IEB mock Paper I — Memorandum

Every answer below was confirmed by an independent verifier at the moment the question was made.

QUESTION 1

(a) Answer: $x = -4p$ or $x = -3p$

1. $x^2 + 7px + 12p^2 = 0$
2. $(x + 4p)(x + 3p) = 0$

(b) Answer: $m = 3$ and $p = -16$

1. $(x - m)^2 + p = x^2 - 2mx + m^2 + p$
2. $-2m = -6$, so $m = 3$
3. $m^2 + p = -7$, so $p = -16$

(c) Answer: -16

1. $(x - m)^2 \geq 0$, so the least value is $p = -16$

(d) Answer: $0 \leq x \leq 1$

1. $x^2 - x \leq 0$
2. $x(x - 1) \leq 0$
3. critical values: $x = 0$ and $x = 1$

(e) Answer: $k = -6$ or $k = 6$

1. $\Delta = k^2 - 4(3)(3) = k^2 - 36$
2. $k^2 - 36 = 0$
3. $k = \pm 6$

QUADRATIC EQUATIONS

QUESTION 2

(a) Answer: $a = 80$ and $b = 1.5$

1. $\mathrm{N}(0) = a \cdot b^0 = a$, so $a = 80$
2. $80 \cdot b^3 = 270$
3. $b^3 = \frac{27}{8}$, so $b = 1.5$

(b) Answer: $\mathrm{N}(4) = 405$

1. $\mathrm{N}(4) = 80 \times (1.5)^4 = 405$

(c) Answer: $y = \log_{1.5}(\frac{x}{80})$

1. swop x and y: $x = 80 \times (1.5)^y$
2. $(1.5)^y = \frac{x}{80}$

(d) Answer: $t = 6.10$ days

1. $80 \times (1.5)^t = 950$
2. $t = \log_{1.5}(\frac{950}{80})$
3. $t = 6.10$

(e) Answer: 1.71 days

1. $(1.5)^t = 2$
2. $t = \log_{1.5} 2 = 1.71$
3. the same from any starting time

QUESTION 3

(a) Answer: $\frac{dy}{dx} = 1 + 18x^{-3}$

1. $y = x - 9x^{-2}$

(b) Answer: $y = -5x - 24$

1. $f'(x) = 6x^2 + 30x + 31$
2. $m = f'(-3) = -5$; $f(-3) = -9$
3. $y + 9 = -5(x + 3)$

(c) Answer: $x = -2$

1. $f'(x) = -5$
2. $6x^2 + 30x + 36 = 0$
3. $x = -3$ or $x = -2$; the other point: $x = -2$

(d) Answer: $k = -16$ or $k = 327$

1. $f'(x) = 67$: $6x^2 + 30x - 36 = 0$
2. $x = 1$ or $x = -6$
3. $k = f(x) - (67)x$: $51 - (67) = -16$ and $-75 - (-402) = 327$

QUESTION 4

(a) Answer: R4 932.86

1. $i = 8.75\%/12$ and $n = 300$
2. $600\,000 = x\frac{1-(1+0.007292)^{-300}}{0.007292}$
3. $x = \mathrm{R}4\ 932.86$

(b) Answer: R508 833.01

1. after 108 payments
2. balance $= 600\,000(1+i)^{108} - 4932.86\frac{(1+i)^{108}-1}{i}$
3. $= \mathrm{R}508\ 833.01$

(c) Answer: 9.11%

1. $1 + i_\text{eff} = (1 + \frac{0.0875}{12})^{12}$
2. $i_\text{eff} = 9.11\%$

(d) Answer: 124 payments

1. $600\,000 = 7400\frac{1-(1+i)^{-n}}{i}$
2. $(1+i)^{-n} = 0.408784$
3. $n = 123.13$

QUESTION 5

(a) Answer: $x = 6$; the terms are 6; 9; 12

1. $\mathrm{T}_2 - \mathrm{T}_1 = \mathrm{T}_3 - \mathrm{T}_2$
2. $2(4x - 15) = (5x - 24) + (2x)$
3. $x - 6 = 0$
4. $x = 6$

(b) Answer: T_{28}

1. $6 + (n - 1)(3) = 87$
2. $n - 1 = 27$
3. $n = 28$

(c) Answer: 28 terms

1. $\frac{n}{2}[2(6) + (n - 1)(3)] = 1302$
2. $3n^2 + 9n - 2604 = 0$
3. $n = 28$ (n is a natural number)

(d) Answer: 803 dots

1. second difference 4: $a = 2$
2. $\mathrm{T}_n = 2n^2 + 3$
3. $\mathrm{T}_{20} = 803$

(e) Answer: picture 54

1. $2n^2 + 3 = 5835$
2. $2n^2 - 5832 = 0$
3. $n = 54$ $(n > 0)$

QUESTION 6

(a) Answer: $y = -\frac{9}{4}(x-2)^2 + 19$

1. $y = a(x-2)^2 + 19$
2. at $(0;\,10)$: $10 = a(-2)^2 + 19$
3. $a = -\frac{9}{4}$

(b) Answer: $\frac{67}{4}\ \text{m}$

1. $y = -\frac{9}{4}(3-2)^2 + 19$
2. $y = \frac{67}{4}\ \text{m}$

(c) Answer: 4.91 m

1. $-\frac{9}{4}(x-2)^2 + 19 = 0$
2. $(x-2)^2 = \frac{76}{9}$
3. $x = 2 + \sqrt{76/9}$
4. $x = 4.91\ \text{m}$

FITTING A GRAPH TO SOMETHING REAL

QUESTION 7

(a) Answer: See the working below.

1. the side along the wall is $120 - 2x$
2. $\mathrm{A} = x(120 - 2x) = 120x - 2x^2$

(b) Answer: 30 m by 60 m; area 1800 m²

1. $\frac{dA}{dx} = 120 - 4x = 0$
2. $x = 30\ \mathrm{m}$
3. the side along the wall is 60 m
4. $\mathrm{A} = 30 \times 60 = 1800\ \mathrm{m^2}$

(c) Answer: -92 m²/m

1. $\frac{dA}{dx}$ at $x = 53$
2. $= -92$ m²/m

OPTIMISATION

QUESTION 8

(a) Answer: 5040

1. $7! = 5040$

(b) Answer: 2400

1. the two ends: 5×4 choices
2. the other 5: 5!
3. $5 \times 4 \times 5! = 2400$

(c) Answer: 3600

1. Thabo and Mia together, as one unit: $6! \times 2! = 1440$
2. not together: $5040 - 1440 = 3600$

(d) Answer: 720

1. the 5 girls as one block: 3 units, 3! ways
2. the girls inside the block: 5! ways
3. $3! \times 5! = 720$

(e) Answer: $\frac{2}{7}$

1. $\mathrm{P}(\text{not next to each other}) = \frac{3600}{5040}$
2. $\mathrm{P}(\text{next to each other}) = 1 - \frac{3600}{5040} = \frac{2}{7}$

QUESTION 9

(a) Answer: $p = -2$ and $q = -4$

1. the axes of symmetry cross where the asymptotes cross
2. $x - 6 = -x - 2$, so $x = 2$ and $y = -4$
3. $x = -p = 2$ gives $p = -2$; $y = q = -4$

(b) Answer: $a = -9$

1. $-5 = \frac{a}{11 - 2} - 4$

(c) Answer: domain: $x \in \mathbb{R}$, $x \neq 2$; range: $y \in \mathbb{R}$, $y \neq -4$

1. the vertical asymptote is $x = 2$; the horizontal one $y = -4$

(d) Answer: $f(x) = 3(x - 2)^2 - 4$

1. turning point $(2; -4)$: $f(x) = k(x - 2)^2 - 4$
2. $23 = k(5 - 2)^2 - 4$, so $k = 3$

(e) Answer: $(-1; -1)$ and $(5; -7)$

1. $-\frac{9}{x - 2} - 4 = -x - 2$
2. $(x - 2)^2 = 9$
3. $x = 2 \pm 3$

QUESTION 10

(a) Answer: $x = -3y - 3$

1. $3^{x+3} \cdot 3^{4y} = 3^{y}$
2. $x + 3 + 4y = y$

(b) Answer: $x = 11$

1. $x + 53 = x^2 - 6x + 9$
2. $x^2 - 7x - 44 = 0$
3. $(x - 11)(x + 4) = 0$
4. check $x = -4$: $\sqrt{-4 + 53} = 7$ but $-4 - 3 = -7$, so $x = -4$ is rejected

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(c) Answer: $x = -3$ or $x = 6$

1. $x^2 - 3x - 16 = 2^1$
2. $x^2 - 3x - 18 = 0$
3. $(x + 3)(x - 6) = 0$

(d) Answer: $x = 1$

1. let $t = 2^x$: $t^2 + t - 6 = 0$
2. $(t - 2)(t + 3) = 0$
3. $2^x = -3$ has no solution
4. $2^x = 2$, so $x = 1$

EXPONENTIAL EQUATIONS AND EXPONENT LAWS

QUESTION 11

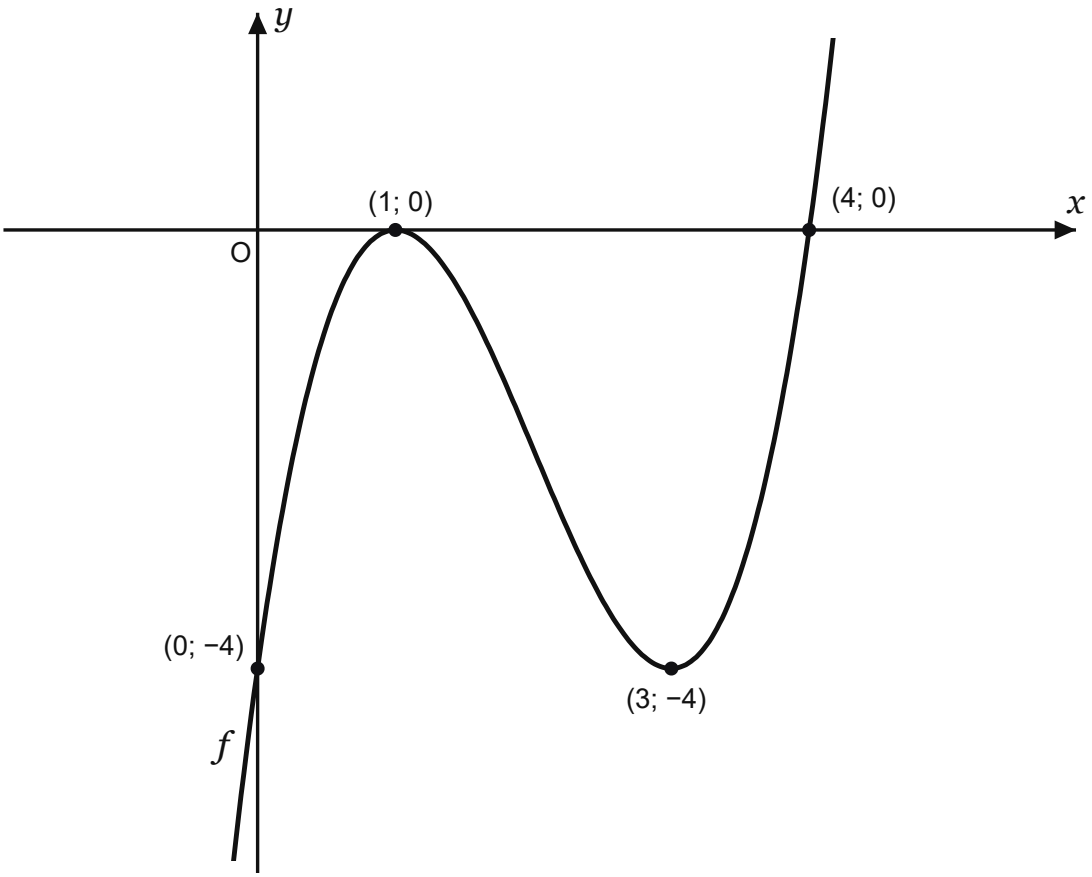
(a) Answer: (1; 0) and (4; 0)

1. $f(4) = 0$, so $(x - 4)$ is a factor
2. $f(x) = (x - 4)(x - 1)^2$

(b) Answer: (1; 0) and $(3; -4)$

1. $f'(x) = 3x^2 - 12x + 9$
2. $3x^2 - 12x + 9 = 0$
3. $x = 1$ or $x = 3$

(c) Answer: See the sketch below.



1. y-intercept $(0; -4)$; x-intercepts (1; 0), (4; 0)
2. turning points $(1; 0)$, $(3; -4)$

(d) Answer: $x > 2$

1. $f''(x) = 6x - 12$

2. $f''(x) > 0$: $6x - 12 > 0$

(e) Answer: $1 < x < 3$

1. $f'(x) < 0$

(f) Answer: $-4 < k < 0$

1. the line $y = k$ must cut the graph three times: between the turning values

STATIONARY POINTS