IEB Analytical geometry practice — Memorandum

Every answer below was confirmed by an independent verifier at the moment the question was made.

The equation of a circle

1

(a) Answer: $\mathrm{M}(-2; 1)$; $r = 5$

1. $(x+2)^2 + (y-1)^2 = 20 + 4 + 1$
2. $(x+2)^2 + (y-1)^2 = 25$
3. $\mathrm{M}(-2; 1)$ and $r = \sqrt{25} = 5$

(b) Answer: See the working below.

1. $(1)^2 + (5)^2 + 4(1) - 2(5) - 20 = 0$
2. so P lies on the circle

(c) Answer: $y = -(\frac{3}{4})x + \frac{23}{4}$

1. $m_{\mathrm{MP}} = \frac{5-(1)}{1-(-2)} = \frac{4}{3}$
2. the tangent is perpendicular to the radius: $m = -\frac{3}{4}$
3. $5 = (-\frac{3}{4})(1) + c$, so $c = \frac{23}{4}$
4. $y = -(\frac{3}{4})x + \frac{23}{4}$

(d) Answer: $\mathrm{QT} = \sqrt{11}$ units

1. $\mathrm{QM}^2 = (-2-(-2))^2 + (-5-(1))^2 = 36$
2. MT ⊥ QT (radius ⊥ tangent), so $\mathrm{QT}^2 = \mathrm{QM}^2 - r^2$
3. $\mathrm{QT}^2 = 36 - 25 = 11$
4. $\mathrm{QT} = \sqrt{11}$

2

(a) Answer: $(x+5)^2+(y-3)^2=9$

1. it touches the x-axis, so $r=|3|=3$
2. $(x+5)^2+(y-3)^2=9$

(b) Answer: $\mathrm{PQ}=10$ units

1. $\mathrm{PQ}=\sqrt{(3-(-5))^2+(-3-(3))^2}$
2. $=10$

(c) Answer: See the working below.

1. $r_{\mathrm{P}}+r_{\mathrm{Q}}=3+7=10=\mathrm{PQ}$
2. $\therefore$ the circles touch externally

(d) Answer: $\mathrm{T}(-\frac{13}{5};\frac{6}{5})$

1. T lies on PQ, 3 units from P: $\frac{3}{10}$ of the way to Q
2. $(-5+(\frac{3}{10})(8);3+(\frac{3}{10})(-6))=(-\frac{13}{5};\frac{6}{5})$

(e) Answer: $y=(\frac{4}{3})x+\frac{14}{3}$

1. $m_{\mathrm{PQ}}=-\frac{3}{4}$, so the tangent has $m=\frac{4}{3}$
2. $\frac{6}{5}=(\frac{4}{3})(-\frac{13}{5})+c$, so $c=\frac{14}{3}$
3. $y=(\frac{4}{3})x+\frac{14}{3}$

The distance between two points

3

(a) Answer: $\mathrm{AB} = \sqrt{10}$ units

1. $\mathrm{AB} = \sqrt{(0-(3))^2 + (0-(-1))^2}$
2. $= \sqrt{10} = \sqrt{10}$

(b) Answer: $y = (\frac{1}{2})x$

1. $m_{\mathrm{DB}} = \frac{0-(2)}{0-(4)} = \frac{1}{2}$
2. $2 = (\frac{1}{2})(4) + c$, so $c = 0$
3. $y = (\frac{1}{2})x$

(c) Answer: $k = 1$

1. C lies on DB: $k = (\frac{1}{2})(2) + (0)$
2. $k = 1$

(d) Answer: See the proof below.

1. $m_{\mathrm{AE}} = \frac{3-(-1)}{1-(3)} = -2$
2. $m_{\mathrm{AE}} \times m_{\mathrm{DB}} = (-2)(\frac{1}{2}) = -1$
3. ∴ AE ⊥ DB

(e) Answer: $\mathrm{A\hat{B}D} = 45.0^{\circ}$

1. the inclination of AB is 161.6° and of DB is 26.6°
2. the lines make 135.0°
3. $\mathrm{A\hat{B}D} = 45.0^{\circ}$ (its supplement)

Two circles: touching, intersecting or apart

4

(a) Answer: $\mathrm{B}(1;-4)$; $r=3$

1. complete the square in x and in y
2. $(x-1)^2+(y+4)^2=9$
3. $\mathrm{B}(1;-4)$ and $r=3$

(b) Answer: $\mathrm{AB}=5$ units

1. $\mathrm{AB}=\sqrt{(1-(-2))^2+(-4-(0))^2}$
2. $=5$

(c) Answer: They cut in two points.

1. sum of the radii: $4+3=7$
2. $\mathrm{AB}=5$ and $7>5$

(d) Answer: $r=1$

1. touching externally: $r1+r=\mathrm{AB}$
2. $r=5-4=1$

The midpoint of a line segment

5

(a) Answer: $\mathrm{M}(-2; -4)$

1. M is the midpoint of the diameter AB
2. $(\frac{-4+(0)}{2}; \frac{-3+(-5)}{2}) = (-2; -4)$

(b) Answer: $(x+2)^2 + (y+4)^2 = 5$

1. $r^2 = \mathrm{MA}^2 = (-4-(-2))^2 + (-3-(-4))^2 = 5$
2. $(x+2)^2 + (y+4)^2 = 5$

(c) Answer: See the working below.

1. $(-3-(-2))^2 + (-6-(-4))^2 = 5 = r^2$
2. $\therefore$ P lies on the circle

(d) Answer: $y = 2x + 5$

1. $m_{\mathrm{MA}} = -\frac{1}{2}$
2. the tangent is perpendicular to the radius MA: $m = 2$
3. $-3 = (2)(-4) + c$, so $c = 5$
4. $y = 2x + 5$

(e) Answer: $y = -5$ or $y = -3$

1. $x = 0$: $(-2)^2 + (y-(-4))^2 = 5$
2. $(y-(-4))^2 = 1$
3. $y = -5$ or $y = -3$

6

(a) Answer: $\mathrm{M}(-8;0)$

1. the diagonals of a parallelogram bisect each other: M is the midpoint of AC
2. $\mathrm{M} = (\frac{-7+-9}{2}; \frac{-4+4}{2}) = (-8;0)$

(b) Answer: $\mathrm{D}(-12;-1)$

1. M is also the midpoint of BD
2. $\mathrm{D} = (2(-8)-(-4); 2(0)-(1)) = (-12;-1)$

(c) Answer: $\mathrm{AB} = \sqrt{34}$ units

1. $\mathrm{AB} = \sqrt{(-7-(-4))^2 + (-4-(1))^2} = \sqrt{34}$
2. $= \sqrt{34}$

(d) Answer: Yes: ABCD is a rectangle

1. $m_{\mathrm{AB}} \times m_{\mathrm{BC}} = (\frac{5}{3})(-\frac{3}{5}) = -1$
2. so AB ⊥ BC and the parallelogram has a right angle

(e) Answer: 17 square units

1. area $= (\frac{1}{2})|x_{\mathrm{A}}(y_{\mathrm{B}} - y_{\mathrm{C}}) + x_{\mathrm{B}}(y_{\mathrm{C}} - y_{\mathrm{A}}) + x_{\mathrm{C}}(y_{\mathrm{A}} - y_{\mathrm{B}})|$
2. $= 17$

Gradient

7

(a) Answer: $m_{\mathrm{AB}} = 4$

1. $m_{\mathrm{AB}} = \frac{4-(-4)}{5-(3)}$
2. $= 4$

(b) Answer: $\theta = 76.0^{\circ}$

1. $\tan\theta = 4$
2. $\theta = 76.0^{\circ}$

(c) Answer: $y = -(\frac{1}{4})x - 2$

1. $m = -\frac{1}{4} = -\frac{1}{4}$
2. $-2 = (-\frac{1}{4})(0) + c$, so $c = -2$
3. $y = -(\frac{1}{4})x - 2$

(d) Answer: $k = 0$

1. $m_{\mathrm{AD}} = m_{\mathrm{AB}}$
2. $\frac{k-(-4)}{4-(3)} = 4$
3. $k = 0$

(e) Answer: $\mathrm{B\hat{A}C} = 70.3^{\circ}$

1. the inclination of AB is 76.0°
2. $m_{\mathrm{AC}} = -\frac{2}{3}$, so the inclination of AC is 146.3°
3. $\mathrm{B\hat{A}C} = 146.3^{\circ} - 76.0^{\circ}$
4. $\mathrm{B\hat{A}C} = 70.3^{\circ}$

8

(a) Answer: $m_{\mathrm{BC}} = -1$

1. $m_{\mathrm{BC}} = \frac{3-(6)}{7-(4)} = -1$

(b) Answer: See the proof below.

1. $m_{\mathrm{AB}} = 1$
2. $m_{\mathrm{AB}} \times m_{\mathrm{BC}} = (1)(-1) = -1$
3. ∴ AB ⊥ BC and $\mathrm{A\hat{B}C} = 90^{\circ}$

(c) Answer: $y = -x + 6$

1. AD ∥ BC, so $m_{\mathrm{AD}} = -1$
2. $4 = (-1)(2) + c$, so $c = 6$
3. $y = -x + 6$

(d) Answer: $\mathrm{D}(6;\,0)$

1. $0 = (-1)x + (6)$
2. $x = 6$, so $\mathrm{D}(6;\,0)$

(e) Answer: 6 square units

1. $\mathrm{AB} = 2\sqrt{2}$ and $\mathrm{BC} = 3\sqrt{2}$
2. area $=$ ½ $\times\ \mathrm{AB} \times \mathrm{BC}$ (the right angle is at B)
3. $= 6$