

IEB-oefening: Trigonometrie — Memorandum

Elke antwoord hieronder is deur 'n onafhanklike verifieerder bevestig op die oomblik toe die vraag gemaak is.

Identiteite, reduksie en saamgestelde hoeke

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(a) Antwoord: $\sin A = \frac{5}{13}$

1. A lê in die tweede kwadrant: $x = -12$, $r = 13$

2. $y^2 = 13^2 - (-12)^2 = 25$, dus $y = 5$

3. $\sin A = \frac{5}{13}$ en $\cos A = -\frac{12}{13}$

(b) Antwoord: $\sin(A + B) = \frac{120}{169}$

1. B lê in die vierde kwadrant: $x = 12$, $r = 13$

2. $y^2 = 13^2 - 12^2 = 25$, dus $y = -5$

3. $\sin B = -\frac{5}{13}$ en $\cos B = \frac{12}{13}$

4. $\sin(A + B) = \sin A \cos B + \cos A \sin B$

5. $= \left(\frac{5}{13}\right)\left(\frac{12}{13}\right) + \left(-\frac{12}{13}\right)\left(-\frac{5}{13}\right)$

6. $= \frac{120}{169}$

(c) Antwoord: $\sin 2A = -\frac{120}{169}$

1. $\sin 2A = 2 \sin A \cos A$

2. $= 2\left(\frac{5}{13}\right)\left(-\frac{12}{13}\right)$

3. $= -\frac{120}{169}$

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(a) Antwoord: $-\cos \theta$

1. $\sin(360^\circ - \theta) = -\sin \theta$

2. $\sin(90^\circ - \theta) = \cos \theta$

3. $\sin(180^\circ - \theta) = \sin \theta$

4. $= -\cos \theta$

(b) Antwoord: $-\frac{\sqrt{3}}{2}$

1. $\cos 89^\circ \cos 61^\circ - \sin 89^\circ \sin 61^\circ = \cos(89^\circ + 61^\circ)$

2. $= \cos 150^\circ = -\frac{\sqrt{3}}{2}$

(c) Antwoord: $\frac{1}{2}$

1. $2 \sin 15^\circ \cos 15^\circ = \sin 30^\circ$

2. $= \frac{1}{2}$

(d) Antwoord: $\theta = -30^\circ + k \cdot 180^\circ, k \in \mathbb{Z}$

1. $\tan \theta = -\frac{1}{\sqrt{3}}$

2. $\theta = -30^\circ + k \cdot 180^\circ, k \in \mathbb{Z}$

(a) Antwoord: $x = -120^\circ + k \cdot 360^\circ$ of $x = 0^\circ + k \cdot 360^\circ$ of $x = 120^\circ + k \cdot 360^\circ$, $k \in \mathbb{Z}$

$$1. 2\sin^2 x + \cos x - 1 = 0$$

$$2. 2(1 - \cos^2 x) + 1 \cos x - 1 = 0$$

$$3. 2\cos^2 x - \cos x - 1 = 0$$

$$4. (2\cos x + 1)(\cos x - 1) = 0$$

$$5. \cos x = -\frac{1}{2} \text{ of } \cos x = 1$$

$$6. x = -120^\circ + k \cdot 360^\circ \text{ of } x = 0^\circ + k \cdot 360^\circ \text{ of } x = 120^\circ + k \cdot 360^\circ, k \in \mathbb{Z}$$

(b) Antwoord: Sien die bewys hieronder.

$$1. \text{LK} = \frac{\cos x(1 + \sin x) - \cos x(1 - \sin x)}{(1 - \sin x)(1 + \sin x)}$$

$$2. = \frac{2 \sin x \cos x}{1 - \sin^2 x}$$

$$3. = \frac{2 \sin x \cos x}{\cos^2 x}$$

$$4. = 2 \tan x$$

$$5. = \text{RK}$$

(c) Antwoord: $x = 60^\circ$ of $x = 240^\circ$

$$1. 2 \tan x = 2\sqrt{3}, \text{ dus } \tan x = \sqrt{3}$$

$$2. x = 60^\circ + k \cdot 180^\circ$$

$$3. \text{In } [0^\circ; 360^\circ]: x = 60^\circ \text{ of } x = 240^\circ$$

(a) Antwoord: Sien die bewys hieronder.

$$1. \text{LK} = \frac{1 - (1 - 2\sin^2 x)}{2 \sin x \cos x}$$

$$2. = \frac{2\sin^2 x}{2 \sin x \cos x}$$

$$3. = \frac{\sin x}{\cos x}$$

$$4. = \tan x$$

$$5. = \text{RK}$$

(b) Antwoord: $x = 0^\circ$ of $x = 90^\circ$ of $x = 180^\circ$ of $x = 270^\circ$ of $x = 360^\circ$

1. Die identiteit is nie geldig waar $\sin 2x = 0$ nie, of waar $\tan x$ ongedefinieer is.

2. $\sin 2x = 0$: $2x = k \cdot 180^\circ$, dus $x = k \cdot 90^\circ$

3. $\tan x$ is ongedefinieer waar $\cos x = 0$, wat onder hierdie waardes is

4. In $[0^\circ; 360^\circ]$: $x = 0^\circ$ of $x = 90^\circ$ of $x = 180^\circ$ of $x = 270^\circ$ of $x = 360^\circ$

(c) Antwoord: geen oplossing nie

$$1. \tan x = 0$$

$$2. x = 0^\circ + k \cdot 180^\circ$$

3. $x = -180^\circ$ of $x = 0^\circ$ of $x = 180^\circ$: die identiteit is nie daar geldig nie, dus is hulle nie oplossings nie

4. In $[-180^\circ; 180^\circ]$: geen oplossing nie

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(a) Antwoord: $\sin 44^\circ = 2p$

1. $\sin 44^\circ = 2 \sin 22^\circ \cos 22^\circ = 2p$

(b) Antwoord: $\cos 46^\circ = 2p$

1. $\cos 46^\circ = \cos(90^\circ - 44^\circ) = \sin 44^\circ = 2p$

(c) Antwoord: $\cos 88^\circ = 1 - 8p^2$

1. $\cos 88^\circ = 1 - 2\sin^2 44^\circ$

2. $= 1 - 2(2p)^2 = 1 - 8p^2$

(d) Antwoord: $\cos 44^\circ = \sqrt{1 - 4p^2}$

1. $\cos^2 44^\circ = 1 - \sin^2 44^\circ = 1 - 4p^2$

2. 44° is 'n skerp hoek, dus $\cos 44^\circ > 0$

3. $\cos 44^\circ = \sqrt{1 - 4p^2}$

(a) Antwoord: $\sin 134^\circ = p$

1. $\sin 134^\circ = \sin(180^\circ - 46^\circ) = \sin 46^\circ = p$

(b) Antwoord: $\cos 46^\circ = \sqrt{1 - p^2}$

1. $\cos^2 46^\circ = 1 - \sin^2 46^\circ = 1 - p^2$

2. 46° is 'n skerp hoek, dus $\cos 46^\circ > 0$

3. $\cos 46^\circ = \sqrt{1 - p^2}$

(c) Antwoord: $\sin 92^\circ = 2p\sqrt{1 - p^2}$

1. $\sin 92^\circ = 2 \sin 46^\circ \cos 46^\circ$

2. $= 2p\sqrt{1 - p^2}$

Trigonometrisiese grafieke

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(a) Antwoord: 180°

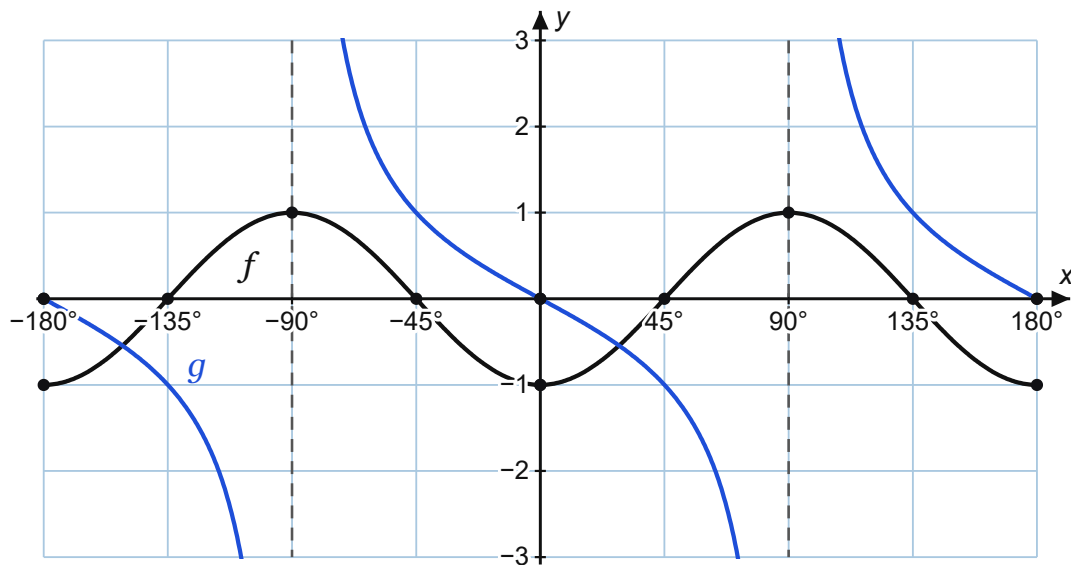
1. $g(x) = -\tan x$ herhaal elke 180°

(b) Antwoord: $x = -90^\circ$ en $x = 90^\circ$

1. $\tan x$ is ongedefinieer waar $\cos x = 0$: $x = 90^\circ + k \cdot 180^\circ$

2. In $[-180^\circ; 180^\circ]$: $x = -90^\circ$ en $x = 90^\circ$

(c) Antwoord: Sien die skets hieronder.



1. $f(x) = -\cos 2x$

2. f : x -afsnitte $(-135^\circ; 0)$, $(-45^\circ; 0)$, $(45^\circ; 0)$, $(135^\circ; 0)$

3. f : y -afsnit $(0^\circ; -1)$

4. f : draaipunte $(-90^\circ; 1)$, $(0^\circ; -1)$, $(90^\circ; 1)$

5. f : eindpunte $(-180^\circ; -1)$, $(180^\circ; -1)$

6. $g(x) = -\tan x$

7. g : x -afsnitte $(-180^\circ; 0)$, $(0^\circ; 0)$, $(180^\circ; 0)$

8. g : y -afsnit $(0^\circ; 0)$

9. g : asimptote $x = -90^\circ$, $x = 90^\circ$

10. g : eindpunten $(-180^\circ; 0)$, $(180^\circ; 0)$

(d) Antwoord: $x = -180^\circ$ of $-135^\circ \leq x < -90^\circ$ of $-45^\circ \leq x \leq 0^\circ$ of $45^\circ \leq x < 90^\circ$ of $135^\circ \leq x \leq 180^\circ$

1. f en g het teenoorgestelde tekens, of een van hulle is 0, van die skets afgelees

2. $x = -180^\circ$ of $-135^\circ \leq x < -90^\circ$ of $-45^\circ \leq x \leq 0^\circ$ of $45^\circ \leq x < 90^\circ$ of $135^\circ \leq x \leq 180^\circ$

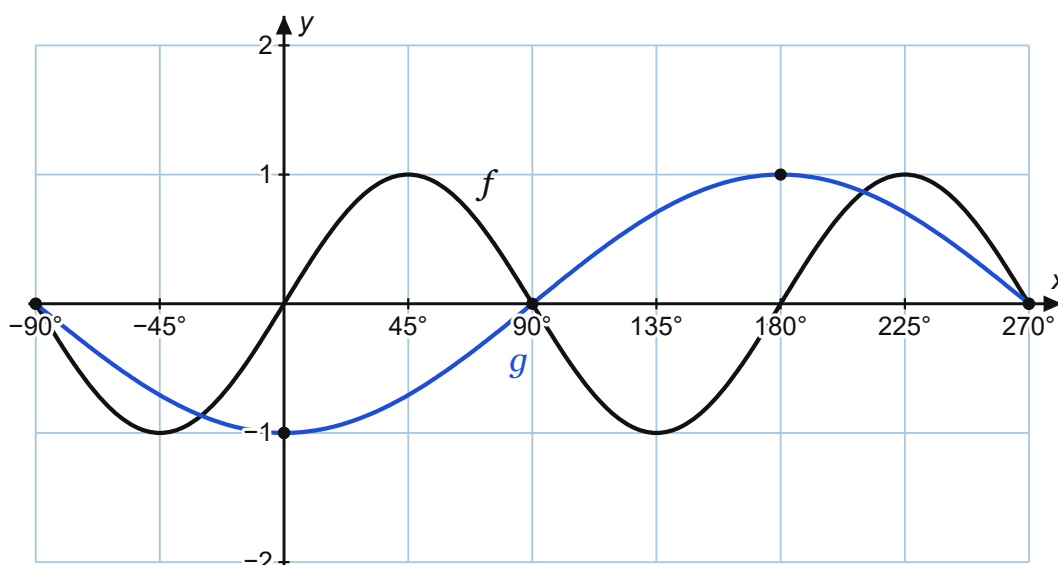
(a) Antwoord: $b = 2$

1. $\sin bx$ het 'n periode van $360^\circ/b$
2. een volledige golf van f strek oor 180°
3. $360^\circ/b = 180^\circ$, dus $b = 2$

(b) Antwoord: 180°

1. $f(x) = \sin 2x$ herhaal elke 180°

(c) Antwoord: Sien die skets hieronder.



1. $g(x) = -\cos x$
2. g : x -afsnitte $(-90^\circ; 0)$, $(90^\circ; 0)$, $(270^\circ; 0)$
3. g : y -afsnit $(0^\circ; -1)$
4. g : draaipunte $(0^\circ; -1)$, $(180^\circ; 1)$
5. g : eindpunte $(-90^\circ; 0)$, $(270^\circ; 0)$

(d) Antwoord: 1

1. die amplitude van $a \cdot \cos x$ is $|a| = 1$

(e) Antwoord: $x = -90^\circ$ of $x = -30^\circ$ of $x = 90^\circ$ of $x = 210^\circ$ of $x = 270^\circ$

1. $\sin 2x = -\cos x$

2. $2 \sin x \cos x + \cos x = 0$

3. $\cos x(2 \sin x + 1) = 0$

4. $\cos x = 0$ of $\sin x = -\frac{1}{2}$

5. $x = 90^\circ + k \cdot 180^\circ$ of $x = -30^\circ + k \cdot 360^\circ$ of $x = 210^\circ + k \cdot 360^\circ$

6. In $[-90^\circ; 270^\circ]$: $x = -90^\circ$ of $x = -30^\circ$ of $x = 90^\circ$ of $x = 210^\circ$ of $x = 270^\circ$

(f) Antwoord: $-90^\circ < x < -30^\circ$ of $90^\circ < x < 210^\circ$

1. f lê onder g , van die skets afgelees

2. $-90^\circ < x < -30^\circ$ of $90^\circ < x < 210^\circ$

(g) Antwoord: $0^\circ < x < 90^\circ$ of $90^\circ < x < 180^\circ$

1. f en g het teenoorgestelde tekens, en $g(x) \neq 0$, van die skets afgelees

2. $0^\circ < x < 90^\circ$ of $90^\circ < x < 180^\circ$

Sinus-, kosinus- en oppervlakttereël (2D)

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(a) Antwoord: Sien die berekening hieronder.

1. $\hat{A}CB = 180^\circ - (\alpha + \beta)$ (som $\angle$ e van Δ)
 2. $\frac{AC}{\sin \beta} = d/(\sin(180^\circ - (\alpha + \beta)))$ (sinusreël)
 3. $\sin(180^\circ - (\alpha + \beta)) = \sin(\alpha + \beta)$
 4. $AC = (d \sin \beta)/(\sin(\alpha + \beta))$
-

(b) Antwoord: Sien die berekening hieronder.

1. In ΔACT , $\hat{A}CT = 90^\circ$: $\tan \gamma = \frac{TC}{AC}$
 2. $TC = AC \tan \gamma = (d \sin \beta \tan \gamma)/(\sin(\alpha + \beta))$
-

(c) Antwoord: $TC = 96.57$ m

1. $TC = \frac{100 \sin 65^\circ \tan 44^\circ}{\sin 115^\circ}$
 2. $= 96.57$ m
-

(d) Antwoord: $BT = 128.33$ m

1. $BC = \frac{100 \sin 50^\circ}{\sin 115^\circ} = 84.52$ m (sinusreël)
 2. $BT^2 = BC^2 + TC^2 = 84.52^2 + 96.57^2$
 3. $BT = 128.33$ m
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(e) Antwoord: 48.81°

1. $\tan \hat{B}C = \frac{TC}{BC} = \frac{96.57}{84.52}$
2. $\hat{B}C = 48.81^\circ$

(a) Antwoord: $QS = 78.02 \text{ m}$

$$1. \text{ In } \triangle PQS: \frac{QS}{\sin \hat{QPS}} = \frac{PQ}{\sin \hat{QSP}}$$

$$2. QS = \frac{100 \sin 45^\circ}{\sin 65^\circ}$$

$$3. = 78.02 \text{ m}$$

(b) Antwoord: $QR = 94.69 \text{ m}$

$$1. \hat{QSR} = 180^\circ - 65^\circ = 115^\circ \text{ (}\angle\text{e op 'n reguit lyn)}$$

$$2. QR^2 = QS^2 + SR^2 - 2(QS)(SR) \cos \hat{QSR}$$

$$3. = 78.02^2 + 30^2 - 2(78.02)(30) \cos 115^\circ$$

$$4. QR = 94.69 \text{ m}$$

(c) Antwoord: oppervlakte = 1060.66 m^2

$$1. \text{ Oppervlakte} = \left(\frac{1}{2}\right)(QS)(SR) \sin \hat{QSR}$$

$$2. = \left(\frac{1}{2}\right)(78.02)(30) \sin 115^\circ$$

$$3. = 1060.66 \text{ m}^2$$

(d) Antwoord: $TQ = 36.38 \text{ m}$

$$1. \text{ In } \triangle TQS, \hat{TQS} = 90^\circ: \tan 25^\circ = \frac{TQ}{QS}$$

$$2. TQ = 78.02 \tan 25^\circ$$

$$3. = 36.38 \text{ m}$$