IEB mock Paper II (second paper) — Memorandum

Every answer below was confirmed by an independent verifier at the moment the question was made.

QUESTION 1

(a) Answer: $\text{P}\hat{\text{T}}\text{A} = 42^\circ$

1. $\text{P}\hat{\text{T}}\text{A} = \text{T}\hat{\text{B}}\text{A} = 42^\circ$ ($\tan$ chord theorem)

(b) Answer: $\text{T}\hat{\text{O}}\text{A} = 84^\circ$

1. $\text{T}\hat{\text{O}}\text{A} = 2 \times \text{T}\hat{\text{B}}\text{A} = 84^\circ$ ($\angle$ at centre $= 2 \times \angle$ at circumference)

(c) Answer: $\text{O}\hat{\text{T}}\text{A} = 48^\circ$

1. $\text{O}\hat{\text{T}}\text{P} = 90^\circ$ ($\tan \perp$ radius)
2. $\therefore \text{O}\hat{\text{T}}\text{A} = 90^\circ - \text{P}\hat{\text{T}}\text{A} = 48^\circ$

(d) Answer: $\text{T}\hat{\text{D}}\text{A} = 138^\circ$

1. $\text{T}\hat{\text{D}}\text{A} = 180^\circ - \text{T}\hat{\text{B}}\text{A} = 138^\circ$ (opp $\angle$s of cyclic quad)

(e) Answer: reflex $\text{T}\hat{\text{O}}\text{A} = 276^\circ$

1. reflex $\text{T}\hat{\text{O}}\text{A} = 360^\circ - 84^\circ = 276^\circ$ ($\angle$s round a pt)

QUESTION 2

(a) Answer: $y = 10.873 + 7.708x$

1. $\mathrm{A} = 10.873$
2. $\mathrm{B} = 7.708$

(b) Answer: $r = 1.00$

(c) Answer: a strong positive correlation

1. $r = 1.00$: $r > 0$ and $|r|$ is at least 0.8

(d) Answer: $y = 280.65$

1. $y = 10.873 + 7.708(35)$

(e) Answer: No: x lies outside the data (extrapolation)

1. the data runs from $x = 7$ to $x = 29$; $x = 35$ lies outside it

THE LEAST-SQUARES LINE OF BEST FIT

QUESTION 3

(a) Answer: $\sin 324^\circ = -k$

1. $\sin 324^\circ = \sin(360^\circ - 36^\circ) = -\sin 36^\circ = -k$

(b) Answer: $\tan 36^\circ = \frac{k}{\sqrt{1-k^2}}$

1. $\cos^2 36^\circ = 1 - \sin^2 36^\circ = 1 - k^2$
2. 36° is acute, so $\cos 36^\circ > 0$
3. $\tan 36^\circ = \frac{\sin 36^\circ}{\cos 36^\circ} = \frac{k}{\sqrt{1-k^2}}$

(c) Answer: $\cos 72^\circ = 1 - 2k^2$

1. $\cos 72^\circ = 1 - 2\sin^2 36^\circ$
2. $= 1 - 2k^2$

(d) Answer: $\sin 77^\circ \cos 41^\circ - \cos 77^\circ \sin 41^\circ = k$

1. $\sin 77^\circ \cos 41^\circ - \cos 77^\circ \sin 41^\circ = \sin(77^\circ - 41^\circ)$
2. $= \sin 36^\circ = k$

QUESTION 4

(a) Answer: $\mathrm{M}(1;\,3)$

1. the diagonals of a square bisect each other: M is the midpoint of AC
2. $\mathrm{M} = (\frac{-1+3}{2};\,\frac{2+4}{2}) = (1;\,3)$

(b) Answer: $y = -2x + 5$

1. $m_{\mathrm{AC}} = \frac{4-2}{3-(-1)} = \frac{1}{2}$
2. the diagonals of a square are perpendicular: $m_{\mathrm{BD}} = -2$
3. through M: $3 = (-2)(1) + c$, so $c = 5$

(c) Answer: $\mathrm{B}(2;\,1)$ and $\mathrm{D}(0;\,5)$

1. the diagonals of a square are equal and bisect each other: $\mathrm{MB} = \mathrm{MD} = \mathrm{MA}$
2. $\mathrm{MA}^2 = (-1-1)^2 + (2-3)^2 = 5$
3. on BD: $(x-1)^2 + (-2x+2)^2 = 5$
4. $x = 2$ or $x = 0$

(d) Answer: 10 square units

1. $\mathrm{AC}^2 = 4 \times \mathrm{MA}^2 = 20$
2. the diagonals are equal and perpendicular: area $= ½ \times \mathrm{AC} \times \mathrm{BD} = ½ \times \mathrm{AC}^2 = 10$

(e) Answer: $\theta = 26.6^\circ$

1. $\tan\theta = m_{\mathrm{AC}} = \frac{1}{2}$

QUESTION 5

(a) Answer: See the proof below.

1. Required to prove: $\mathrm{B\hat{A}T} = \mathrm{A\hat{C}B}$
2. Construction: Draw the diameter AD and join DB.
3. $\mathrm{D\hat{A}T} = 90^\circ$ ($\tan \perp$ radius)
4. $\mathrm{A\hat{B}D} = 90^\circ$ ($\angle$ in semi-circle)
5. $\therefore \mathrm{B\hat{A}T} = 90^\circ - \mathrm{D\hat{A}B}$
6. $\mathrm{A\hat{D}B} = 90^\circ - \mathrm{D\hat{A}B}$ (sum $\angle$s of Δ)
7. $\therefore \mathrm{B\hat{A}T} = \mathrm{A\hat{D}B}$
8. $\mathrm{A\hat{D}B} = \mathrm{A\hat{C}B}$ ($\angle$s in the same seg)
9. $\therefore \mathrm{B\hat{A}T} = \mathrm{A\hat{C}B}$

(b) Answer: $\mathrm{P\hat{A}D} = \mathrm{A\hat{E}D}$

1. $\mathrm{P\hat{A}B} = \mathrm{A\hat{C}B}$ ($\tan$ chord theorem)
2. $\mathrm{A\hat{C}B} = \mathrm{A\hat{E}D}$ (corresp $\angle$s; DE $\parallel$ BC)
3. $\therefore \mathrm{P\hat{A}D} = \mathrm{A\hat{E}D}$

(c) Answer: PQ is a tangent to the circle through A, D and E

1. $\mathrm{P\hat{A}D} = \mathrm{A\hat{E}D}$ (proved)
2. $\therefore \mathrm{PQ}$ is a tangent to the circle through A, D and E (converse $\tan$ chord theorem)

(d) Answer: $\mathrm{AE} = 6$ units

1. $\frac{\mathrm{AE}}{\mathrm{EC}} = \frac{\mathrm{AD}}{\mathrm{DB}} = 2$ (line $\parallel$ one side of Δ; DE $\parallel$ BC)
2. $\therefore \mathrm{AE} = (\frac{4}{6}) \times 9 = 6$ units

PROVING THE THEOREM ITSELF, NOT USING IT

QUESTION 6

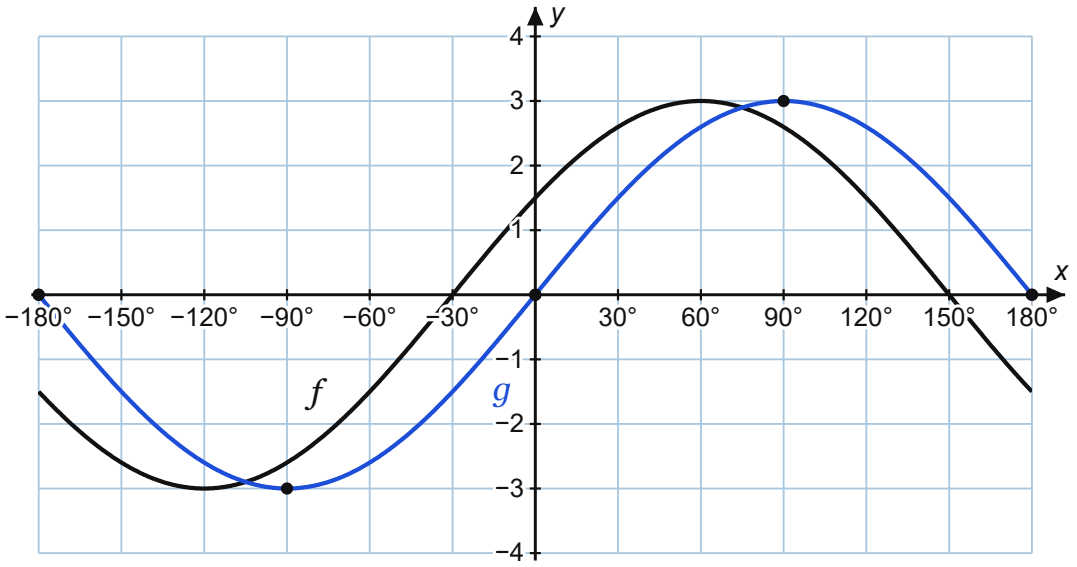
(a) Answer: $a = 3$ and $p = -60^\circ$

1. the greatest value of f is 3, so $a = 3$
2. $\cos(x + p)$ is greatest where $x + p = 0^\circ$
3. $60^\circ + p = 0^\circ$, so $p = -60^\circ$

(b) Answer: $-3 \leq y \leq 3$

1. f runs from -3 to 3

(c) Answer: See the sketch below.



1. $g(x) = 3 \sin x$
2. g: x-intercepts $(-180^\circ; 0), (0^\circ; 0), (180^\circ; 0)$
3. g: y-intercept (0°; 0)
4. g: turning points $(-90^\circ; -3), (90^\circ; 3)$
5. g: end points $(-180^\circ; 0), (180^\circ; 0)$

(d) Answer: $x = -105^\circ$ or $x = 75^\circ$

1. $3 \cos(x - 60^\circ) = 3 \sin x$
2. $\cos(x - 60^\circ) = \cos(90^\circ - x)$
3. $x - 60^\circ = 90^\circ - x + k{\cdot}360^\circ$, so $x = 75^\circ + k{\cdot}180^\circ$

4. or $x - 60^\circ = -(90^\circ - x) + k{\cdot}360^\circ$, which gives $-60^\circ = -90^\circ + k{\cdot}360^\circ$: no solution

5. In $[-180^\circ; 180^\circ]$: $x = -105^\circ$ or $x = 75^\circ$

(e) Answer: $-30^\circ < x < 0^\circ$ or $150^\circ < x < 180^\circ$

1. f and g have opposite signs, read from the sketch

TRIGONOMETRIC GRAPHS

QUESTION 7

(a) Answer: $\mathrm{FG} : \mathrm{GC} = 5 : 4$

1. In ΔCBF: DG ∥ BF
2. $\therefore \mathrm{FG} : \mathrm{GC} = 5 : 4$ (line ∥ one side of Δ; DG ∥ BF)

(b) Answer: $\mathrm{AF} = \mathrm{FG}$

1. In $\Delta\mathrm{ADG}$: $\mathrm{AE} = \mathrm{ED}$ and EF ∥ DG
2. $\therefore \mathrm{AF} = \mathrm{FG}$ (line through midpt ∥ to second side)

(c) Answer: $\mathrm{AF} : \mathrm{FC} = 5 : 9$

1. $\mathrm{AF} = \mathrm{FG} = 5k$ and $\mathrm{GC} = 4k$ (proved)
2. $\therefore \mathrm{AF} : \mathrm{FC} = 5 : 9$

(d) Answer: 5 : 9

1. area ΔABF : area $\Delta\mathrm{CBF} = 5 : 9$ (same height)

PRODUCTS AND RATIOS OF LENGTHS

QUESTION 8

(a) Answer: $\mathrm{M}(0;\,4);\, r = \sqrt{17}$

1. $x^2 + (y-4)^2 = 1 + 16$
2. $x^2 + (y-4)^2 = 17$
3. $\mathrm{M}(0;\,4)$ and $r = \sqrt{17} = \sqrt{17}$

(b) Answer: See the working below.

1. $(1)^2 + (8)^2 + 0(1) - 8(8) - 1 = 0$
2. so P lies on the circle

(c) Answer: $y = -\frac{1}{4}x + \frac{33}{4}$

1. $m_{\mathrm{MP}} = \frac{8-4}{1-0} = 4$
2. the tangent is perpendicular to the radius: $m = -\frac{1}{4}$
3. $8 = (-\frac{1}{4})(1) + c$, so $c = \frac{33}{4}$

(d) Answer: $\mathrm{QT} = 9$ units

1. $\mathrm{QM}^2 = (-7-0)^2 + (11-4)^2 = 98$
2. MT ⊥ QT (radius ⊥ tangent), so $\mathrm{QT}^2 = \mathrm{QM}^2 - r^2$
3. $\mathrm{QT}^2 = 98 - 17 = 81$
4. $\mathrm{QT} = 9$

QUESTION 9

(a) Answer: See the working below.

1. $m_{\text{AB}} = \frac{-4-2}{-4-(-2)} = 3$
2. $m_{\text{BC}} = \frac{-6-(-4)}{2-(-4)} = -\frac{1}{3}$
3. $m_{\text{AB}} \times m_{\text{BC}} = (3)(-\frac{1}{3}) = -1$, so AB ⊥ BC

(b) Answer: $\text{M}(0; -2)$

1. $\text{A}\hat{\text{B}}\text{C} = 90^\circ$, so AC subtends a right angle at B: AC is a diameter (converse ∠ in semi-circle)
2. M is the midpoint of AC: $(\frac{-2+2}{2}; \frac{2+(-6)}{2}) = (0; -2)$

(c) Answer: $x^2 + (y+2)^2 = 20$

1. $r^2 = \text{MA}^2 = (-2-0)^2 + (2-(-2))^2 = 20$

(d) Answer: $y = \frac{1}{2}x - 7$

1. $m_{\text{MC}} = -2$
2. the tangent is perpendicular to the radius MC: $m = \frac{1}{2}$
3. $-6 = (\frac{1}{2})(2) + c$, so $c = -7$

(e) Answer: 20 square units

1. $\text{AB} = 2\sqrt{10}$ and $\text{BC} = 2\sqrt{10}$
2. area $= ½ \times \text{AB} \times \text{BC}$ (the right angle is at B) $= 20$

QUESTION 10

(a) Answer: CDHE is a cyclic quadrilateral

1. $\mathrm{H\hat{D}C} = 90^\circ$ (given)
2. $\mathrm{H\hat{E}C} = 90^\circ$ (given)
3. $\therefore \mathrm{H\hat{D}C} + \mathrm{H\hat{E}C} = 180^\circ$
4. $\therefore \mathrm{CDHE}$ is a cyclic quad (opp ∠s of quad supp)

(b) Answer: ABDE is a cyclic quadrilateral

1. $\mathrm{A\hat{D}B} = \mathrm{A\hat{E}B} = 90^\circ$ (given)
2. $\therefore \mathrm{ABDE}$ is a cyclic quad (line subtends equal ∠s)

(c) Answer: $\mathrm{D\hat{A}C} = 90^\circ - x$

1. $\mathrm{A\hat{D}C} = 90^\circ$ (given)
2. $\therefore \mathrm{D\hat{A}C} = 90^\circ - x$ (sum ∠s of Δ)

(d) Answer: $\hat{\mathrm{D}}_1 = \mathrm{B\hat{A}C}$

1. $\hat{\mathrm{D}}_1 = \mathrm{B\hat{A}E}$ (ext ∠ of cyclic quad)

(e) Answer: ΔCDE ||| ΔCAB

1. In ΔCDE and ΔCAB:
2. $\mathrm{D\hat{C}E} = \mathrm{A\hat{C}B}$ (common)
3. $\hat{\mathrm{D}}_1 = \mathrm{C\hat{A}B}$ (proved)
4. $\therefore \Delta\mathrm{CDE} \,|||\, \Delta\mathrm{CAB}$ $(\angle\angle\angle)$

(f) Answer: $\mathrm{CB} = 30$ units

1. $\frac{\mathrm{CD}}{\mathrm{CA}} = \frac{\mathrm{CE}}{\mathrm{CB}}$ $(|||\ \Delta\mathrm{s})$
2. $\therefore \mathrm{CB} = \frac{18 \cdot 25}{15} = 30$ units

PROVING A QUADRILATERAL IS CYCLIC

QUESTION 11

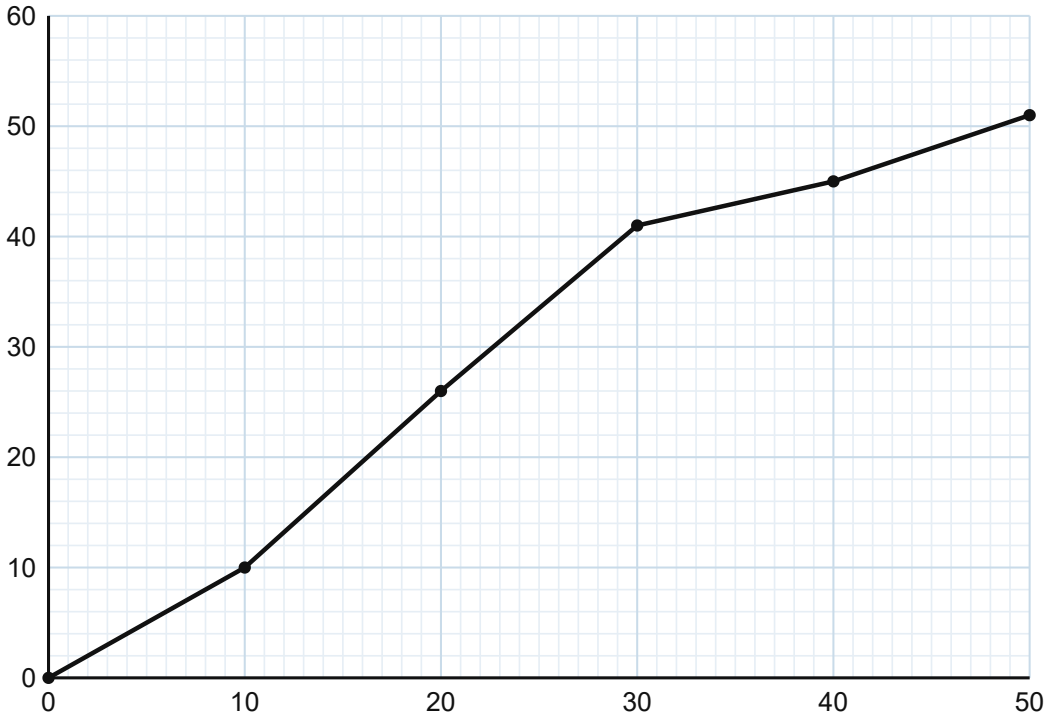
(a) Answer: 21.08

1. midpoints: 5, 15, 25, 35, 45
2. $\Sigma\ f{\cdot}x = 1075$ and $n = 51$
3. mean $= \frac{1075}{51} = 21.08$

(b) Answer: $10 \leq x < 20$

1. the highest frequency is 16

(c) Answer: See the ogive below.



1. cumulative frequencies: 10, 26, 41, 45, 51
2. plot each against the upper limit of its class, starting at (0; 0)

(d) Answer: about 19.7

1. the median is at a cumulative frequency of $\frac{51}{2} = 25.5$
2. read across to the ogive and down: about 19.7

(e) Answer: about 16.5

1. Q1 at 12.75: about 11.7; Q3 at 38.25: about 28.2

2. $\mathrm{IQR} = 28.2 - 11.7 = 16.5$

(f) Answer: Skewed to the right (positively)

1. mean 21.08 and median about 19.7
2. mean $>$ median

THE ESTIMATED MEAN OF GROUPED DATA

QUESTION 12

(a) Answer: See the working below.

1. AT makes an angle α and the road an angle θ with the horizontal: $\mathrm{P\hat{A}T} = \alpha - \theta$
2. the horizontal at P makes 90° with PT, and PA lies θ below it: $\mathrm{A\hat{P}T} = 90^\circ + \theta$
3. $\mathrm{A\hat{T}P} = 180^\circ - (\alpha - \theta) - (90^\circ + \theta) = 90^\circ - \alpha$ (sum ∠s of Δ)

(b) Answer: See the working below.

1. In $\Delta\mathrm{APT}$: $\dfrac{\mathrm{PT}}{\sin \mathrm{P\hat{A}T}} = \dfrac{\mathrm{AP}}{\sin \mathrm{A\hat{T}P}}$ (sine rule)
2. $\mathrm{PT} = \dfrac{d \sin(\alpha - \theta)}{\sin(90^\circ - \alpha)}$
3. $\sin(90^\circ - \alpha) = \cos\alpha$, so $\mathrm{PT} = \dfrac{d \sin(\alpha - \theta)}{\cos\alpha}$

(c) Answer: $\mathrm{PT} = 9.81\ \mathrm{m}$

1. $\mathrm{PT} = \dfrac{26 \sin 20^\circ}{\cos 25^\circ}$
2. $= 9.81\ \mathrm{m}$

(d) Answer: $\mathrm{BT} = 47.86\ \mathrm{m}$

1. $\mathrm{AT} = \dfrac{d \sin(90^\circ + \theta)}{\sin(90^\circ - \alpha)} = \dfrac{26 \cos 5^\circ}{\cos 25^\circ} = 28.58\ \mathrm{m}$ (sine rule)
2. $\mathrm{B\hat{A}T} = 180^\circ - 20^\circ = 160^\circ$ (∠s on a str line)
3. $\mathrm{BT}^2 = 20^2 + 28.58^2 - 2(20)(28.58)\cos 160^\circ$ (cosine rule)

(e) Answer: 16.78°

1. $\dfrac{\sin \mathrm{A\hat{B}T}}{\mathrm{AT}} = \dfrac{\sin \mathrm{B\hat{A}T}}{\mathrm{BT}}$ (sine rule)
2. $\mathrm{A\hat{B}T} = 11.78^\circ$
3. the road rises at 5°: the angle of elevation is $11.78^\circ + 5^\circ = 16.78^\circ$