IEB Euclidean geometry practice — Memorandum

Every answer below was confirmed by an independent verifier at the moment the question was made.

The angle at the centre

1

(a) Answer: $\mathrm{A\hat{O}B} = 92^\circ$

1. $\mathrm{A\hat{O}B} = 92^\circ$ ($\angle$ at centre $= 2 \times \angle$ at circumference)

(b) Answer: $\mathrm{P\hat{B}O} = 46^\circ$

1. $\mathrm{OB} = \mathrm{OP}$ (radii)
2. $\mathrm{P\hat{B}O} = 46^\circ$ (∠s opp equal sides)

(c) Answer: $\mathrm{P\hat{O}B} = 88^\circ$

1. $\mathrm{P\hat{O}B} = 88^\circ$ (sum ∠s of Δ)

(d) Answer: $\mathrm{P\hat{A}B} = 57^\circ$

1. $\mathrm{PA} = \mathrm{PB}$ (Tans from common pt)
2. $\mathrm{P\hat{A}B} = \mathrm{P\hat{B}A}$ (∠s opp equal sides)
3. $\mathrm{P\hat{A}B} = 57^\circ$ (sum ∠s of Δ)

(e) Answer: $\mathrm{P\hat{B}A} = 57^\circ$

1. $\mathrm{P\hat{B}A} = 57^\circ$ (∠s opp equal sides)

(f) Answer: $\mathrm{B\hat{A}U} = 123^\circ$

1. $\mathrm{B\hat{A}U} = 123^\circ$ (∠s on a str line)

(g) Answer: $\mathrm{V\hat{B}A} = 123^\circ$

1. $\mathrm{V\hat{B}A} = 123^\circ$ (∠s on a str line)

2

(a) Answer: $\mathrm{A\hat{O}B} = 62^\circ$

1. $\mathrm{A\hat{O}B} = 62^\circ$ (∠ at centre $= 2 \times \angle$ at circumference)

(b) Answer: $\mathrm{O\hat{A}B} = 59^\circ$

1. $\mathrm{OA} = \mathrm{OB}$ (radii)
2. $\mathrm{O\hat{A}B} = \mathrm{O\hat{B}A}$ (∠s opp equal sides)
3. $\mathrm{O\hat{A}B} = 59^\circ$ (sum ∠s of Δ)

(c) Answer: $\mathrm{O\hat{B}A} = 59^\circ$

1. $\mathrm{O\hat{B}A} = 59^\circ$ (∠s opp equal sides)

(d) Answer: $\mathrm{D\hat{C}B} = 107^\circ$

1. $\mathrm{D\hat{C}B} = 107^\circ$ (opp ∠s of cyclic quad)

(e) Answer: $\mathrm{E\hat{C}D} = 73^\circ$

1. $\mathrm{E\hat{C}D} = 73^\circ$ (ext ∠ of cyclic quad)

Proving the theorem itself, not using it

3

(a) Answer: See the proof below.

1. Given: ABCD is a cyclic quadrilateral in a circle with centre O.
2. To prove: $\hat{\text{A}} + \hat{\text{C}} = 180^\circ$
3. Construction: Join OB and OD.
4. Let $\hat{\text{O}}_1$ be the angle at the centre on C's side of BD, and $\hat{\text{O}}_2$ the other one.
5. $\hat{\text{O}}_1 = 2 \times \hat{\text{A}}$ ($\angle$ at centre $= 2 \times \angle$ at circumference)
6. $\hat{\text{O}}_2 = 2 \times \hat{\text{C}}$ ($\angle$ at centre $= 2 \times \angle$ at circumference)
7. $\hat{\text{O}}_1 + \hat{\text{O}}_2 = 360^\circ$ ($\angle$s round a pt)
8. $\therefore 2\hat{\text{A}} + 2\hat{\text{C}} = 360^\circ$
9. $\therefore \hat{\text{A}} + \hat{\text{C}} = 180^\circ$

(b) Answer: See the proof below.

1. $\text{A}\hat{\text{B}}\text{C} = 180^\circ - 51^\circ - 52^\circ = 77^\circ$ (sum $\angle$s of Δ)
2. $\text{A}\hat{\text{B}}\text{C} + \text{A}\hat{\text{D}}\text{C} = 77^\circ + 103^\circ = 180^\circ$
3. $\therefore$ ABCD is a cyclic quadrilateral (opp $\angle$s of quad supp)

4

(a) Answer: See the proof below.

1. Given: ΔABC with D on AB and E on AC, and DE ∥ BC.
2. To prove: $\frac{\mathrm{AD}}{\mathrm{DB}} = \frac{\mathrm{AE}}{\mathrm{EC}}$
3. Construction: Join BE and CD. Draw $\mathrm{EX} \perp \mathrm{AB}$ and $\mathrm{DY} \perp \mathrm{AC}$.
4. area $\Delta\mathrm{ADE}$ / area $\Delta\mathrm{BDE} = \frac{\mathrm{AD}}{\mathrm{DB}}$ (same height)
5. area $\Delta\mathrm{ADE}$ / area $\Delta\mathrm{CDE} = \frac{\mathrm{AE}}{\mathrm{EC}}$ (same height)
6. area $\Delta\mathrm{BDE} =$ area ΔCDE (same base; same height)
7. $\therefore \frac{\mathrm{AD}}{\mathrm{DB}} = \frac{\mathrm{AE}}{\mathrm{EC}}$

(b) Answer: See the proof below.

1. $\mathrm{AD} = \mathrm{DB}$ (given)
2. DE ∥ BC (given)
3. $\therefore \mathrm{AE} = \mathrm{EC}$ (line from midpoint ∥ to one side)

(c) Answer: $\mathrm{DE} = 5$ units

1. $\mathrm{DE} =$ ½BC (Midpt Theorem)
2. $\mathrm{DE} = ½ \times 10 = 5$

(d) Answer: $\mathrm{DK} = 2$ units

1. $\mathrm{BH} = \frac{2}{5} \times 10 = 4$
2. In $\Delta\mathrm{ABH}$: $\mathrm{AD} = \mathrm{DB}$ and DK ∥ BH (given)
3. $\therefore \mathrm{AK} = \mathrm{KH}$ (line from midpoint ∥ to one side)
4. $\mathrm{DK} = ½\mathrm{BH} = 2$ (Midpt Theorem)

(e) Answer: $\mathrm{DK} : \mathrm{KE} = 2 : 3$

1. In $\Delta\mathrm{AHC}$: $\mathrm{KE} =$ ½HC (Midpt Theorem)
2. $\therefore \mathrm{DK} : \mathrm{KE} =$ ½BH : $½\mathrm{HC} = 2 : 3$

5

(a) Answer: See the proof below.

1. Given: Circle with centre O. AB is a chord. $\mathrm{OM} \perp \mathrm{AB}$.
2. To prove: $\mathrm{AM} = \mathrm{MB}$
3. Construction: Join OA and OB.
4. In ΔOMA and ΔOMB:
5. $\mathrm{OA} = \mathrm{OB}$ (radii)
6. $\mathrm{OM} = \mathrm{OM}$ (common)
7. $\mathrm{O\hat{M}A} = \mathrm{O\hat{M}B} = 90^\circ$ (given)
8. $\therefore \Delta\mathrm{OMA} \equiv \Delta\mathrm{OMB}$ (RHS)
9. $\therefore \mathrm{AM} = \mathrm{MB}$ $(\equiv \Delta\mathrm{s})$

(b) Answer: $\mathrm{O\hat{M}A} = 90^\circ$

1. $\mathrm{O\hat{M}A} = 90^\circ$ (line from centre ⊥ to chord)

(c) Answer: $\mathrm{O\hat{M}B} = 90^\circ$

1. $\mathrm{O\hat{M}B} = 90^\circ$ (line from centre ⊥ to chord)

(d) Answer: $\mathrm{A\hat{O}M} = 56^\circ$

1. $\mathrm{A\hat{O}M} = 56^\circ$ (sum ∠s of Δ)

Tangents, and tangents from a point outside

6

(a) Answer: $\mathrm{B\hat{A}S} = 123^\circ$

1. $\mathrm{B\hat{A}S} = 123^\circ$ (∠s on a str line)

(b) Answer: $\mathrm{O\hat{A}S} = 90^\circ$

1. $\mathrm{O\hat{A}S} = 90^\circ$ $(\tan \perp$ radius)

(c) Answer: $\mathrm{O\hat{A}T} = 90^\circ$

1. $\mathrm{O\hat{A}T} = 90^\circ$ $(\tan \perp$ radius)

(d) Answer: $\mathrm{O\hat{B}A} = 33^\circ$

1. $\mathrm{B\hat{A}O} = 33^\circ$
2. $\mathrm{OA} = \mathrm{OB}$ (radii)
3. $\mathrm{O\hat{B}A} = 33^\circ$ (∠s opp equal sides)

(e) Answer: $\mathrm{B\hat{O}A} = 114^\circ$

1. $\mathrm{B\hat{O}A} = 114^\circ$ (sum ∠s of Δ)

(f) Answer: $\mathrm{P\hat{B}A} = 25^\circ$

1. $\mathrm{P\hat{B}A} = 25^\circ$ (sum ∠s of Δ)

(g) Answer: $\mathrm{D\hat{P}A} = 112^\circ$

1. $\mathrm{D\hat{P}A} = 112^\circ$ (∠s on a str line)

Similar triangles

7

(a) Answer: See the proof below.

1. In ΔDET and ΔCBT:
2. $\mathrm{E\hat{D}T} = \mathrm{B\hat{C}T}$ (alt ∠s; DE ∥ BC)
3. $\mathrm{D\hat{E}T} = \mathrm{C\hat{B}T}$ (alt ∠s; DE ∥ BC)
4. $\mathrm{D\hat{T}E} = \mathrm{B\hat{T}C}$ (vert opp ∠s $=$)
5. $\therefore \Delta\mathrm{DET} \ ||| \ \Delta\mathrm{CBT} \ (\angle\angle\angle)$

(b) Answer: $\mathrm{BC} = 33$ units

1. $\frac{\mathrm{BC}}{\mathrm{DE}} = \frac{\mathrm{CT}}{\mathrm{DT}} \ (|||||| \ \Delta\mathrm{s})$
2. $\mathrm{BC} = 11 \times \frac{21}{7} = 33$

(c) Answer: $\mathrm{BT} = 27$ units

1. $\frac{\mathrm{BT}}{\mathrm{ET}} = \frac{\mathrm{CT}}{\mathrm{DT}} \ (|||||| \ \Delta\mathrm{s})$
2. $\mathrm{BT} = 9 \times \frac{21}{7} = 27$

(d) Answer: $\mathrm{AD} : \mathrm{DB} = 1 : 2$

1. In ΔADE and ΔABC:
2. $\hat{\mathrm{A}} = \hat{\mathrm{A}}$ (common)
3. $\mathrm{A\hat{D}E} = \mathrm{A\hat{B}C}$ (corresp ∠s equal; DE ∥ BC)
4. $\therefore \Delta\mathrm{ADE} \ ||| \ \Delta\mathrm{ABC} \ (\angle\angle\angle)$
5. $\frac{\mathrm{AD}}{\mathrm{AB}} = \frac{\mathrm{DE}}{\mathrm{BC}} = \frac{11}{33} = \frac{1}{3} \ (|||||| \ \Delta\mathrm{s})$
6. $\therefore \mathrm{AD} : \mathrm{DB} = 1 : 2$

8

(a) Answer: See the proof below.

1. In ΔDET and ΔCBT:
2. $\mathrm{E\hat{D}T} = \mathrm{B\hat{C}T}$ (alt ∠s; DE ∥ BC)
3. $\mathrm{D\hat{E}T} = \mathrm{C\hat{B}T}$ (alt ∠s; DE ∥ BC)
4. $\mathrm{D\hat{T}E} = \mathrm{B\hat{T}C}$ (vert opp ∠s $=$)
5. $\therefore \Delta\mathrm{DET} \,|||\, \Delta\mathrm{CBT}\ (\angle\angle\angle)$

(b) Answer: $\mathrm{BC} = 21$ units

1. $\frac{\mathrm{BC}}{\mathrm{DE}} = \frac{\mathrm{CT}}{\mathrm{DT}}\ (|||||\ \Delta\mathrm{s})$
2. $\mathrm{BC} = 7 \times \frac{12}{4} = 21$

(c) Answer: $\mathrm{BT} = 15$ units

1. $\frac{\mathrm{BT}}{\mathrm{ET}} = \frac{\mathrm{CT}}{\mathrm{DT}}\ (|||||\ \Delta\mathrm{s})$
2. $\mathrm{BT} = 5 \times \frac{12}{4} = 15$

(d) Answer: $\mathrm{AD} : \mathrm{DB} = 1 : 2$

1. In ΔADE and ΔABC:
2. $\hat{\mathrm{A}} = \hat{\mathrm{A}}$ (common)
3. $\mathrm{A\hat{D}E} = \mathrm{A\hat{B}C}$ (corresp ∠s equal; DE ∥ BC)
4. $\therefore \Delta\mathrm{ADE} \,|||\, \Delta\mathrm{ABC}\ (\angle\angle\angle)$
5. $\frac{\mathrm{AD}}{\mathrm{AB}} = \frac{\mathrm{DE}}{\mathrm{BC}} = \frac{7}{21} = \frac{1}{3}\ (|||||\ \Delta\mathrm{s})$
6. $\therefore \mathrm{AD} : \mathrm{DB} = 1 : 2$

The angle in a semi-circle

9

(a) Answer: $\mathrm{A\hat{P}B} = 90^{\circ}$

1. $\mathrm{A\hat{P}B} = 90^{\circ}$ (∠s in semi-circle)

(b) Answer: $\mathrm{B\hat{Q}A} = 90^{\circ}$

1. $\mathrm{B\hat{Q}A} = 90^{\circ}$ (∠s in semi-circle)

(c) Answer: $\mathrm{P\hat{B}O} = 60^{\circ}$

1. $\mathrm{P\hat{B}O} = 60^{\circ}$ (sum ∠s of Δ)

(d) Answer: $\mathrm{B\hat{A}T} = 62^{\circ}$

1. $\mathrm{B\hat{A}T} = 62^{\circ}$ (∠s on a str line)

(e) Answer: $\mathrm{A\hat{C}B} = 62^{\circ}$

1. $\mathrm{A\hat{C}B} = 62^{\circ}$ ($\tan$ chord theorem)

Chords, and the line from the centre

10

(a) Answer: $O\hat{M}A = 90^\circ$

1. $O\hat{M}A = 90^\circ$ (line from centre $\perp$ to chord)

(b) Answer: $O\hat{M}B = 90^\circ$

1. $O\hat{M}B = 90^\circ$ (line from centre $\perp$ to chord)

(c) Answer: $A\hat{O}M = 56^\circ$

1. $A\hat{O}M = 56^\circ$ (sum ∠s of Δ)

(d) Answer: $A\hat{P}B = 90^\circ$

1. $A\hat{P}B = 90^\circ$ (∠s in semi-circle)

(e) Answer: $B\hat{Q}A = 90^\circ$

1. $B\hat{Q}A = 90^\circ$ (∠s in semi-circle)

(f) Answer: $P\hat{B}O = 54^\circ$

1. $P\hat{B}O = 54^\circ$ (sum ∠s of Δ)

The proportion theorem

11

(a) Answer: $\mathrm{AF} : \mathrm{FG} = 2 : 1$

1. $\frac{\mathrm{AF}}{\mathrm{FG}} = \frac{\mathrm{AE}}{\mathrm{EC}} = \frac{2}{1}$ (line ∥ one side of Δ; FE ∥ GC)
2. $\mathrm{AF} : \mathrm{FG} = 2 : 1$

(b) Answer: $\mathrm{FG} : \mathrm{GB} = 1 : 1$

1. Let $\mathrm{AF} = 2k$ and $\mathrm{FB} = 2k$ (given)
2. $\mathrm{FG} = \frac{1}{2} \times 2k = 1k$
3. $\mathrm{GB} = \mathrm{FB} - \mathrm{FG} = 2k - 1k = 1k$
4. $\mathrm{FG} : \mathrm{GB} = 1 : 1$

(c) Answer: $\mathrm{BC} : \mathrm{CD} = 1 : 1$

1. $\frac{\mathrm{BC}}{\mathrm{CD}} = \frac{\mathrm{BG}}{\mathrm{GF}}$ (line ∥ one side of Δ; GC ∥ FD)
2. $\frac{\mathrm{BC}}{\mathrm{CD}} = \frac{1k}{1k}$
3. $\mathrm{BC} : \mathrm{CD} = 1 : 1$

Angles in the same segment

12

(a) Answer: $\mathrm{P\hat{B}A} = 57^\circ$

1. $\mathrm{P\hat{B}A} = 57^\circ$ (sum ∠s of Δ)

(b) Answer: $\mathrm{D\hat{P}A} = 81^\circ$

1. $\mathrm{D\hat{P}A} = 81^\circ$ (∠s on a str line)

(c) Answer: $\mathrm{B\hat{P}C} = 81^\circ$

1. $\mathrm{B\hat{P}C} = 81^\circ$ (∠s on a str line)

(d) Answer: $\mathrm{P\hat{D}C} = 24^\circ$

1. $\mathrm{P\hat{D}C} = 24^\circ$ (∠s in the same seg)

(e) Answer: $\mathrm{D\hat{P}C} = 99^\circ$

1. $\mathrm{D\hat{P}C} = 99^\circ$ (vert opp ∠s =)

(f) Answer: $\mathrm{A\hat{C}B} = 43^\circ$

1. $\mathrm{A\hat{C}B} = 43^\circ$ (sum ∠s of Δ)

(g) Answer: $\mathrm{D\hat{C}A} = 137^\circ$

1. $\mathrm{D\hat{C}A} = 137^\circ$ (∠s on a str line)