IEB mock Paper II (second paper)

Time: 3 hours · 150 marks

Answer all the questions. Show all calculations. You may use an approved calculator unless a question says otherwise. Diagrams are not necessarily drawn to scale.

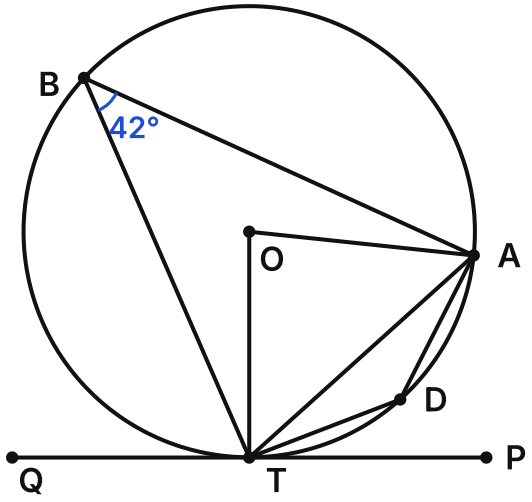
Section A

[72 marks]

QUESTION 1

In the diagram:

- O is the centre of the circle.
- A, B, D and T lie on the circle.
- PTQ is a tangent to the circle at T.
- $\hat{TBA} = 42^\circ$



State all relevant reasons with your statements.

(a) Determine $P\hat{T}A$.

(2)

(b) Determine $T\hat{O}A$.

(2)

(c) Determine $O\hat{T}A$.

(2)

(d) Determine $T\hat{D}A$.

(2)

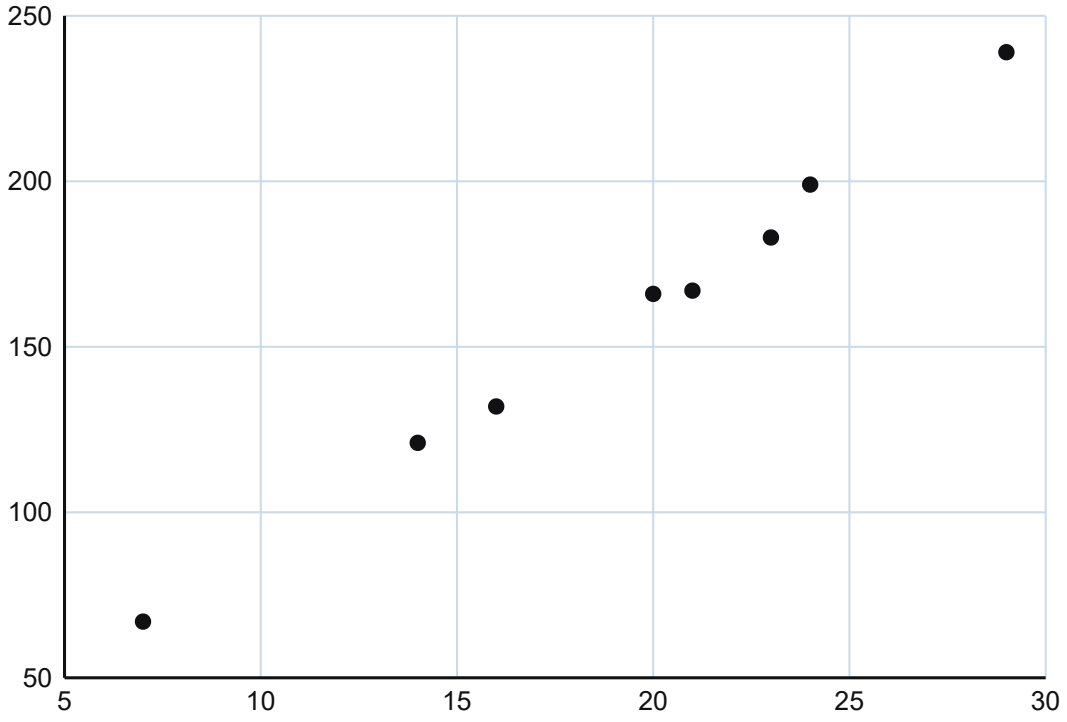
(e) Determine the size of reflex $T\hat{O}A$.

(1)

[9]

QUESTION 2

The table and scatter plot show the distance of a taxi trip in km (x) and the fare in rands (y): $(7; 67)$; $(14; 121)$; $(16; 132)$; $(20; 166)$; $(21; 167)$; $(23; 183)$; $(24; 199)$; $(29; 239)$.



(a) Use your calculator to determine the equation of the least squares regression line in the form $y = \mathrm{A} + \mathrm{Bx}$. Give A and B correct to three decimal places.

(3)

(b) Determine the correlation coefficient r, correct to two decimal places.

(1)

(c) Describe the strength and direction of the correlation.

(2)

(d) Use your equation to predict y when $x = 35$.

(2)

(e) Is this prediction reliable? Explain.

(2)

[10]

QUESTION 3

Given: $\sin 36^\circ = k$. Determine, without the use of a calculator, each of the following in terms of k.

(a) $\sin 324^\circ$

(2)

(b) $\tan 36^\circ$

(3)

(c) $\cos 72^\circ$

(3)

(d) $\sin 77^\circ \cos 41^\circ - \cos 77^\circ \sin 41^\circ$

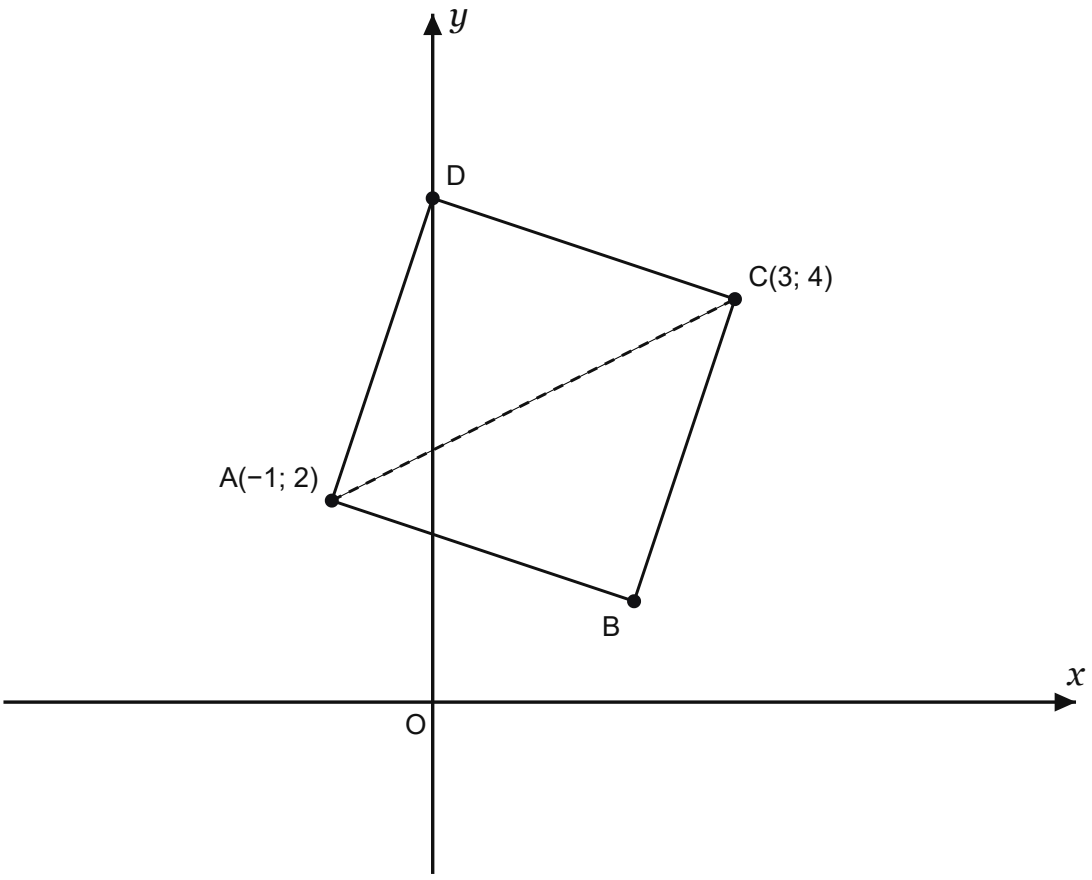
(3)

[11]

QUESTION 4

In the diagram:

- ABCD is a square.
- $\mathrm{A}(-1;\ 2)$ and $\mathrm{C}(3;\ 4)$ are opposite vertices.



(a) Determine the coordinates of M, the point where the diagonals of the square meet.

(2)

(b) Determine the equation of the diagonal BD.

(3)

(c) Determine the coordinates of B and D, given that B has the greater x-coordinate.

(4)

(d) Calculate the area of the square ABCD.

(2)

(e) Determine the inclination of AC, correct to one decimal place.

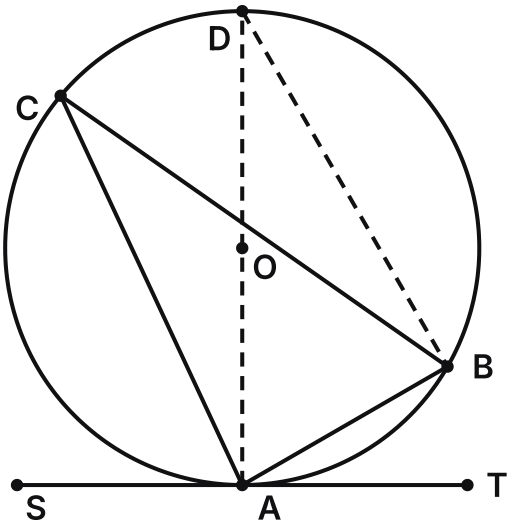
(2)

[13]

QUESTION 5

In the diagram:

- O is the centre of the circle, and SAT is a tangent to the circle at A.
- B and C lie on the circle.

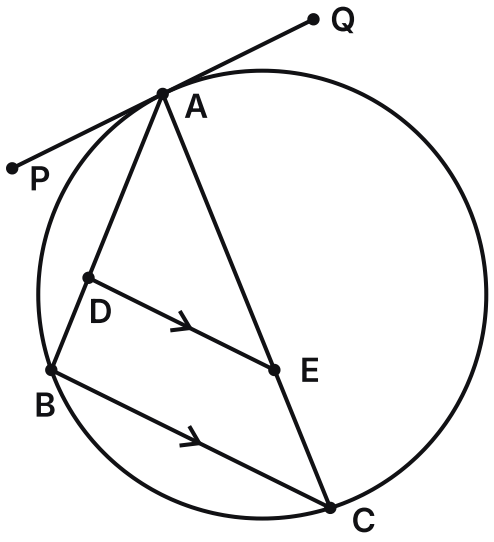


State all relevant reasons with your statements.

(a) Use the diagram to prove the theorem that states that the angle between a tangent to a circle and a chord drawn from the point of contact is equal to the angle in the alternate segment.

(6)

In the diagram below, A, B and C lie on the circle and PAQ is a tangent to the circle at A. D lies on AB and E on AC, with DE ∥ BC.



(b) Prove that $\mathrm{P\hat{A}D} = \mathrm{A\hat{E}D}$.

(3)

(c) Hence prove that PQ is a tangent to the circle through A, D and E.

(2)

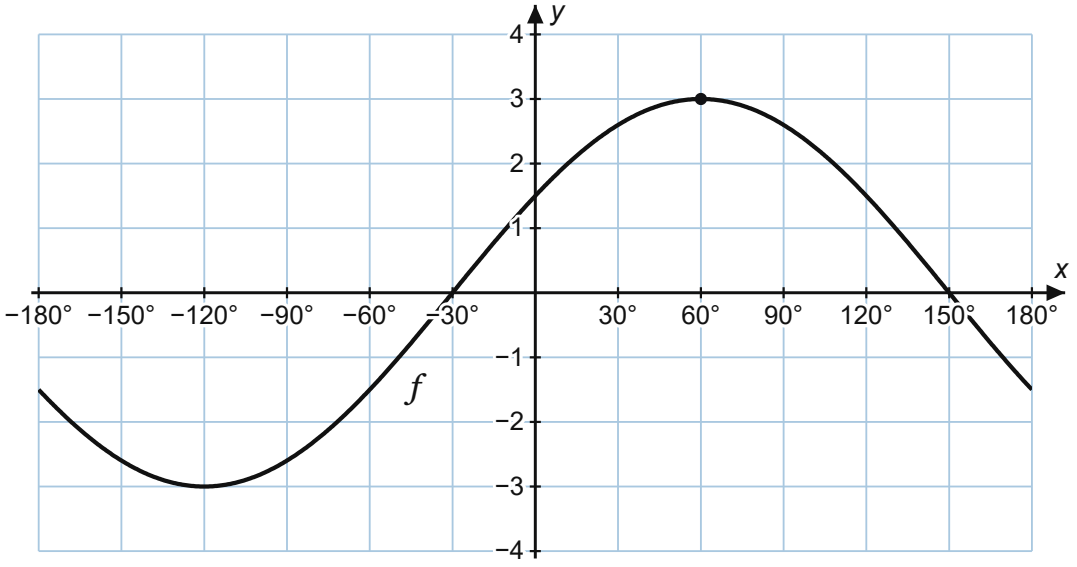
(d) If $\mathrm{AD} = 4$ units, $\mathrm{DB} = 2$ units and $\mathrm{AC} = 9$ units, calculate the length of AE.

(2)

[13]

QUESTION 6

The graph of $f(x) = a\cos(x + p)$, where $a > 0$ and $-90^\circ < p < 90^\circ$, is drawn below for $x \in [-180^\circ; 180^\circ]$. $\mathrm{A}(60^\circ; 3)$ is a turning point of f.



(a) Write down the values of a and p.

(3)

(b) Write down the range of f.

(1)

(c) Sketch the graph of $g(x) = 3\sin x$ on the same set of axes as f. Label all intercepts with the axes, turning points and end points.

(4)

(d) Calculate the values of $x \in [-180^\circ; 180^\circ]$ for which $f(x) = g(x)$.

(5)

(e) Use your graph to determine the values of $x \in [-180^\circ; 180^\circ]$ for which $f(x) \cdot g(x) < 0$.

(3)

[16]

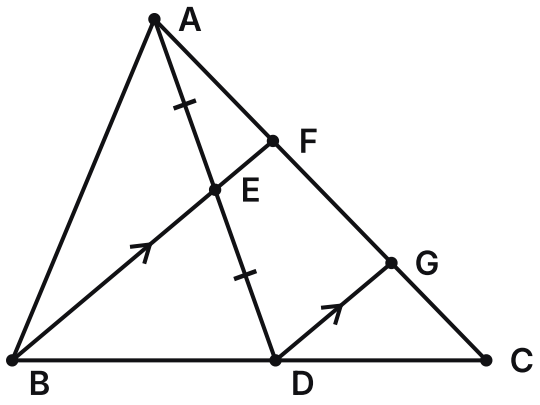
Section B

[78 marks]

QUESTION 7

In the diagram:

- In ΔABC, D lies on BC with $\mathrm{BD} : \mathrm{DC} = 5 : 4$.
- E is the midpoint of AD, and BE produced meets AC at F.
- G lies on FC such that DG ∥ BF.



State all relevant reasons with your statements.

(a) Determine FG : GC.

(2)

(b) Prove that $\mathrm{AF} = \mathrm{FG}$.

(2)

(c) Hence determine AF : FC.

(2)

(d) Determine area ΔABF : area ΔCBF.

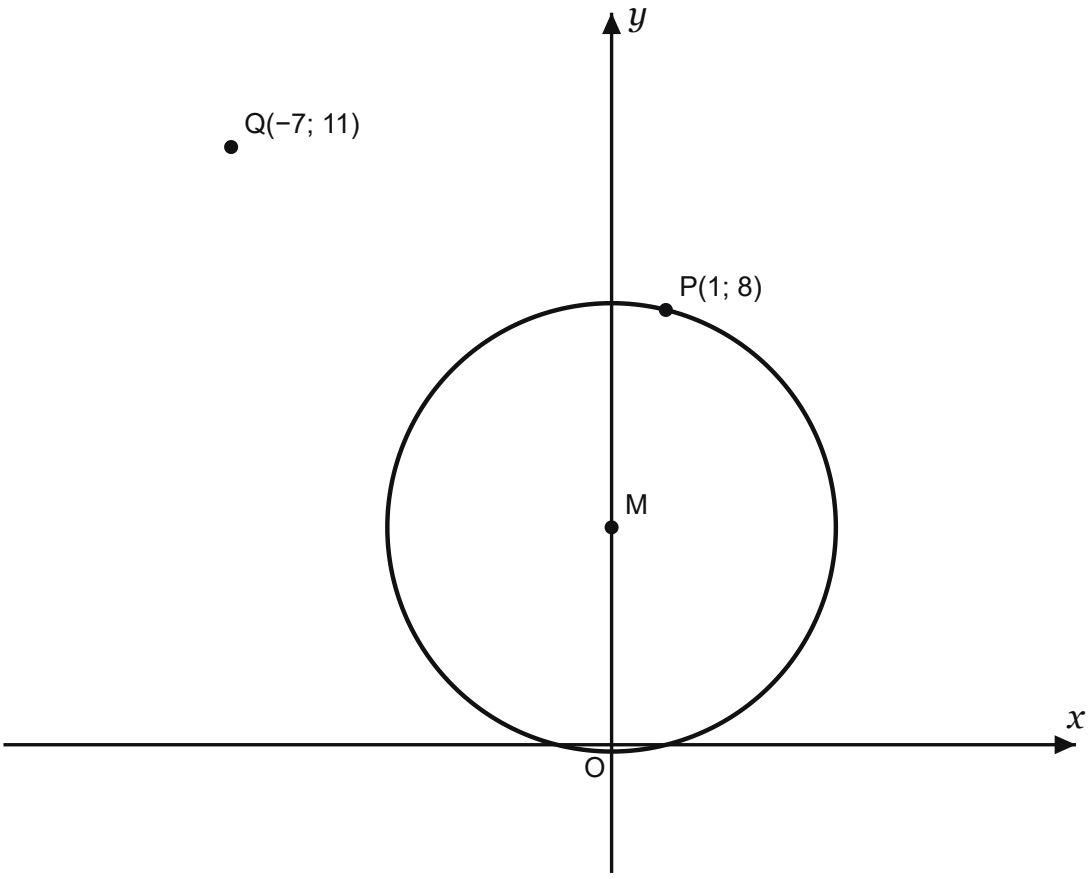
(2)

[8]

QUESTION 8

In the diagram:

- The circle with centre M has the equation $x^2 + y^2 - 8y - 1 = 0$.



(a) Determine the coordinates of the centre M and the radius of the circle.

(3)

(b) Show that $\mathrm{P}(1; 8)$ lies on the circle.

(2)

(c) Determine the equation of the tangent to the circle at P.

(4)

(d) A tangent is drawn from $\mathrm{Q}(-7; 11)$ to the circle, touching it at T. Determine the length of QT.

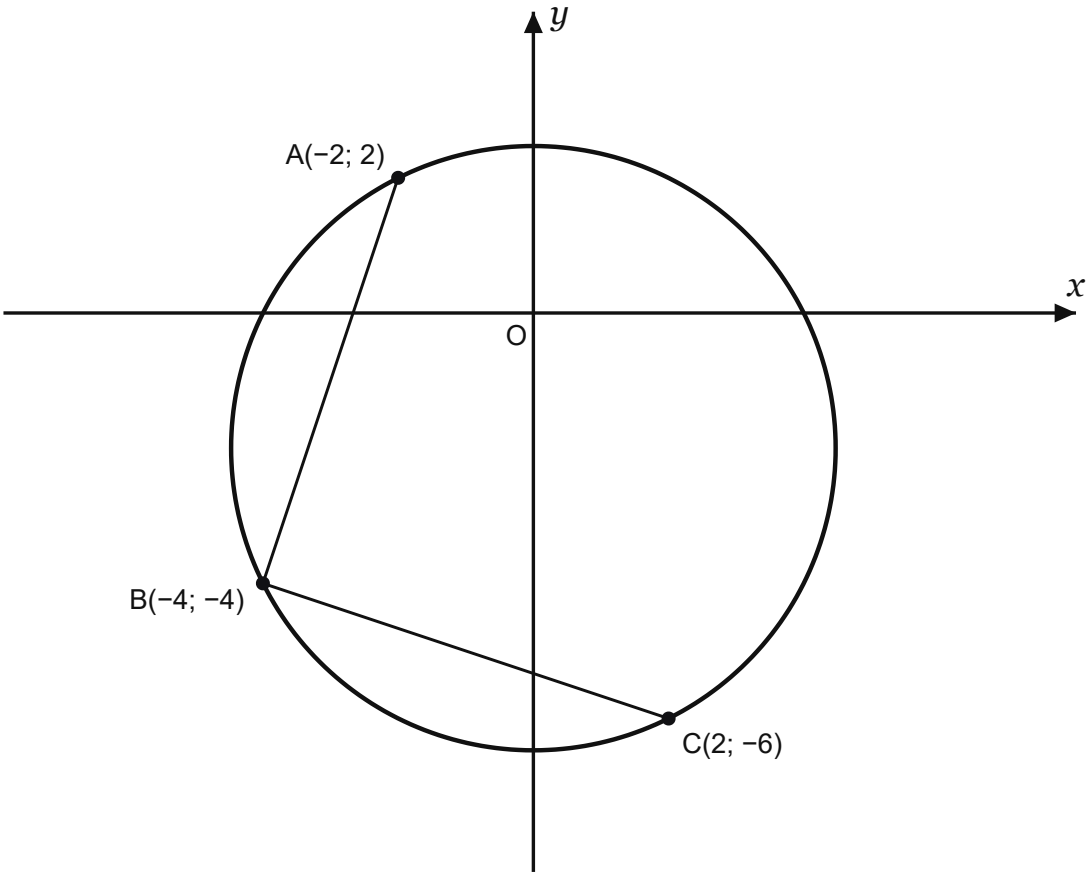
(3)

[12]

QUESTION 9

In the diagram:

- $\mathrm{A}(-2;\ 2)$, $\mathrm{B}(-4;\ -4)$ and $\mathrm{C}(2;\ -6)$ lie on the circle with centre M.



(a) Show that AB ⊥ BC.

(3)

(b) Hence explain why AC is a diameter of the circle through A, B and C, and determine the coordinates of its centre M.

(3)

(c) Determine the equation of the circle.

(2)

(d) Determine the equation of the tangent to the circle at C.

(3)

(e) Calculate the area of ΔABC.

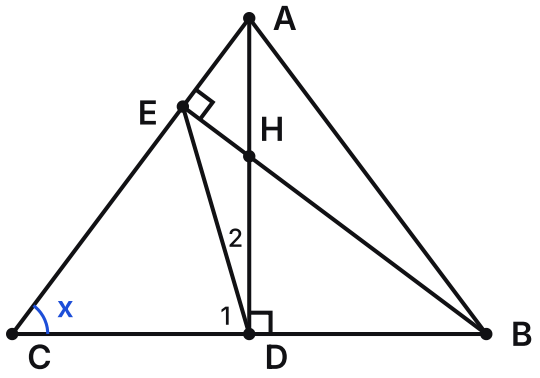
(2)

[13]

QUESTION 10

In the diagram:

- In ΔABC, D lies on BC and E lies on AC.
- $\mathrm{AD} \perp \mathrm{BC}$ and $\mathrm{BE} \perp \mathrm{AC}$.
- AD and BE intersect at H.
- $\hat{\mathrm{C}} = x$



State all relevant reasons with your statements.

(a) Prove that CDHE is a cyclic quadrilateral.

(3)

(b) Prove that ABDE is a cyclic quadrilateral.

(3)

(c) Express DÂC in terms of x.

(2)

(d) Prove that $\hat{\mathrm{D}}_1 = \mathrm{B}\hat{\mathrm{A}}\mathrm{C}$.

(2)

(e) Hence prove that ΔCDE ||| ΔCAB.

(3)

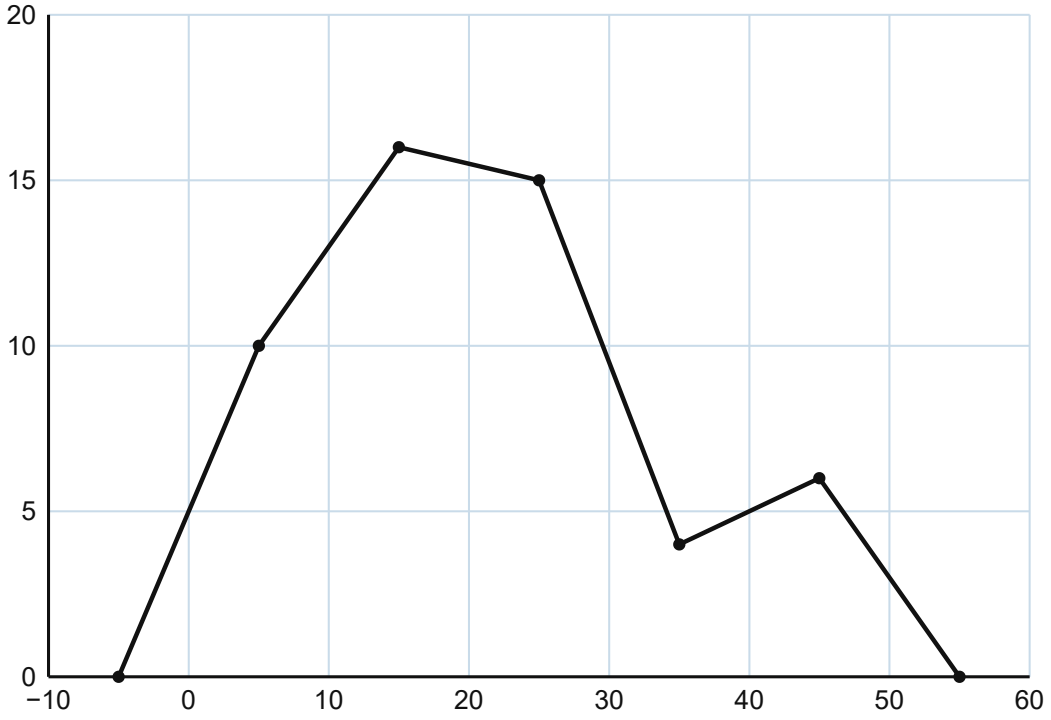
(f) If $\mathrm{CD} = 15$ units, $\mathrm{CA} = 25$ units and $\mathrm{CE} = 18$ units, calculate the length of CB.

(2)

[15]

QUESTION 11

The frequency polygon shows the time in minutes that 51 people waited at a clinic, grouped in the classes $0 \leq x < 10$; $10 \leq x < 20$; $20 \leq x < 30$; $30 \leq x < 40$; $40 \leq x < 50$.



(a) Estimate the mean of the data.

(3)

(b) Write down the modal class.

(1)

(c) Draw an ogive (cumulative frequency curve) for the data.

(4)

(d) Use your ogive to estimate the median.

(2)

(e) Use your ogive to estimate the interquartile range.

(3)

(f) Is the data skewed? Explain, using the mean and the median.

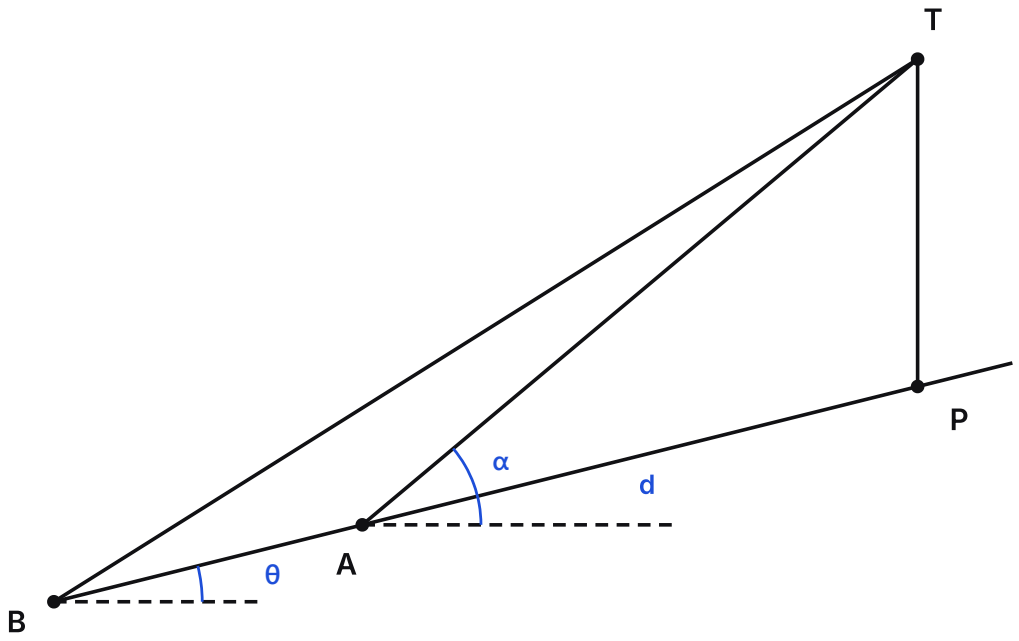
(2)

[15]

QUESTION 12

In the diagram below (answers correct to two decimal places):

- A straight road rises at an angle θ to the horizontal.
- PT is a vertical street light at the point P on the road.
- A is a point on the road, d metres down the road from P.
- The angle of elevation of T from A is α.



(a) Show that $\mathrm{A\hat{T}P} = 90^\circ - \alpha$.

(3)

(b) Hence show that $\mathrm{PT} = \dfrac{d\sin(\alpha - \theta)}{\cos\alpha}$.

(3)

(c) Calculate PT if $d = 26\ \mathrm{m}$, $\theta = 5^\circ$ and $\alpha = 25^\circ$.

(2)

(d) B is a point on the road, 20 m further down the road than A. Calculate the length of BT.

(4)

(e) Calculate the angle of elevation of T from B.

(3)

[15]