IEB mock Paper I

Time: 3 hours · 150 marks

Answer all the questions. Show all calculations. You may use an approved calculator unless a question says otherwise. Diagrams are not necessarily drawn to scale.

Section A

[73 marks]

QUESTION 1

(a) Solve for x in terms of p: $x^2 + 12p^2 = -7px$

(3)

(b) If $x^2 - 6x - 7 = (x - m)^2 + p$, determine the values of m and p.

(3)

(c) Hence write down the minimum value of $x^2 - 6x - 7$.

(1)

(d) Solve for x: $x(x - 1) \leq 0$

(3)

(e) Determine the value(s) of k for which $3x^2 + kx + 3 = 0$ has equal roots.

(3)

[13]

QUESTION 2

The number of people who have heard a rumour t days after the start is given by $\mathrm{N}(t) = a \cdot b^t$. At the start it is 80, and after 3 days it is 270.

(a) Determine the values of a and b.

(3)

(b) Calculate $\mathrm{N}(4)$.

(2)

(c) Determine the equation of the inverse of N in the form $y = \ldots$

(3)

(d) After how many days will N first be more than 950? Give your answer correct to two decimal places.

(3)

(e) How long does it take N to double? Give your answer correct to two decimal places.

(2)

[13]

QUESTION 3

(a) Determine $\frac{dy}{dx}$ if $y = \frac{x^3 - 9}{x^2}$.

(4)

(b) Given $f(x) = 2x^3 + 15x^2 + 31x + 3$. Determine the equation of the tangent to f at the point where $x = -3$.

(4)

(c) Determine the x-coordinate of the other point on f at which the tangent is parallel to the tangent at $x = -3$.

(3)

(d) The line $y = 67x + k$ is a tangent to f. Determine the possible values of k.

(4)

[15]

QUESTION 4

A home loan of R600 000 is repaid in equal monthly payments over 25 years. The interest rate is 8.75% per annum compounded monthly, and the first payment is made one month after the loan is granted. Give answers correct to two decimal places.

(a) Calculate the monthly repayment.

(4)

(b) Calculate the balance outstanding at the end of year 9, immediately after the payment at that time.

(4)

(c) Determine the effective annual interest rate of the loan.

(3)

(d) If you pay R7 400 per month instead, how many payments will it take to pay off the loan? (The last payment may be smaller.)

(4)

[15]

QUESTION 5

(a) $5x - 24$; $4x - 15$ and $2x$ are the first three terms of an arithmetic sequence. Determine the value of x and the three terms.

(3)

(b) Which term of this sequence is equal to 87?

(3)

(c) How many terms of this sequence must be added to give a sum of 1302?

(4)

(d) A pattern of dots is built picture by picture. Pictures 1 to 4 have 5; 11; 21; 35 dots, and the numbers of dots form a quadratic sequence. How many dots are in picture 20?

(4)

(e) Which picture has 5835 dots?

(3)

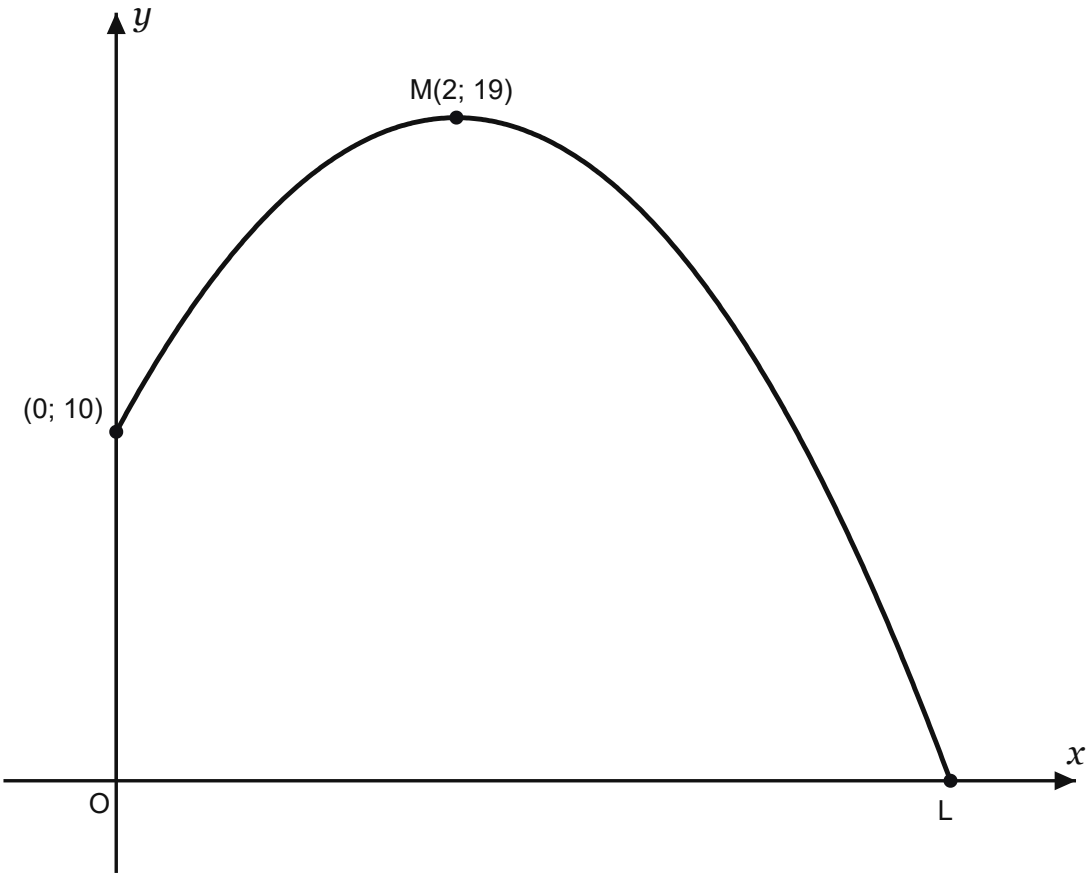
[17]

Section B

[77 marks]

QUESTION 6

A ball is thrown from the top of a building 10 m high. It reaches its greatest height of 19 m at a horizontal distance of 2 m from the building. In the diagram the foot of the building is at the origin and the ground lies along the x-axis.



(a) Determine the equation of the path of the ball in the form $y = a(x - p)^2 + q$.

(3)

(b) Calculate the height of the ball 3 m horizontally from the building.

(2)

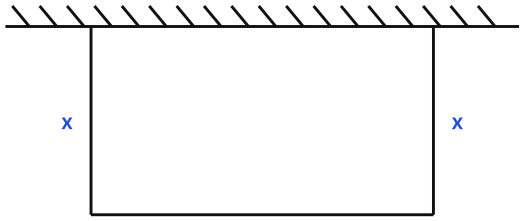
(c) Calculate, correct to two decimal places, how far from the building the ball lands at L.

(3)

[8]

QUESTION 7

A rectangular garden is built against a straight wall. The other three sides are fenced with 120 m of fencing. The two sides at right angles to the wall are each x m long.



(a) Show that the area of the garden is $\mathrm{A} = 120x - 2x^2$.

(2)

(b) Determine the dimensions of the garden with the largest area, and that area.

(5)

(c) Determine the rate at which the area changes with respect to x when $x = 53$.

(2)

[9]

QUESTION 8

2 boys and 5 girls are to sit in a row of 7 chairs. Thabo is one of the boys and Mia one of the girls.

(a) In how many different ways can the 7 learners be arranged?

(1)

(b) In how many ways can they be arranged if there must be a girl at each end of the row?

(3)

(c) In how many ways can they be arranged if Thabo and Mia may not be next to each other?

(3)

(d) In how many ways can they be arranged if all the girls must be next to one another?

(3)

(e) Hence, if the learners take their places in a random order, what is the probability that Thabo and Mia are next to each other?

(3)

[13]

QUESTION 9

The graph of $g(x) = \frac{a}{x+p} + q$ has the lines $y = x - 6$ and $y = -x - 2$ as its axes of symmetry, and passes through the point $(11; -5)$.

(a) Determine the values of p and q.

(3)

(b) Determine the value of a.

(2)

(c) Write down the domain and the range of g.

(2)

(d) The parabola f has its turning point where the asymptotes of g meet, and passes through (5; 23). Determine the equation of f.

(3)

(e) Determine the coordinates of the points where the axis of symmetry $y = -x - 2$ cuts the graph of g.

(4)

[14]

QUESTION 10

(a) Solve for x in terms of y: $3^{x+3} \cdot 81^{y} = 3^{y}$

(3)

(b) Solve for x: $\sqrt{x + 53} = x - 3$

(5)

(c) Solve for x: $\log_2 (x^2 - 3x - 16) = 1$

(4)

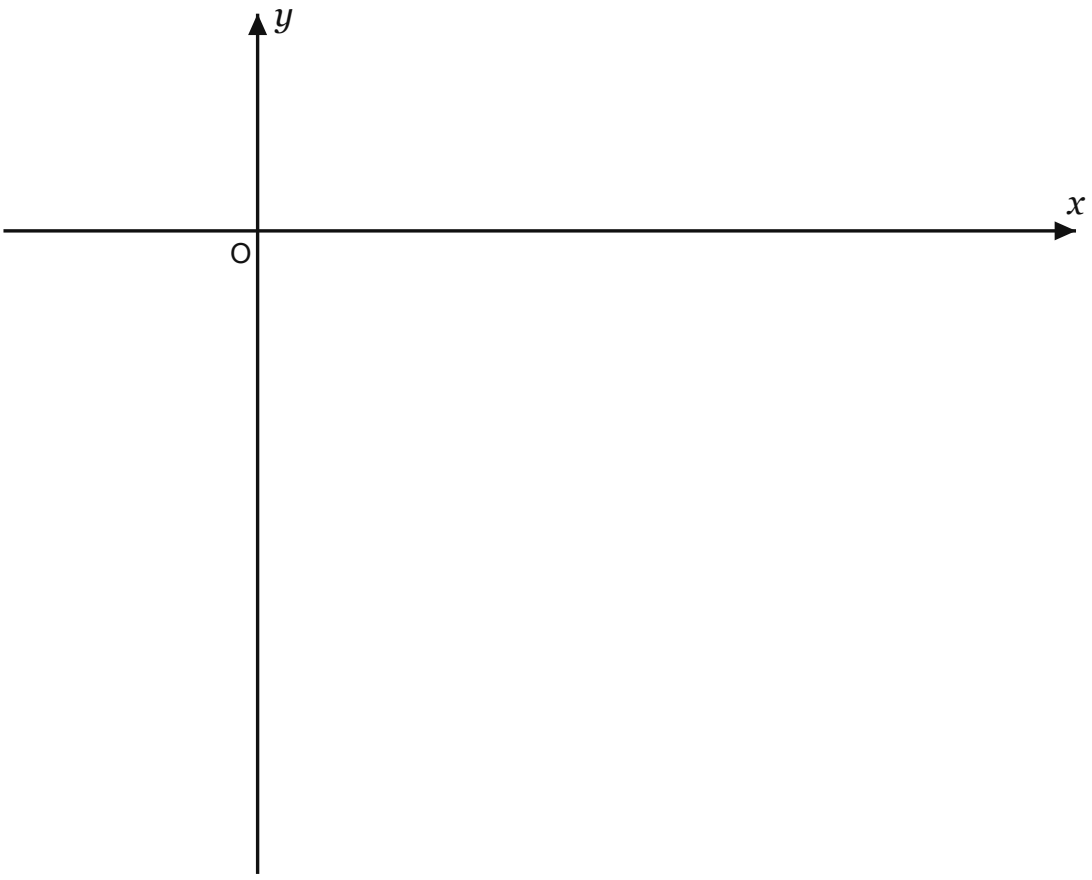
(d) Solve for x: $2^{2x} + 2^{x} - 6 = 0$

(4)

[16]

QUESTION 11

Given: $f(x) = x^3 - 6x^2 + 9x - 4$



(a) Show that $x = 4$ is a root of $f(x) = 0$, and hence determine the x-intercepts of f.

(3)

(b) Determine the coordinates of the turning points of f.

(4)

(c) Sketch the graph of f on the axes provided. Show all intercepts with the axes and the turning points.

(4)

(d) For which values of x is the graph of f concave up?

(2)

(e) For which values of x is f decreasing?

(2)

(f) For which values of k will $f(x) = k$ have three different real roots?

(2)

[17]