

IEB Algebra practice — Memorandum

Every answer below was confirmed by an independent verifier at the moment the question was made.

Exponential equations and exponent laws

1

(a) Answer: $x = 4 - y$

1. $5^{x-4} \cdot 5^{2y} = 5^y$

2. $x - 4 + 2y = y$

3. $x = 4 - y$

(b) Answer: $x = 3$

1. $x + 22 = x^2 + 4x + 4$

2. $x^2 + 3x - 18 = 0$

3. $(x - 3)(x + 6) = 0$

4. check $x = -6$: $\sqrt{-6 + 22} = 4$ but $-6 + 2 = -4$, so $x = -6$ is rejected

5. $x = 3$

(c) Answer: $x = -3$ or $x = 5$

1. $x^2 - 2x + 12 = 3^3$

2. $x^2 - 2x - 15 = 0$

3. $(x + 3)(x - 5) = 0$

4. $x = -3$ or $x = 5$

(d) Answer: $x = 1$ or $x = 2$

1. let $t = 5^x$: $t^2 - 30t + 125 = 0$

2. $(t - 5)(t - 25) = 0$

3. $5^x = 5$, so $x = 1$

4. $5^x = 25$, so $x = 2$

2

(a) Answer: $x = 0.21$

1. $3x = \log_3 2$

2. $x = \frac{\log 2}{3 \log 3}$

3. $x = 0.21$

(b) Answer: $x = 0$ or $x = 5$

1. $x^2 - 5x + 3 = 3^1$

2. $x^2 - 5x = 0$

3. $(x)(x - 5) = 0$

4. $x = 0$ or $x = 5$

(c) Answer: $x = 9$

1. $\sqrt{x - 5} = x - 7$

2. $x - 5 = x^2 - 14x + 49$

3. $x^2 - 15x + 54 = 0$

4. $(x - 9)(x - 6) = 0$

5. check $x = 6$: $\sqrt{6 - 5} = 1$ but $6 - 7 = -1$, so $x = 6$ is rejected

6. $x = 9$

Quadratic equations

3

(a) Answer: $x = -\frac{7}{3}$ or $x = 0$ or $x = 4$

1. $x = 0$ or $x - 4 = 0$ or $3x + 7 = 0$

2. $x = -\frac{7}{3}$ or $x = 0$ or $x = 4$

(b) Answer: $x = -9.47$ or $x = -0.53$

1. $x^2 + 10x = -5$

2. $x^2 + 10x + 25 = -5 + 25$

3. $(x + 5)^2 = 20$

4. $x + 5 = \pm\sqrt{20}$

5. $x = -5 \pm \sqrt{20}$

6. $x = -9.47$ or $x = -0.53$

(c) Answer: $x \leq -4$ or $x \geq 3$

1. $x^2 + x - 12 \geq 0$

2. $(x + 4)(x - 3) \geq 0$

3. critical values: $x = -4$ and $x = 3$

4. $x \leq -4$ or $x \geq 3$

(d) Answer: $x = -2p$ or $x = -p$

1. $x^2 + 3px + 2p^2 = 0$

2. $(x + 2p)(x + p) = 0$

3. $x = -2p$ or $x = -p$

(a) Answer: $x = -4p$ or $x = 2p$

$$1. x^2 + 2px - 8p^2 = 0$$

$$2. (x + 4p)(x - 2p) = 0$$

$$3. x = -4p \text{ or } x = 2p$$

(b) Answer: $m = 1$ and $p = 19$

$$1. (x - m)^2 + p = x^2 - 2mx + m^2 + p$$

$$2. -2m = -2, \text{ so } m = 1$$

$$3. m^2 + p = 20, \text{ so } p = 19$$

(c) Answer: 19

$$1. (x - m)^2 \geq 0, \text{ so the least value is } p = 19$$

(d) Answer: $x \leq -3$ or $x \geq 6$

$$1. x^2 - 3x - 18 \geq 0$$

$$2. (x + 3)(x - 6) \geq 0$$

$$3. \text{critical values: } x = -3 \text{ and } x = 6$$

$$4. x \leq -3 \text{ or } x \geq 6$$

(e) Answer: $k > \frac{49}{4}$

$$1. \text{non-real roots: } \Delta < 0$$

$$2. \Delta = (-7)^2 - 4(1)(k) = 49 - 4k$$

$$3. 49 - 4k < 0$$

$$4. k > \frac{49}{4}$$

Simultaneous equations

5

(a) Answer: $x = -1, y = 3$

1. $5^{x+y} = 5^2$, so $x + y = 2$

2. $2^{x-y} = 2^{-4}$, so $x - y = -4$

3. adding: $2x = -2$, so $x = -1$

4. $y = 2 - (-1) = 3$

(b) Answer: $k < 4$

1. $x^2 - x + 5 = -3x + k$

2. $x^2 + 2x + 5 - k = 0$

3. no point of intersection: $\Delta < 0$

4. $(2)^2 - 4(1)(5 - k) < 0$

5. $k < 4$

(c) Answer: $x = 17$

1. $\frac{391}{x} - \frac{391}{x+6} = 6$

2. $391(x+6) - 391x = 6x(x+6)$

3. $x^2 + 6x - 391 = 0$

4. $(x-17)(x+23) = 0$

5. $x = 17$ ($x = -23$ is rejected: a rate is positive)

(a) Answer: $x = -4, y = 10$ or $x = 3, y = -11$

1. $y = -3x - 2$

2. $-3x - 2 = x^2 - 2x - 14$

3. $x^2 + x - 12 = 0$

4. $(x - 3)(x + 4) = 0$

5. $x = -4$ or $x = 3$

6. $y = 10$ or $y = -11$

(b) Answer: $k < -3$

1. $x^2 + 2x + 1 = -2x + k$

2. $x^2 + 4x + 1 - k = 0$

3. no point of intersection: $\Delta < 0$

4. $(4)^2 - 4(1)(1 - k) < 0$

5. $k < -3$

(c) Answer: 22 people

1. let there be n people: $\frac{19800}{n} - \frac{19800}{n+2} = 75$

2. $19800(n+2) - 19800n = 75n(n+2)$

3. $n^2 + 2n - 528 = 0$

4. $(n - 22)(n + 24) = 0$

5. $n = 22$ ($n = -24$ is rejected: the number of people is positive)