IEB Trigonometry practice — Memorandum

Every answer below was confirmed by an independent verifier at the moment the question was made.

Identities, reduction and compound angles

1

(a) Answer: $\sin \mathrm{A} = \frac{5}{13}$

1. A lies in the second quadrant: $x = -12$, $r = 13$
2. $y^2 = 13^2 - (-12)^2 = 25$, so $y = 5$
3. $\sin \mathrm{A} = \frac{5}{13}$ and $\cos \mathrm{A} = -\frac{12}{13}$

(b) Answer: $\sin(\mathrm{A} + \mathrm{B}) = \frac{120}{169}$

1. B lies in the fourth quadrant: $x = 12$, $r = 13$
2. $y^2 = 13^2 - 12^2 = 25$, so $y = -5$
3. $\sin \mathrm{B} = -\frac{5}{13}$ and $\cos \mathrm{B} = \frac{12}{13}$
4. $\sin(\mathrm{A} + \mathrm{B}) = \sin \mathrm{A} \cos \mathrm{B} + \cos \mathrm{A} \sin \mathrm{B}$
5. $= (\frac{5}{13})(\frac{12}{13}) + (-\frac{12}{13})(-\frac{5}{13})$
6. $= \frac{120}{169}$

(c) Answer: $\sin 2\mathrm{A} = -\frac{120}{169}$

1. $\sin 2\mathrm{A} = 2 \sin \mathrm{A} \cos \mathrm{A}$
2. $= 2(\frac{5}{13})(-\frac{12}{13})$
3. $= -\frac{120}{169}$

2

(a) Answer: $-\cos\theta$

1. $\sin(360^\circ - \theta) = -\sin\theta$
2. $\sin(90^\circ - \theta) = \cos\theta$
3. $\sin(180^\circ - \theta) = \sin\theta$
4. $= -\cos\theta$

(b) Answer: $-\frac{\sqrt{3}}{2}$

1. $\cos 89^\circ \cos 61^\circ - \sin 89^\circ \sin 61^\circ = \cos(89^\circ + 61^\circ)$
2. $= \cos 150^\circ = -\frac{\sqrt{3}}{2}$

(c) Answer: $\frac{1}{2}$

1. $2 \sin 15^\circ \cos 15^\circ = \sin 30^\circ$
2. $= \frac{1}{2}$

(d) Answer: $\theta = -30^\circ + k{\cdot}180^\circ,\ k \in \mathbb{Z}$

1. $\tan\theta = -\frac{1}{\sqrt{3}}$
2. $\theta = -30^\circ + k{\cdot}180^\circ,\ k \in \mathbb{Z}$

3

(a) Answer: $x = -120^\circ + k{\cdot}360^\circ$ or $x = 0^\circ + k{\cdot}360^\circ$ or $x = 120^\circ + k{\cdot}360^\circ$, $k \in \mathbb{Z}$

1. $2\sin^2 x + \cos x - 1 = 0$
2. $2(1 - \cos^2 x) + 1\cos x - 1 = 0$
3. $2\cos^2 x - \cos x - 1 = 0$
4. $(2\cos x + 1)(\cos x - 1) = 0$
5. $\cos x = -\frac{1}{2}$ or $\cos x = 1$
6. $x = -120^\circ + k{\cdot}360^\circ$ or $x = 0^\circ + k{\cdot}360^\circ$ or $x = 120^\circ + k{\cdot}360^\circ$, $k \in \mathbb{Z}$

(b) Answer: See the proof below.

1. $\text{LHS} = \dfrac{\cos x(1 + \sin x) - \cos x(1 - \sin x)}{(1 - \sin x)(1 + \sin x)}$
2. $= \dfrac{2\sin x \cos x}{1 - \sin^2 x}$
3. $= \dfrac{2\sin x \cos x}{\cos^2 x}$
4. $= 2\tan x$
5. $= \text{RHS}$

(c) Answer: $x = 60^\circ$ or $x = 240^\circ$

1. $2\tan x = 2\sqrt{3}$, so $\tan x = \sqrt{3}$
2. $x = 60^\circ + k{\cdot}180^\circ$
3. In $[0^\circ; 360^\circ]$: $x = 60^\circ$ or $x = 240^\circ$

4

(a) Answer: See the proof below.

1. $\text{LHS} = \dfrac{1-(1-2\sin^2 x)}{2\sin x\cos x}$
2. $= \dfrac{2\sin^2 x}{2\sin x\cos x}$
3. $= \dfrac{\sin x}{\cos x}$
4. $= \tan x$
5. $= \text{RHS}$

(b) Answer: $x = 0^\circ$ or $x = 90^\circ$ or $x = 180^\circ$ or $x = 270^\circ$ or $x = 360^\circ$

1. The identity is not valid where $\sin 2x = 0$, or where $\tan x$ is undefined.
2. $\sin 2x = 0$: $2x = k{\cdot}180^\circ$, so $x = k{\cdot}90^\circ$
3. $\tan x$ is undefined where $\cos x = 0$, which is among these values
4. In $[0^\circ; 360^\circ]$: $x = 0^\circ$ or $x = 90^\circ$ or $x = 180^\circ$ or $x = 270^\circ$ or $x = 360^\circ$

(c) Answer: no solution

1. $\tan x = 0$
2. $x = 0^\circ + k{\cdot}180^\circ$
3. $x = -180^\circ$ or $x = 0^\circ$ or $x = 180^\circ$: the identity is not valid there, so these are not solutions
4. In $[-180^\circ; 180^\circ]$: no solution

5

(a) Answer: $\sin 44^\circ = 2p$

1. $\sin 44^\circ = 2 \sin 22^\circ \cos 22^\circ = 2p$

(b) Answer: $\cos 46^\circ = 2p$

1. $\cos 46^\circ = \cos(90^\circ - 44^\circ) = \sin 44^\circ = 2p$

(c) Answer: $\cos 88^\circ = 1 - 8p^2$

1. $\cos 88^\circ = 1 - 2\sin^2 44^\circ$
2. $= 1 - 2(2p)^2 = 1 - 8p^2$

(d) Answer: $\cos 44^\circ = \sqrt{1 - 4p^2}$

1. $\cos^2 44^\circ = 1 - \sin^2 44^\circ = 1 - 4p^2$
2. 44° is acute, so $\cos 44^\circ > 0$
3. $\cos 44^\circ = \sqrt{1 - 4p^2}$

6

(a) Answer: $\sin 134^\circ = p$

1. $\sin 134^\circ = \sin(180^\circ - 46^\circ) = \sin 46^\circ = p$

(b) Answer: $\cos 46^\circ = \sqrt{1 - p^2}$

1. $\cos^2 46^\circ = 1 - \sin^2 46^\circ = 1 - p^2$
2. 46° is acute, so $\cos 46^\circ > 0$
3. $\cos 46^\circ = \sqrt{1 - p^2}$

(c) Answer: $\sin 92^\circ = 2p\sqrt{1 - p^2}$

1. $\sin 92^\circ = 2 \sin 46^\circ \cos 46^\circ$
2. $= 2p\sqrt{1 - p^2}$

Trigonometric graphs

7

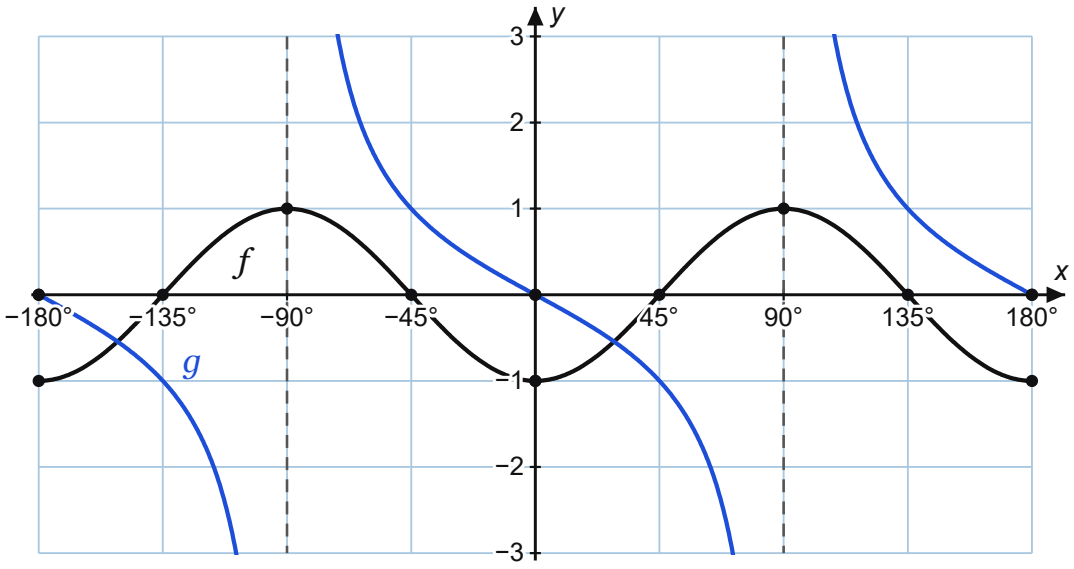
(a) Answer: 180°

1. $g(x) = -\tan x$ repeats every 180°

(b) Answer: $x = -90^\circ$ and $x = 90^\circ$

1. $\tan x$ is undefined where $\cos x = 0$: $x = 90^\circ + k{\cdot}180^\circ$
2. In $[-180^\circ; 180^\circ]$: $x = -90^\circ$ and $x = 90^\circ$

(c) Answer: See the sketch below.



1. $f(x) = -\cos 2x$
2. f: x-intercepts $(-135^\circ; 0)$, $(-45^\circ; 0)$, $(45^\circ; 0)$, $(135^\circ; 0)$
3. f: y-intercept $(0^\circ; -1)$
4. f: turning points $(-90^\circ; 1)$, $(0^\circ; -1)$, $(90^\circ; 1)$
5. f: end points $(-180^\circ; -1)$, $(180^\circ; -1)$
6. $g(x) = -\tan x$
7. g: x-intercepts $(-180^\circ; 0)$, $(0^\circ; 0)$, $(180^\circ; 0)$
8. g: y-intercept (0°; 0)
9. g: asymptotes $x = -90^\circ$, $x = 90^\circ$

10. g: end points $(-180^\circ; 0), (180^\circ; 0)$

(d) Answer: $x = -180^\circ$ or $-135^\circ \leq x < -90^\circ$ or $-45^\circ \leq x \leq 0^\circ$ or $45^\circ \leq x < 90^\circ$ or $135^\circ \leq x \leq 180^\circ$

1. f and g have opposite signs, or one of them is 0, read from the sketch
2. $x = -180^\circ$ or $-135^\circ \leq x < -90^\circ$ or $-45^\circ \leq x \leq 0^\circ$ or $45^\circ \leq x < 90^\circ$ or $135^\circ \leq x \leq 180^\circ$

8

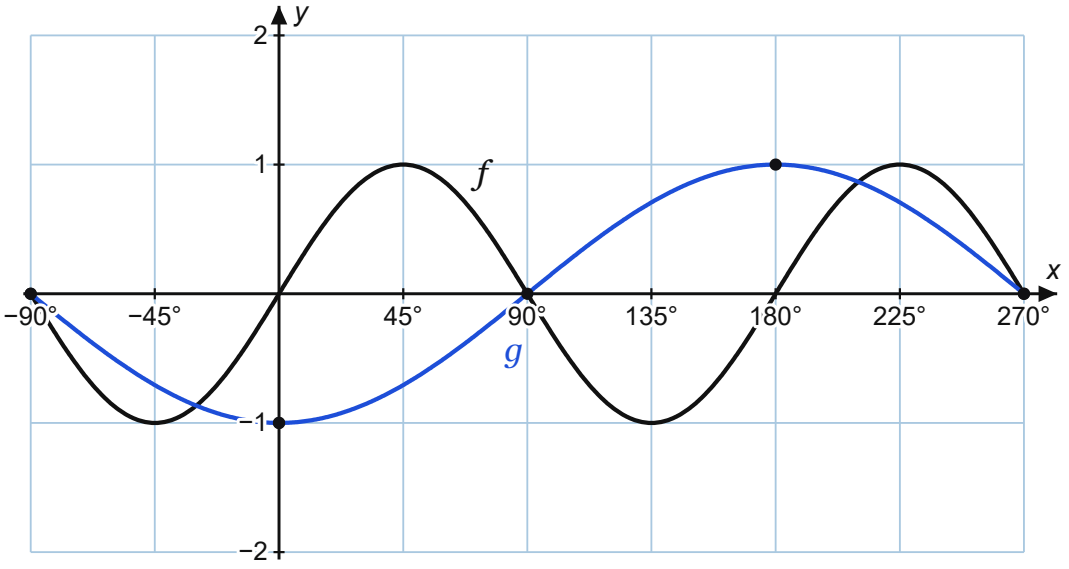
(a) Answer: $b = 2$

1. $\sin bx$ has period $360^\circ / b$
2. one full wave of f takes 180°
3. $360^\circ / b = 180^\circ$, so $b = 2$

(b) Answer: 180°

1. $f(x) = \sin 2x$ repeats every 180°

(c) Answer: See the sketch below.



1. $g(x) = -\cos x$
2. g: x-intercepts $(-90^\circ; 0)$, $(90^\circ; 0)$, $(270^\circ; 0)$
3. g: y-intercept $(0^\circ; -1)$
4. g: turning points $(0^\circ; -1)$, $(180^\circ; 1)$
5. g: end points $(-90^\circ; 0)$, $(270^\circ; 0)$

(d) Answer: 1

1. the amplitude of $a \cdot \cos x$ is $|a| = 1$

(e) Answer: $x = -90^\circ$ or $x = -30^\circ$ or $x = 90^\circ$ or $x = 210^\circ$ or $x = 270^\circ$

1. $\sin 2x = -\cos x$
2. $2\sin x \cos x + \cos x = 0$
3. $\cos x(2\sin x + 1) = 0$
4. $\cos x = 0$ or $\sin x = -\frac{1}{2}$
5. $x = 90^\circ + k{\cdot}180^\circ$ or $x = -30^\circ + k{\cdot}360^\circ$ or $x = 210^\circ + k{\cdot}360^\circ$
6. In $[-90^\circ; 270^\circ]$: $x = -90^\circ$ or $x = -30^\circ$ or $x = 90^\circ$ or $x = 210^\circ$ or $x = 270^\circ$

(f) Answer: $-90^\circ < x < -30^\circ$ or $90^\circ < x < 210^\circ$

1. f lies below g, read from the sketch
2. $-90^\circ < x < -30^\circ$ or $90^\circ < x < 210^\circ$

(g) Answer: $0^\circ < x < 90^\circ$ or $90^\circ < x < 180^\circ$

1. f and g have opposite signs, and $g(x) \neq 0$, read from the sketch
2. $0^\circ < x < 90^\circ$ or $90^\circ < x < 180^\circ$

Sine, cosine and area rules (2D)

9

(a) Answer: See the working below.

1. $\mathrm{A\hat{C}B} = 180^\circ - (\alpha + \beta)$ (sum ∠s of Δ)
2. $\frac{\mathrm{AC}}{\sin\beta} = d/(\sin(180^\circ - (\alpha + \beta)))$ (sine rule)
3. $\sin(180^\circ - (\alpha + \beta)) = \sin(\alpha + \beta)$
4. $\mathrm{AC} = (d \sin\beta)/(\sin(\alpha + \beta))$

(b) Answer: See the working below.

1. In $\Delta\mathrm{ACT}$, $\mathrm{A\hat{C}T} = 90^\circ$: $\tan\gamma = \frac{\mathrm{TC}}{\mathrm{AC}}$
2. $\mathrm{TC} = \mathrm{AC}\tan\gamma = (d \sin\beta \tan\gamma)/(\sin(\alpha + \beta))$

(c) Answer: $\mathrm{TC} = 96.57\ \mathrm{m}$

1. $\mathrm{TC} = \frac{100 \sin 65^\circ \tan 44^\circ}{\sin 115^\circ}$
2. $= 96.57\ \mathrm{m}$

(d) Answer: $\mathrm{BT} = 128.33\ \mathrm{m}$

1. $\mathrm{BC} = \frac{100 \sin 50^\circ}{\sin 115^\circ} = 84.52\ \mathrm{m}$ (sine rule)
2. $\mathrm{BT}^2 = \mathrm{BC}^2 + \mathrm{TC}^2 = 84.52^2 + 96.57^2$
3. $\mathrm{BT} = 128.33\ \mathrm{m}$

(e) Answer: 48.81°

1. $\tan\mathrm{T\hat{B}C} = \frac{\mathrm{TC}}{\mathrm{BC}} = \frac{96.57}{84.52}$
2. $\mathrm{T\hat{B}C} = 48.81^\circ$

10

(a) Answer: $\mathrm{QS} = 78.02\ \mathrm{m}$

1. In $\Delta \mathrm{PQS}$: $\dfrac{\mathrm{QS}}{\sin \mathrm{Q\hat{P}S}} = \dfrac{\mathrm{PQ}}{\sin \mathrm{Q\hat{S}P}}$
2. $\mathrm{QS} = \dfrac{100 \sin 45^\circ}{\sin 65^\circ}$
3. $= 78.02\ \mathrm{m}$

(b) Answer: $\mathrm{QR} = 94.69\ \mathrm{m}$

1. $\mathrm{Q\hat{S}R} = 180^\circ - 65^\circ = 115^\circ$ (∠s on a str line)
2. $\mathrm{QR}^2 = \mathrm{QS}^2 + \mathrm{SR}^2 - 2(\mathrm{QS})(\mathrm{SR}) \cos \mathrm{Q\hat{S}R}$
3. $= 78.02^2 + 30^2 - 2(78.02)(30) \cos 115^\circ$
4. $\mathrm{QR} = 94.69\ \mathrm{m}$

(c) Answer: area $= 1060.66\ \mathrm{m^2}$

1. Area $= (\frac{1}{2})(\mathrm{QS})(\mathrm{SR}) \sin \mathrm{Q\hat{S}R}$
2. $= (\frac{1}{2})(78.02)(30) \sin 115^\circ$
3. $= 1060.66\ \mathrm{m^2}$

(d) Answer: $\mathrm{TQ} = 36.38\ \mathrm{m}$

1. In $\Delta \mathrm{TQS}$, $\mathrm{T\hat{Q}S} = 90^\circ$: $\tan 25^\circ = \dfrac{\mathrm{TQ}}{\mathrm{QS}}$
2. $\mathrm{TQ} = 78.02 \tan 25^\circ$
3. $= 36.38\ \mathrm{m}$