

IEB Sequences and series practice —

Memorandum

Every answer below was confirmed by an independent verifier at the moment the question was made.

Geometric sequences

1

(a) Answer: 0.71 m

1. a geometric sequence of heights with $r = \frac{3}{4}$

$$2. 3 \times \left(\frac{3}{4}\right)^5 = 0.71 \text{ m}$$

(b) Answer: 18.60 m

1. down 3 m, then up and down again after each of 7 bounces

$$2. 3 + 2 \times 3 \left(\frac{3}{4}\right) \frac{1 - (3/4)^7}{1 - 3/4}$$

$$3. = 18.60 \text{ m}$$

(c) Answer: 21.00 m

1. $-1 < r = \frac{3}{4} < 1$, so the sum converges

$$2. 3 + 2 \times 3 \frac{3/4}{1 - 3/4}$$

$$3. = 21.00 \text{ m}$$

(d) Answer: after bounce 11

$$1. 3\left(\frac{3}{4}\right)^n < 0.15$$

$$2. n > \frac{\log(0.15/3)}{\log(3/4)} = 10.41$$

$$3. n = 11$$

2

(a) Answer: $a = -2$ and $r = 3$

1. $ar^3 = -54$ and $ar^6 = -1458$

2. $r^3 = -1458 / -54 = 27$

3. $r = 3$

4. $a = -\frac{54}{3} = -2$

(b) Answer: $S_8 = -6560$

1. $S_8 = -2 \frac{(3)^8 - 1}{3 - 1}$

2. $= -6560$

(c) Answer: $T_8 = -4374$

1. $T_8 = -2(3)^7$

2. $= -4374$

(d) Answer: T_{10}

1. $|-2| \cdot 3^{n-1} > 30000$

2. $n - 1 > \frac{\log(30000/2)}{\log 3}$

3. $n = 10$

(e) Answer: $-7 < x < -1$

1. a geometric series with $r = \frac{x+4}{3}$

2. $-1 < \frac{x+4}{3} < 1$

3. $-7 < x < -1$

(f) Answer: $S_\infty = \frac{1}{2}$

1. $r = \frac{-3+4}{3} = \frac{1}{3}$

2. $a = T_1 = \frac{1}{3}$

$$3. S_{\infty} = \frac{a}{1-r} = \frac{1}{2}$$

Arithmetic sequences

3

(a) Answer: $x = 2$; the terms are 5; 9; 13

1. $T_2 - T_1 = T_3 - T_2$

2. $2(4x + 1) = (2x + 1) + (x + 11)$

3. $5x - 10 = 0$

4. $x = 2$

(b) Answer: T_{24}

1. $5 + (n - 1)(4) = 97$

2. $n - 1 = 23$

3. $n = 24$

(c) Answer: 14 terms

1. $\frac{n}{2}[2(5) + (n - 1)(4)] = 434$

2. $4n^2 + 6n - 868 = 0$

3. $n = 14$ (n is a natural number)

(d) Answer: 1261 dots

1. second difference 6: $a = 3$

2. $T_n = 3n^2 + 3n + 1$

3. $T_{20} = 1261$

(e) Answer: picture 27

1. $3n^2 + 3n + 1 = 2269$

2. $3n^2 + 3n - 2268 = 0$

3. $n = 27$ ($n > 0$)

(a) Answer: $T_n = 3n + 16$

$$1. a = 19 \text{ and } d = 22 - (19) = 3$$

$$2. T_n = 19 + (n - 1)(3) = 3n + 16$$

(b) Answer: 47 terms

$$1. 3n + 16 = 157$$

$$2. 3n = 141$$

$$3. n = 47$$

(c) Answer: $S_{47} = 4136$

$$1. S_n = \frac{n}{2}(a + l)$$

$$2. S_{47} = \frac{47}{2}(19 + 157)$$

$$3. = 4136$$

(d) Answer: $\sum_{n=1}^{47} (3n + 16)$

$$1. \text{ the terms are } T_n = 3n + 16, \text{ from } n = 1 \text{ to } n = 47$$

$$2. \sum_{n=1}^{47} (3n + 16)$$

(e) Answer: $T_n = -2n^2 - 6n - 6$

$$1. \text{ first differences: } -12; -16; -20; -24$$

$$2. \text{ second difference: } -4, \text{ so } 2a = -4 \text{ and } a = -2$$

$$3. 3a + b = -12, \text{ so } b = -6$$

$$4. a + b + c = -14, \text{ so } c = -6$$

$$5. T_n = -2n^2 - 6n - 6$$

(f) Answer: Yes: it is T_{18}

$$1. -2n^2 - 6n - 6 = -762$$

$$2. -2n^2 - 6n + 756 = 0$$

3. $n = 18$, a natural number