IEB Functions and graphs practice — Memorandum

Every answer below was confirmed by an independent verifier at the moment the question was made.

The parabola

1

(a) Answer: $f(x) = -x^2 + 8x - 12$

1. $f(x) = a(x-2)(x-6)$
2. $\text{C: } -12 = a(-2)(-6)$, so $a = -1$
3. $f(x) = -x^2 + 8x - 12$

(b) Answer: (4; 4)

1. $x = \frac{2+6}{2} = 4$
2. $f(4) = 4$

(c) Answer: $y \leq 4$

1. the turning value is 4
2. $y \leq 4$

(d) Answer: $k < 4$

1. the line $y = k$ must cut f twice
2. $k < 4$

(e) Answer: (1; 5)

1. $(4-3; 4+1)$
2. (1; 5)

2

(a) Answer: $f(x) = -(x+1)^2 + 5$

1. $f(x) = a(x+1)^2 + 5$
2. substitute $(0; 4)$: $4 = a(1)^2 + 5$
3. $a = -1$
4. $f(x) = -(x+1)^2 + 5$

(b) Answer: $\mathrm{A}(-3; 1)$ and $\mathrm{B}(4; -20)$

1. $-(x+1)^2 + 5 = -3x - 8$
2. $-x^2 + x + 12 = 0$
3. $-(x+3)(x-4) = 0$
4. $x = -3$ or $x = 4$
5. $\mathrm{A}(-3; 1)$ and $\mathrm{B}(4; -20)$

(c) Answer: $-3 \leq x \leq 4$

1. read from the graph at A and B
2. $-3 \leq x \leq 4$

(d) Answer: $y \leq 5$

1. the turning point is a maximum: $y = 5$

(e) Answer: $t > 5$

1. the line $y = t$ must miss the parabola entirely
2. $t > 5$

The hyperbola

3

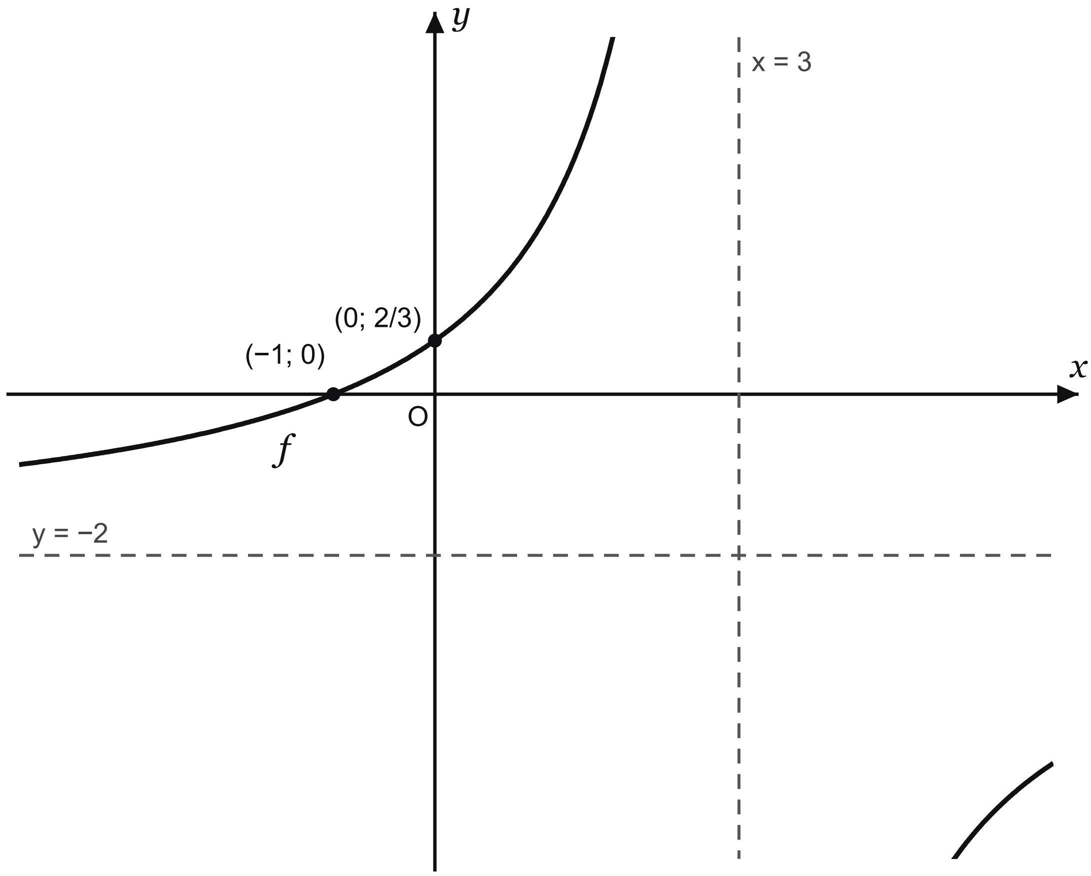
(a) Answer: $f(0) = \frac{2}{3}$

1. $f(0) = -\frac{8}{-3} - 2 = \frac{2}{3}$

(b) Answer: $x = -1$

1. $-\frac{8}{x-3} = 2$
2. $-8 = 2(x-3)$
3. $x = -1$

(c) Answer: See the sketch below.



1. asymptotes: $x = 3$ and $y = -2$
2. y-intercept $(0; \frac{2}{3})$; x-intercept $(-1; 0)$

(d) Answer: $x \in \mathbb{R}, x \neq 3$

1. f is undefined only at $x = 3$

(e) Answer: $-1 < x < 3$

1. read from the graph: the x-intercept is $x = -1$ and the asymptote $x = 3$ is never included
2. $-1 < x < 3$

(f) Answer: $y = \frac{8}{x-3} + 2$

1. $y = -f(x) = \frac{8}{x-3} + 2$

4

(a) Answer: $p = 1$ and $q = 2$

1. the asymptotes are $x = -1$ and $y = 2$
2. $x + p = 0$ at $x = -1$, so $p = 1$; $q = 2$

(b) Answer: $a = 3$

1. $3 = \frac{a}{2+1} + 2$
2. $a = (3 - 2)(2 + 1) = 3$

(c) Answer: $y = x + 3$ and $y = -x + 1$

1. both pass through the point where the asymptotes meet, $(-1; 2)$
2. $y = (x + 1) + 2$ and $y = -(x + 1) + 2$

(d) Answer: $(-1 - \sqrt{3}; 2 - \sqrt{3})$ and $(-1 + \sqrt{3}; 2 + \sqrt{3})$

1. $\frac{3}{x+1} + 2 = x + 3$
2. $(x + 1)^2 = 3$
3. $x = -1 \pm \sqrt{3}$
4. $(-1 - \sqrt{3}; 2 - \sqrt{3})$ and $(-1 + \sqrt{3}; 2 + \sqrt{3})$

(e) Answer: $x \in \mathbb{R}$, $x \neq -1$

1. f is undefined only at $x = -1$

(f) Answer: $x \leq -\frac{5}{2}$ or $x > -1$

1. read from the graph: the x-intercept is $x = -\frac{5}{2}$ and the asymptote $x = -1$ is never included
2. $x \leq -\frac{5}{2}$ or $x > -1$

(g) Answer: $g(x) = \frac{3}{x+2} + 0$

1. $g(x) = f(x + 1) - 2$

2. $g(x) = \frac{3}{x + 2} + 0$

Exponential equations and exponent laws

5

(a) Answer: $y > -7$

1. the asymptote is $y = -7$ and h lies above it

(b) Answer: $p = 1$ and $q = -7$

1. $q = -7$, from the asymptote
2. $2 = 3^{1 + p} - 7$
3. $3^{1 + p} = 9 = 3^2$
4. $p = 1$

(c) Answer: (0.77; 0)

1. $3^{x + 1} - 7 = 0$
2. $3^{x + 1} = 7$
3. $x = \log_3 7 - 1 = 0.77$

(d) Answer: $k(x) = 3^{-x + 1} - 7$

1. a reflection in the y-axis replaces x by $-x$
2. $k(x) = 3^{-x + 1} - 7$

(e) Answer: $x > 0.77$

1. h is increasing and cuts the x-axis at $x = 0.77$
2. $x > 0.77$

6

(a) Answer: $b = \frac{1}{2}$

1. P lies on f: $b^2 = \frac{1}{4}$
2. $b = \frac{1}{2}$

(b) Answer: $y = \log_{1/2} x$

1. g is the reflection of f in $y = x$: $x = (\frac{1}{2})^y$
2. $y = \log_{1/2} x$

(c) Answer: $x > 0$

1. the domain of g is the range of f

(d) Answer: $0 < x < 2$

1. $g(x) = -1$ when $x = (\frac{1}{2})^{-1} = 2$
2. g is decreasing $(0 < b < 1)$, so the inequality sign is reversed
3. $0 < x < 2$

(e) Answer: $\mathrm{AB} = \frac{5}{4}$ units

1. $\mathrm{A}(2;\, \frac{1}{4})$: $f(2) = (\frac{1}{2})^2 = \frac{1}{4}$
2. $\mathrm{B}(2;\, -1)$: $g(2) = \log_{1/2} 2 = -1$
3. $\mathrm{AB} = \frac{1}{4} - (-1) = \frac{5}{4}$

(f) Answer: $y = -3$; x-intercept $(-1.58;\, 0)$

1. f's asymptote $y = 0$ moves down 3: $y = -3$
2. $(\frac{1}{2})^x - 3 = 0$, so $(\frac{1}{2})^x = 3$
3. $x = \log_{1/2} 3 = -1.58$

Fitting a graph to something real

7

(a) Answer: $y = -\frac{1}{12}(x-6)^2 + 7$

1. $y = a(x-6)^2 + 7$
2. at $(0; 4)$: $4 = a(-6)^2 + 7$
3. $a = -\frac{1}{12}$
4. $y = -\frac{1}{12}(x-6)^2 + 7$

(b) Answer: $\frac{59}{12}$ m

1. $y = -\frac{1}{12}(11-6)^2 + 7$
2. $y = \frac{59}{12}$ m

(c) Answer: 15.17 m

1. $-\frac{1}{12}(x-6)^2 + 7 = 0$
2. $(x-6)^2 = 84$
3. $x = 6 + \sqrt{84}$
4. $x = 15.17$ m

8

(a) Answer: $y = \frac{7}{64}(x-8)^2 + 2$

1. $y = a(x-8)^2 + 2$
2. at $(0; 9)$: $9 = a(-8)^2 + 2$
3. $a = \frac{7}{64}$
4. $y = \frac{7}{64}(x-8)^2 + 2$

(b) Answer: $\frac{135}{64}\ \mathrm{m}$

1. $y = \frac{7}{64}(9-8)^2 + 2$
2. $y = \frac{135}{64}\ \mathrm{m}$

(c) Answer: 1.95 m and 14.05 m

1. $\frac{7}{64}(x-8)^2 + 2 = 6$
2. $(x-8)^2 = \frac{256}{7}$
3. $x = 8 \pm \sqrt{256/7}$
4. $x = 1.95\ \mathrm{m}$ or $x = 14.05\ \mathrm{m}$