IEB mock Paper I (second paper)

Time: 3 hours · 150 marks

Answer all the questions. Show all calculations. You may use an approved calculator unless a question says otherwise. Diagrams are not necessarily drawn to scale.

Section A

[73 marks]

QUESTION 1

Give answers correct to two decimal places.

(a) You deposit R1 900 at the end of every month for 10 years into an account earning 7% per annum compounded monthly. You then stop the deposits, and for the next 3 years the account earns 6% per annum compounded quarterly. Calculate the amount in the account at the end of these 13 years.

(5)

(b) From then on the account earns 5.5% per annum compounded monthly, and you withdraw R3 100 at the end of every month. How many full withdrawals of R3 100 can you make?

(4)

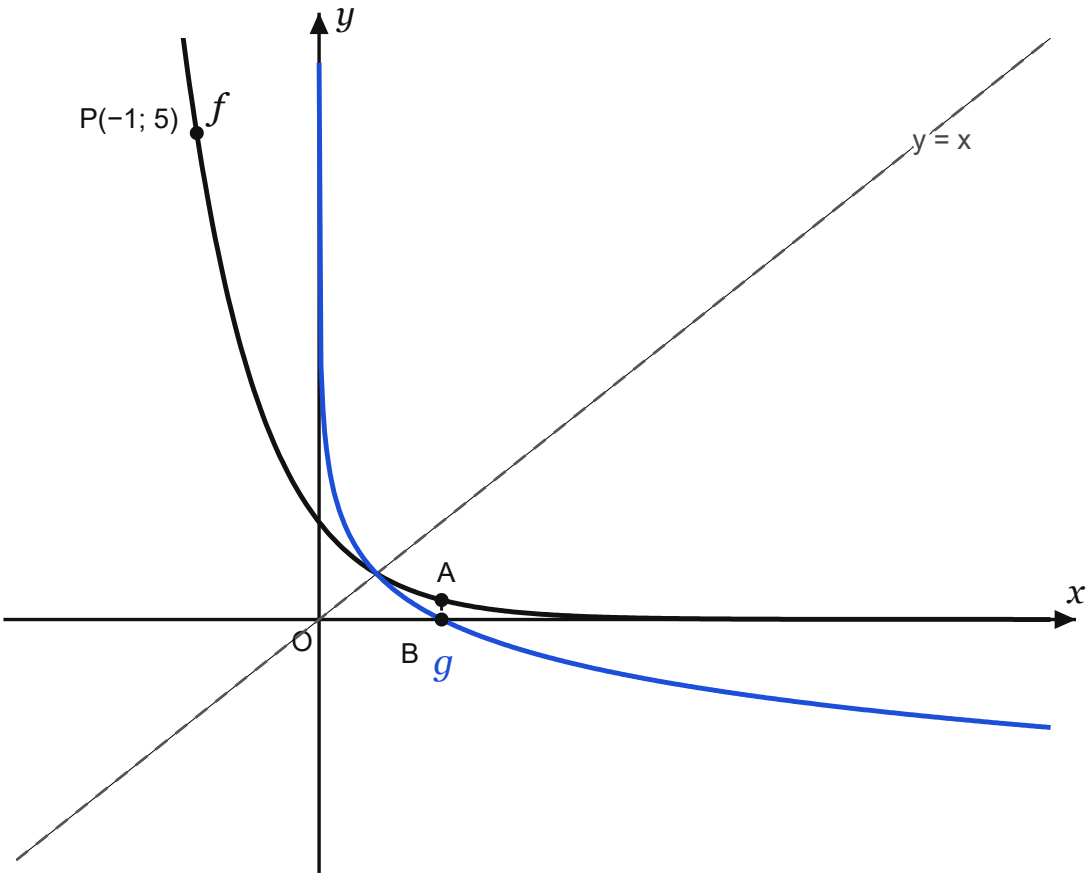
(c) A savings account earns 13% per annum compounded half-yearly. The first deposit of R6 500 is made six months after the account is opened, and the same amount is deposited at the end of every six months after that. How much is in the account 3 years after it was opened, just after the deposit made then?

(4)

[13]

QUESTION 2

In the diagram below, $f(x) = b^x$ and g, the reflection of f in the line $y = x$, are drawn. $\mathrm{P}(-1; 5)$ lies on f.



(a) Determine the value of b.

(2)

(b) Write down the equation of g in the form $y = \ldots$

(2)

(c) Write down the domain of g.

(1)

(d) For which values of x is $g(x) > 2$?

(2)

(e) The line $x = 1$ meets f at A and g at B. Determine the length of AB.

(3)

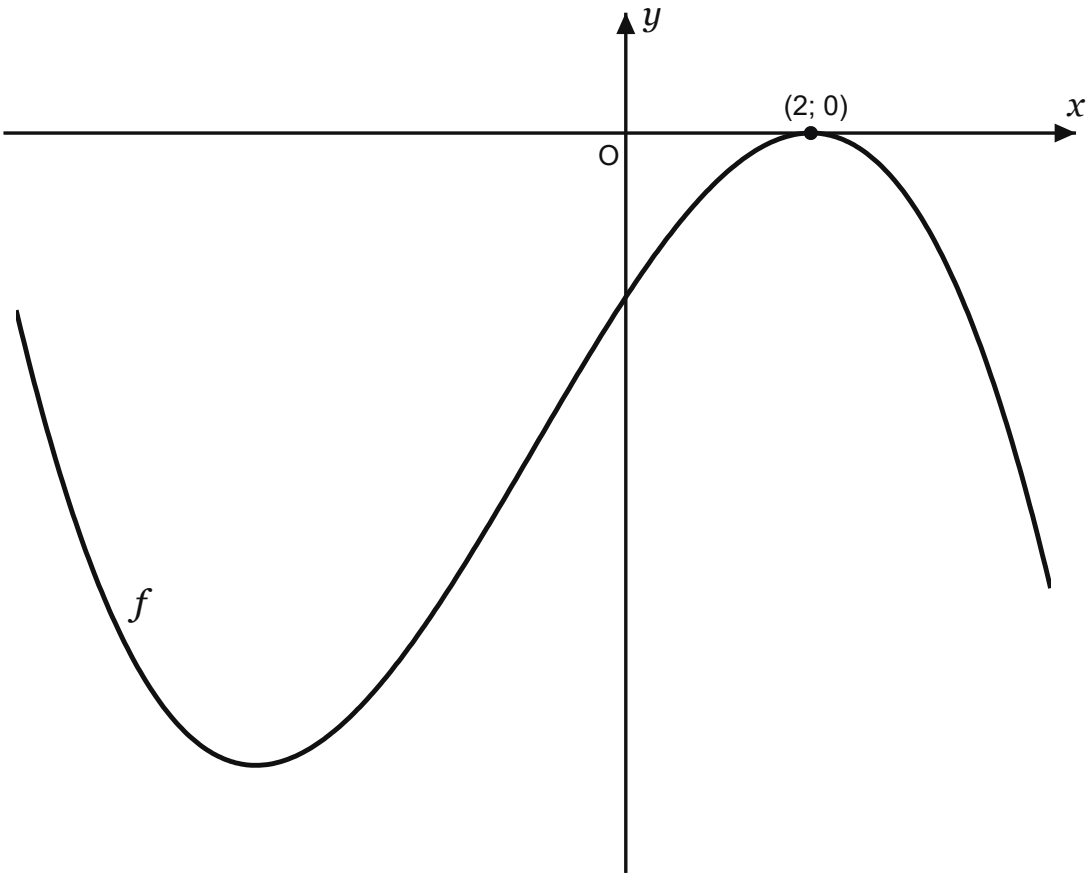
(f) $h(x) = f(x) - 25$. Write down the equation of the asymptote of h and determine the x-intercept of h.

(3)

[13]

QUESTION 3

In the diagram below, the graph of $f(x) = -x^3 - 3x^2 + 24x + d$ is drawn, where d is a constant. (2; 0) is a stationary point of f.



(a) Determine the value of d.

(2)

(b) Determine the coordinates of the other stationary point of f.

(4)

(c) Determine the x-coordinate of the point of inflection of f.

(2)

(d) For which values of k will $f(x) = k$ have exactly one real root?

(3)

(e) Determine the equation of the tangent to f at $x = 0$.

(3)

[14]

QUESTION 4

(a) Solve for x: $(5x - 7)(x^2 + 6x + 9) = 0$

(3)

(b) Use the quadratic formula to solve for x, correct to two decimal places, if $4x^2 = 9x + 4$. Show all working.

(4)

(c) Hence, solve for x if $4x^2 - 9x - 4 > 0$.

(2)

(d) Show that the roots of $3x^2 + px = 2$ are real and unequal for all real values of p.

(3)

(e) Determine the values of p for which $x^2 - (p - 1)x + 36 = 0$ has equal roots.

(4)

[16]

QUESTION 5

Consider the arithmetic series $7 + 11 + 15 + 19 + \ldots + 223$

(a) Determine a formula for T_n, the general term of the series.

(2)

(b) Determine the number of terms in the series.

(3)

(c) Calculate the sum of the series.

(3)

(d) Write the series in sigma notation.

(2)

(e) The first five terms of a quadratic sequence are 17; 28; 43; 62; 85; ... Determine T_n, the nth term.

(4)

(f) Determine whether 1184 is a term of this quadratic sequence.

(3)

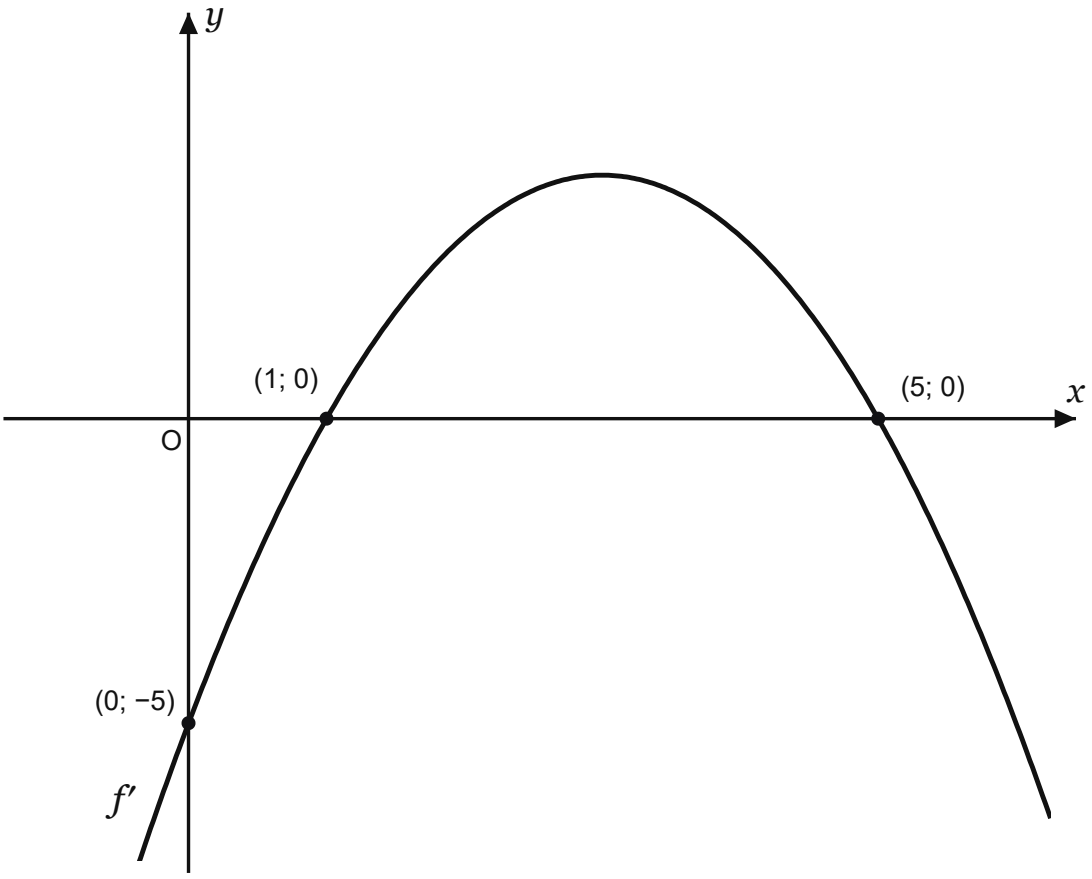
[17]

Section B

[77 marks]

QUESTION 6

The graph of $y = f'(x)$, the derivative of a cubic function f, is drawn below. It cuts the x-axis at (1; 0) and (5; 0) and the y-axis at $(0; -5)$.



(a) For which values of x is f increasing?

(2)

(b) Write down the x-coordinates of the turning points of f, and say which one is a local maximum.

(3)

(c) Determine the x-coordinate of the point of inflection of f.

(2)

(d) For which values of x is f concave up?

(2)

(e) Write down the gradient of the tangent to f at $x = 0$.

(1)

[10]

QUESTION 7

(a) Solve for x and y if $x - y = 0$ and $xy = 1$.

(6)

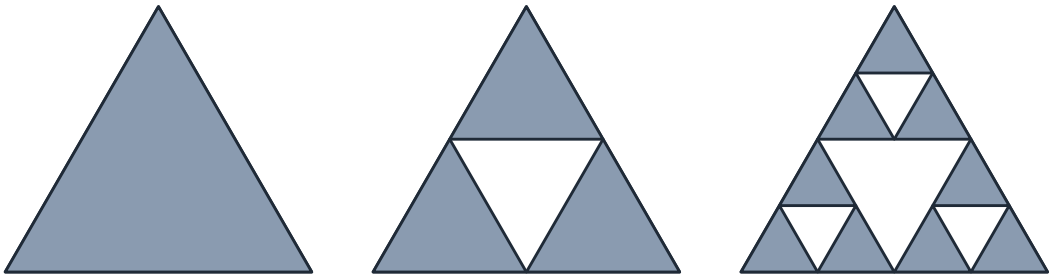
(b) The 288 chairs in a school hall are set out in equal rows. If there were 3 fewer rows, each row would need 8 more chairs. How many rows are there?

(5)

[11]

QUESTION 8

Stage 1 of a pattern is a shaded equilateral triangle with sides of 36 cm. To make each next stage, every shaded triangle is divided into four equal triangles by joining the midpoints of its sides, and the middle one is removed. Stages 1, 2 and 3 are shown.



1 2 3

(a) Show that the area of the triangle in stage 1 is $324\sqrt{3}$ cm².

(2)

(b) How many shaded triangles are there in stage 6?

(2)

(c) Show that the areas removed in making stages 2, 3, 4, ... form a geometric series, and calculate the total area removed in making stages 2 to 7, correct to two decimal places.

(3)

(d) If the pattern could go on for ever, what total area would be removed? What does this tell you about the shaded area?

(2)

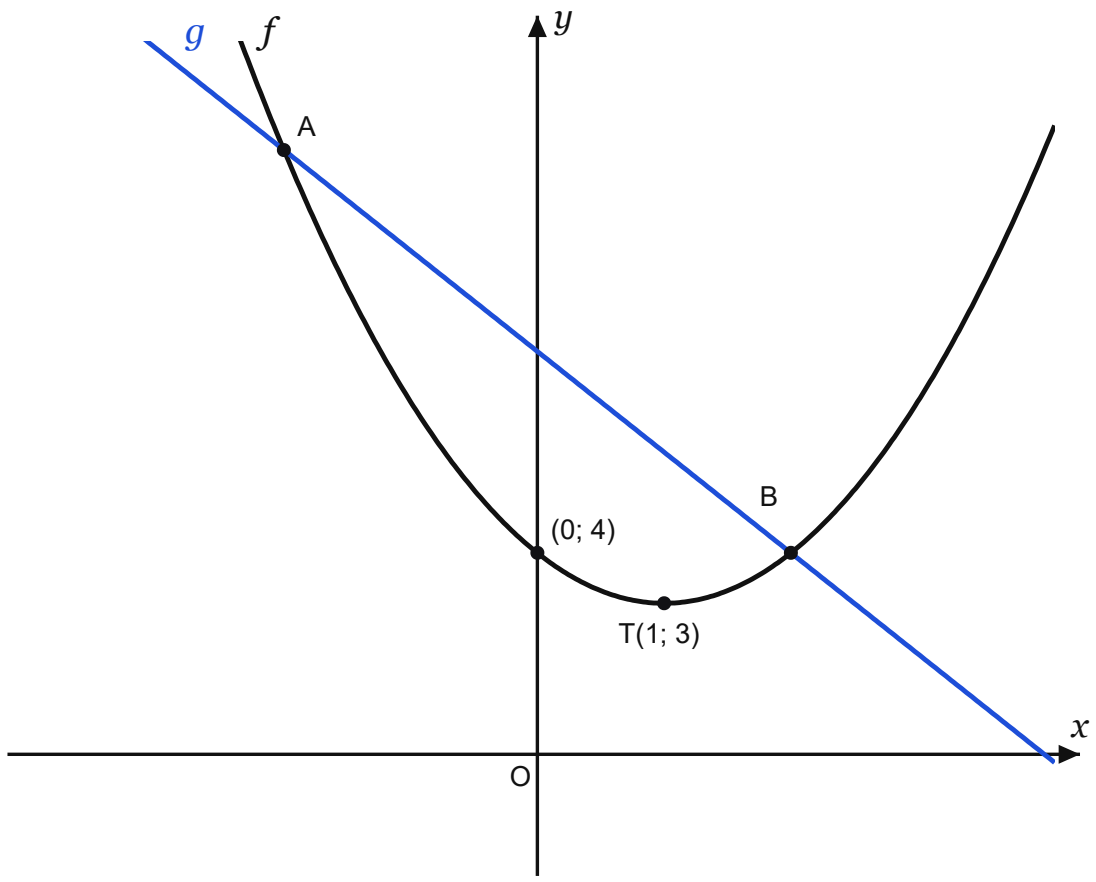
(e) In which stage is the total perimeter of the shaded triangles for the first time more than 2550 cm?

(3)

[12]

QUESTION 9

In the diagram below, the parabola f with turning point $\mathrm{T}(1;\ 3)$ and y-intercept (0; 4), and the straight line g, are drawn. The graphs meet at A and B.



(a) Determine the equation of f in the form $f(x) = a(x - p)^2 + q$.

(3)

(b) $g(x) = -2x + 8$. Determine the coordinates of A and B, the points where f and g meet.

(5)

(c) For which values of x is $f(x) > g(x)$?

(2)

(d) Write down the range of f.

(1)

(e) For which values of t will $f(x) = t$ have no real roots?

(2)

[13]

QUESTION 10

(a) A bag holds 4 red and 3 blue balls. Two balls are taken out one after the other, without replacement. Draw a tree diagram to show all the outcomes and their probabilities.

(3)

(b) Calculate the probability that the two balls are the same colour.

(2)

(c) For two events A and B, $\mathrm{P(A)} = 0.45$, $\mathrm{P(B')} = 0.55$ and $\mathrm{P(A' \text{ and } B')} = 0.1$. Determine $\mathrm{P(A \text{ or } B)}$ and $\mathrm{P(A \text{ and } B)}$.

(3)

(d) Are A and B mutually exclusive? Are they independent? Give reasons.

(3)

(e) In a group of 30 learners, 19 take Life Sciences, 12 take Physical Sciences and 3 take neither. How many take both?

(2)

(f) A learner is chosen at random. Calculate the probability that the learner takes exactly one of the two subjects.

(2)

[15]

QUESTION 11

(a) Given $f(x) = -2x^2 - 3x + 1$. Determine $f'(x)$ from first principles.

(5)

(b) Determine $\frac{dy}{dx}$ if $y = \frac{x^2 - 3x + 2}{\sqrt{x}}$.

(4)

(c) Determine the equation of the tangent to f at $x = -2$.

(4)

(d) Determine the average gradient of f between $x = -1$ and $x = 3$.

(3)

[16]