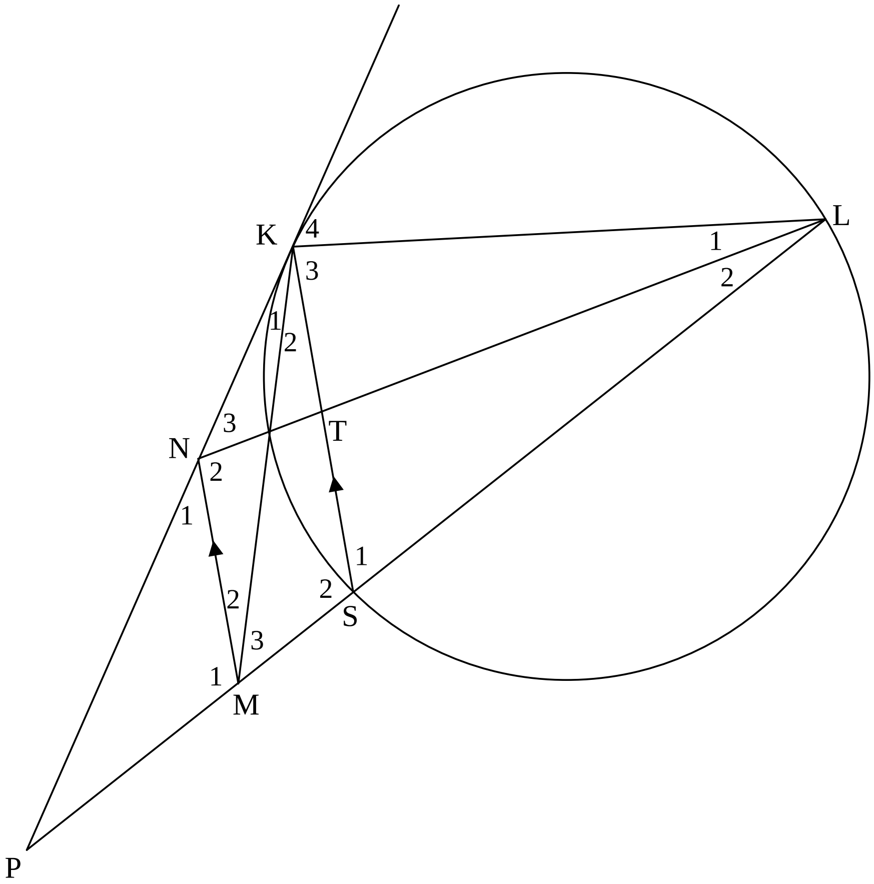
	Construction: Draw radii DO and OF	✓ construction
	Konstruksie: Trek radii DO en OF	
	$O\hat{F}K = 90°$ or $D\hat{F}K = 90° - O\hat{F}D$ radius⊥ tangent/*raaklyn*]	✓ S ✓R
	$O\hat{D}F = O\hat{F}D$ [∠s opp = sides/∠*e teenoor = sye*]	✓ S
	$\therefore D\hat{O}F = 180° - 2O\hat{F}D$ [∠s of Δ/∠*e van Δ*]	
	$D\hat{E}F = 90° - O\hat{F}D$ [∠ at centre = 2×∠ circumf/ *midpts∠ = 2× omtreks∠*]	✓ S/R
	$\therefore D\hat{F}K = D\hat{E}F$	(5)

OR/OF



	Construction: Draw diameter BF and join BD.	✓ construction
	Konstruksie: Trek middellyn BF en verbind BD.	
	$B\hat{F}K = 90°$ or $D\hat{F}K = 90° - B\hat{F}D$ [radius⊥ tangent/*raaklyn*]	✓ S ✓/R
	$F\hat{D}B = 90°$ [∠ in half circle/*semi-sirkel*]	✓ S
	$\hat{B} = 90° - B\hat{F}D$	
	$\therefore D\hat{F}K = \hat{B}$	
	but $\hat{B} = \hat{E}$ [∠s same segment/∠*e dieselfde segment*]	✓ S/R
	$\therefore D\hat{F}K = \hat{E}$	(5)

11.2



11.2.1(a)	$\hat{K}_4 = \hat{S}_1$ [tan chord theorem/*raaklynkoordstelling*] $\hat{M}_2 + \hat{M}_3 = \hat{S}_1$ [corresp ∠s; / *ooreenk ∠s;* MN ∥ KS] $\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3 = N\hat{M}L$	✓ S ✓ R ✓ S ✓ R (4)
11.2.1(b)	$\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3 = N\hat{M}L$ $\therefore$ KLMN is a cyclic quad [ext ∠ of quad = opp int ∠ / *buite ∠ van vh = teenoorst binne ∠*] **OR/OF** $N_1 = \hat{K}_1 + \hat{K}_2 = N\hat{K}S$ [corresp ∠s; / *ooreenk ∠s;* MN ∥ KS] $N\hat{K}S = K\hat{L}S$ [tan chord theorem / *raaklynkoordstelling*] $\hat{N}_1 = K\hat{L}S$ $\therefore$ KLMN is a cyclic quad [ext ∠ of quad = opp int ∠ / *buite ∠ van vh = teenoorst binne ∠*] **OR/OF**	✓ R (1) ✓ R (1)

	$\text{NKL} = 180^\circ - \text{K}_4$ [adj. suppl.] $\therefore \text{NKL} = 180^\circ - \text{NML}$ [proved] $\therefore$ KLMN is a cyclic quad [opp.∠s supplementary]	✓ R (1)
11.2.2	In $\Delta\text{LKN} \parallel\!\mid \Delta\text{KSM}$: $\hat{\text{N}}_3 = \hat{\text{M}}_3$ [∠s in the same seg / *∠e in dieselfde sirkel segm*] $\hat{\text{L}}_1 = \hat{\text{M}}_2$ [∠s in the same seg / *∠e in dieselfde sirkel segm*] $= \hat{\text{K}}_2$ [alt ∠s; / *verw∠e;* MN $\parallel$ KS] $\text{N}\hat{\text{K}}\text{L} = \text{M}\hat{\text{S}}\text{K}$ [∠s of Δ / *∠e van Δ*] $\Delta\text{LKN} \parallel\!\mid \Delta\text{KSM}$ **OR/OF** In $\Delta\text{LKN} \parallel\!\mid \Delta\text{KSM}$: $\hat{\text{N}}_3 = \hat{\text{M}}_3$ [∠s in the same seg / *∠e in dieselfde sirkel segm*] $\text{N}\hat{\text{K}}\text{L} = \hat{\text{M}}_1$ [ext ∠ of cyclic quad/*buite∠ van koordevh*] $= \hat{\text{S}}_2$ [corresp ∠s/*ooreenk ∠e*; KS $\parallel$ NM] $\Delta\text{LKN} \parallel\!\mid \Delta\text{KSM}$ [∠,∠,∠] **OR/OF** In $\Delta\text{LKN} \parallel\!\mid \Delta\text{KSM}$: $\hat{\text{N}}_3 = \hat{\text{M}}_3$ [∠s in the same seg / *∠e in dieselfde sirkel segm*] $\hat{\text{K}}_4 + \text{N}\hat{\text{K}}\text{L} = \hat{\text{S}}_1 + \hat{\text{S}}_2$ [∠s on straight line/*∠e op reguitlyn*] $\therefore\ \text{N}\hat{\text{K}}\text{L} = \hat{\text{S}}_2$ [$\hat{\text{K}}_4 = \hat{\text{S}}_1$] $\Delta\text{LKN} \parallel\!\mid \Delta\text{KSM}$ [∠,∠,∠]	✓ S ✓ R ✓ S ✓ S/R ✓ S (5) ✓ S ✓ R ✓ S/R ✓ S ✓ R (5) ✓ S ✓ R ✓ S/R ✓ S ✓ R (5)
11.2.3	$\dfrac{\text{LK}}{\text{KS}} = \dfrac{\text{KN}}{\text{SM}}$ [$\Delta\text{LKN} \parallel\!\mid \Delta\text{KSM}$] $\therefore \dfrac{12}{\text{KS}} = \dfrac{4}{3}$ $\text{KS} = 9$ units	✓ S ✓ R ✓ substitution ✓ answer (4)
11.2.4	4SM = 3KN $\text{SM} = \dfrac{3(8)}{4}$ SM = 6 $\dfrac{\text{LT}}{\text{NL}} = \dfrac{\text{LS}}{\text{ML}}$ [line $\parallel$ one side of Δ / *lyn* $\parallel$ *een sy v* Δ] $\dfrac{\text{LT}}{16} = \dfrac{13}{19}$ $\text{LT} = \dfrac{208}{19} = 10{,}95$	✓ SM = 6 ✓ S ✓ R ✓ answer (4)
		[23]

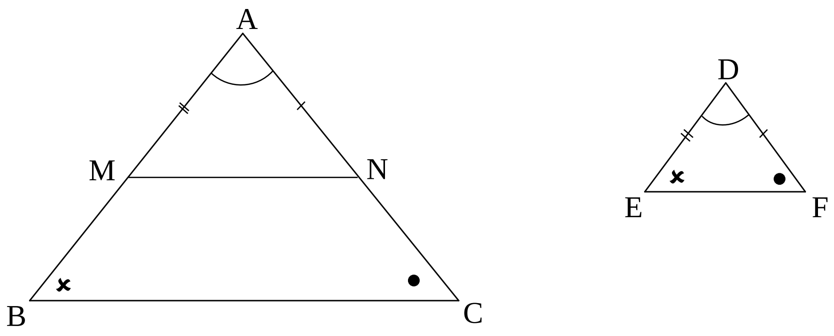
TOTAL/*TOTAAL*: 150

22. Equiangular triangles, then the longest rider in the book

Official DBE marking guideline · May/June 2022 · Q10 · 25 marks

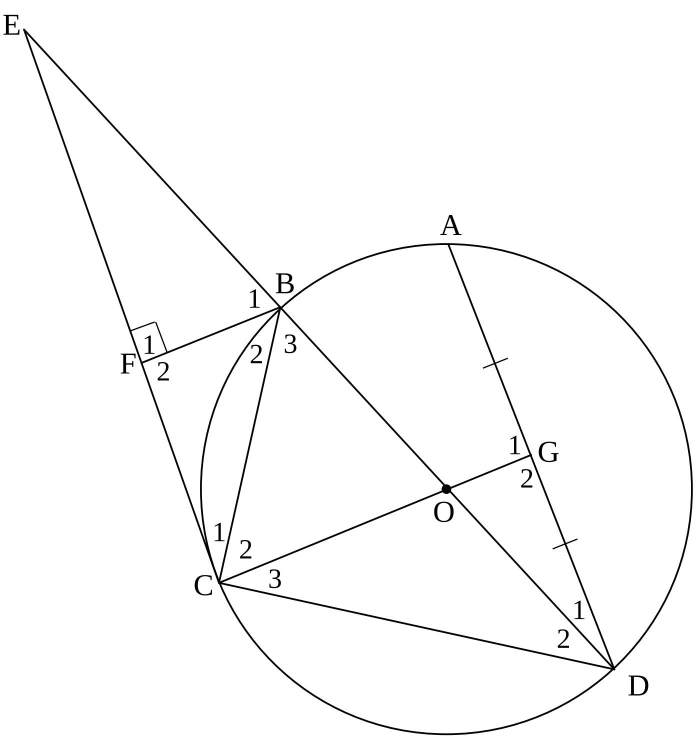
QUESTION/*VRAAG* 10

10.1



10.1	Constr: Let M and N lie on AB and AC respectively such that AM = DE and AN = DF. Draw MN.	✓Constr
	Proof: In Δ AMN and Δ DEF	
	AM = DE [Constr / *Konstruksie*]	
	AN = DF [Constr / *Konstruksie*]	
	$\hat{A} = \hat{D}$ [Given /*Gegee*]	
	$\therefore \Delta AMN \equiv \Delta DEF$ $[s, \angle, s]$	✓S ✓R
	$\therefore A\hat{M}N = \hat{E} = \hat{B}$	
	MN ∥ BC [corresp ∠'s are equal/ *ooreenk.* ∠ *e gelyk*]	✓S /R
	$\frac{AB}{AM} = \frac{AC}{AN}$ [line ∥ one side of Δ **OR/*OF*** prop theorem; MN ∥BC / *Lyn* ∥ *een sy v* Δ]	✓S ✓R
	$\therefore \frac{AB}{DE} = \frac{AC}{DF}$ [AM = DE and AN = DF]	(6)

10.2



10.2.1(a)	$F\hat{C}O = 90°$ [tan ⊥ radius / *raaklyn* ⊥ *radius*] $\hat{F}_1 = 90°$ [BF ⊥ EC] $\therefore F\hat{C}O = \hat{F}_1 = 90°$ FB ‖ CG [corresp ∠s = / *ooreenk.* ∠ *gelyk*]	✓ S / R ✓ S ✓ R (3)
10.2.1(b)	In ΔFCB and ΔCDB $B\hat{C}D = 90°$ [∠ in semi-circle / ∠ $\frac{1}{2}$ ⊙] $\hat{F}_2 = 90°$ [BF ⊥ EC] $\therefore \hat{F}_2 = B\hat{C}D = 90°$ $\hat{C}_1 = \hat{D}_2$ [tan chord theorem / ∠ *tussen rkl en koord*] $\hat{B}_2 = \hat{B}_3$ [sum of ∠s in Δ / ∠*e van* Δ] ∴ ΔFCB ||| ΔCDB **OR/*OF*** In ΔFCB and ΔCDB $B\hat{C}D = 90°$ [∠ in semi-circle / ∠ $\frac{1}{2}$ ⊙] $\hat{F}_2 = 90°$ [BF ⊥ EC] $\therefore \hat{F}_2 = B\hat{C}D = 90°$ $\hat{C}_1 = \hat{D}_2$ [tan chord theorem / ∠ *tussen rkl en koord*] ∴ ΔFCB ||| ΔCDB [∠,∠,∠]	✓ S /R ✓ S ✓ S ✓ R ✓ S ✓ S / R ✓ S ✓ S ✓ R ✓ R (5)

10.2.2	$\hat{G}_1 = 90°$ [line from centre to midpt of chord / *midpt.* ⊙; *midpt. koord*]	✓ R (1)
10.2.3	In ΔGCD and ΔCDB $\hat{G}_2 = B\hat{C}D = 90°$ $\hat{C}_3 = \hat{D}_2$ [$\angle$s opp equal sides /$\angle$*e teenoor gelyke sye*] $G\hat{D}C = \hat{B}_3$ [sum of $\angle$s in Δ /$\angle$*e van* Δ] $\therefore \Delta GCD \mid\mid\mid \Delta CDB$ [$\angle,\angle,\angle$] $\therefore \frac{CD}{DB} = \frac{CG}{CD}$ [||| Δs] $\therefore CD^2 = CG.DB$	✓ identifying Δs ✓ S ✓ S / R ✓ S **OR** ✓ R ✓ S (5)
10.2.4	$\frac{BC}{DB} = \frac{FB}{BC}$ [$\Delta FCB \mid\mid\mid \Delta CDB$] $\therefore BC^2 = DB.FB$ $CD^2 + BC^2 = CG.DB + DB.FB$ $DB^2 = DB(CG + FB)$ $DB = CG + FB$	✓ S ✓ R ✓ S ✓ sum ✓ $DB^2 = CD^2 + BC^2$ (5)
		[25]

TOTAL/*TOTAAL*: 150