

IEB Differential calculus practice — Questions

8 newly generated questions across all 11 differential calculus topics the IEB examines in Paper I — 100 marks in all. Every answer is worked in the memorandum booklet. Nothing here is taken from a past paper.

The derivative from first principles

1

[16]

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- (a) Given $f(x) = -2x^2 + 4x + 6$. Determine $f'(x)$ from first principles. (5)

Answer: _____

-
- (b) Determine $\frac{dy}{dx}$ if $y = \frac{x^2 - 6x - 4}{\sqrt{x}}$. (4)

Answer: _____

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- (c) Determine the equation of the tangent to f at $x = -2$. (4)

Answer: _____

-
- (d) Determine the average gradient of f between $x = 1$ and $x = 2$. (3)

Answer: _____

-
- (a) Given $f(x) = 2x^2 - 2x + 1$. Determine $\frac{f(x+h) - f(x)}{h}$, the average rate of change of f over the interval $[x; x+h]$, in its simplest form. (3)

Answer: _____

- (b) Hence determine $f'(x)$. (2)

Answer: _____

- (c) The line $y = -7x + k$ is a tangent to the graph of $g(x) = x^2 - 3x + 7$. Determine the coordinates of the point where the line touches g , and the value of k . (5)

Answer: _____

- (d) At which point on f is the tangent parallel to the line $y = -6x$? (3)

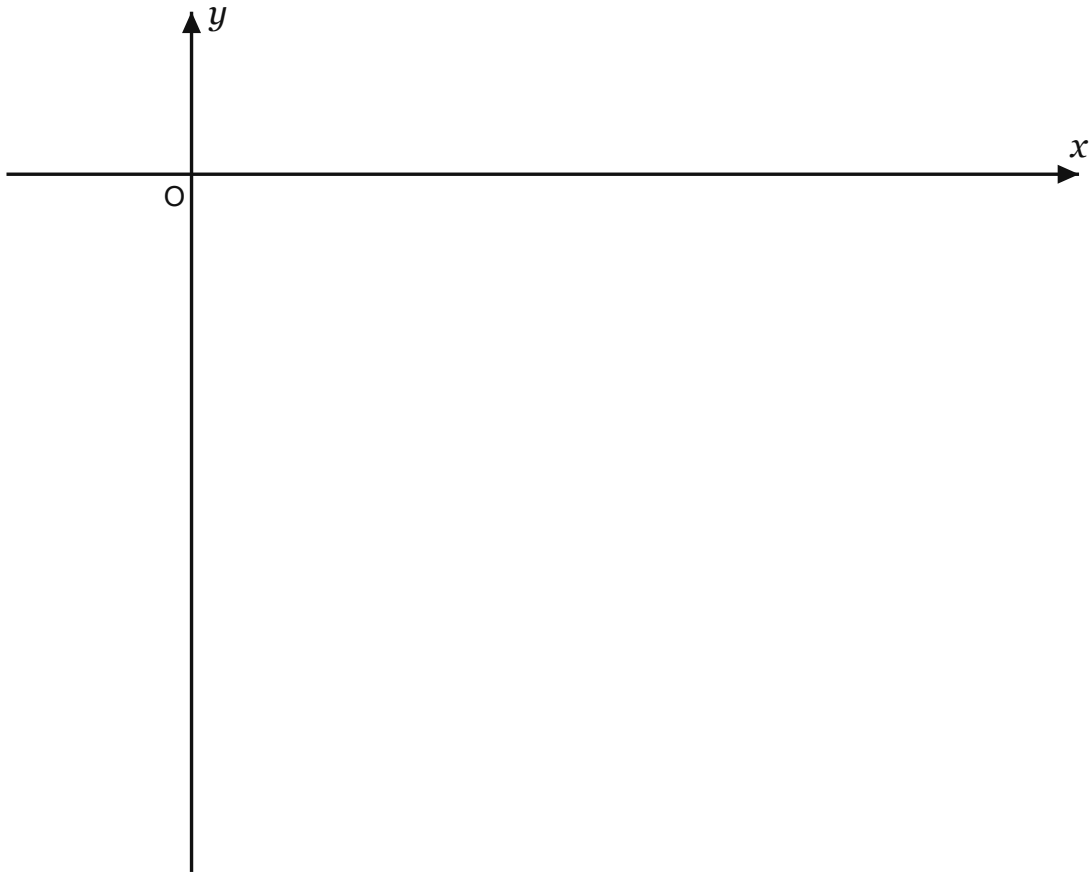
Answer: _____

Stationary points

3

[17]

Given: $f(x) = x^3 - 9x^2 + 24x - 20$



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- (a) Show that $x = 5$ is a root of $f(x) = 0$, and hence determine the x -intercepts of f . (3)

Answer: _____

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- (b) Determine the coordinates of the turning points of f . (4)

Answer: _____

-
- (c) Sketch the graph of f on the axes provided. Show all intercepts with the axes and the turning points. (4)

Answer: _____

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- (d) For which values of x is the graph of f concave up? (2)

Answer: _____

(e) For which values of x is f increasing?

(2)

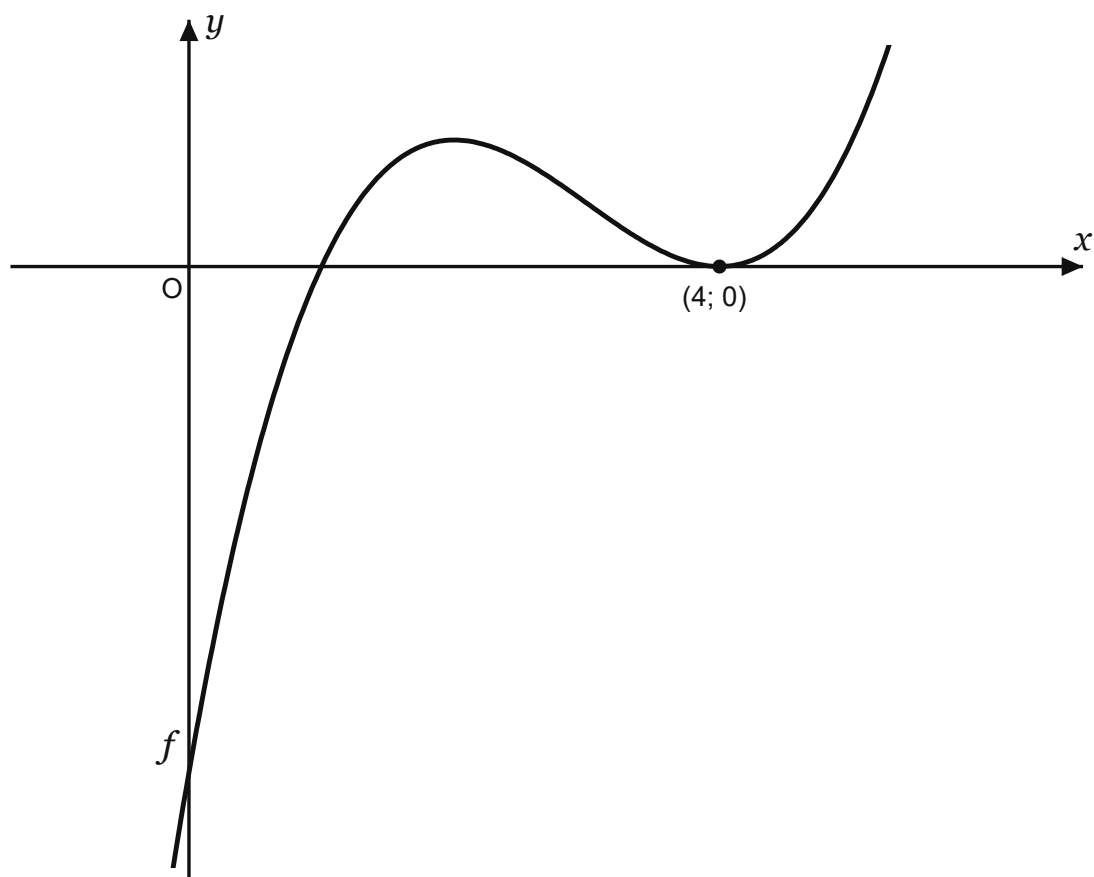
Answer: _____

(f) For which values of k will $f(x) = k$ have three different real roots?

(2)

Answer: _____

In the diagram below, the graph of $f(x) = x^3 - 9x^2 + 24x + d$ is drawn, where d is a constant. $(4; 0)$ is a stationary point of f .



- (a) Determine the value of d . (2)

Answer: _____

- (b) Determine the coordinates of the other stationary point of f . (4)

Answer: _____

- (c) Determine the x -coordinate of the point of inflection of f . (2)

Answer: _____

- (d) For which values of k will $f(x) = k$ have exactly one real root? (3)

Answer: _____

- (e) Determine the equation of the tangent to f at $x = 0$.

(3)

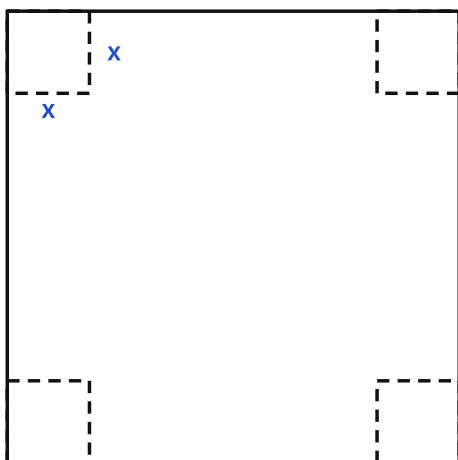
Answer: _____

Optimisation

5

[9]

An open box is made from a square sheet of cardboard 36 cm by 36 cm by cutting a square of side x cm from each corner and folding up the sides.



- (a) Show that the volume of the box is $V = 4x^3 - 144x^2 + 1296x$.

(2)

Answer: _____

- (b) Determine the value of x for which the volume is a maximum, and the maximum volume.

(5)

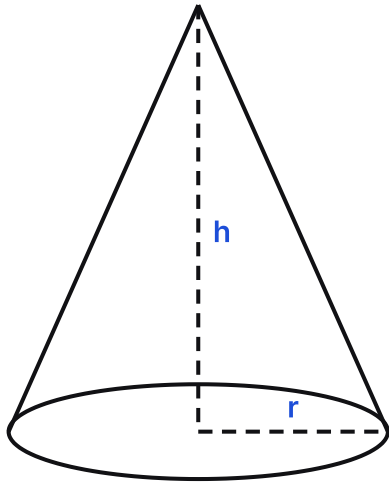
Answer: _____

- (c) Determine the rate at which the volume changes with respect to x when $x = 17$.

(2)

Answer: _____

A cone has radius r cm and height h cm, and $r + h = 12$. ($V = (\frac{1}{3})\pi r^2 h$)



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- (a) Show that the volume of the cone is $V = (\pi/3)(12r^2 - r^3)$. (2)

Answer: _____

-
- (b) Determine the radius that gives the largest volume, and that volume in terms of π . (5)

Answer: _____

-
- (c) Determine the rate at which the volume changes with respect to r when $r = 6$. (2)

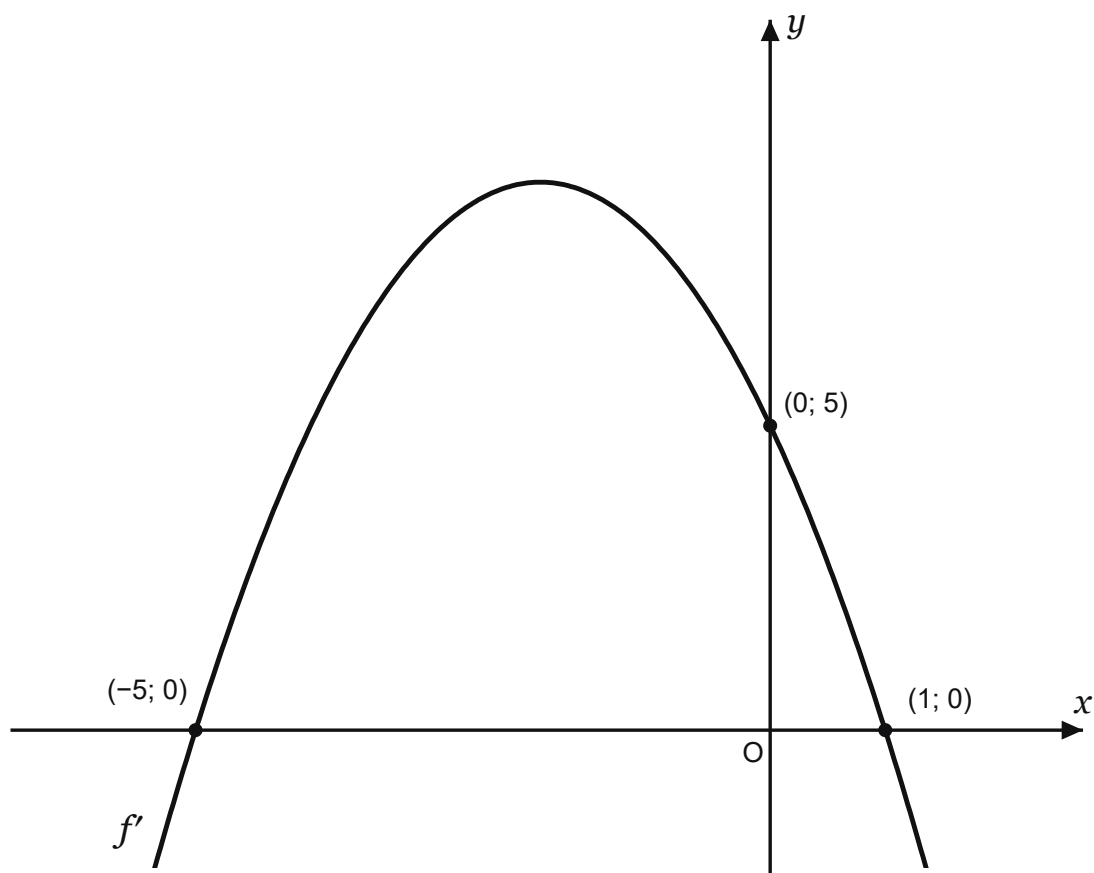
Answer: _____

Reading the graph of f'

7

[10]

The graph of $y = f'(x)$, the derivative of a cubic function f , is drawn below. It cuts the x -axis at $(-5; 0)$ and $(1; 0)$ and the y -axis at $(0; 5)$.



- (a) For which values of x is f increasing? (2)

Answer: _____

- (b) Write down the x -coordinates of the turning points of f , and say which one is a local maximum. (3)

Answer: _____

- (c) Determine the x -coordinate of the point of inflection of f . (2)

Answer: _____

(d) For which values of x is f concave up?

(2)

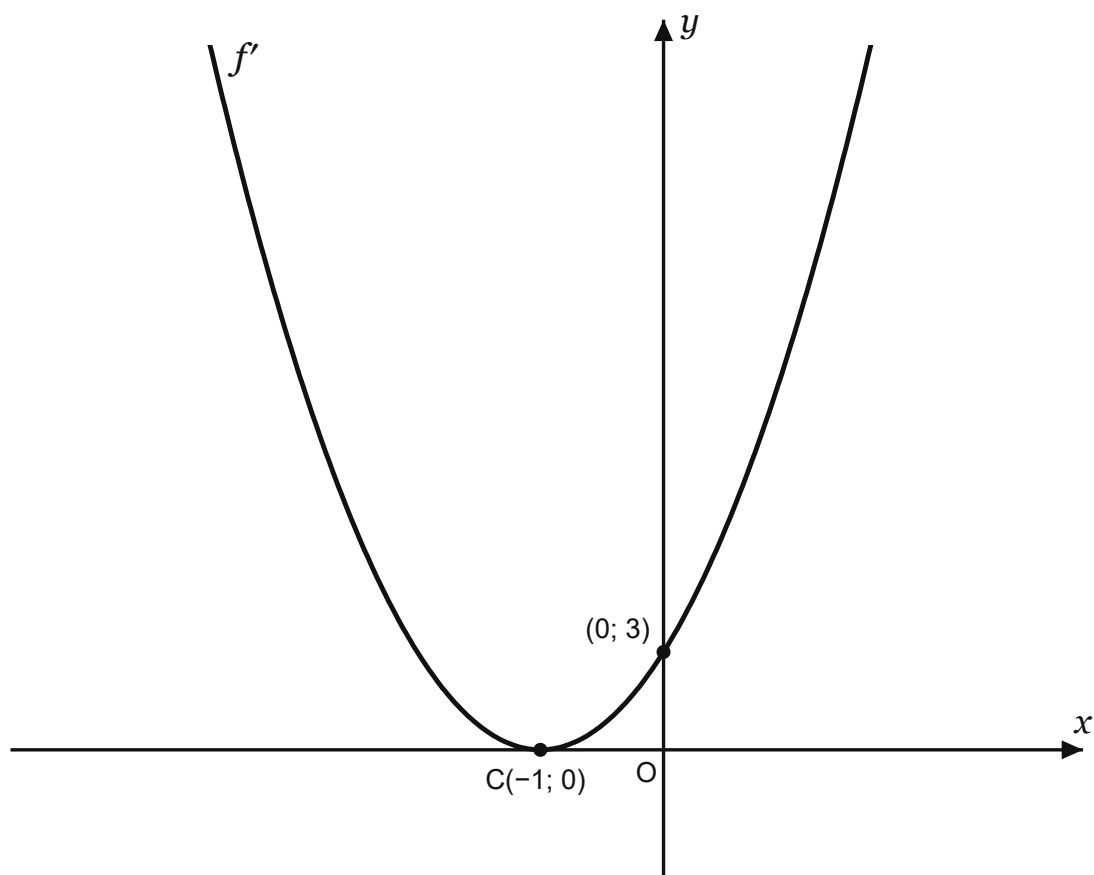
Answer: _____

(e) Write down the gradient of the tangent to f at $x = 0$.

(1)

Answer: _____

The graph of $y = f'(x)$, the derivative of a cubic function f , is drawn below. It touches the x -axis at $C(-1; 0)$ and cuts the y -axis at $(0; 3)$.



- (a) Determine the equation of f' . (3)

Answer: _____

- (b) Given that $f(0) = -1$, determine $f(x)$. (4)

Answer: _____

- (c) Is $x = -1$ a turning point of f , or a point of inflection? Give a reason. (2)

Answer: _____

- (d) Determine the x -values at which the tangent to f has a gradient of 12. (3)

Answer: _____