IEB mock Paper II — Memorandum

Every answer below was confirmed by an independent verifier at the moment the question was made.

QUESTION 1

(a) Answer: (4; 0)

1. (4; 0) lies far from the trend of the other points

(b) Answer: $y = 5.892 + 2.737x$

1. $\mathrm{A} = 5.892$
2. $\mathrm{B} = 2.737$

(c) Answer: $r = 0.99$

(d) Answer: y increases by about 2.737

1. $\mathrm{B} = 2.737$ is the change in y for each 1 added to x

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(e) Answer: Weaker

1. with the outlier $r = 0.91$; without it $r = 0.99$
2. $|r|$ is smaller with the outlier: it weakened the correlation

SCATTER PLOTS

QUESTION 2

(a) Answer: $\mathrm{P\hat{B}A} = x$

1. $\mathrm{P\hat{B}A} = \mathrm{A\hat{C}B} = x$ (tan chord theorem)

(b) Answer: $\mathrm{A\hat{P}B} = 180^\circ - 2x$

1. $\mathrm{PA} = \mathrm{PB}$ (tans from common pt)
2. $\mathrm{P\hat{A}B} = \mathrm{P\hat{B}A} = x$ (∠s opp equal sides)
3. $\therefore \mathrm{A\hat{P}B} = 180^\circ - 2x$ (sum ∠s of Δ)

(c) Answer: Yes: $\mathrm{BA} = \mathrm{BC}$, so ΔABC is isosceles

1. $\mathrm{C\hat{A}B} = \mathrm{A\hat{B}P} = x$ (alt ∠s; AC ∥ PB)
2. $\mathrm{A\hat{C}B} = x$ (given)
3. $\therefore \mathrm{C\hat{A}B} = \mathrm{A\hat{C}B}$
4. $\therefore \mathrm{BA} = \mathrm{BC}$ (sides opp equal ∠s)

(d) Answer: PAOB is a cyclic quadrilateral

1. $\mathrm{O\hat{A}P} = 90^\circ$ (tan $\perp$ radius)
2. $\mathrm{O\hat{B}P} = 90^\circ$ (tan $\perp$ radius)
3. $\therefore \mathrm{O\hat{A}P} + \mathrm{O\hat{B}P} = 180^\circ$
4. $\therefore \mathrm{PAOB}$ is a cyclic quad (opp ∠s of quad supp)

(e) Answer: $\mathrm{A\hat{O}B} = 76^\circ$

1. $\mathrm{A\hat{O}B} = 2 \times \mathrm{A\hat{C}B} = 76^\circ$ (∠ at centre $= 2 \times \angle$ at circumference)

QUESTION 3

(a) Answer: $-\cos\theta$

1. $\cos(180^\circ + \theta) = -\cos\theta$
2. $\cos(90^\circ + \theta) = -\sin\theta$
3. $\sin(360^\circ - \theta) = -\sin\theta$
4. $= -\cos\theta$

(b) Answer: $-\frac{\sqrt{3}}{2}$

1. $\cos 113^\circ \cos 37^\circ - \sin 113^\circ \sin 37^\circ = \cos(113^\circ + 37^\circ)$
2. $= \cos 150^\circ = -\frac{\sqrt{3}}{2}$

(c) Answer: $-\frac{\sqrt{3}}{2}$

1. $\cos^2 75^\circ - \sin^2 75^\circ = \cos 150^\circ$
2. $= -\frac{\sqrt{3}}{2}$

(d) Answer: $\theta = 30^\circ + k{\cdot}180^\circ,\ k \in \mathbb{Z}$

1. $\tan\theta = \frac{1}{\sqrt{3}}$

QUESTION 4

(a) Answer: $y = 3x - 15$

1. $m = \frac{6 - (-9)}{7 - 2} = 3$
2. $-9 = (3)(2) + c$, so $c = -15$

(b) Answer: $\frac{75}{2}$ square units

1. $\mathrm{C}(5;\, 0)$ and $\mathrm{D}(0;\, -15)$
2. area $= ½ \times 5 \times 15 = \frac{75}{2}$

(c) Answer: $y = -\frac{1}{3}x + \frac{5}{3}$

1. PF $\perp$ AB: $m_{\mathrm{PF}} = -\frac{1}{3} = -\frac{1}{3}$
2. $3 = (-\frac{1}{3})(-4) + c$, so $c = \frac{5}{3}$

(d) Answer: $\mathrm{F}(5;\, 0)$

1. $3x - 15 = -\frac{1}{3}x + \frac{5}{3}$
2. $x = 5$, $y = 0$

(e) Answer: $\mathrm{PF} = 3\sqrt{10}$ units

1. $\mathrm{PF}^2 = (-4 - 5)^2 + (3 - 0)^2 = 90$
2. $\mathrm{PF} = 3\sqrt{10}$

(f) Answer: 75 square units

1. $\mathrm{AB} = 5\sqrt{10}$ and the height is $\mathrm{PF} = 3\sqrt{10}$
2. area $= ½ \times \mathrm{AB} \times \mathrm{PF} = 75$

QUESTION 5

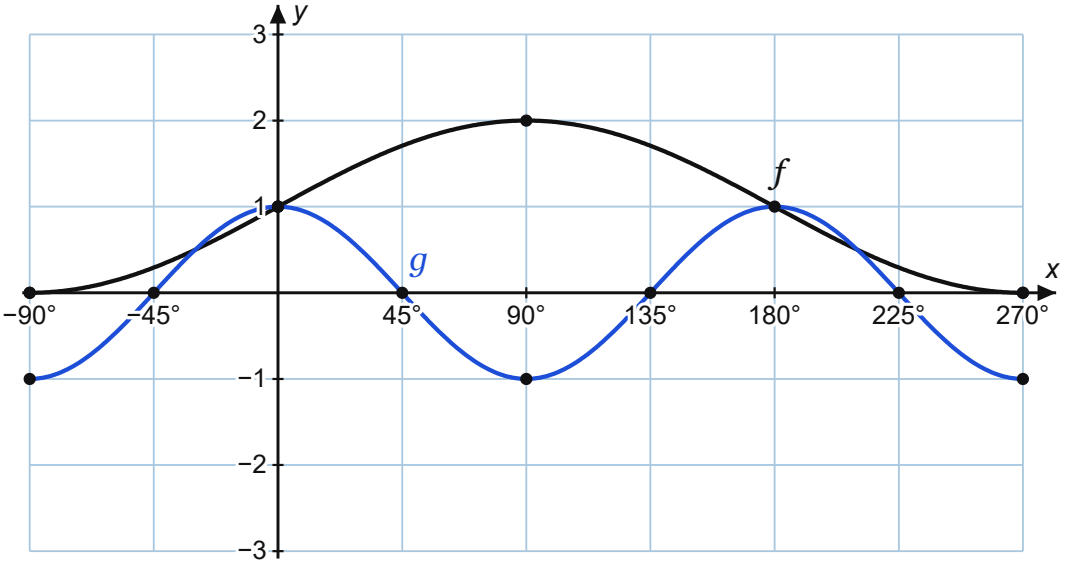
(a) Answer: $x = k{\cdot}180^\circ$ or $x = -30^\circ + k{\cdot}360^\circ$ or $x = 210^\circ + k{\cdot}360^\circ$

1. $\sin x + 1 = \cos 2x$
2. $1 + \sin x = 1 - 2\sin^2 x$
3. $2\sin^2 x + \sin x = 0$
4. $\sin x(2 \sin x + 1) = 0$
5. $\sin x = 0$: $x = k{\cdot}180^\circ$
6. or $\sin x = -\frac{1}{2}$: $x = -30^\circ + k{\cdot}360^\circ$ or $x = 210^\circ + k{\cdot}360^\circ$

(b) Answer: $x = -30^\circ$ or $x = 0^\circ$ or $x = 180^\circ$ or $x = 210^\circ$

1. $k = \ldots, -1, 0, 1, \ldots$ in $x = k{\cdot}180^\circ$ or $x = -30^\circ + k{\cdot}360^\circ$ or $x = 210^\circ + k{\cdot}360^\circ$
2. In $[-90^\circ; 270^\circ]$: $x = -30^\circ$ or $x = 0^\circ$ or $x = 180^\circ$ or $x = 210^\circ$

(c) Answer: See the sketch below.



1. $f(x) = \sin x + 1$
2. f: x-intercepts $(-90^\circ; 0)$, $(270^\circ; 0)$
3. f: y-intercept (0°; 1)
4. f: turning points (90°; 2)
5. f: end points $(-90^\circ; 0)$, $(270^\circ; 0)$
6. $g(x) = \cos 2x$

7. g: x-intercepts $(-45^\circ; 0)$, $(45^\circ; 0)$, $(135^\circ; 0)$, $(225^\circ; 0)$
8. g: y-intercept (0°; 1)
9. g: turning points $(0^\circ; 1)$, $(90^\circ; -1)$, $(180^\circ; 1)$
0. g: end points $(-90^\circ; -1)$, $(270^\circ; -1)$

(d) Answer: $x = -90^\circ$ or $-45^\circ \leq x \leq 45^\circ$ or $135^\circ \leq x \leq 225^\circ$ or $x = 270^\circ$

1. f and g have the same sign, or one of them is 0, read from the sketch

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(e) Answer: $-3 \leq y \leq 1$

1. $-1 \leq g(x) \leq 1$
2. $-3 \leq 2g(x) - 1 \leq 1$

QUESTION 6

(a) Answer: $(x-3)^2+(y-4)^2=25$

1. it passes through O: $r=\text{PO}=\sqrt{(3)^2+(4)^2}=5$

(b) Answer: $\text{PQ}=10$ units

1. $\text{PQ}=\sqrt{(-5-3)^2+(-2-4)^2}$
2. $=10$

(c) Answer: See the working below.

1. $r_{\text{P}}+r_{\text{Q}}=5+5=10=\text{PQ}$
2. $\therefore$ the circles touch externally

(d) Answer: $\text{T}(-1;\,1)$

1. T lies on PQ, 5 units from P: $\frac{5}{10}$ of the way to Q
2. $(3+(\frac{5}{10})(-8);\,4+(\frac{5}{10})(-6))=(-1;\,1)$

(e) Answer: $y=-\frac{4}{3}x-\frac{1}{3}$

1. $m_{\text{PQ}}=\frac{3}{4}$, so the tangent has $m=-\frac{4}{3}$
2. $1=(-\frac{4}{3})(-1)+c$, so $c=-\frac{1}{3}$

QUESTION 7

(a) Answer: See the proof below.

1. Required to prove: $\mathrm{B\hat{A}D} + \mathrm{B\hat{C}D} = 180^\circ$
2. Construction: Join OB and OD.
3. $\hat{\mathrm{O}}_1 = 2 \times \mathrm{B\hat{A}D}$ (∠ at centre $= 2 \times \angle$ at circumference)
4. reflex $\mathrm{B\hat{O}D} = 2 \times \mathrm{B\hat{C}D}$ (∠ at centre $= 2 \times \angle$ at circumference)
5. $\hat{\mathrm{O}}_1$ + reflex $\mathrm{B\hat{O}D} = 360^\circ$ (∠s round a pt)
6. $\therefore 2 \times \mathrm{B\hat{A}D} + 2 \times \mathrm{B\hat{C}D} = 360^\circ$
7. $\therefore \mathrm{B\hat{A}D} + \mathrm{B\hat{C}D} = 180^\circ$

(b) Answer: $\mathrm{A\hat{E}F} = \mathrm{A\hat{B}C}$

1. $\mathrm{A\hat{E}F} = \mathrm{A\hat{B}C}$ (corresp ∠s; BC ∥ EF)

(c) Answer: ADFE is a cyclic quadrilateral

1. $\mathrm{A\hat{B}C} + \mathrm{A\hat{D}C} = 180^\circ$ (opp ∠s of cyclic quad)
2. $\mathrm{A\hat{E}F} = \mathrm{A\hat{B}C}$ (proved)
3. $\therefore \mathrm{A\hat{E}F} + \mathrm{A\hat{D}F} = 180^\circ$
4. $\therefore \mathrm{ADFE}$ is a cyclic quad (opp ∠s of quad supp)

(d) Answer: $\mathrm{A\hat{E}F} = 101^\circ$

1. $\mathrm{A\hat{B}C} = 180^\circ - \mathrm{A\hat{D}C} = 101^\circ$ (opp ∠s of cyclic quad)
2. $\therefore \mathrm{A\hat{E}F} = 101^\circ$ (proved)

PROVING THE THEOREM ITSELF, NOT USING IT

QUESTION 8

(a) Answer: See the explanation below.

1. $\mathrm{A\hat{O}B} = 90^\circ$: the axes are perpendicular
2. AB subtends a right angle at O, a point on the circle, so AB is a diameter (converse ∠ in semi-circle)

(b) Answer: $\mathrm{M}(-3;\ -5);\ r = \sqrt{34}$

1. M is the midpoint of AB: $(\frac{-6+0}{2};\ \frac{0+(-10)}{2}) = (-3;\ -5)$
2. $r = \mathrm{MO} = \sqrt{(-3)^2 + (-5)^2} = \sqrt{34}$

(c) Answer: See the working below.

1. $(x+3)^2 + (y+5)^2 = 34$
2. $x^2 + 6x + 9 + y^2 + 10y + 25 = 34$
3. $x^2 + y^2 + 6x + 10y = 0$

(d) Answer: $y = -\frac{3}{5}x$

1. $m_{\mathrm{MO}} = \frac{-5-0}{-3-0} = \frac{5}{3}$
2. the tangent is perpendicular to the radius MO: $m = -\frac{3}{5}$
3. it passes through O: $y = -\frac{3}{5}x$

(e) Answer: $\mathrm{C}(2;\ -2)$

1. substitute $y = -x$: $x^2 + (-x)^2 + 6x - 10x = 0$
2. $2x(x-2) = 0$
3. $x = 0$ (the point O) or $x = 2$

THE MIDPOINT OF A LINE SEGMENT

QUESTION 9

(a) Answer: $\mathrm{C\hat{A}B} = x$

1. $\mathrm{C\hat{A}B} = \hat{\mathrm{C}}_3 = x$ (tan chord theorem)

(b) Answer: $\hat{\mathrm{C}}_2 = 90^\circ - x$

1. $\mathrm{A\hat{C}B} = 90^\circ$ (∠ in semi-circle)
2. $\hat{\mathrm{C}}_1 = \mathrm{C\hat{A}O} = x$ (∠s opp equal sides)
3. $\therefore \hat{\mathrm{C}}_2 = 90^\circ - x$

(c) Answer: ΔDCB ||| ΔDAC

1. In ΔDCB and ΔDAC:
2. $\mathrm{C\hat{D}B} = \mathrm{A\hat{D}C}$ (common)
3. $\hat{\mathrm{C}}_3 = \mathrm{C\hat{A}B} = x$ (proved)
4. $\therefore \Delta\mathrm{DCB} \,|||\, \Delta\mathrm{DAC}$ $(\angle\angle\angle)$

(d) Answer: $\mathrm{DC}^2 = \mathrm{DB} \cdot \mathrm{DA}$

1. $\frac{\mathrm{DC}}{\mathrm{DA}} = \frac{\mathrm{DB}}{\mathrm{DC}}$ $(|||\ \Delta\mathrm{s})$
2. $\therefore \mathrm{DC}^2 = \mathrm{DB} \cdot \mathrm{DA}$

(e) Answer: $r = 3$ units

1. $4^2 = 2(2 + 2r)$ (proved)
2. $16 = 4 + 4r$
3. $\therefore r = 3$ units

THE ANGLE IN A SEMI-CIRCLE

QUESTION 10

(a) Answer: 60

1. the ogive ends at a cumulative frequency of 60

(b) Answer: about 17.9

1. the median is at a cumulative frequency of $\frac{60}{2} = 30$
2. read across to the ogive and down: about 17.9

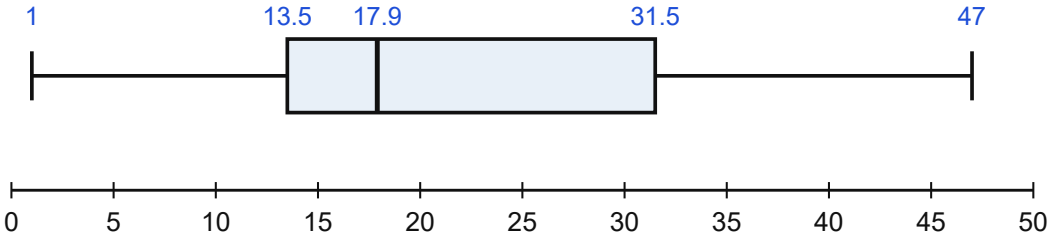
(c) Answer: about 18.0

1. Q1 at 15: about 13.5; Q3 at 45: about 31.5
2. $\mathrm{IQR} = 31.5 - 13.5 = 18.0$

(d) Answer: 19

1. read the ogive at 20 and at 40: 37 and 56
2. $56 - 37 = 19$

(e) Answer: See the box-and-whisker plot below.



1. minimum 1, Q1 ≈ 13.5, median ≈ 17.9, Q3 ≈ 31.5, maximum 47

(f) Answer: 95%

1. 3 values lie below 10, so 57 values are 10 or more
2. $\frac{57}{60} \times 100 = 95\%$

(g) Answer: Skewed to the right (positively)

1. $\mathrm{Q3} -$ median ≈ 13.6 and median $- \mathrm{Q1} \approx 4.4$
2. the upper half of the box is longer

QUESTION 11

(a) Answer: $\mathrm{BD} = 7.37$ cm

1. $\mathrm{BD}^2 = \mathrm{AB}^2 + \mathrm{AD}^2 - 2(\mathrm{AB})(\mathrm{AD})\cos \mathrm{B\hat{A}D}$ (cosine rule)
2. $= 5^2 + 9^2 - 2(5)(9)\cos 55^\circ$

(b) Answer: $\mathrm{B\hat{C}D} = 125^\circ$ (opp $\angle s$ of cyclic quad)

1. $\mathrm{B\hat{C}D} = 180^\circ - 55^\circ = 125^\circ$ (opp $\angle s$ of cyclic quad)

(c) Answer: $\mathrm{B\hat{D}C} = 41.80^\circ$

1. $\frac{\sin \mathrm{B\hat{D}C}}{\mathrm{BC}} = \frac{\sin \mathrm{B\hat{C}D}}{\mathrm{BD}}$ (sine rule)
2. $\sin \mathrm{B\hat{D}C} = \frac{6 \sin 125^\circ}{7.37}$

(d) Answer: $\mathrm{CD} = 2.06$ cm

1. $\mathrm{C\hat{B}D} = 180^\circ - 125^\circ - 41.80^\circ = 13.20^\circ$ (sum $\angle s$ of Δ)
2. $\frac{\mathrm{CD}}{\sin \mathrm{C\hat{B}D}} = \frac{\mathrm{BD}}{\sin \mathrm{B\hat{C}D}}$ (sine rule)
3. $\mathrm{CD} = \frac{7.37 \sin 13.20^\circ}{\sin 125^\circ} = 2.06$ cm

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(e) Answer: area $\mathrm{ABCD} = 23.48$ cm²

1. area $\Delta\mathrm{ABD} = (\frac{1}{2})(5)(9)\sin 55^\circ$ (area rule)
2. area $\Delta\mathrm{BCD} = (\frac{1}{2})(6)(7.37)\sin 13.20^\circ$ (area rule)