IEB mock Paper I (second paper) — Memorandum

Every answer below was confirmed by an independent verifier at the moment the question was made.

QUESTION 1

(a) Answer: R393 192.35

1. after 120 deposits: $1900\dfrac{(1+7/1200)^{120}-1}{7/1200} = \mathrm{R}328\ 861.13$
2. then 12 quarters: $\mathrm{R}328\ 861.13(1+\frac{6}{400})^{12} = \mathrm{R}393\ 192.35$

(b) Answer: 190 withdrawals

1. $393\,192.35 = 3100\dfrac{1-(1+0.004583)^{-n}}{0.004583}$
2. $(1+0.004583)^{-n} = 0.418667$
3. $n = 190.40$
4. 190 full withdrawals

(c) Answer: R45 914.23

1. deposits at 6, 12, …, 36 months: 6 of them
2. $\mathrm{FV} = 6500\dfrac{(1+0.065)^{6}-1}{0.065}$
3. $= \mathrm{R}45\ 914.23$

SAVING: THE FUTURE-VALUE ANNUITY

QUESTION 2

(a) Answer: $b = \frac{1}{5}$

1. P lies on f: $b^{-1} = 5$

(b) Answer: $y = \log_{1/5} x$

1. g is the reflection of f in $y = x$: $x = (\frac{1}{5})^y$

(c) Answer: $x > 0$

1. the domain of g is the range of f

(d) Answer: $0 < x < \frac{1}{25}$

1. $g(x) = 2$ when $x = (\frac{1}{5})^2 = \frac{1}{25}$
2. g is decreasing $(0 < b < 1)$, so the inequality sign is reversed

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(e) Answer: $\mathrm{AB} = \frac{1}{5}$ units

1. $\mathrm{A}(1; \frac{1}{5})$: $f(1) = (\frac{1}{5})^1 = \frac{1}{5}$
2. $\mathrm{B}(1; 0)$: $g(1) = \log_{1/5} 1 = 0$
3. $\mathrm{AB} = \frac{1}{5} - 0 = \frac{1}{5}$

(f) Answer: $y = -25$; x-intercept $(-2; 0)$

1. f's asymptote $y = 0$ moves down 25: $y = -25$
2. $(\frac{1}{5})^x - 25 = 0$, so $(\frac{1}{5})^x = 25$
3. $x = \log_{1/5} 25 = -2$

QUESTION 3

(a) Answer: $d = -28$

1. $f(2) = 0$
2. $28 + d = 0$

(b) Answer: $(-4;\ -108)$

1. $f'(x) = -3x^2 - 6x + 24 = 0$
2. $-3(x - (-4))(x - (2)) = 0$
3. $x = -4$
4. $f(-4) = -108$

(c) Answer: $x = -1$

1. $f''(x) = -6x - 6 = 0$

(d) Answer: $k < -108$ or $k > 0$

1. the turning values are -108 and 0
2. one root when the line $y = k$ passes above the maximum or below the minimum

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(e) Answer: $y = 24x - 28$

1. $f(0) = -28$ and $f'(0) = 24$

STATIONARY POINTS

QUESTION 4

(a) Answer: $x = -3$ or $x = \frac{7}{5}$

1. $(5x - 7)(x + 3)^2 = 0$
2. $5x - 7 = 0$ or $x + 3 = 0$

(b) Answer: $x = -0.38$ or $x = 2.63$

1. $4x^2 - 9x - 4 = 0$
2. $x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(4)(-4)}}{2(4)}$
3. $x = \frac{9 \pm \sqrt{145}}{8}$

(c) Answer: $x < -0.38$ or $x > 2.63$

1. the critical values are $x = -0.38$ and $x = 2.63$

(d) Answer: See the proof below.

1. $3x^2 + px - 2 = 0$
2. $\Delta = (p)^2 - 4(3)(-2) = p^2 + 24$
3. $p^2 \geq 0$, so $\Delta \geq 24 > 0$
4. so the roots are real and unequal for all real values of p

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(e) Answer: $p = -11$ or $p = 13$

1. $\Delta = (p - 1)^2 - 4(1)(36) = (p - 1)^2 - 144$
2. $\Delta = 0$ for equal roots: $(p - 1)^2 = 144$
3. $p - 1 = \pm 12$

QUESTION 5

(a) Answer: $\mathrm{T}_n = 4n + 3$

1. $a = 7$ and $d = 11 - (7) = 4$
2. $\mathrm{T}_n = 7 + (n - 1)(4) = 4n + 3$

(b) Answer: 55 terms

1. $4n + 3 = 223$
2. $4n = 220$
3. $n = 55$

(c) Answer: $\mathrm{S}_{55} = 6325$

1. $\mathrm{S}_n = \frac{n}{2}(a + l)$
2. $\mathrm{S}_{55} = \frac{55}{2}(7 + 223)$
3. $= 6325$

(d) Answer: $\Sigma_{n=1}^{55}\,(4n + 3)$

1. the terms are $\mathrm{T}_n = 4n + 3$, from $n = 1$ to $n = 55$

(e) Answer: $\mathrm{T}_n = 2n^2 + 5n + 10$

1. first differences: 11; 15; 19; 23
2. second difference: 4, so $2a = 4$ and $a = 2$
3. $3a + b = 11$, so $b = 5$
4. $a + b + c = 17$, so $c = 10$

(f) Answer: No: 1184 is not a term

1. $2n^2 + 5n + 10 = 1184$
2. $2n^2 + 5n - 1174 = 0$
3. $n = 23.01$, not a natural number

QUESTION 6

(a) Answer: $1 < x < 5$

1. f is increasing where $f'(x) > 0$: where the graph of f' lies above the x-axis

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(b) Answer: $x = 5$ (local maximum) and $x = 1$ (local minimum)

1. $f'(x) = 0$ at $x = 1$ and $x = 5$
2. f' changes from positive to negative at $x = 5$: a maximum

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(c) Answer: $x = 3$

1. $f''(x) = 0$ where f' has its turning point
2. $x = \frac{1+5}{2} = 3$

(d) Answer: $x < 3$

1. f is concave up where $f''(x) > 0$: where f' is increasing

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(e) Answer: -5

1. the gradient of f at $x = 0$ is $f'(0) = -5$

READING THE GRAPH OF F'

QUESTION 7

(a) Answer: $x = -1, y = -1$ or $x = 1, y = 1$

1. $y = x$
2. $x(x) = 1$
3. $x^2 - 1 = 0$
4. $(x - 1)(x + 1) = 0$
5. $x = -1$ or $x = 1$
6. $y = -1$ or $y = 1$

(b) Answer: 12 rows

1. let there be r rows: $\frac{288}{r-3} - \frac{288}{r} = 8$
2. $288r - 288(r - 3) = 8r(r - 3)$
3. $r^2 - 3r - 108 = 0$
4. $(r - 12)(r + 9) = 0$
5. $r = 12$ ($r = -9$ is rejected: the number of rows is positive)

SIMULTANEOUS EQUATIONS

QUESTION 8

(a) Answer: $324\sqrt{3}$ cm²

1. area $= ½ \times 36 \times 36 \times \sin 60^\circ$
2. $= ½ \times 1296 \times \frac{\sqrt{3}}{2} = 324\sqrt{3}$ cm²

(b) Answer: 243

1. stages 1, 2 and 3 have 1, 3 and 9: every shaded triangle leaves three
2. stage 6: $3^5 = 243$

(c) Answer: 461.31 cm²

1. stage 2 removes ¼ of $324\sqrt{3}$, which is $81\sqrt{3}$ cm²
2. each next stage removes 3 times as many triangles, each ¼ of the size: $r = \frac{3}{4}$
3. $\mathrm{S}_6 = 81\sqrt{3}\frac{1 - 0.75^6}{1 - 0.75} = 461.31$ cm²

(d) Answer: $324\sqrt{3}$ cm², all of stage 1: the shaded area tends to 0

1. $-1 < r = 0.75 < 1$, so $\mathrm{S}\infty = 81\frac{\sqrt{3}}{1 - 0.75} = 324\sqrt{3}$ cm²
2. that is the whole area of stage 1, so the shaded area tends to 0

(e) Answer: stage 9

1. stage 1: 108 cm; each next stage has 3 times as many triangles, each with half the perimeter: $r = 1.5$
2. $108(1.5)^{n-1} > 2550$
3. $n - 1 > \frac{\log(2550/108)}{\log 1.5} = 7.80$
4. $n - 1 = 8$, so stage 9

GEOMETRIC SEQUENCES

QUESTION 9

(a) Answer: $f(x) = (x-1)^2 + 3$

1. $f(x) = a(x-1)^2 + 3$
2. substitute $(0; 4)$: $4 = a(-1)^2 + 3$
3. $a = 1$

(b) Answer: $\mathrm{A}(-2; 12)$ and $\mathrm{B}(2; 4)$

1. $(x-1)^2 + 3 = -2x + 8$
2. $x^2 - 4 = 0$
3. $(x+2)(x-2) = 0$
4. $x = -2$ or $x = 2$

(c) Answer: $x < -2$ or $x > 2$

1. read from the graph at A and B

(d) Answer: $y \geq 3$

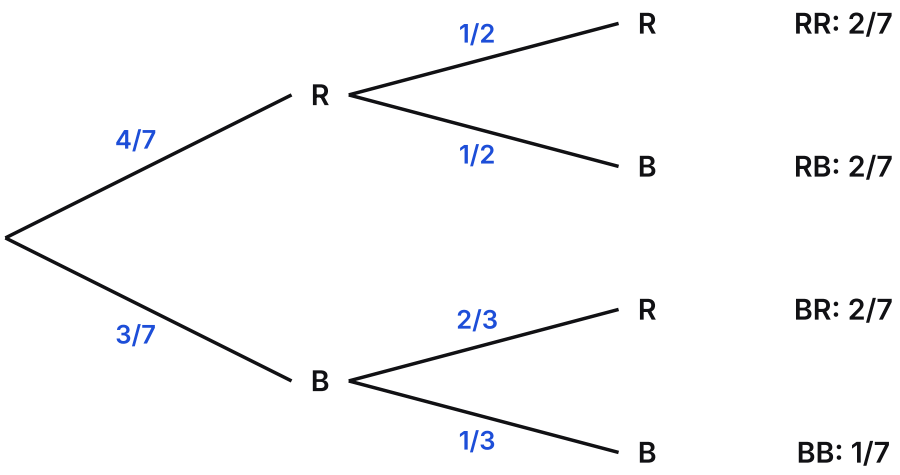
1. the turning point is a minimum: $y = 3$

(e) Answer: $t < 3$

1. the line $y = t$ must miss the parabola entirely

QUESTION 10

(a) Answer: See the tree below.



1. first draw: red $\frac{4}{7}$, blue $\frac{3}{7}$
2. second draw: 6 balls are left, one fewer of the colour taken

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(b) Answer: $\frac{3}{7}$

1. $\mathrm{P}(\mathrm{RR}) + \mathrm{P}(\mathrm{BB}) = \frac{2}{7} + \frac{1}{7}$
2. $= \frac{3}{7}$

(c) Answer: $\mathrm{P}(\mathrm{A \text{ or } B}) = 0.9$; $\mathrm{P}(\mathrm{A \text{ and } B}) = 0$

1. $\mathrm{P}(\mathrm{A \text{ or } B}) = 1 - \mathrm{P}(\mathrm{A}' \text{ and } \mathrm{B}') = 1 - 0.1 = 0.9$
2. $\mathrm{P}(\mathrm{B}) = 1 - \mathrm{P}(\mathrm{B}') = 0.45$
3. $\mathrm{P}(\mathrm{A \text{ and } B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - \mathrm{P}(\mathrm{A \text{ or } B}) = 0.45 + 0.45 - 0.9 = 0$

(d) Answer: Mutually exclusive, and not independent

1. $\mathrm{P}(\mathrm{A \text{ and } B}) = 0$: they cannot happen together
2. $\mathrm{P}(\mathrm{A}) \times \mathrm{P}(\mathrm{B}) = 0.45 \times 0.45 = 0.2025 \neq \mathrm{P}(\mathrm{A \text{ and } B})$
3. mutually exclusive, and not independent

(e) Answer: 4

1. $19 + 12 - x + 3 = 30$
2. $x = 4$

(f) Answer: $\frac{23}{30}$

1. only Life Sciences: 15; only Physical Sciences: 8
2. $\mathrm{P} = \frac{23}{30} = \frac{23}{30}$

THE PROBABILITY RULES

QUESTION 11

(a) Answer: $f'(x) = -4x - 3$

1. $f(x + h) = -2(x + h)^2 - 3(x + h) + 1$
2. $= -2x^2 - 4xh - 2h^2 - 3x - 3h + 1$
3. $f(x + h) - f(x) = -4xh - 2h^2 - 3h$
4. $f'(x) = \lim_{h \to 0} \frac{-4xh - 2h^2 - 3h}{h}$
5. $= \lim_{h \to 0} (-4x - 2h - 3)$
6. $= -4x - 3$

(b) Answer: $\frac{dy}{dx} = \frac{3}{2}x^{1/2} - \frac{3}{2}x^{-1/2} - x^{-3/2}$

1. $y = x^{3/2} - 3x^{1/2} + 2x^{-1/2}$

(c) Answer: $y = 5x + 9$

1. $f(-2) = -1$, so the point is $(-2; -1)$
2. $m = f'(-2) = 5$
3. $y + 1 = 5(x + 2)$

(d) Answer: -7

1. $f(-1) = 2$ and $f(3) = -26$
2. $\frac{-26 - (2)}{3 - (-1)} = -7$

THE DERIVATIVE FROM FIRST PRINCIPLES