IEB mock Paper II

Time: 3 hours · 150 marks

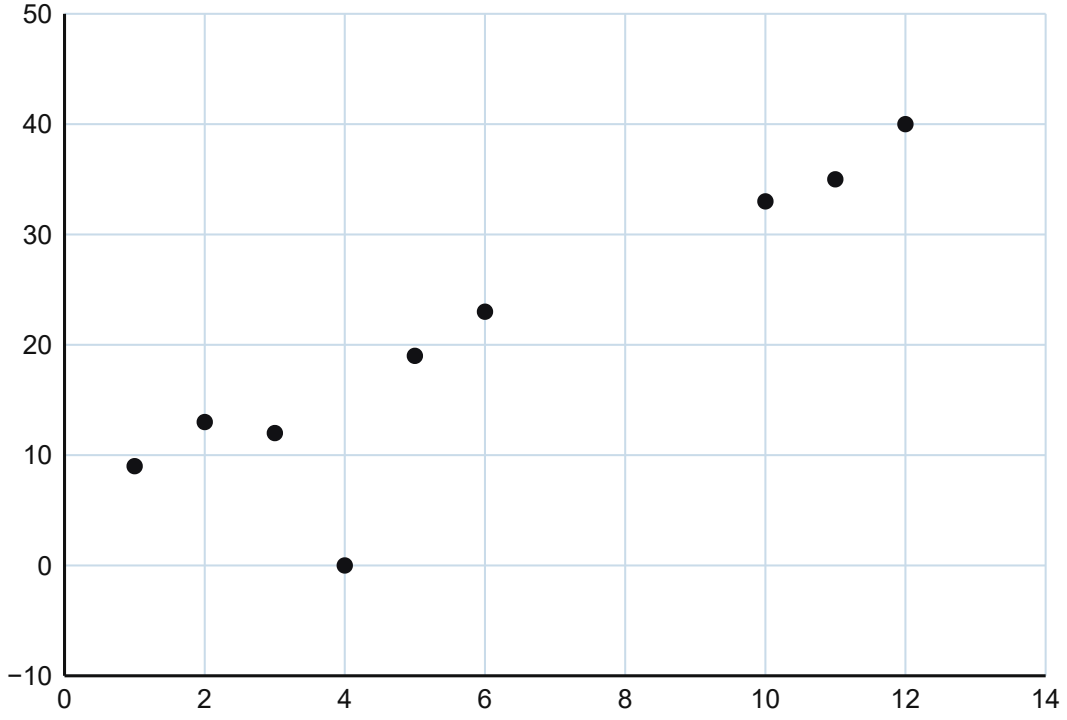
Answer all the questions. Show all calculations. You may use an approved calculator unless a question says otherwise. Diagrams are not necessarily drawn to scale.

Section A

[70 marks]

QUESTION 1

The scatter plot shows the number of hours (x) a learner studied and the learner's test mark out of 50 (y): $(1; 9)$; $(2; 13)$; $(3; 12)$; $(4; 0)$; $(5; 19)$; $(6; 23)$; $(10; 33)$; $(11; 35)$; $(12; 40)$.



(a) Identify the outlier in the data.

(1)

(b) The outlier is removed. Determine the equation of the least squares regression line for the remaining data, in the form $y = \mathrm{A} + \mathrm{Bx}$, with A and B correct to three decimal places.

(3)

(c) Determine the correlation coefficient r of the remaining data, correct to two decimal places.

(1)

(d) By how much does y change, on average, for each increase of 1 in x? Use your equation.

(2)

(e) Did the outlier make the correlation stronger or weaker? Explain.

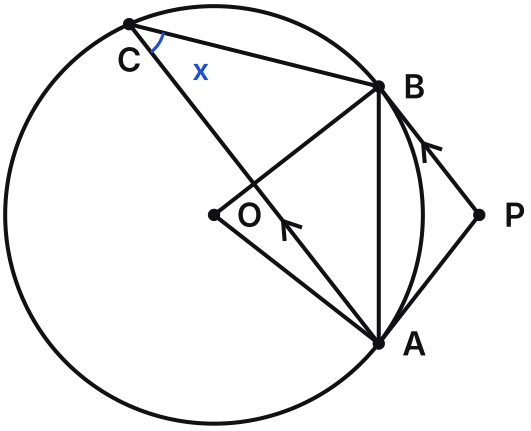
(2)

[9]

QUESTION 2

In the diagram:

- O is the centre of the circle.
- PA and PB are tangents to the circle at A and B respectively.
- C is a point on the circle such that AC ∥ PB.
- $\mathrm{A\hat{C}B} = x$



State all relevant reasons with your statements.

(a) Express P$\hat{B}$A in terms of x.

(2)

(b) Express A$\hat{P}$B in terms of x.

(3)

(c) Is ΔABC an isosceles triangle? Explain.

(4)

(d) Prove that PAOB is a cyclic quadrilateral.

(3)

(e) If $x = 38^\circ$, determine the size of A$\hat{O}$B.

(2)

[14]

QUESTION 3

(a) Simplify fully to a single trigonometric ratio: $\frac{\cos(180^\circ + \theta) \cdot \cos(90^\circ + \theta)}{\sin(360^\circ - \theta)}$

(6)

(b) Determine, without the use of a calculator, the value of $\cos 113^\circ \cos 37^\circ - \sin 113^\circ \sin 37^\circ$.

(3)

(c) Determine, without the use of a calculator, the value of $\cos^2 75^\circ - \sin^2 75^\circ$.

(2)

(d) Determine the general solution of $\sqrt{3} \sin \theta = \cos \theta$.

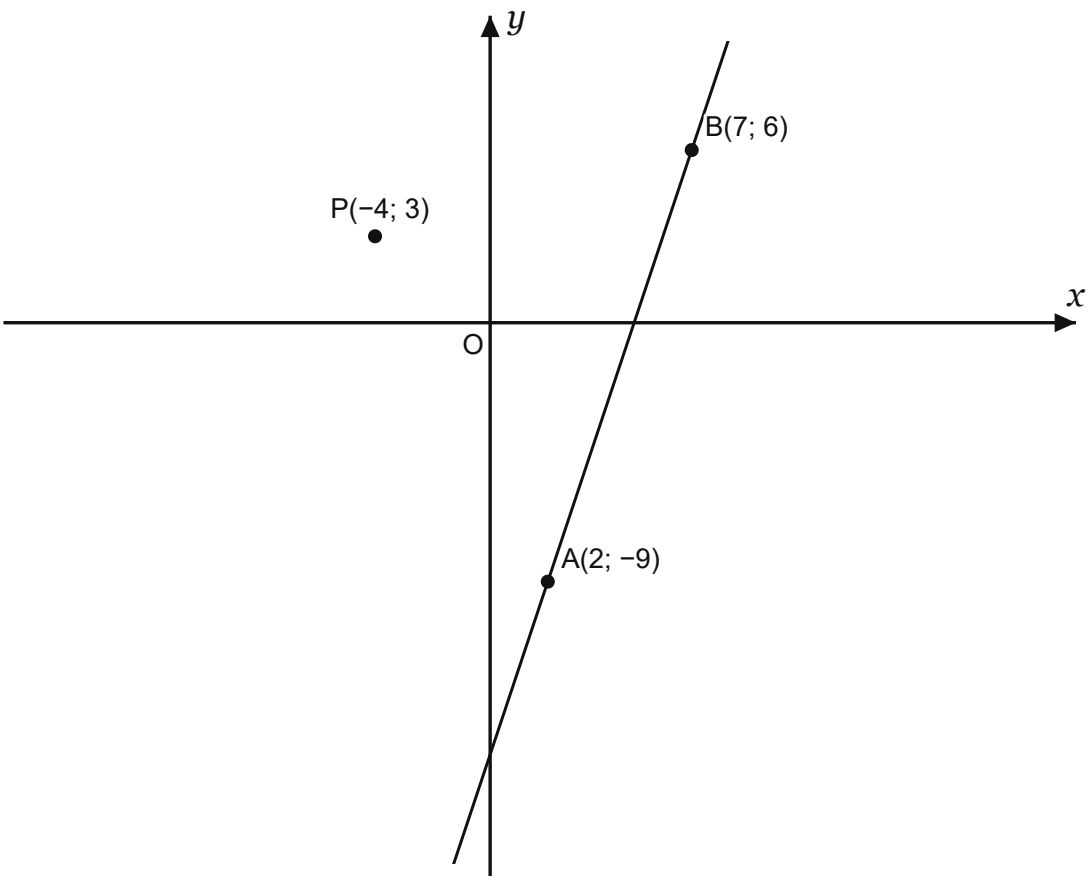
(3)

[14]

QUESTION 4

In the diagram:

- The straight line through $\mathrm{A}(2;\ -9)$ and $\mathrm{B}(7;\ 6)$ is drawn.
- $\mathrm{P}(-4;\ 3)$ is a point that does not lie on AB.



(a) Determine the equation of the line AB.

(2)

(b) The line AB cuts the x-axis at C and the y-axis at D. Calculate the area of ΔOCD, where O is the origin.

(3)

(c) F is the foot of the perpendicular from P to AB. Determine the equation of PF.

(3)

(d) Determine the coordinates of F.

(3)

(e) Determine the shortest distance from P to the line AB, in simplest surd form.

(2)

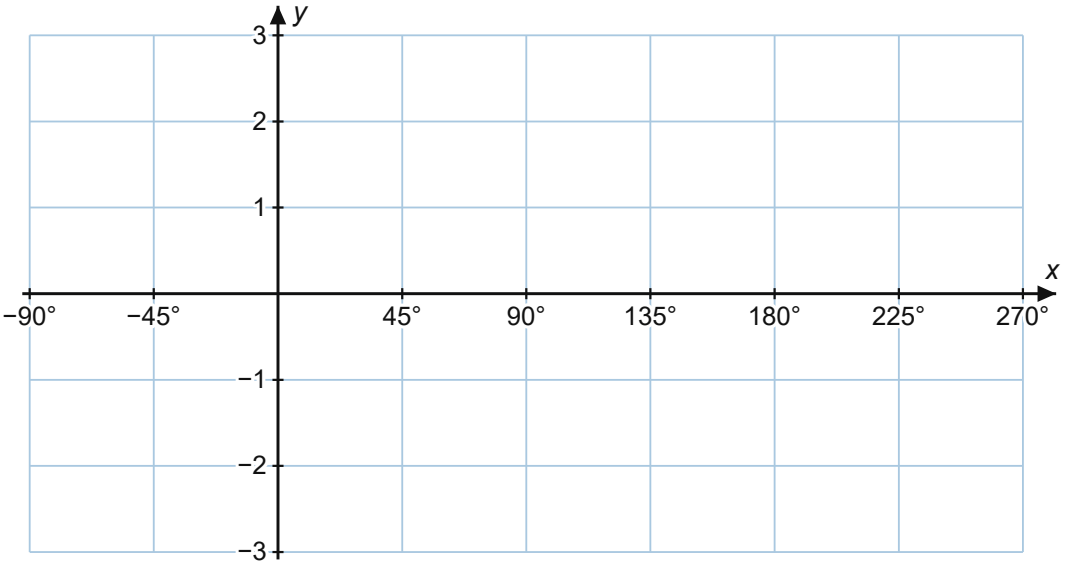
(f) Calculate the area of ΔPAB.

(2)

[15]

QUESTION 5

Given: $f(x) = \sin x + 1$ and $g(x) = \cos 2x$ for $x \in [-90^\circ; 270^\circ]$.



(a) Determine the general solution of $f(x) = g(x)$.

(5)

(b) Hence write down the values of $x \in [-90^\circ; 270^\circ]$ for which $f(x) = g(x)$.

(2)

(c) Sketch the graphs of f and g on the set of axes provided. Label all intercepts with the axes, turning points and end points.

(6)

(d) Use your graph to determine the values of $x \in [-90^\circ; 270^\circ]$ for which $f(x) \cdot g(x) \geq 0$.

(3)

(e) Write down the range of $h(x) = 2g(x) - 1$.

(2)

[18]

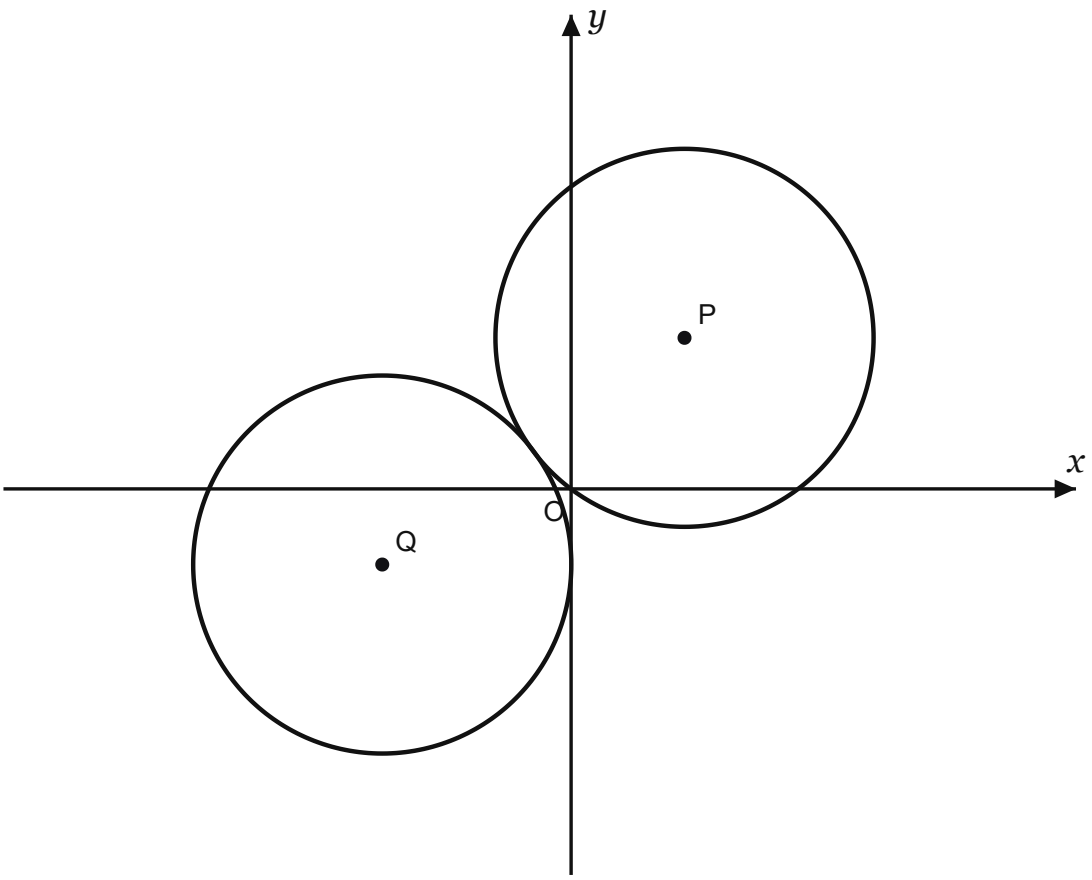
Section B

[80 marks]

QUESTION 6

In the diagram:

- Circle P has centre $\mathrm{P}(3;\ 4)$ and passes through the origin.
- Circle Q has the equation $(x+5)^2+(y+2)^2=25$.



(a) Determine the equation of circle P.

(2)

(b) Calculate the distance PQ between the centres.

(2)

(c) Show that the circles touch externally.

(2)

(d) Determine the coordinates of T, the point where the circles touch.

(3)

(e) Determine the equation of the common tangent to the circles at T.

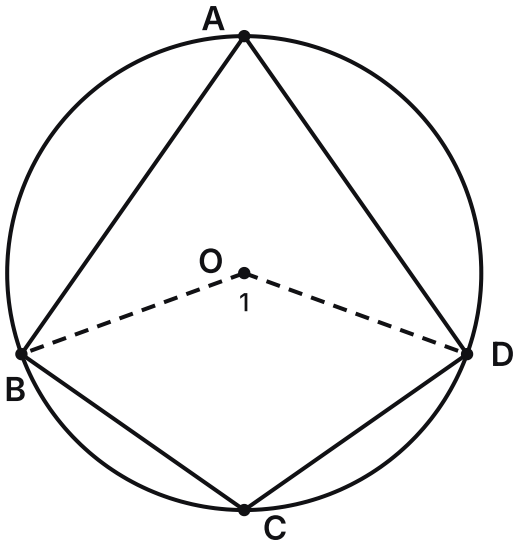
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[12]

QUESTION 7

In the diagram:

- ABCD is a cyclic quadrilateral in the circle with centre O.

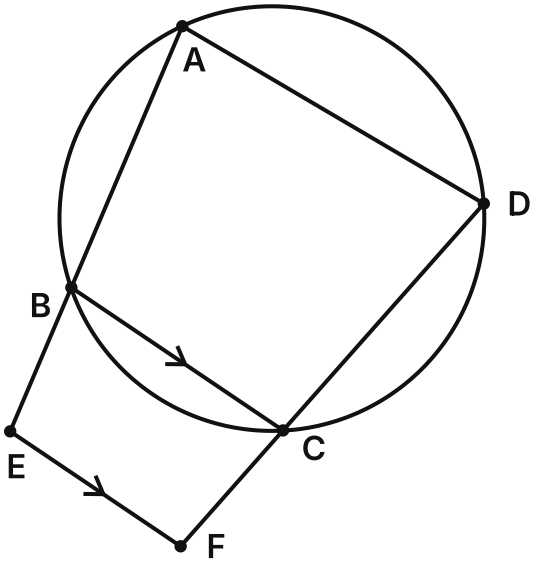


State all relevant reasons with your statements.

(a) Use the diagram to prove the theorem that states that the opposite angles of a cyclic quadrilateral are supplementary.

(5)

In the diagram below, ABCD is a cyclic quadrilateral. AB is produced to E and DC is produced to F so that EF ∥ BC.



(b) Prove that $\mathrm{A\hat{E}F} = \mathrm{A\hat{B}C}$.

(1)

(c) Hence prove that ADFE is a cyclic quadrilateral.

(4)

(d) If $\mathrm{A\hat{D}C} = 79^\circ$, determine the size of AÊF.

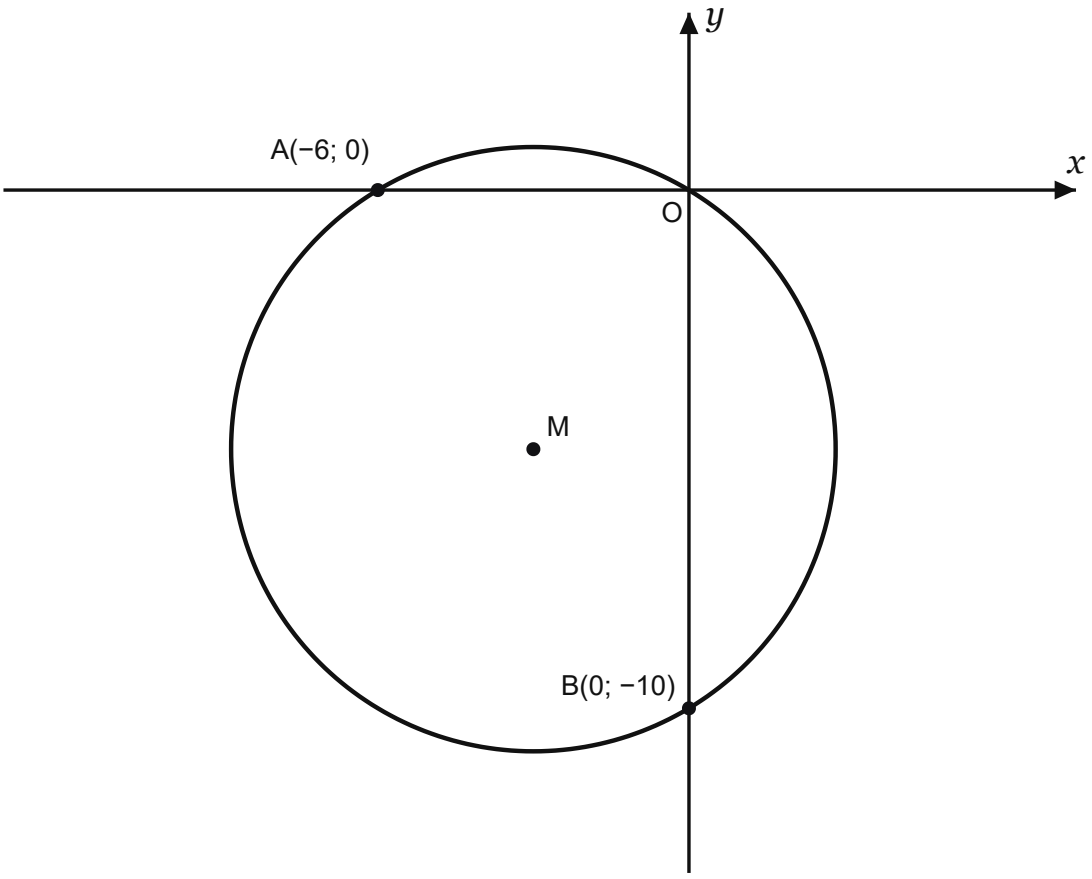
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[12]

QUESTION 8

In the diagram:

- The circle with centre M passes through the origin O.
- The circle cuts the x-axis at $\mathrm{A}(-6;\ 0)$ and the y-axis at $\mathrm{B}(0;\ -10)$.



(a) Explain why AB is a diameter of the circle.

(2)

(b) Determine the coordinates of the centre M and the radius of the circle.

(3)

(c) Show that the equation of the circle can be written as $x^2 + y^2 + 6x + 10y = 0$.

(2)

(d) Determine the equation of the tangent to the circle at O.

(3)

(e) The line $y = -x$ cuts the circle at O and at C. Determine the coordinates of C.

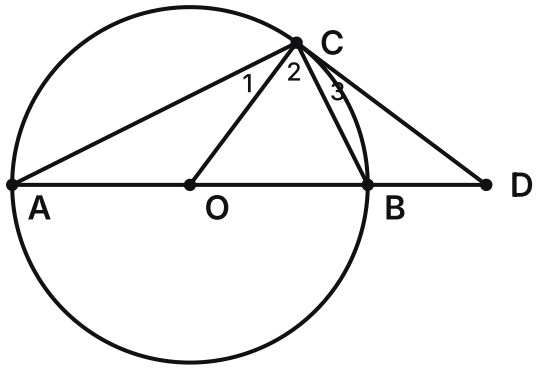
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[13]

QUESTION 9

In the diagram:

- AB is a diameter of the circle with centre O.
- The tangent to the circle at C meets AB produced at D.
- $\hat{C}_3 = x$



State all relevant reasons with your statements.

(a) Express CÂB in terms of x.

(2)

(b) Express $\hat{C}_2$ in terms of x.

(3)

(c) Prove that ΔDCB ||| ΔDAC.

(3)

(d) Hence prove that $DC^2 = DB \cdot DA$.

(2)

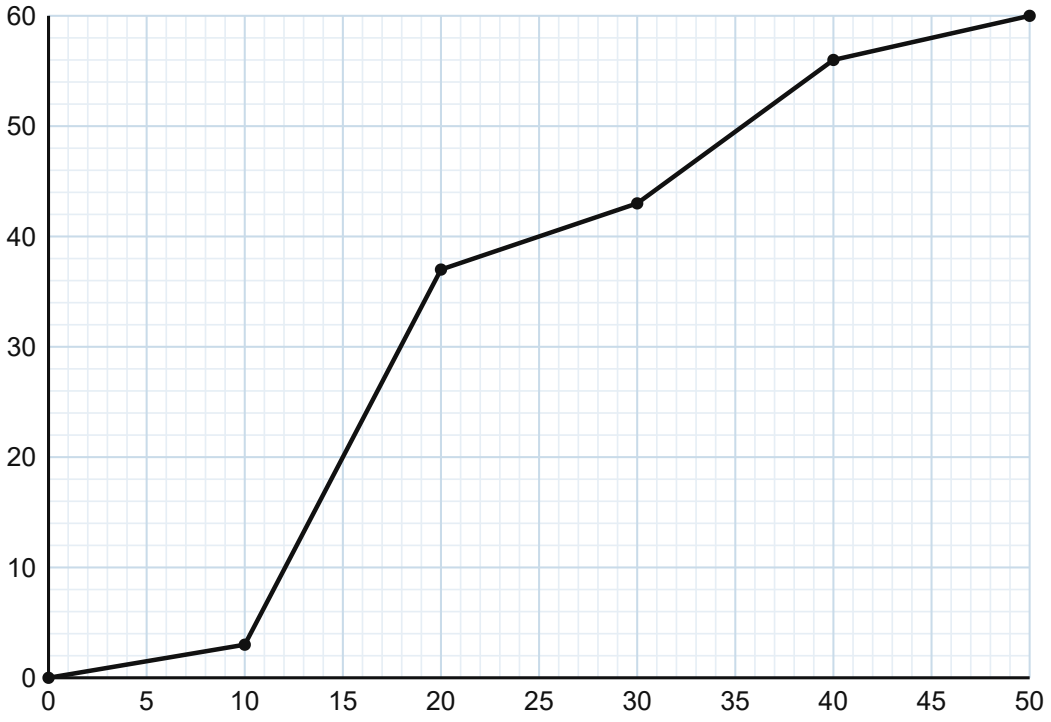
(e) If $DB = 2$ units and $DC = 4$ units, calculate the radius of the circle.

(3)

[13]

QUESTION 10

The ogive (cumulative frequency curve) below shows the time in minutes that 60 people waited at a clinic.



(a) How many data values does the ogive represent?

(1)

(b) Estimate the median.

(2)

(c) Estimate the interquartile range.

(3)

(d) How many of the data values lie between 20 and 40?

(2)

(e) The lowest value is 1 and the highest is 47. Draw a box-and-whisker plot of the data.

(3)

(f) What percentage of the data values are 10 or more?

(2)

(g) Is the data skewed? Use the quartiles to explain.

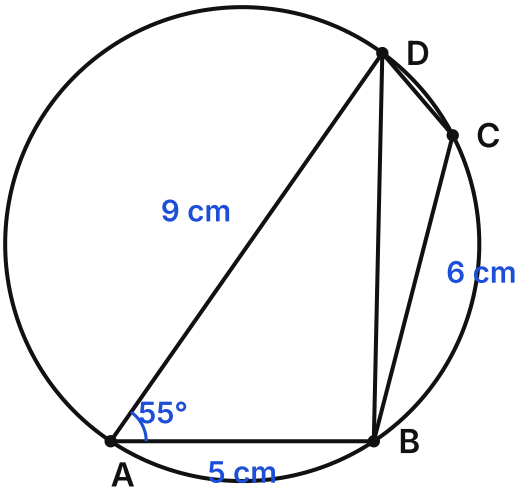
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[15]

QUESTION 11

In the diagram (answers correct to two decimal places):

- A, B, C and D lie on the circle.
- $\mathrm{AB} = 5$ cm, $\mathrm{AD} = 9$ cm and $\mathrm{BC} = 6$ cm.
- $\mathrm{B\hat{A}D} = 55^\circ$.



(a) Calculate the length of BD.

(3)

(b) Write down the size of $\mathrm{B\hat{C}D}$. Give a reason.

(2)

(c) Calculate the size of $\mathrm{B\hat{D}C}$.

(3)

(d) Calculate the length of CD.

(3)

(e) Calculate the area of ABCD.

(4)

[15]