The Complete ACT Mathematics Practice Book

Eben Roux

The Complete ACT Mathematics Practice Book

Eben Roux

Every question in this book was written from scratch and solved independently by computer algebra before it was published — including the wrong options, so the explanation of a distractor is never hand-waved.

The book's structure comes from the eight official College Board practice tests: what is examined, in what proportion, and where the questions with no options sit. No question from those tests is reproduced here.

Read it online at mathbench.academy/act/

Licence and disclaimer

Confirm test dates, fees and registration arrangements with ACT. This book cannot do that for you.

Contents

ACT Mathematics — Complete Study Guide

Read this first. It is written for you.

If maths has never quite clicked, this guide is built for exactly that. Nothing here assumes you already understand it. Every rule is explained in plain English *before* the formula appears, every symbol is spelled out in words, and every method comes with a worked example where **every step is shown** — nothing is left for you to "just know".

You do not have to be naturally good at maths to do well on this test. The ACT Mathematics section examines a **fixed and published** list of ideas. It is not trying to surprise you with new mathematics; it is checking whether you can use familiar mathematics accurately, at speed, on a problem worded in an unfamiliar way. That is a skill, and skills are built by practice.

The test changed in 2025. Make sure you are studying the right one.

This matters more than anything else on this page. In April 2025 ACT replaced its maths paper, and from spring 2026 the old one is not given at all. A great deal of the ACT practice material in circulation — including books still being sold — is for the paper that no longer exists.

	the old paper	the paper you will sit
questions	60	**45**
minutes	60	**50**
time per question	60 seconds	**67 seconds**
answer choices	**five** (A–E / F–K)	**four** (A–D / F–J)
questions that count	60	**41** — four are unscored trials
calculator	allowed throughout	allowed throughout
formula sheet	none	**none**
score range	1–36	1–36

ACT's own wording, from its guide for educators:

> There are 44 fewer questions overall. [...] **The number of answer choices in math questions has been reduced from five to four.**

How to tell instantly whether a practice test is out of date: look at the answer choices. If you see an **E** or a **K**, it is the old paper. Its mathematics is still perfectly good practice — the *content* did not change — but its shape, its timing and its pacing are wrong.

Four choices, not five — and why that is good news

Five choices meant a blind guess paid off one time in five. Four choices means it pays off **one time in four**. Combined with the extra seven seconds per question, a question you have to guess on is now worth more than it used to be.

There is no penalty for a wrong answer. None. A blank is worth exactly zero and a guess is worth a quarter of a mark on average. **Never leave anything blank**, even if you have to fill something in blindly in the last ten seconds.

The letters alternate, and that is deliberate

Odd-numbered questions are lettered **A, B, C, D**. Even-numbered questions are lettered **F, G, H, J**.

This is not decoration. It is a safety net: if you slip a row on the answer sheet, you will find yourself trying to shade a "B" on a row that only offers F, G, H and J, and you will catch the mistake immediately. Every question in this book is lettered the same way, so you build the habit here.

The one thing the ACT does that the SAT does not

The ACT gives you no formula sheet.

The SAT prints a page of formulas at the start of every maths module — areas, volumes, the special right triangles, the circle equation. The ACT prints nothing. Everything you need must be in your head when you sit down.

That single fact changes how you must prepare, and it is why this book has **Chapter 8**, which contains nothing but the formulas the ACT expects you to know, with the reason each one is true so it stays learned rather than memorised and forgotten.

The other ACT-only topics, none of which appear on the SAT at all:

- **matrices** — adding them, multiplying by a number, determinants
- **complex numbers** — the number i, and its powers
- **vectors** — magnitude and components
- **sequences** — arithmetic and geometric, and the nth term
- **logarithms**

- **trigonometry beyond right triangles** — the law of sines and the law of cosines
- **radians**, and the graphs of sine and cosine

If you are moving across from SAT preparation, that list is your gap.

What is on it, and how much of each

ACT publishes the blueprint. There are two reporting categories, and the first splits into five:

category	share of the test	roughly, out of 45
Preparing for Higher Math	**80%**	36
— Algebra	17–20%	9
— Functions	17–20%	8
— Geometry	17–20%	8
— Statistics & Probability	12–15%	6
— Number & Quantity	10–12%	5
Integrating Essential Skills	**20%**	9

Integrating Essential Skills is the category people ignore, because its mathematics is the easiest — rates, percentages, ratios, averages, area. It is also **a fifth of the paper**, which is more than Statistics and Number & Quantity put together. Chapter 1 is devoted to it, and it comes first on purpose.

Modeling is also reported, but it is not separate content: a question counts as modeling **and** as one of the categories above.

⚠ A warning about ACT's own free practice booklet

ACT publishes a free booklet called *Preparing for the ACT*. It is genuinely useful and you should do it. But its maths paper is **not built to the blueprint above**.

We counted it, question by question, from ACT's own scoring key. Of its 41 scored questions, 25 are Preparing for Higher Math and 16 are Integrating Essential Skills — a **61% / 39%** split, not 80% / 20%. That is almost exactly the *old* paper's mix, which the three official pre-2020 forms confirm at 58% / 42%.

Independent analysis of four released enhanced forms puts the real tests at 80/20, matching the published blueprint.

What that means for you: the free booklet gives you about twice as much easy foundational material as the real test will, and correspondingly less algebra, functions and geometry. If you practise only on it, you will be over-prepared for the easy fifth and under-prepared for the rest. **This book is built to 80/20.**

How it is scored

You get one mark per correct answer out of the 41 that count. Nothing is subtracted for a wrong one. That total — your **raw score** — is converted to the familiar **1–36 scale score**.

The conversion is not a percentage. From ACT's own table for the official enhanced form:

raw score (out of 41)	scale score
41	36
38	34
33	30
28	27
23	22
18	18
13	16
8	14
3	9

Read the middle of that table carefully. Around the middle of the range **one extra correct answer is worth about one scale point**. Five more right answers is roughly five points. That is a very large return for a modest amount of study, and it is concentrated in the questions you *nearly* get right — which is what the worked solutions in this book are for.

The mock papers in Chapter 9 are scored with this exact table.

Time, and how to spend it

50 minutes, 45 questions — 67 seconds each. That is the whole difficulty of the ACT in one number. The mathematics is not harder than the SAT's; there is simply less time per question.

Three consequences worth taking seriously:

1. **The questions get harder as you go.** Not strictly, but strongly. The first ten are mostly one-step; the last ten are the ones that take real thought. So a minute spent stuck on question 34 costs you a question 12 you could have had. **Go through in order, and skip anything that does not open up within about 30 seconds.** Come back to it.
2. **Answer every question you skip anyway.** Before you move on, put *something* in the box. If you come back, change it. If you run out of time, you have a one-in-four chance instead of nothing.
3. **A calculator is allowed on every question**, so arithmetic speed is not what is being tested — setting the problem up correctly is. Most wrong options in this book are a perfectly correct calculation of something the question did not ask.

About the calculator

Bring one you already know. The permitted-calculator rules are ACT's, they change occasionally, and **you must check them on ACT's own site before test day** — a prohibited calculator can cost you the whole section.

What matters more than the model: a calculator will happily give you a precise answer to the wrong equation. Every "silly mistake" you have ever made was a setup mistake, not an arithmetic one. This book puts its effort there.

How to use this book

Every question here has been checked by machine. The answer to each one is solved independently, by computer algebra, from the same source that prints the options on the page — so the answer key cannot disagree with the question. And **every wrong option is there because of a specific, named mistake**, which the solution tells you. When you pick a wrong answer, you can find out exactly which slip you made.

A workable order:

1. **Chapter 0** — this guide. Once.
2. **Chapter 8** — the formulas. Start learning them now, a few at a time, and keep coming back. They are the single cheapest marks in the book, and the ACT will not give them to you.
3. **Chapter 1** — Integrating Essential Skills. A fifth of the test, and the most improvable part of it.
4. **Chapters 2 to 7** — the five higher-maths categories. In any order; do the one you are weakest at first, not last.
5. **Chapter 9** — the two full papers. **Time them properly**: 50 minutes, one sitting, no interruptions. An untimed mock paper measures your mathematics; a timed one measures your ACT score, and they are not the same number.

Do the second paper only when you have worked through what the first one showed you. A mock paper you have not learned from is a mock paper wasted.

Topics covered in this guide: the 2025 format change · four answer choices · alternating answer letters · no formula sheet · the published blueprint · the warning about ACT's own booklet · scoring and the conversion table · timing · the calculator · a study order

Chapter 1 — Integrating Essential Skills

This chapter is **a fifth of the whole test** — nine questions out of forty-five, more than Statistics and Number & Quantity put together. It is also the category almost every ACT book treats as a warm-up, because its mathematics is the easiest thing on the paper: rates, percentages, ratios, averages, area.

That combination — a large share of the marks, and material you have met before — is why this chapter comes first. It is the most improvable part of the ACT. The questions are not hard; they are **fiddly**, and they are placed early in the paper where you are still settling down. Nearly every mark lost here is lost to one of four things:

- answering a different question from the one asked
- forgetting to convert a unit
- taking a percentage of the wrong number
- stopping one step early

Every wrong option in this chapter is one of those four, named.

A note about the calculator. You may use one on every ACT question. That means the arithmetic is not what is being examined — **the setup is**. A calculator will give you a beautifully precise answer to the wrong equation.

Topics covered: rates and unit conversion · percentages · percentage change · ratios and proportion · area, perimeter and volume in context · averages and medians · expressing numbers in different ways

---

Section 1 — Rates and Unit Conversion

A **rate** compares two quantities with different units: miles per hour, pages per minute, dollars per kilogram. The word "per" means **divided by**.

The whole skill is keeping track of which unit is on top. Write the rate as a fraction with its units attached, and arrange the multiplication so the units you do not want **cancel**:

$$30\ \frac{\text{miles}}{\text{hour}} \times \frac{1\ \text{hour}}{60\ \text{minutes}} = \frac{1}{2}\ \frac{\text{miles}}{\text{minute}}$$

If you set it up and the wrong unit survives, you have multiplied where you should have divided. That check costs two seconds and catches almost every conversion error.

Q1 — Basic

An office printer runs at a steady rate, producing 3 pages every 4 seconds without pausing between jobs. How many pages does the printer produce in one minute?

A) 180

B) 80

C) 12

D) 45

Q2 — Medium

A boat is travelling at a steady 30 miles per hour, and an engineer needs that same speed expressed in feet per second for a design calculation. One mile is 5280 feet and one hour is 3600 seconds. What is the speed in feet per second?

F) $\frac{225}{11}$

G) 44

H) 5280

J) $\frac{22}{15}$

Q3 — Medium

A plumber charges a fixed call-out fee of $45 for turning up, and then $28 for each hour spent on the job. One repair was invoiced at $171 in total. How many hours did the repair take?

A) 6

B) $\frac{171}{28}$

C) $\frac{54}{7}$

D) $\frac{9}{2}$

Q4 — Medium

A shop sells the same coffee in two sizes. A 250 gram bag costs \$6 and a 400 gram bag costs \$9. A customer wants whichever size gives more coffee per dollar spent. How many **more** grams per dollar does the better value size give?

F) $\frac{125}{3}$

G) $\frac{400}{9}$

H) 150

J) $\frac{25}{9}$

Q5 — Medium

A concentrate is mixed at a rate of 2 scoops for every 4 cups of water, and a technician has 5 gallons of water to use up. There are 16 cups in a gallon. How many scoops of concentrate are needed?

A) 160

B) 40

C) $\frac{5}{2}$

D) 20

Q6 — Hard

A storage tank has developed a steady leak, losing 3 litres every minute and showing no sign of slowing. The maintenance report has to state that loss per **day**. What is the rate of loss in litres per day?

F) 180

G) 4320

H) 72

J) 30240

Q7 — Medium

A cyclist rides for 40 minutes at a steady 18 kilometres per hour along a flat road. The distance covered is wanted in kilometres. How far does the cyclist travel?

A) 720

B) 12

C) $\frac{9}{20}$

D) 27

Q8 — Hard

One pump can empty a tank in 6 hours and a second pump can empty the same tank in 12 hours. Both are switched on at the same time and run together until the tank is empty. How many hours does it take the two pumps working together?

F) 18

G) 4

H) 9

J) 72

Section 2 — Percentages

A percentage is a fraction with 100 on the bottom. "17%" means $\frac{17}{100}$, or 0.17. Every percentage question on the ACT is one of four shapes:

the question	what to do
what is $p\%$ of N?	multiply: $N \times \frac{p}{100}$
A is what percent of N?	divide, then multiply by 100: $\frac{A}{N} \times 100$
increase N by $p\%$	multiply by $\left(1 + \frac{p}{100}\right)$
decrease N by $p\%$	multiply by $\left(1 - \frac{p}{100}\right)$

The two rows that cost marks are the last two. **Increasing by 15% is multiplying by 1.15, in one step** — not "find 15%, then add it", which is the same thing but twice as many chances to slip. And the reverse of increasing by 15% is dividing by 1.15, **not** decreasing by 15%.

Q9 — Basic

A jacket in a shop is priced at \$80. At the start of the new season the shop raises the price of everything in that range by 15%, calculated on the current price. What is the new price of the jacket?

A) 12

B) 68

C) 95

D) 92

Q10 — Medium

A coat is offered in a sale at 20% off its original price, and after the reduction a customer pays \$96 for it. What was the original price?

F) 116

G) $\frac{576}{5}$

H) 76

J) 120

Q11 — Hard

A share price rises by 25% during one year and then falls by 20% during the next, each change being calculated on the price at the start of that year rather than on the original price. Over the two years taken together, the price has:

A) risen by 5%

B) returned to exactly its original value

C) fallen by 5%

D) risen by 45%

Q12 — Medium

In a town, 60% of the residents own a bicycle, and of those bicycle owners, 25% ride one every day. What percent of **all** residents ride every day?

F) 15

G) 85

H) 35

J) 25

Q13 — Medium

A shop buys a chair from its supplier for $60 and sells it at a 35% profit calculated on that cost. What is the selling price?

A) 21

B) 95

C) 39

D) 81

Q14 — Basic

A quality report needs one figure written as a percentage of another: 40 items out of a batch of 250 were set aside for inspection. What percent of 250 is 40?

F) 625

G) 40

H) 84

J) 16

Q15 — Hard

A shop reduces a coat by 20%, and then takes a further 10% off the already reduced price at the till. A customer wants to know what single percentage reduction off the original price this pair of discounts is equivalent to. What is that single reduction?

A) 72

B) 30

C) 28

D) 2

Q16 — Hard

Tanya earned $34000 in her first year in a job, and in each following year her salary went up by the **same number of dollars** as it had the year before. By her fourth year she was earning $43000. What was her salary in the second year?

F) 37000

G) 40000

H) 36250

J) 43000

Section 3 — Ratios and Proportion

A **ratio** compares parts of the same whole. "The ratio of red to blue is $4:7$" does not mean 4 red and 7 blue; it means that for every 4 red there are 7 blue, in some multiple.

The reliable method is the **multiplier**. Write the ratio as parts, add them to get the total number of parts, and find what one part is worth:

$$4:7 \quad \longrightarrow \quad 4+7=11 \text{ parts}$$

If there are 44 objects altogether, one part is $44 \div 11 = 4$, so red is $4 \times 4 = 16$ and blue is $7 \times 4 = 28$.

The single most common error is using **4 out of 7** instead of **4 out of 11** — confusing a part-to-part ratio with a part-to-whole fraction.

Q17 — Basic

A bag contains only red and blue marbles, and the ratio of red to blue is $4 : 7$. There are 44 marbles altogether. How many marbles are red?

A) $\frac{176}{7}$

B) 16

C) 28

D) 4

Q18 — Medium

Three lengths of timber are cut from one plank in the ratio $5 : 3 : 2$, and the plank is 120 centimetres long altogether. How long is the **longest** piece, in centimetres?

F) 12

G) 24

H) 36

J) 60

Q19 — Medium

A biscuit recipe uses 3 cups of flour for every 8 biscuits it makes, and the proportion of flour to biscuits stays the same however many are baked. A baker needs to make 20 biscuits for an order. How many cups of flour are needed?

A) $\frac{160}{3}$

B) 60

C) $\frac{15}{2}$

D) $\frac{6}{5}$

Q20 — Medium

Leavonne bought 30 raffle tickets and 75 snack vouchers to share out equally among the members of a club, with no tickets and no vouchers left over. What is the **largest** number of members the club can have?

F) 5

G) 15

H) 30

J) 150

Q21 — Medium

A map is drawn to a scale of 1 centimetre to 25 kilometres, and two towns are 7.5 centimetres apart on it. How far apart are the towns in reality?

A) 32

B) $\frac{3}{10}$

C) 25

D) $\frac{375}{2}$

Q22 — Hard

At a bakery, one third of the pies sold on a particular day were apple, and a quarter of the remaining pies were cherry. Exactly 60 cherry pies were sold. How many pies were sold altogether?

F) 360

G) 240

H) 720

J) 180

Section 4 — Area, Perimeter and Volume in Context

These questions are not really geometry. They are arithmetic wearing a shape: work out an area or a volume, then multiply by a cost or divide into a supply.

The ACT gives you no formula sheet, so the formulas themselves have to be known. The ones this section needs:

shape	formula
rectangle	$\text{area} = \text{length} \times \text{width}$
rectangle	$\text{perimeter} = 2(\text{length} + \text{width})$
triangle	$\text{area} = \frac{1}{2} \times \text{base} \times \text{height}$
circle	$\text{area} = \pi r^2$, $\text{circumference} = 2\pi r$
box	$\text{volume} = \text{length} \times \text{width} \times \text{height}$
cylinder	$\text{volume} = \pi r^2 h$

Chapter 8 has the complete list, with the reason each one is true. What matters here is the second half of the question: **the units**. An area in square metres multiplied by a cost per square metre gives dollars; anything else means something has gone wrong.

Q23 — Basic

A rectangular room measures 6 metres by 4 metres, and carpet for it costs $18 for each square metre laid. What is the total cost of carpeting the room?

A) 24

B) 360

C) 432

D) 42

Q24 — Medium

A rectangular trough measures 2 metres long, 1.5 metres wide and 0.4 metres deep, and it is to be filled to the brim with water. One cubic metre holds 1000 litres. How many litres of water does the trough hold?

F) $\frac{6}{5}$

G) 1200

H) 3900

J) 12

Q25 — Medium

A rectangular vegetable bed has an area of 45 square metres, and one of its sides measures 5 metres. The gardener wants to run a border all the way around the outside of the bed. What is the perimeter of the bed, in metres?

A) 28

B) 9

C) 50

D) 90

Q26 — Hard

A rectangular driveway measuring 12 metres by 3 metres is to be covered with gravel to a uniform depth of 5 centimetres. Gravel is sold by the cubic metre. How many cubic metres of gravel are needed?

F) $\frac{9}{5}$

G) 180

H) 36

J) $\frac{36}{5}$

Section 5 — Averages and Medians

The **mean** is the total divided by how many there are. Rearranged, that gives the fact almost every ACT average question turns on:

$$\text{total} = \text{mean} \times \text{how many}$$

Knowing the mean and the count gives you the **total**, and the total is what lets you recover a missing value or add a new one.

The **median** is the middle value once the data is **in order**. With an even number of values there is no single middle, so it is the mean of the middle two. Forgetting to sort first is the most common median error there is.

Q27 — Medium

A teacher recorded five test results and found their mean was 20. Four of the five values were 12, 22, 15 and 25, and the fifth was mislaid. What was the fifth number?

A) 74

B) 20

C) 26

D) 100

Q28 — Medium

A researcher recorded the number of birds visiting a feeder on each of six mornings.

Morning	Mon	Tue	Wed	Thu	Fri	Sat
Birds	4	9	2	11	6	8

What is the median of these values?

F) $\frac{20}{3}$

G) 6

H) 9

J) 7

Q29 — Hard

One class of 20 students averaged 68 on a test, and a second class of 30 students averaged 78 on the same test. What is the mean score across both classes together?

A) 73

B) 146

C) 74

D) 50

Q30 — Medium

A set of readings is $12,\ 14,\ 15,\ 13,\ 16$ — five values grouped closely together. A sixth reading of 60 is then recorded, far outside the range of the others. Which statement about the effect of that new value is true?

F) The mean rises by much more than the median does

G) The median rises by much more than the mean does

H) Both rise by the same amount

J) Neither the mean nor the median changes

Section 6 — Expressing Numbers in Different Ways

The last part of this category is small but reliable: the same number written as a fraction, a decimal, a percentage, or in scientific notation, and questions about which of several values is largest.

$$\frac{3}{8} = 0.375 = 37.5\% \qquad 4\,500\,000 = 4.5 \times 10^6$$

In scientific notation the first number is always **at least 1 and less than 10**. If you write 45×10^5 you have not finished.

Q31 — Basic

A country's annual output is recorded as 4 500 000 units, and it has to be entered into a form in scientific notation. Which of the following is 4 500 000 written that way?

A) 45×10^5

B) 4.5×10^6

C) 0.45×10^7

D) 4.5×10^5

Q32 — Medium

A design rule requires a whole number of units strictly greater than $\frac{75}{2}$ and strictly less than $\frac{131}{3}$. How many integers satisfy the rule?

F) 4

G) 7

H) 6

J) 5

Q33 — Medium

A wall measures 8 metres long and 3 metres high, and one tin of paint covers 6 square metres. How many tins are needed to cover the wall once?

A) 4

B) 24

C) 22

D) 6

Q34 — Hard

A rectangular slab measures 4 metres by 2.5 metres and is poured to a uniform depth of 20 centimetres. Concrete is ordered by the cubic metre. How many cubic metres are needed?

F) 200

G) 10

H) 2

J) $\frac{1}{5}$

Q35 — Basic

A report needs the fraction $\dfrac{3}{8}$ expressed as a percentage. What is $\dfrac{3}{8}$ as a percentage?

A) $\frac{75}{2}$

B) 38

C) $\frac{800}{3}$

D) 3

Q36 — Medium

Four values are to be put in order: $\dfrac{5}{8}$, 0.6, 63% and $\dfrac{3}{5}$. Which is the **largest**?

F) $\dfrac{5}{8}$

G) 0.6

H) 63%

J) $\dfrac{3}{5}$

Q37 — Medium

A measurement of 4.6472 metres must be recorded to two decimal places. What is the measurement rounded to two decimal places?

A) 4.65

B) 4.64

C) 4.6

D) 4.7

Q38 — Medium

A room's four walls have a combined area of 54 square metres, and one tin of paint covers 12 square metres. How many tins must be bought to give the walls one coat?

F) 4

G) 6

H) 5

J) 12

Q39 — Basic

A measurement of 0.35 has to be recorded as a fraction in its lowest terms. Which of the following is 0.35 written that way?

A) $\frac{7}{20}$

B) $\frac{7}{2}$

C) $\frac{1}{35}$

D) $\frac{3}{5}$

Q40 — Medium

In the number 4825 the digit 8 occupies a particular place. What is the value of the digit 8 in 4825?

F) 8

G) 80

H) 800

J) 8000

Q41 — Medium

A rectangular kitchen floor measures 3.5 metres by 4 metres, and each tile covers 0.25 square metres. Tiles are sold singly, and the floor must be fully covered. How many tiles are needed?

A) 56

B) 14

C) 28

D) 224

Q42 — Medium

Four values are to be ordered: $\frac{7}{20}$, 0.36, 34% and $\frac{1}{3}$. Which is the **smallest**?

F) $\frac{7}{20}$

G) 0.36

H) 34%

J) $\frac{1}{3}$

Q43 — Hard

A cylindrical pillar has a radius of 0.5 metres and a height of 4 metres, and only its curved side is to be painted. What is that area, in square metres, in terms of π?

A) π

B) 2π

C) 4π

D) 8π

Q44 — Medium

A measurement of 0.004736 metres must be recorded to **two significant figures**. What is the measurement to two significant figures?

F) 0.0047

G) 0.0048

H) 0.0

J) 0.005

Q45 — Medium

A rectangular garden measures 12 metres by 7 metres and is to be fenced all the way round, at $9 per metre of fencing. What is the total cost?

A) 38

B) 342

C) 756

D) 171

Q46 — Medium

A report states a rate of 6%, and it must be entered elsewhere as a decimal. What is 6% as a decimal?

F) 0.6

G) 6

H) $\frac{3}{50}$

J) $\frac{3}{5000}$

Q47 — Medium

A calculation gives $\frac{5}{11}$, and the result must be recorded as a decimal to three places. What is $\frac{5}{11}$ to three decimal places?

A) 0.455

B) 0.454

C) 0.045

D) 0.545

Answers and Explanations

Section 1 — Rates and Unit Conversion

Q1 — Basic — Answer: D

Topic: A unit rate over a longer period

The rate is 3 pages per 4 seconds, so per **second** it is

$$\frac{3}{4} = 0.75 \text{ pages per second}$$

One minute is 60 seconds, so

$$0.75 \times 60 = 45$$

Check the units: $\dfrac{\text{pages}}{\text{second}} \times$ seconds leaves pages. Correct.

Why each wrong option is wrong:

- **B,** 80 — used $\frac{4}{3}$ instead of $\frac{3}{4}$, that is, 4 pages per 3 seconds. Turning a rate upside down is the most common slip in this topic. Ask yourself which number is bigger and whether the answer should be.
- **C,** 12 — multiplied 3 by 4. Those are the two halves of one rate, not two quantities to combine.
- **A,** 180 — read the rate as 3 pages **per second** and ignored the 4.

> **Takeaway:** write the rate as a fraction with units attached, then multiply so the unwanted unit cancels. If the surviving unit is wrong, so is the arithmetic.

Q2 — Medium — Answer: G

Topic: Converting a rate between two pairs of units

Change both units in one chain, and let them cancel:

$$30\,\frac{\text{miles}}{\text{hour}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} \times \frac{1 \text{ hour}}{3600 \text{ seconds}}$$

Miles cancel against miles, hours against hours, and feet over seconds survives — which is what was asked for.

$$\frac{30 \times 5280}{3600} = \frac{158400}{3600} = 44$$

A sanity check worth remembering: **60 mph is 88 feet per second**. So 30 mph should be half of that, 44. It is.

Why each wrong option is wrong:

- **F,** $\frac{225}{11}$ — multiplied by 3600 and divided by 5280, the two conversions the wrong way round.
- **H,** 5280 — converted correctly to feet per **minute** and stopped there.
- **J,** $\frac{22}{15}$ — converted 1 mile per hour instead of 30.

> **Takeaway:** when both units change, write both conversions as fractions in one line and cancel. And keep "60 mph = 88 ft/s" in your head as a check.

Q3 — Medium — Answer: D

Topic: A rate with a fixed charge on top

Let h be the number of hours. The bill is the fixed part plus the hourly part:

$$45 + 28h = 171$$

Take the call-out fee off first, because it is charged once and is not part of the rate:

$$28h = 126$$

$$h = \frac{126}{28} = \frac{9}{2} = 4.5 \text{ hours}$$

Check: $28 \times 4.5 = 126$, and $126 + 45 = 171$. Correct.

Why each wrong option is wrong:

- **A,** 6 — subtracted the fee correctly but then divided by 21 instead of 28.
- **B,** $\frac{171}{28}$ — divided the whole bill by the hourly rate, as though there were no call-out fee. This is the trap the question is built around.
- **C,** $\frac{54}{7}$ — added the call-out fee instead of subtracting it.

> **Takeaway:** a fixed charge is not part of the rate. Remove it **before** you divide, never after.

Q4 — Medium — Answer: J

Topic: Comparing two rates to find better value

"Value" here means grams per dollar, so divide grams by dollars for each:

$$\text{250 g bag: } \frac{250}{6} = 41\tfrac{2}{3} \text{ g per dollar}$$
$$\text{400 g bag: } \frac{400}{9} = 44\tfrac{4}{9} \text{ g per dollar}$$

The larger bag is better value. The question asks for the **difference**:

$$\frac{400}{9} - \frac{250}{6} = \frac{800}{18} - \frac{750}{18} = \frac{50}{18} = \frac{25}{9}$$

Why each wrong option is wrong:

- **F,** $\frac{125}{3}$ and **G,** $\frac{400}{9}$ — each gives one bag's rate rather than the difference between them. Both are steps on the way; neither is the answer.
- **H,** 150 — compared the weights, $400 - 250$, ignoring price altogether.

> **Takeaway:** "better value" always means a rate, never a raw size or a raw price. And read the last line: this question wanted the *difference*, not the winner.

Q5 — Medium — Answer: B

Topic: Converting a volume before using a rate

Convert first:

$$5 \text{ gallons} \times 16 \frac{\text{cups}}{\text{gallon}} = 80 \text{ cups}$$

The rate is 2 scoops per 4 cups, which is $\frac{2}{4} = \frac{1}{2}$ scoop per cup:

$$80 \times \frac{1}{2} = 40 \text{ scoops}$$

Why each wrong option is wrong:

- **A,** 160 — multiplied the 80 cups by 2 instead of by $\frac{2}{4}$.
- **C,** $\frac{5}{2}$ — applied the rate to 5 **gallons** without converting to cups. The unit check catches this instantly: the answer would be in scoops per gallon, not scoops.
- **D,** 20 — divided by 4 but never multiplied by the 2.

> **Takeaway:** convert to a common unit **before** applying a rate, not after.

Q6 — Hard — Answer: G

Topic: A two-step conversion of a rate

Go one step at a time, and never skip a unit:

$$3\frac{\text{litres}}{\text{minute}} \times 60\frac{\text{minutes}}{\text{hour}} = 180\frac{\text{litres}}{\text{hour}}$$
$$180\frac{\text{litres}}{\text{hour}} \times 24\frac{\text{hours}}{\text{day}} = 4320\frac{\text{litres}}{\text{day}}$$

Why each wrong option is wrong:

- **F,** 180 — stopped after the first conversion, giving litres per hour. Stopping one step early is the single most common error in this whole chapter.
- **H,** 72 — jumped from minutes straight to days by multiplying by 24, which skips the factor of 60.
- **J,** 30240 — went one conversion too far and gave litres per week.

> **Takeaway:** do multi-step conversions one unit at a time and write each one down. The two failure modes are stopping early and going one step too far, and both are invisible unless the units are on the page.

Q7 — Medium — Answer: B

Topic: Distance, speed and time in a single journey

Distance is speed multiplied by time, but only when the units agree. The speed is per **hour**, so convert 40 minutes to hours:

$$40 \text{ minutes} = \frac{40}{60} = \frac{2}{3} \text{ hour}$$
$$18 \times \frac{2}{3} = 12 \text{ km}$$

A quick sanity check: in a full hour the cyclist would do 18 km, and 40 minutes is two thirds of an hour, so the answer must be less than 18. It is.

Why each wrong option is wrong:

- **A,** 720 — multiplied 18 by 40 directly, mixing hours with minutes. The result, 720 km in 40 minutes, fails the sanity check by a mile.
- **C,** $\frac{9}{20}$ — divided instead of multiplying.
- **D,** 27 — used $\frac{60}{40}$ instead of $\frac{40}{60}$, turning the time fraction upside down, which makes the answer bigger than 18.

> **Takeaway:** before multiplying a speed by a time, check they use the same unit of time. Then check the answer is on the right side of "one whole hour".

Q8 — Hard — Answer: G

Topic: Two rates working together

The mistake this question exists to catch is adding the **times**. Times do not add — think about it: two pumps together must be *faster* than either alone, so the answer has to be **less than 6**. That single check eliminates three of the four options before any arithmetic.

What does add is the **rates**. In one hour:

$$\text{pump 1 empties } \frac{1}{6} \text{ of the tank} \qquad \text{pump 2 empties } \frac{1}{12}$$

$$\frac{1}{6} + \frac{1}{12} = \frac{2}{12} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$$

Together they empty a quarter of the tank per hour, so the whole tank takes

$$1 \div \frac{1}{4} = 4 \text{ hours}$$

Why each wrong option is wrong:

- **F,** 18 — added the two times. Two pumps cannot be slower than one.
- **H,** 9 — averaged the two times, which is the same error in disguise: the answer is still not less than 6.
- **J,** 72 — multiplied the times.

> **Takeaway:** for combined-work questions, add the **rates** ($\frac{1}{t}$), then invert at the end. And check the answer is smaller than the fastest worker alone — it always must be.

Section 2 — Percentages

Q9 — Basic — Answer: D

Topic: A percentage increase

Increasing by 15% means multiplying by 1.15:

$$80 \times 1.15 = 92$$

If you would rather do it in two steps: 15% of 80 is $80 \times 0.15 = 12$, and $80 + 12 = 92$. Same answer, more steps, more chances to go wrong.

Why each wrong option is wrong:

- **A,** 12 — gave the increase itself, 12, and stopped. The question asked for the new price.
- **B,** 68 — subtracted the 15% instead of adding it.
- **C,** 95 — added 15 **dollars**. A percentage is a proportion of something, never a fixed amount.

> **Takeaway:** increase by $p\%$ in one multiplication, by $\left(1 + \frac{p}{100}\right)$. Then check you answered the question that was asked — the increase, or the new total?

Q10 — Medium — Answer: J

Topic: Working backwards from a percentage

After a 20% reduction the customer pays 80% of the original. Call the original price P:

$$0.80 \times P = 96$$

$$P = \frac{96}{0.80} = 120$$

Check: 20% of 120 is 24, and $120 - 24 = 96$. Correct.

The trap here is worth dwelling on. Adding 20% back onto 96 gives $96 \times 1.2 = 115.20$, not 120. Taking 20% off and then putting 20% back on does not return you to where you started, because the second percentage is taken from a smaller number. To undo a multiplication, you **divide**.

Why each wrong option is wrong:

- **F,** 116 and **H,** 76 — treated the 20% as 20 dollars.
- **G,** $\frac{576}{5}$ — added 20% back on instead of dividing by 0.80. This is the intended trap, and it is very close to right.

> **Takeaway:** to undo a percentage change, **divide** by the multiplier. Off by 20% means divide by 0.8; up by 20% means divide by 1.2.

Q11 — Hard — Answer: B

Topic: Two percentage changes one after the other

Percentages do not add. Each change is a **multiplication**, so two changes one after the other multiply together.

Take a convenient starting price of 100:

$$100 \times 1.25 = 125 \qquad \text{then} \qquad 125 \times 0.80 = 100$$

Back exactly where it started. In one line:

$$1.25 \times 0.80 = 1.00$$

A multiplier of exactly 1 means no overall change.

The reason it works out so neatly: a 20% fall undoes a 25% rise because $\frac{1}{1.25} = 0.8$. They are inverses.

Why each wrong option is wrong:

- **A, risen by 5%** and **C, fallen by 5%** — subtracted the percentages, in one direction or the other. $25 - 20$ is not how successive changes combine.
- **D, risen by 45%** — added them.

> **Takeaway:** successive percentage changes **multiply**. Start from 100, apply each multiplier in turn, and read the answer off directly.

Q12 — Medium — Answer: F

Topic: A percentage of a percentage

"Of those bicycle owners" is the key phrase: the 25% applies to the 60%, not to the town.

Take a town of 100 people:

$$\text{bicycle owners} = 60 \qquad \text{daily riders} = 25\% \text{ of } 60 = 15$$

15 out of 100 is 15%.

In one line, "of" means multiply:

$$0.60 \times 0.25 = 0.15 = 15\%$$

Why each wrong option is wrong:

- **G,** 85 — added the percentages.
- **H,** 35 — subtracted them.
- **J,** 25 — applied the 25% to the whole town, ignoring the fact that only bicycle owners were being described.

> **Takeaway:** in percentage questions the word **"of" means multiply**. A percentage of a percentage is always *smaller* than either one.

Q13 — Medium — Answer: D

Topic: Cost, profit and selling price

A 35% profit on the cost means the selling price is 135% of the cost:

$$60 \times 1.35 = 81$$

In two steps: 35% of 60 is $60 \times 0.35 = 21$, and $60 + 21 = 81$.

Why each wrong option is wrong:

- **A,** 21 — gave the profit, 21, not the selling price.
- **B,** 95 — added 35 dollars rather than 35 percent.
- **C,** 39 — subtracted the profit, which would be selling at a loss.

> **Takeaway:** profit "on cost" means multiply the cost by $\left(1 + \frac{\text{profit }\%}{100}\right)$. Then read the last line again: profit, or selling price?

Q14 — Basic — Answer: J

Topic: Expressing one number as a percentage of another

"What percent of 250" tells you 250 is the **whole**, so it goes on the bottom:

$$\frac{40}{250} \times 100 = 16\%$$

Check it the other way: 16% of 250 is $250 \times 0.16 = 40$. Correct.

Why each wrong option is wrong:

- **F,** 625 — divided 250 by 40, giving 625%. A part cannot be more than 100% of its whole, so this fails an instant sanity check.
- **G,** 40 — repeated the 40 with a percent sign attached.
- **H,** 84 — gave the percentage that was *not* set aside.

> **Takeaway:** the number after "percent **of**" is the whole and goes underneath. If your answer is over 100%, you have divided upside down.

Q15 — Hard — Answer: C

Topic: Two successive discounts as a single one

Discounts do not add, because the second is taken from the reduced price.

Start from 100. After 20% off you pay 80% of it; after a further 10% off you pay 90% of *that*:

$$100 \times 0.80 \times 0.90 = 72$$

So the customer pays 72% of the original, which means the reduction is

$$100 - 72 = 28\%$$

Why each wrong option is wrong:

- **B,** 30 — added the two discounts. If that were right, ten successive 10% discounts would make everything free.
- **A,** 72 — gave the percentage **paid**, not the percentage taken off. A very near miss, and the reason the last line of the question says "reduction".
- **D,** 2 — multiplied 20 by 10 and divided by 100.

> **Takeaway:** chain the multipliers, then subtract from 100 if the question asks for the discount rather than the price.

Q16 — Hard — Answer: F

Topic: A repeated fixed rise, not a percentage rise

The same **dollar amount** each year, not the same percentage — so this is addition, not multiplication.

From year 1 to year 4 there are **three** rises, not four. That off-by-one is the whole question:

$$43000 - 34000 = 9000 \text{ over three rises}$$

$$\text{each rise} = \frac{9000}{3} = 3000$$

So year 2 is $34000 + 3000 = 37000$.

Check the whole run: 34000, 37000, 40000, 43000. Four years, three steps. Correct.

Why each wrong option is wrong:

- **H,** 36250 — divided the 9000 by **four** instead of three: one for each year rather than one for each rise. This is the error the question is built around, and it is very easy to make quickly.

- **G,** 40000 — subtracted one rise from the fourth year, which gives year 3.
- **J,** 43000 — added the entire 9000 in one go.

> **Takeaway:** between year 1 and year n there are $n-1$ steps. Count the gaps, not the years. Write the whole sequence out if there is any doubt — it takes five seconds and removes the error entirely.

Section 3 — Ratios and Proportion

Q17 — Basic — Answer: B

Topic: Part-to-whole from a part-to-part ratio

Add the parts to find how many the whole is divided into:

$$4+7=11 \text{ parts}$$
$$\text{one part}=\frac{44}{11}=4 \text{ marbles}$$

Red is 4 parts, so

$$4\times 4=16$$

Check: blue is $7\times 4=28$, and $16+28=44$. Correct.

Why each wrong option is wrong:

- **A,** $\frac{176}{7}$ — used $\frac{4}{7}$, treating a part-to-part ratio as a fraction of the whole. This is the error the question is built around.
- **C,** 28 — worked correctly and gave the **blue** marbles.
- **D,** 4 — gave the value of one part and stopped.

> **Takeaway:** add the parts first. $4:7$ means $\frac{4}{11}$ of the whole, not $\frac{4}{7}$. Then check your two answers add back to the total.

Q18 — Medium — Answer: J

Topic: A ratio in three parts

$$5 + 3 + 2 = 10 \text{ parts}$$

$$\text{one part} = \frac{120}{10} = 12 \text{ cm}$$

The longest piece is 5 parts:

$$5 \times 12 = 60 \text{ cm}$$

Check: $60 + 36 + 24 = 120$. Correct.

Why each wrong option is wrong:

- **F,** 12 — gave the size of one part.
- **G,** 24 — gave the **shortest** piece, 2 parts.
- **H,** 36 — gave the middle piece, 3 parts.

> **Takeaway:** the method does not change with more parts — add them all, divide, then multiply by the part you were asked for. Read which one that is.

Q19 — Medium — Answer: C

Topic: Scaling a recipe by proportion

Set the two ratios equal, keeping flour on top in both:

$$\frac{3 \text{ cups}}{8 \text{ biscuits}} = \frac{c}{20 \text{ biscuits}}$$

Cross-multiply:

$$8c = 60 \qquad c = \frac{60}{8} = \frac{15}{2} = 7.5 \text{ cups}$$

Sanity check: 20 biscuits is two and a half times 8, so the flour should be two and a half times 3, which is 7.5. Correct.

Why each wrong option is wrong:

- **A,** $\frac{160}{3}$ — set the proportion up with biscuits over cups on one side and cups over biscuits on the other.
- **B,** 60 — multiplied 3×20 and never divided by 8.
- **D,** $\frac{6}{5}$ — multiplied by 8 and divided by 20, the wrong way round.

> **Takeaway:** in a proportion, put the same quantity on top of both fractions. Then check the direction: more biscuits must mean more flour.

Q20 — Medium — Answer: G

Topic: Dividing two different quantities evenly

Every member gets a whole number of tickets and a whole number of vouchers, so the number of members must divide **both** 30 and 75. The largest such number is the **greatest common factor**.

$$30 = 2 \times 3 \times 5 \qquad 75 = 3 \times 5 \times 5$$

The factors they share are $3 \times 5 = 15$.

Check: $30 \div 15 = 2$ tickets each and $75 \div 15 = 5$ vouchers each, both whole. Correct.

Why each wrong option is wrong:

- **F,** 5 — 5 divides both, but it is not the **largest** number that does.
- **H,** 30 — 30 does not divide 75.
- **J,** 150 — gave the lowest common **multiple**, 150. A multiple is bigger than both numbers; the number of members has to be smaller.

> **Takeaway:** "shared out with none left over, as many groups as possible" is the greatest common factor. "Happens again at the same time" is the lowest common multiple. Decide which before you calculate.

Q21 — Medium — Answer: D

Topic: Reading a distance off a scale

Each centimetre on the map stands for 25 real kilometres, so

$$7.5 \times 25 = 187.5 \text{ km}$$

The direction matters: going from the map to the world, the number gets **bigger**, so you multiply.

Why each wrong option is wrong:

- **B,** $\frac{3}{10}$ — divided by the scale, which is the conversion the other way round, from the world to the map.
- **C,** 25 — repeated the scale factor.
- **A,** 32 — added the two numbers.

> **Takeaway:** decide which way the conversion goes before touching a calculator. Map to world, the number grows; world to map, it shrinks.

Q22 — Hard — Answer: F

Topic: A fraction of a total, worked backwards

Let n be the total. Apple pies take a third, so what remains is

$$n - \frac{1}{3}n = \frac{2}{3}n$$

Cherry pies are a quarter of **that**:

$$\frac{1}{4} \times \frac{2}{3}n = \frac{1}{6}n = 60$$
$$n = 360$$

Check: 360 pies, 120 apple, 240 remaining, and a quarter of 240 is 60. Correct.

Why each wrong option is wrong:

- **G,** 240 — took the quarter from the whole day's sales, forgetting that the apple pies had already been removed.
- **H,** 720 — divided by both fractions separately instead of by their product.
- **J,** 180 — used the third rather than the quarter.

> **Takeaway:** "of the remaining" means the fraction applies to what is left, not to the original. Write the leftover as a fraction of the total first.

Section 4 — Area, Perimeter and Volume in Context

Q23 — Basic — Answer: C

Topic: Area, then a cost per unit area

Area first:

$$6 \times 4 = 24 \text{ square metres}$$

Then the cost:

$$24 \times 18 = 432$$

Check the units: square metres $\times$ dollars per square metre gives dollars. Correct.

Why each wrong option is wrong:

- **A,** 24 — gave the area, 24 square metres, and stopped. Stopping one step early is the commonest error in this chapter.
- **B,** 360 — used the perimeter, $2(6 + 4) = 20$. Carpet covers a surface, so it is an area; perimeter would be for skirting board.
- **D,** 42 — added the cost rather than multiplying by it.

> **Takeaway:** covering a floor is area; going round an edge is perimeter. Then finish the question — the area is rarely the answer.

Q24 — Medium — Answer: G

Topic: Volume in context, with a conversion

Volume of a box is length $\times$ width $\times$ height:

$$2 \times 1.5 \times 0.4 = 1.2 \text{ cubic metres}$$

Then convert:

$$1.2 \times 1000 = 1200 \text{ litres}$$

Why each wrong option is wrong:

- **F,** $\frac{6}{5}$ — gave the volume in cubic metres without converting to litres.
- **H,** 3900 — added the three dimensions, $2 + 1.5 + 0.4 = 3.9$, before converting. Volume multiplies lengths; it never adds them.
- **J,** 12 — the right shape of answer with a decimal point in the wrong place. Multiplying by 0.4 makes a number smaller, not ten times smaller.

> **Takeaway:** volume multiplies three lengths. Then read the unit the answer is wanted in — it is often not the unit the measurements came in.

Q25 — Medium — Answer: A

Topic: Working back from an area to a missing side

Find the missing side from the area:

$$5 \times w = 45 \qquad w = \frac{45}{5} = 9 \text{ m}$$

Now the perimeter, which is twice the sum of the two different sides:

$$2 \times (5 + 9) = 28 \text{ m}$$

Why each wrong option is wrong:

- **B,** 9 — found the missing side, 9, and stopped there.
- **C,** 50 — added an area to a length, which is not a meaningful operation — the units alone rule it out.
- **D,** 90 — doubled the area.

> **Takeaway:** an area divided by one side gives the other side. Watch the units: you cannot add square metres to metres.

Q26 — Hard — Answer: F

Topic: Volume of a layer spread over an area

Every measurement must be in the same unit before multiplying. The depth is 5 centimetres, and there are 100 centimetres in a metre:

$$5 \text{ cm} = \frac{5}{100} = 0.05 \text{ m}$$

$$12 \times 3 \times 0.05 = 1.8 \text{ cubic metres}$$

Why each wrong option is wrong:

- **G,** 180 — multiplied by 5 without converting, treating the depth as 5 metres. A driveway five metres deep is a swimming pool; a sanity check catches this.
- **H,** 36 — gave the area of the driveway, forgetting the depth entirely.
- **J,** $\frac{36}{5}$ — divided by the depth instead of multiplying.

> **Takeaway:** convert every length to the same unit **before** you multiply, and then ask whether the answer is a sensible size for the thing described.

Section 5 — Averages and Medians

Q27 — Medium — Answer: C

Topic: Recovering a missing value from a mean

Five numbers with a mean of 20 must total

$$5 \times 20 = 100$$

The four known values add to

$$12 + 22 + 15 + 25 = 74$$

So the fifth is

$$100 - 74 = 26$$

Check: $12 + 22 + 15 + 25 + 26 = 100$, and $100 \div 5 = 20$. Correct.

Why each wrong option is wrong:

- **A,** 74 — gave the total of the four known values.
- **B,** 20 — assumed the missing value must equal the mean. It only does when the others happen to balance exactly, which is not the case here.
- **D,** 100 — gave the total of all five.

> **Takeaway:** mean $\times$ count $=$ total. Get the total first, then everything else is subtraction.

Q28 — Medium — Answer: J

Topic: The median of an even number of values

Sort them first — this is the step people skip:

$$2,\ 4,\ 6,\ 8,\ 9,\ 11$$

Six values, so there is no single middle one. The median is the mean of the third and fourth:

$$\frac{6+8}{2} = 7$$

Why each wrong option is wrong:

- **F,** $\frac{20}{3}$ — gave the mean, $40 \div 6$, rather than the median.
- **G,** 6 — picked a value from the middle of the **unsorted** list.
- **H,** 9 — gave the range, $11 - 2$.

> **Takeaway:** sort, then find the middle. With an even count, average the middle two — and notice the median need not be one of the values in the list.

Q29 — Hard — Answer: C

Topic: A weighted average of two groups

Go back to totals. The mean of a combined group is the combined total divided by the combined count:

$$\text{class 1 total} = 20 \times 68 = 1360$$
$$\text{class 2 total} = 30 \times 78 = 2340$$
$$\text{overall mean} = \frac{1360 + 2340}{20 + 30} = \frac{3700}{50} = 74$$

Why each wrong option is wrong:

- **A,** 73 — averaged the two averages, giving 73. That would only be right if the classes were the same size. The bigger class has the higher average, so the true answer must be pulled **above** 73 — which is a useful check.
- **B,** 146 — added the two averages.
- **D,** 50 — gave the total number of students.

> **Takeaway:** averages cannot be averaged unless the groups are equal in size. Convert each to a total, add the totals, divide by the combined count.

Q30 — Medium — Answer: F

Topic: What an extreme value does to the mean and the median

Work out both, before and after.

Before, the five values total 70, so the mean is $70 \div 5 = 14$. Sorted, they are $12, 13, 14, 15, 16$, so the median is also 14.

After adding 60, the six values total 130, so the mean is $130 \div 6 = 21.67$. Sorted, they are $12, 13, 14, 15, 16, 60$, so the median is $\frac{14+15}{2} = 14.5$.

The mean moved by nearly 8; the median moved by a half.

The reason is structural: the mean uses the **size** of every value, so one huge number drags it a long way. The median only cares about **position**, so a huge number counts for no more than a slightly-bigger-than-average one.

Why each wrong option is wrong:

- **G, The median rises by much more than the mean does** — has it exactly backwards.
- **H, Both rise by the same amount** — they do not move together; that is the whole point of the question.
- **J, Neither the mean nor the median changes** — both change, just by very different amounts.

> **Takeaway:** an outlier drags the **mean** and barely moves the **median**. That is why house prices and salaries are reported as medians.

Section 6 — Expressing Numbers in Different Ways

Q31 — Basic — Answer: B

Topic: Writing a large number in scientific notation

Move the decimal point until exactly one non-zero digit stands in front of it:

$$4\,500\,000 \longrightarrow 4.5$$

Count how many places it moved: 6. So the number is

$$4.5 \times 10^6$$

Check by expanding: $4.5 \times 1\,000\,000 = 4\,500\,000$. Correct.

Why each wrong option is wrong:

- **A,** 45×10^5 — the right size, but not scientific notation: the leading number must be below 10.
- **C,** 0.45×10^7 — also the right size, but the leading number must be at least 1.
- **D,** 4.5×10^5 — counted one place too few, giving 450 000.

> **Takeaway:** scientific notation needs exactly one non-zero digit before the decimal point. Two answers can be the same *number* and still only one of them is in the right *form*.

Q32 — Medium — Answer: H

Topic: Counting the integers between two values

Convert both bounds:

$$\frac{75}{2} = 37.5 \qquad \frac{131}{3} = 43.67$$

The integers strictly between 37.5 and 43.67 are

$$38,\ 39,\ 40,\ 41,\ 42,\ 43$$

That is **6** of them.

The reliable way to count a run of consecutive integers is last $-$ first $+ 1$:

$$43 - 38 + 1 = 6$$

The $+1$ is the fencepost: a fence from post 38 to post 43 has six posts, not five.

Why each wrong option is wrong:

- **F,** 4 and **J,** 5 — fencepost errors, counting the gaps between the numbers rather than the numbers themselves.
- **G,** 7 — included 37 or 44, which lie outside the range.

> **Takeaway:** convert the bounds to decimals, write down the first and last integer that fit, then use last $-$ first $+ 1$.

Q33 — Medium — Answer: A

Topic: Area of a wall, and how much paint it needs

Area first:

$$8 \times 3 = 24 \text{ square metres}$$

Then divide by what one tin covers:

$$\frac{24}{6} = 4 \text{ tins}$$

Here it divides exactly. When it does not, you must **round up** — three and a bit tins means buying four, because half a tin is not sold.

Why each wrong option is wrong:

- **B,** 24 — gave the area and stopped.
- **C,** 22 — used the perimeter. Paint covers a surface, so it is an area.

- **D,** 6 — repeated the coverage figure.

> **Takeaway:** area ÷ coverage, then **round up** for anything sold whole.

Q34 — Hard — Answer: H

Topic: How much concrete a slab needs

Convert the depth: $20 \text{ cm} = 0.2 \text{ m}$.

$$4 \times 2.5 \times 0.2 = 2 \text{ cubic metres}$$

Why each wrong option is wrong:

- **F,** 200 — used 20 metres as the depth. A slab twenty metres deep is a quarry; the sanity check catches this instantly.
- **G,** 10 — gave the area of the slab, forgetting the depth.
- **J,** $\frac{1}{5}$ — gave the depth on its own.

> **Takeaway:** convert every length to the same unit **before** multiplying, then ask whether the answer is a sensible size.

Q35 — Basic — Answer: A

Topic: The same value written as a fraction, a decimal and a percent

Divide, then multiply by 100:

$$\frac{3}{8} = 0.375 \qquad 0.375 \times 100 = 37.5\%$$

Why each wrong option is wrong:

- **B,** 38 — read the digits 3 and 8 straight off as "38%".
- **C,** $\frac{800}{3}$ — divided 8 by 3, giving over 250%. A proper fraction is always less than 100%.
- **D,** 3 — gave the numerator.

> **Takeaway:** fraction → decimal → percent, in that order. A fraction below 1 must give a percentage below 100.

Q36 — Medium — Answer: H

Topic: Ordering values written in different forms

Put everything into decimals:

$$\frac{5}{8} = 0.625 \qquad 0.6 = 0.600 \qquad 63\% = 0.630 \qquad \frac{3}{5} = 0.600$$

The largest is 0.63.

Note how close 0.625 and 0.63 are — this is why converting all four is necessary rather than eyeballing them.

Why each wrong option is wrong:

- **F,** $\frac{5}{8}$ — 0.625, second largest by five thousandths.
- **G,** 0.6 and **J,** $\frac{3}{5}$ — both are 0.6, and they are equal to each other.

> **Takeaway:** convert everything to decimals before comparing, and use the same number of decimal places so the comparison is a straight read.

Q37 — Medium — Answer: A

Topic: Rounding to a stated place value

Two decimal places means keeping 4.64 and looking at the **next** digit, which is 7. Seven is 5 or more, so the last kept digit rounds up:

$$4.6472 \longrightarrow 4.65$$

Only the digit immediately after matters. The 2 at the end plays no part.

Why each wrong option is wrong:

- **B,** 4.64 — cut the number off instead of rounding.
- **C,** 4.6 — rounded to one decimal place.
- **D,** 4.7 — rounded the 4 up to 5 and then carried on rounding, which compounds one rounding into another.

> **Takeaway:** look at **one** digit past the place you are keeping. Round once, not repeatedly.

Q38 — Medium — Answer: H

Topic: How much paint a room needs

$$\frac{54}{12} = 4.5 \text{ tins}$$

Four tins would leave part of the wall bare, and half a tin cannot be bought, so **round up** to 5.

Why each wrong option is wrong:

- **F,** 4 — rounded down. Four tins cover only 48 of the 54 square metres.
- **G,** 6 — rounded up further than needed.
- **J,** 12 — repeated the coverage figure.

> **Takeaway:** for anything sold whole — tins, buses, boxes — divide and then round **up**, whatever the decimal says.

Q39 — Basic — Answer: A

Topic: A decimal written as a fraction in lowest terms

Two decimal places means hundredths:

$$0.35 = \frac{35}{100}$$

Both parts divide by 5:

$$\frac{35}{100} = \frac{7}{20}$$

Check: $7 \div 20 = 0.35$. ✓

Why each wrong option is wrong:

- **B,** $\frac{7}{2}$ — used tenths, giving 3.5, ten times too big.
- **C,** $\frac{1}{35}$ — inverted the fraction.
- **D,** $\frac{3}{5}$ — read only the 3, giving 0.6.

> **Takeaway:** count the decimal places to fix the power of ten — one place is tenths, two is hundredths — then cancel. Always divide back to check.

Q40 — Medium — Answer: H

Topic: Place value in a large number

Reading from the right: 5 is units, 2 is tens, 8 is **hundreds**, 4 is thousands.

$$8 \times 100 = 800$$

Why each wrong option is wrong:

- **F,** 8 — gave the digit rather than its value.
- **G,** 80 and **J,** 8000 — counted the places wrongly by one in each direction.

> **Takeaway:** a digit's **value** is the digit multiplied by its place. Count places from the right: units, tens, hundreds.

Q41 — Medium — Answer: A

Topic: The area of a floor, and the tiles it takes

Area first:

$$3.5 \times 4 = 14 \text{ square metres}$$

Each tile covers 0.25 m^2, so

$$\frac{14}{0.25} = 56 \text{ tiles}$$

Dividing by a number below 1 makes the answer **bigger**, which is the check that catches most errors here — you need more tiles than square metres.

Why each wrong option is wrong:

- **B,** 14 — gave the area and stopped.
- **C,** 28 and **D,** 224 — divided by the wrong tile size.

> **Takeaway:** area ÷ coverage. Dividing by a fraction increases the count, so a smaller answer than the area means something has gone wrong.

Q42 — Medium — Answer: J

Topic: Comparing a fraction, a decimal and a percentage

Convert everything to decimals:

$$\frac{7}{20} = 0.350 \qquad 0.36 = 0.360 \qquad 34\% = 0.340 \qquad \frac{1}{3} = 0.333$$

The smallest is $\frac{1}{3}$.

Write them all to the same number of decimal places — the four values span only 0.027, so a rough comparison will not separate them.

Why each wrong option is wrong:

- **H,** 34% — 0.34, and the closest rival, but still above 0.333.
- **F,** $\dfrac{7}{20}$ — 0.35.
- **G,** 0.36 — the largest of the four.

> **Takeaway:** convert to decimals, pad to equal length, then compare digit by digit. $\frac{1}{3} = 0.333\ldots$ is worth knowing on sight.

Q43 — Hard — Answer: C

Topic: How much paint a cylinder's curved surface needs

Unroll the side: it is a rectangle as tall as the pillar and as wide as the circle around it.

$$\text{width} = 2\pi r = 2\pi(0.5) = \pi \qquad \text{height} = 4$$

$$\text{area} = \pi \times 4 = 4\pi \text{ square metres}$$

Why each wrong option is wrong:

- **A,** π — used $\pi r^2 h$, which is a **volume**. Check the units: an area needs two lengths, not three.
- **B,** 2π — gave the circumference alone.
- **D,** 8π — used 1 as the radius, which is the diameter.

> **Takeaway:** a cylinder's curved surface is $2\pi r h$ — circumference times height. Picture unrolling it, and the formula stops needing to be remembered.

Q44 — Medium — Answer: F

Topic: Rounding to a significant figure

The first significant figure is the **4** — leading zeros never count. So two significant figures means keeping 0.0047, and the next digit decides whether the 7 rounds up.

That next digit is 3, which is below 5, so the 7 stays:

$$0.004736 \longrightarrow 0.0047$$

Why each wrong option is wrong:

- **G,** 0.0048 — rounded up, but the deciding digit is 3, not the 6 further along.
- **H,** 0.0 — counted **decimal places** rather than significant figures.
- **J,** 0.005 — gave one significant figure.

> **Takeaway:** significant figures start at the first non-zero digit. Look at the **one** digit past where you stop, and ignore everything after it.

Q45 — Medium — Answer: B

Topic: Fencing a garden, and the cost of it

The fence follows the perimeter:

$$2(12 + 7) = 38 \text{ metres}$$

$$38 \times 9 = 342$$

Why each wrong option is wrong:

- **A,** 38 — gave the perimeter and stopped.
- **C,** 756 — used the area. A fence goes round an edge; area would be for turfing it.
- **D,** 171 — added the two sides once and forgot the garden has two of each.

> **Takeaway:** round an edge is **perimeter**; covering a surface is **area**. Decide which before choosing a formula.

Q46 — Medium — Answer: H

Topic: A percentage written as a decimal and a fraction

$$6\% = \frac{6}{100} = 0.06$$

Moving the decimal point two places to the left is the same operation, and it is quicker: $6.0 \to 0.06$.

Why each wrong option is wrong:

- **F,** 0.6 — moved the point one place, giving 60%.
- **G,** 6 — left the number unchanged, so it now means 600%.
- **J,** $\frac{3}{5000}$ — divided by 100 twice.

> **Takeaway:** percent to decimal divides by 100 — two places left. Check by converting back.

Q47 — Medium — Answer: A

Topic: A fraction written as a repeating decimal

$$\frac{5}{11} = 0.454545\ldots$$

The digits 45 repeat. To three decimal places, look at the fourth digit, which is 5 — that is 5 or more, so the third place rounds up:

$$0.4545\ldots \longrightarrow 0.455$$

Why each wrong option is wrong:

- **B,** 0.454 — cut the number off at three places instead of rounding.
- **C,** 0.045 — a decimal-place slip; $\frac{5}{11}$ is close to a half, not to a twentieth.
- **D,** 0.545 — divided the wrong way round.

> **Takeaway:** a repeating decimal still rounds by the ordinary rule — look at the next digit. And sanity-check the size: $\frac{5}{11}$ is a little under $\frac{1}{2}$.

Chapter 2 — Number and Quantity

This is the smallest of the five higher-maths categories — about 10 to 12% of the test, so five questions — but it is the one most likely to contain something you have simply never been taught. **Four of its six topics do not appear on the SAT at all**: complex numbers, matrices, vectors, and sequences.

That makes this chapter unusually cheap to improve on. The mathematics is not deep. Multiplying a matrix by a number is easier than factorising a quadratic. The marks are lost to unfamiliarity, not difficulty, and unfamiliarity is fixed by reading one page.

Topics covered: exponents and roots · radicals · scientific notation · factors, multiples and primes · complex numbers and powers of i · matrices · vectors · arithmetic and geometric sequences

---

Section 1 — Exponents and Roots

Every exponent question on the ACT comes from these rules. They are worth knowing cold, because the ACT will not give them to you.

rule	in words
$x^a \cdot x^b = x^{a+b}$	multiplying, **add** the powers
$\frac{x^a}{x^b} = x^{a-b}$	dividing, **subtract** the powers
$(x^a)^b = x^{ab}$	a power of a power, **multiply**
$(xy)^a = x^a y^a$	a power spreads over a product
$x^0 = 1$	anything (except 0) to the power 0
$x^{-a} = \frac{1}{x^a}$	a negative power means "one over"
$x^{1/n} = \sqrt[n]{x}$	a fractional power is a root
$x^{m/n} = \sqrt[n]{x^m}$	top is the power, bottom is the root

The last one is the one people get backwards. In $8^{2/3}$, the **3** on the bottom is the cube root and the **2** on top is the square: $\left(\sqrt[3]{8}\right)^2 = 2^2 = 4$.

Take the root first when you can — the numbers stay small.

Q1 — Basic

A computer science calculation counts operations as $\frac{(x^3)^4}{x^5}$, where x is a positive integer describing the size of the input. Which of the following is equivalent to it?

A) x^{12}

B) x^{60}

C) x^2

D) x^7

Q2 — Medium

An engineer's model contains the quantity $8^{2/3}$, and the value must be written as an ordinary whole number before the rest of the calculation can go ahead. What is the value of $8^{2/3}$?

F) 16

G) 4

H) $\frac{16}{3}$

J) 512

Q3 — Medium

A formula produces the value 4^{-2}, and it must be rewritten without a negative exponent before being entered into a table of results. What is the value of 4^{-2}?

A) -16

B) $\frac{1}{16}$

C) $-\frac{1}{16}$

D) $\frac{1}{8}$

Q4 — Medium

A length in a design comes out as $\sqrt{72}$ centimetres, and the specification requires it in the simplest radical form $a\sqrt{b}$. Which of the following is $\sqrt{72}$ in that form?

F) $8\sqrt{2}$

G) $2\sqrt{18}$

H) $6\sqrt{2}$

J) 36

Q5 — Hard

A model requires the value of x for which $\sqrt[4]{x} = 3$, where x is a real number. What is the value of x?

A) 12

B) 81

C) 64

D) $\frac{3}{4}$

Q6 — Medium

A calculation reduces to $3^0 + 2^{-1}$. What is the value of the expression?

F) $\frac{1}{2}$

G) $\frac{3}{2}$

H) 0

J) $\frac{5}{2}$

Section 2 — Number Properties and Scientific Notation

A handful of definitions that the ACT uses without explaining:

- a **factor** of n divides into n exactly; a **multiple** of n is n times a whole number
- a **prime** has exactly two factors, itself and 1. **2 is prime; 1 is not**
- the **greatest common factor** is the largest number dividing both
- the **lowest common multiple** is the smallest number both divide into

"Shared out with nothing left over" is a common **factor**. "Happens together again" is a common **multiple**.

Q7 — Medium

A number-theory question asks for the prime factorisation of 84 in order to compare it with another number's factors. Which of the following is the prime factorisation of 84?

A) $2^2 \times 3 \times 7$

B) 2×42

C) 4×21

D) $2 \times 3 \times 7$

Q8 — Basic

A measurement of 0.00042 metres has to be entered in scientific notation. Which of the following is 0.00042 written that way?

F) 4.2×10^{-3}

G) 42×10^{-5}

H) 4.2×10^{-5}

J) 4.2×10^{-4}

Q9 — Medium

Two lighthouses flash at regular intervals: one every 12 seconds and the other every 18 seconds. They have just flashed together. After how many seconds will they next flash at the same time?

A) 6

B) 30

C) 36

D) 216

Section 3 — Complex Numbers

The ACT tests this and the SAT does not, so it is worth reading carefully even if you have never met it.

There is no real number whose square is negative. So mathematicians defined one:

$$i = \sqrt{-1} \qquad \text{which means} \qquad i^2 = -1$$

That single fact is nearly the whole topic. Everything else follows from it, and the powers of i cycle every four:

$$i^1 = i \qquad i^2 = -1 \qquad i^3 = -i \qquad i^4 = 1 \qquad i^5 = i \ldots$$

To find any power of i, divide the exponent by 4 and use the remainder.

A **complex number** is written $a + bi$: a real part and an imaginary part. Add them like algebra, keeping real with real. Multiply them like brackets, then **replace every i^2 with** -1.

Q10 — Medium

A calculation produces i^{27}, where i is defined by $i^2 = -1$. What is i^{27}?

F) 1

G) i

H) $-i$

J) -1

Q11 — Medium

Two complex numbers $(3+2i)$ and $(4-i)$ are multiplied together, where i satisfies $i^2=-1$. What is the product, in the form $a+bi$?

A) $12+5i$

B) $10+5i$

C) $14-5i$

D) $14+5i$

Q12 — Hard

An expression requires $\sqrt{-9}+\sqrt{-16}$, where i is defined by $i^2=-1$. What is the value?

F) $7i$

G) $5i$

H) -7

J) 5

Section 4 — Matrices

A **matrix** is a rectangular block of numbers, described by its size: a 2×3 matrix has 2 rows and 3 columns. **Rows first, always.**

The ACT asks for very little:

- **Adding** — add matching entries. Both matrices must be the same size.
- **Multiplying by a number** (a *scalar*) — multiply **every** entry.
- **The determinant of a** 2×2 — for $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ it is $ad-bc$: the product of the main diagonal minus the product of the other one.

That is essentially the whole syllabus. The commonest error is multiplying only the first entry by the scalar.

Q13 — Basic

A transformation is scaled by multiplying the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ by 5. What is the resulting matrix?

A) $\begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}$

B) $\begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$

C) $\begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$

D) $\begin{bmatrix} 5 & 10 \\ 3 & 4 \end{bmatrix}$

Q14 — Medium

A system's behaviour depends on the determinant of $\begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix}$. What is the determinant of this matrix?

F) 24

G) 10

H) 0

J) 12

Q15 — Medium

Two sets of readings are stored as matrices and must be combined entry by entry:

$$\begin{pmatrix} 2 & -1 \\ 0 & 5 \end{pmatrix} + \begin{pmatrix} 3 & 4 \\ -2 & 1 \end{pmatrix}$$

What is the sum?

A) $\begin{bmatrix} 5 & 3 \\ -2 & 6 \end{bmatrix}$

B) $\begin{bmatrix} 5 & 3 \\ 2 & 6 \end{bmatrix}$

C) $\begin{bmatrix} 6 & -4 \\ 0 & 5 \end{bmatrix}$

D) $\begin{bmatrix} -1 & -5 \\ 2 & 4 \end{bmatrix}$

Section 5 — Vectors

A **vector** has both a size and a direction, written by its components: $\langle 3, 4\rangle$ means 3 across and 4 up.

- **Add** two vectors by adding matching components.
- Its **magnitude** — its length — comes from Pythagoras:

$$|\langle a, b\rangle| = \sqrt{a^2 + b^2}$$

That is the entire ACT vector syllabus. Notice that magnitude is Pythagoras wearing different notation, so if you can find the hypotenuse of a right triangle you can already do this.

Q16 — Medium

A force is recorded as the vector $\langle 5, -12\rangle$, and its magnitude is needed for a stress calculation. What is the magnitude of the vector?

F) -7

G) 13

H) 7

J) 17

Q17 — Medium

Two displacements are recorded as $\langle 2, -3\rangle$ and $\langle -5, 8\rangle$, and the combined displacement is their sum. What is the sum of the two vectors?

A) $(7,\ -11)$

B) $(-10,\ -24)$

C) $(-3,\ -5)$

D) $(-3,\ 5)$

Section 6 — Sequences

A **sequence** is a list of numbers following a rule. The ACT tests two kinds, and the first job is always to work out which one you have.

Arithmetic — you **add** the same amount each time. That amount is the *common difference*, d.

$$n\text{th term} = a + (n-1)d$$

Geometric — you **multiply** by the same amount each time. That amount is the *common ratio*, r.

$$n\text{th term} = a \cdot r^{n-1}$$

In both, a is the first term. And in both, the exponent and the bracket contain $n-1$, **not** n — because getting from term 1 to term n takes $n-1$ steps. That off-by-one is the single most common sequence error.

How to tell them apart: subtract consecutive terms. If you always get the same number, it is arithmetic. If not, divide consecutive terms instead; the same number means geometric.

Q18 — Medium

A sequence begins $7,\ 21,\ 35,\ 49, \ldots$, and each term is obtained by adding the same fixed amount to the one before it. What is the 20th term?

F) 287

G) 140

H) 133

J) 273

Q19 — Medium

A culture is measured as $3,\ 12,\ 48,\ 192, \ldots$ where each value is obtained from the one before by multiplying by a fixed amount. The 7th term is required. What is it?

A) 49152

B) 27

C) 12288

D) 84

Q20 — Hard

The nth term of an arithmetic sequence is $5n + 3$. A researcher needs the sum of the first three terms of that sequence. What is that sum?

F) 39

G) 8

H) 18

J) 33

Q21 — Medium

A force is recorded as the vector $\langle -2, 5 \rangle$ and is then tripled in size while keeping its direction. What is the resulting vector?

A) $(-2,\ 15)$

B) $(-6,\ 5)$

C) $(-6,\ 15)$

D) $(1,\ 8)$

Q22 — Hard

Two displacements are recorded as $\mathbf{u} = \langle 7, -2 \rangle$ and $\mathbf{v} = \langle 3, 4 \rangle$. What is $\mathbf{u} - \mathbf{v}$?

F) $(4,\ -6)$

G) $(-4,\ 6)$

H) $(10,\ 2)$

J) $(4,\ 2)$

Q23 — Medium

A displacement is recorded as $\langle -6, 8 \rangle$, and its magnitude is required. What is the magnitude?

A) 10

B) 2

C) 14

D) $\sqrt{14}$

Q24 — Medium

Two complex numbers $(4 - 3i)$ and $(-1 + 7i)$ are added, where i satisfies $i^2 = -1$. What is the sum?

F) $3 - 4i$

G) $5 + 4i$

H) $3 + 4i$

J) $3 + 10i$

Answers and Explanations

Section 1 — Exponents and Roots

Q1 — Basic — Answer: D

Topic: Multiplying and dividing powers of the same base

Work from the inside out. A power of a power multiplies:

$$(x^3)^4 = x^{3\times 4} = x^{12}$$

Dividing subtracts:

$$\frac{x^{12}}{x^5} = x^{12-5} = x^7$$

Why each wrong option is wrong:

- **A,** x^{12} — did the top correctly and then forgot to divide.
- **B,** x^{60} — multiplied all three numbers together.
- **C,** x^2 — **added** 3 and 4 instead of multiplying them, giving x^7 on top and then x^2 after dividing. The rule for a power of a power is multiply; the rule for a product is add. Mixing them up is the most common exponent error there is.

> **Takeaway:** power of a power **multiplies**; multiplying like bases **adds**; dividing **subtracts**. Say which one you are using before you write anything.

Q2 — Medium — Answer: G

Topic: A fractional exponent

$$8^{2/3} = \left(\sqrt[3]{8}\right)^2$$

The 3 on the bottom says cube root; the 2 on top says square.

$$\sqrt[3]{8} = 2 \qquad 2^2 = 4$$

Do the root **first**. Going the other way, $8^2 = 64$ and then $\sqrt[3]{64} = 4$, gives the same answer but with much bigger numbers.

Why each wrong option is wrong:

- **F,** 16 — squared to get 64 and then halved rather than taking a cube root.
- **H,** $\frac{16}{3}$ — multiplied 8 by $\frac{2}{3}$. An exponent is not a multiplier.
- **J,** 512 — used the 3 as the power and ignored the 2.

> **Takeaway:** in $x^{m/n}$ the **bottom is the root** and the **top is the power**. Take the root first to keep the arithmetic small.

Q3 — Medium — Answer: B

Topic: A negative exponent

A negative exponent means the reciprocal:

$$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

The minus sign belongs to the **exponent**, and its job is to flip the fraction over. It never makes the value negative. A positive base raised to any power whatsoever stays positive.

Why each wrong option is wrong:

- **A,** -16 — treated the minus as belonging to the answer.
- **C,** $-\frac{1}{16}$ — inverted correctly *and* kept a minus sign, doing the job twice.
- **D,** $\frac{1}{8}$ — multiplied 4 by 2.

> **Takeaway:** a negative exponent flips, it does not negate. $x^{-a} = \frac{1}{x^a}$, and the sign of the answer is unchanged.

Q4 — Medium — Answer: H

Topic: Simplifying a radical

Find the largest **square** number that divides 72. The squares are 4, 9, 16, 25, 36, 49... and 36 divides 72.

$$\sqrt{72} = \sqrt{36 \times 2} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$$

Check with a calculator: $6 \times 1.414 = 8.485$, and $\sqrt{72} = 8.485$. Correct.

Why each wrong option is wrong:

- **F,** $8\sqrt{2}$ — used 64, which is a square but does **not** divide 72.

- **G,** $2\sqrt{18}$ — took out the factor 4 rather than 36, giving $2\sqrt{18}$. That equals the right value, but 18 still contains the square factor 9, so it is not in simplest form. This is why "simplest" is stated in the question.
- **J,** 36 — halved 72.

> **Takeaway:** pull out the **largest** square factor, then check what is left has no square factor of its own. If it does, you have not finished.

Q5 — Hard — Answer: B

Topic: A radical equation with a fourth root

To undo a fourth root, raise both sides to the fourth power:

$$\left(\sqrt[4]{x}\right)^4 = 3^4$$
$$x = 81$$

Check: $\sqrt[4]{81} = 3$, because $3 \times 3 \times 3 \times 3 = 81$. Correct.

Why each wrong option is wrong:

- **A,** 12 — multiplied 3 by 4. A root is not a division and its inverse is not a multiplication.
- **C,** 64 — computed 4^3 instead of 3^4. The base and the exponent are not interchangeable: $4^3 = 64$ but $3^4 = 81$.
- **D,** $\frac{3}{4}$ — divided by 4.

> **Takeaway:** an nth root is undone by an nth power. And keep the base and the exponent the right way round — they give different answers.

Q6 — Medium — Answer: G

Topic: Zero and negative exponents together

Two separate rules:

$$3^0 = 1 \qquad \text{(anything except 0, to the power 0, is 1)}$$
$$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$$
$$1 + \frac{1}{2} = \frac{3}{2}$$

Why each wrong option is wrong:

- **F,** $\frac{1}{2}$ — took 3^0 to be 0. The exponent being zero does not make the term zero; it makes it **one**.
- **H,** 0 — took both terms as zero.
- **J,** $\frac{5}{2}$ — used 2 for 3^0.

> **Takeaway:** $x^0 = 1$, always. A zero exponent means "no copies of the base multiplied together", and an empty product is 1, not 0.

Section 2 — Number Properties and Scientific Notation

Q7 — Medium — Answer: A

Topic: Prime factors of a number

Divide by primes until nothing composite is left:

$$84 = 2 \times 42 = 2 \times 2 \times 21 = 2 \times 2 \times 3 \times 7$$

So

$$84 = 2^2 \times 3 \times 7$$

Check by multiplying back: $4 \times 3 \times 7 = 84$. Correct.

Why each wrong option is wrong:

- **B,** 2×42 and **C,** 4×21 — correct products, but 42, 4 and 21 are not prime, so neither is a prime factorisation. Multiply them out and both give 84, which is exactly why the word "prime" in the question matters.
- **D,** $2 \times 3 \times 7$ — dropped one of the two factors of 2, giving 42.

> **Takeaway:** keep dividing until **every** factor is prime, then check by multiplying back. Getting the right product is not enough; the form is what is being asked for.

Q8 — Basic — Answer: J

Topic: Writing a number in scientific notation

Move the decimal point so that exactly one non-zero digit stands in front of it:

$$0.00042 \longrightarrow 4.2$$

The point moved **4 places to the right**, and moving right makes the number bigger, so the power of ten must make it smaller again:

$$4.2 \times 10^{-4}$$

Check: $4.2 \times 0.0001 = 0.00042$. Correct.

Why each wrong option is wrong:

- **F,** 4.2×10^{-3}, **G,** 42×10^{-5} and **H,** 4.2×10^{-5} — all have the right leading number and the wrong power. Counting the places is the whole question, so count them twice, and always check by expanding the answer back out.

> **Takeaway:** a number **smaller than 1** has a **negative** power of ten. Count the places the point moves, then expand your answer to check.

Q9 — Medium — Answer: C

Topic: The lowest common multiple in context

They coincide at times that are multiples of **both** 12 and 18, so the first is the lowest common multiple.

$$12 = 2^2 \times 3 \qquad 18 = 2 \times 3^2$$

Take the highest power of each prime that appears:

$$2^2 \times 3^2 = 36$$

Check: $36 = 3 \times 12$ and $36 = 2 \times 18$. Both whole. Correct.

Why each wrong option is wrong:

- **A,** 6 — gave the greatest common **factor**, 6. That is the wrong tool: a factor is smaller than both intervals, and they cannot coincide before either has flashed.
- **B,** 30 — added the intervals.
- **D,** 216 — multiplied them. 216 *is* a common multiple, but it is not the **lowest** one, and the question asks when they next coincide.

> **Takeaway:** "when do they happen together again" is the lowest common multiple. Multiplying always gives *a* common multiple, but rarely the lowest.

Section 3 — Complex Numbers

Q10 — Medium — Answer: H

Topic: A high power of the imaginary unit

Divide the exponent by 4 and keep the remainder:

$$27 \div 4 = 6 \text{ remainder } 3$$

So $i^{27} = i^3$, and from the cycle

$$i^3 = -i$$

Why it works: $i^{27} = (i^4)^6 \times i^3 = 1^6 \times i^3 = i^3$, because $i^4 = 1$ and 1 to any power is 1.

Why each wrong option is wrong:

- **F,** 1, **G,** i and **J,** -1 — each uses a different remainder. Only the arithmetic of $27 \div 4$ separates them, so do that division carefully; it is the entire question.

> **Takeaway:** powers of i cycle $i, -1, -i, 1$ with period 4. Divide the exponent by 4 and use the **remainder**: 1, 2, 3, 0 give $i, -1, -i, 1$.

Q11 — Medium — Answer: D

Topic: Multiplying two complex numbers

Expand as ordinary brackets:

$$(3 + 2i)(4 - i) = 12 - 3i + 8i - 2i^2$$

Collect the i terms: $-3i + 8i = 5i$.

Now the key step — replace i^2 with -1:

$$-2i^2 = -2(-1) = +2$$

So

$$12 + 2 + 5i = 14 + 5i$$

Why each wrong option is wrong:

- **A,** $12 + 5i$ — expanded correctly but left the $-2i^2$ alone, so the real part stayed 12. This is the mistake the question exists to catch.
- **B,** $10 + 5i$ — subtracted the 2 rather than adding it, forgetting that -2×-1 is positive.
- **C,** $14 - 5i$ — sign slip when collecting $-3i + 8i$.

> **Takeaway:** expand normally, then hunt for every i^2 and replace it with -1. A $-i^2$ becomes $+1$ — two negatives, and it is easy to keep only one.

Q12 — Hard — Answer: F

Topic: Adding roots of negative numbers

Convert each root separately first:

$$\sqrt{-9} = \sqrt{9}\sqrt{-1} = 3i \qquad \sqrt{-16} = 4i$$

Now add, treating i like a variable:

$$3i + 4i = 7i$$

Why you must convert first. The rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ only holds when a and b are not negative. Combining under one root here would give $\sqrt{-25} = 5i$, which is wrong — and it is option **G**.

Why each wrong option is wrong:

- **G,** $5i$ — combined the two roots into $\sqrt{-25}$.
- **H,** -7 — dropped the i and kept a minus sign.
- **J,** 5 — ignored the minus signs and worked with $\sqrt{9} + \sqrt{16}$.

> **Takeaway:** rewrite each $\sqrt{\text{negative}}$ as a multiple of i **before** doing anything else. The usual root rules do not survive a negative under the sign.

Section 4 — Matrices

Q13 — Basic — Answer: B

Topic: Multiplying a matrix by a number

Multiply every one of the four entries by 5:

$$5\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix} = \begin{pmatrix}5\times 1 & 5\times 2\\ 5\times 3 & 5\times 4\end{pmatrix} = \begin{pmatrix}5 & 10\\ 15 & 20\end{pmatrix}$$

Why each wrong option is wrong:

- **A,** $\begin{bmatrix}5 & 2\\ 3 & 4\end{bmatrix}$ — multiplied only the top-left entry.

- **C,** $\begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$ — added 5 to each entry instead of multiplying.
- **D,** $\begin{bmatrix} 5 & 10 \\ 3 & 4 \end{bmatrix}$ — multiplied the first row and left the second alone.

> **Takeaway:** a scalar reaches **every** entry. Count them: a 2×2 has four, and all four must change.

Q14 — Medium — Answer: H

Topic: The determinant of a 2 by 2 matrix

For $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ the determinant is $ad - bc$:

$$(6 \times 2) - (3 \times 4) = 12 - 12 = 0$$

A determinant of zero is not a mistake — it is a meaningful answer, telling you the two rows are multiples of one another. Here the first row is exactly 1.5 times the second.

Why each wrong option is wrong:

- **F,** 24 — added the two products instead of subtracting.
- **G,** 10 — multiplied down the **columns**, $6imes3 - 4imes2$, instead of along the diagonals. Note that subtracting the right products in the wrong order, $bc - ad$, would give 0 here as well — with this matrix that particular slip is invisible, which is a good reason to fix the order by habit rather than by checking the answer.
- **J,** 12 — used only the main diagonal.

> **Takeaway:** $ad - bc$, main diagonal first. A determinant of 0 is a real answer, and it means the rows are proportional.

Q15 — Medium — Answer: A

Topic: Adding two matrices

Add each pair in matching positions:

$$\begin{pmatrix} 2+3 & -1+4 \\ 0+(-2) & 5+1 \end{pmatrix} = \begin{pmatrix} 5 & 3 \\ -2 & 6 \end{pmatrix}$$

Why each wrong option is wrong:

- **B,** $\begin{bmatrix} 5 & 3 \\ 2 & 6 \end{bmatrix}$ — got $+2$ in the bottom left, ignoring the minus sign on -2.
- **C,** $\begin{bmatrix} 6 & -4 \\ 0 & 5 \end{bmatrix}$ — multiplied matching entries rather than adding them.
- **D,** $\begin{bmatrix} -1 & -5 \\ 2 & 4 \end{bmatrix}$ — subtracted the first matrix from the second.

> **Takeaway:** matrix addition is entry-by-entry, and the negative signs are where the marks go. Work position by position, not left to right along the whole block.

Section 5 — Vectors

Q16 — Medium — Answer: G

Topic: The magnitude of a vector

Magnitude is Pythagoras on the two components:

$$\sqrt{5^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$$

Notice that squaring removes the minus sign, which is why a magnitude can never come out negative. A length has no direction.

Why each wrong option is wrong:

- **F,** -7 — added the components, giving a negative "length".
- **H,** 7 and **J,** 17 — subtracted or added the sizes without squaring. Both would give the right answer only if the two components pointed the same way, and they do not — they are at right angles.

> **Takeaway:** magnitude is $\sqrt{a^2 + b^2}$. Square first, so the signs disappear, and expect a positive answer every time. (5, 12, 13 is a Pythagorean triple worth memorising — see Chapter 8.)

Q17 — Medium — Answer: D

Topic: Adding two vectors

Add the matching components:

$$\langle 2 + (-5),\ -3 + 8 \rangle = \langle -3,\ 5 \rangle$$

Take the two separately and slowly. $2 + (-5) = -3$; $-3 + 8 = 5$.

Why each wrong option is wrong:

- **A,** $(7,\ -11)$ — subtracted the second vector from the first.
- **B,** $(-10,\ -24)$ — multiplied the components.
- **C,** $(-3,\ -5)$ — got the first component right and slipped a sign on the second.

> **Takeaway:** vector addition is component by component. Write the two calculations on separate lines — running them together in your head is where the sign errors come from.

Section 6 — Sequences

Q18 — Medium — Answer: J

Topic: A term of an arithmetic sequence

First check which kind of sequence it is. Subtract consecutive terms:

$$21 - 7 = 14 \qquad 35 - 21 = 14 \qquad 49 - 35 = 14$$

Always 14, so it is arithmetic with $a = 7$ and $d = 14$.

$$20\text{th term} = a + (n-1)d = 7 + 19 \times 14 = 7 + 266 = 273$$

Why each wrong option is wrong:

- **F,** 287 — used $a + nd$ instead of $a + (n-1)d$, adding one difference too many. This is the off-by-one the formula is designed to prevent.
- **G,** 140 — multiplied the first term by 20, which would only work if the sequence started at 0 and had difference 7.
- **H,** 133 — used 7 as the common difference.

> **Takeaway:** $a + (n-1)d$. There are $n - 1$ steps to reach the nth term, not n — you are already standing on the first one.

Q19 — Medium — Answer: C

Topic: A term of a geometric sequence

Subtracting gives $12 - 3 = 9$ and $48 - 12 = 36$ — not constant, so it is not arithmetic. Divide instead:

$$\frac{12}{3} = 4 \qquad \frac{48}{12} = 4 \qquad \frac{192}{48} = 4$$

Constant, so it is geometric with $a = 3$ and $r = 4$.

$$7\text{th term} = a \cdot r^{\,n-1} = 3 \times 4^6 = 3 \times 4096 = 12288$$

Why each wrong option is wrong:

- **A,** 49152 — used r^n instead of r^{n-1}, giving a term four times too big.
- **B,** 27 — treated the sequence as arithmetic with a difference of 4.
- **D,** 84 — multiplied the three numbers together.

> **Takeaway:** subtract to test for arithmetic; if that fails, divide to test for geometric. Then use $a \cdot r^{\,n-1}$ — the exponent is one less than the term number.

Q20 — Hard — Answer: F

Topic: Working backwards to the first term

Substitute $n = 1, 2, 3$ in turn:

$$n = 1: \;\; 5(1) + 3 = 8$$

$$n = 2: \;\; 5(2) + 3 = 13$$

$$n = 3: \;\; 5(3) + 3 = 18$$

$$8 + 13 + 18 = 39$$

Notice the terms go up by 5 each time, which matches the coefficient of n — that is a useful check that the substitution was done correctly.

Why each wrong option is wrong:

- **G,** 8 and **H,** 18 — gave a single term rather than the sum.
- **J,** 33 — added $1 + 2 + 3$ first and applied the rule once, to $n = 6$. The rule has to be applied to each term separately, then the results added.

> **Takeaway:** an nth-term expression is a machine — feed it each n in turn. Do not put a sum of n values into it.

Q21 — Medium — Answer: C

Topic: Scalar multiplication of a vector

Multiply both components by 3:

$$3\langle -2, 5\rangle = \langle -6, 15\rangle$$

The sign is carried through: $3 \times (-2) = -6$.

Why each wrong option is wrong:

- **A,** $(-2,\ 15)$ and **B,** $(-6,\ 5)$ — multiplied one component and left the other.
- **D,** $(1,\ 8)$ — added 3 to each rather than multiplying.

> **Takeaway:** a scalar reaches **every** component, signs included. The direction is unchanged; only the length scales.

Q22 — Hard — Answer: F

Topic: Subtracting one vector from another

Subtract matching components, keeping the order the question gives:

$$\langle 7-3,\ -2-4\rangle = \langle 4,\ -6\rangle$$

Do the two on separate lines. The second is $-2-4$, which is -6, not 2 — running them together in your head is where the sign goes.

Why each wrong option is wrong:

- **G,** $(-4,\ 6)$ — computed $\mathbf{v} - \mathbf{u}$, which is the negative of the answer.
- **H,** $(10,\ 2)$ — added the vectors.
- **J,** $(4,\ 2)$ — got the first component right and slipped the sign on the second.

> **Takeaway:** vector subtraction is component by component and is **not** symmetric: $\mathbf{u} - \mathbf{v}$ and $\mathbf{v} - \mathbf{u}$ point opposite ways.

Q23 — Medium — Answer: A

Topic: A vector's magnitude from its components

$$\sqrt{(-6)^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$$

Another 6-8-10 triangle, which is 3-4-5 doubled.

Why each wrong option is wrong:

- **B,** 2 and **C,** 14 — combined the components without squaring. Both would only be right if the two pointed along the same line; they are at right angles.
- **D,** $\sqrt{14}$ — added before squaring.

> **Takeaway:** magnitude is $\sqrt{a^2 + b^2}$ — square first, so the signs vanish, then root at the end.

Q24 — Medium — Answer: H

Topic: Adding two complex numbers

Add the real parts and the imaginary parts separately:

$$\text{real: } 4 + (-1) = 3 \qquad \text{imaginary: } -3 + 7 = 4$$
$$(4 - 3i) + (-1 + 7i) = 3 + 4i$$

There is no i^2 here, so nothing needs replacing — that only happens when complex numbers are **multiplied**.

Why each wrong option is wrong:

- **F,** $3 - 4i$ — subtracted the imaginary parts.
- **G,** $5 + 4i$ — subtracted the real parts, giving $4 - (-1) = 5$.
- **J,** $3 + 10i$ — computed $7 - (-3) = 10$ rather than $-3 + 7$.

> **Takeaway:** real with real, imaginary with imaginary. Adding complex numbers never produces an i^2, so no substitution is needed.

Chapter 3 — Algebra

Algebra is the largest of the five higher-maths categories: **17 to 20% of the test**, so eight or nine questions. Together with Functions in the next chapter it accounts for well over a third of everything you will be asked.

The good news is how narrow it is. Almost every question here is one of six things: solve for a letter, solve two equations at once, expand, factor, rearrange a formula, or handle a power. This chapter takes each in turn.

A warning carried over from Chapter 0. The ACT gives you no formula sheet, so the quadratic formula is not printed anywhere on the paper. It is in Chapter 8 with the rest, and you need it from memory.

Topics covered: linear equations · linear inequalities · systems of equations · expanding and factoring · quadratic equations · the discriminant · radical and rational equations · exponential relationships · rearranging a formula

---

Section 1 — Linear Equations and Inequalities

To solve a linear equation, undo what was done to the variable, in the reverse order it was done, doing the same to both sides.

For an **inequality** there is exactly one extra rule, and it is the one that costs marks:

> **Multiplying or dividing by a negative number reverses the inequality sign.**

Adding and subtracting never do. Only multiplying or dividing by a negative.

Q1 — Basic

A technician's two readings agree only at one value of x, where $5(2x-3)=3(x+4)+7$. What is the value of x?

A) $\frac{4}{7}$

B) 2

C) $\frac{20}{7}$

D) $\frac{34}{7}$

Q2 — Medium

A machine setting x must be chosen so that the quantity $-4x+7$ stays at or above the safety threshold of 19. Which of the following gives all values of x that satisfy $-4x+7 \geq 19$?

F) $x \leq -3$

G) $x \geq -3$

H) $x \leq 3$

J) $x \geq 3$

Q3 — Medium

In the equation $4x+9=kx-3$, k is a constant. The equation has **no** solution. What is the value of k ?

A) -4

B) 4

C) 9

D) -3

Q4 — Medium

A process runs correctly only while the quantity $3x+4$ stays above -5 and at the same time does not exceed 13, where x is a measured input. If $-5 < 3x+4 \leq 13$, which of the following gives all possible values of x?

F) $-3 \leq x \leq 3$

G) $-3 < x < 3$

H) $-3 \leq x < 3$

J) $-3 < x \leq 3$

Section 2 — Systems of Equations

Two equations, two unknowns. Two methods, and the choice between them is worth making deliberately:

Substitution — best when one equation already gives a variable on its own. Put that expression into the other equation.

Elimination — best when neither does. Add or subtract the two equations so one variable disappears, scaling one or both first if needed.

The ACT also asks about systems with **no solution** (parallel lines: same slope, different intercept) and **infinitely many** (the same line twice).

Q5 — Basic

Two orders placed with the same supplier satisfy $3x + 2y = 19$ and $5x - 2y = 13$, where x and y are quantities of two items. What is the value of x?

A) $\frac{7}{2}$

B) 4

C) 32

D) 8

Q6 — Medium

Two conditions on the same pair of quantities are recorded as $2x + 5y = 24$ and $3x - 2y = -2$. What is the value of y?

F) 4

G) 76

H) 2

J) -2

Q7 — Hard

The system $\begin{cases} 6x - 2y = 10 \\ 9x + ky = 3 \end{cases}$, where k is a constant, has **no** solution. What is the value of k?

A) 3

B) -3

C) $-\frac{4}{3}$

D) -2

Section 3 — Expanding and Factoring

Expanding and factoring are the same operation in opposite directions, and the ACT asks for both.

Three patterns are worth knowing on sight, because spotting them saves the time that the ACT is really testing:

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$(a+b)(a-b) = a^2 - b^2$$

The middle term in the first two is what people forget. $(a+b)^2$ is **not** $a^2 + b^2$.

To factor $x^2 + bx + c$, find two numbers that **multiply to** c and **add to** b.

Q8 — Basic

A cost model contains the expression $(2x-3)^2$, and it must be written out without brackets before the terms can be collected. Which of the following is equivalent to $(2x-3)^2$?

F) $4x^2 + 9$

G) $4x^2 - 9$

H) $4x^2 - 6x + 9$

J) $4x^2 - 12x + 9$

Q9 — Medium

A quadratic model is written as $2x^2 + 5x - 3$, and one of its linear factors is needed in order to find a zero. Which of the following is a factor of $2x^2 + 5x - 3$?

A) $x - 3$

B) $2x + 1$

C) $2x - 1$

D) $x + 1$

Q10 — Medium

An expression in a design formula is $9x^2 - 49$. Which of the following is equivalent to $9x^2 - 49$?

F) $(3x - 7)(3x + 7)$

G) $(3x - 7)^2$

H) $(x + 7)(9x - 7)$

J) $(3x - 49)(3x + 1)$

Section 4 — Quadratic Equations

A quadratic equation is one with an x^2 and no higher power. Three ways to solve one, in the order you should try them:

1. **Factor**, if it factors easily. Fastest by far.
2. **Take square roots**, if there is no middle term.
3. **The quadratic formula**, which always works:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The ACT does not print that formula. Learn it.

The part under the root, $b^2 - 4ac$, is the **discriminant**, and it tells you how many real solutions there are before you find any of them:

discriminant	real solutions
positive	two
zero	exactly one
negative	none

Q11 — Medium

The height of a projectile returns to zero at the times satisfying $x^2 - 5x - 14 = 0$, where x is the number of seconds since launch. What is that solution?

A) -2

B) 14

C) 5

D) 7

Q12 — Hard

A designer needs the parabola $y = x^2 + kx + 16$ to touch the x-axis at exactly one point. The constant k is known to be positive. What is the value of k?

F) 4

G) 16

H) 64

J) 8

Q13 — Medium

A design constraint gives $(x - 3)^2 = 25$. What are the solutions?

A) $(-8,\ 2)$

B) $(-2,\ 8)$

C) $(5,\ 5)$

D) $(2,\ 8)$

Section 5 — Radical, Rational and Exponential Equations

Three equation types that each carry one specific hazard.

Radical — square both sides to clear the root, then **check every answer in the original equation**. Squaring can manufacture solutions that were never there.

Rational (a variable in a denominator) — multiply through by the denominator. Any value that would make a denominator zero is excluded.

Exponential — if both sides can be written with the **same base**, the exponents must be equal.

Q14 — Hard

A model requires the value of x for which $\sqrt{x+6} = x$, where the square root sign means the non-negative root. What is the solution?

F) -2

G) 1

H) 6

J) 3

Q15 — Medium

An equation in a growth model reduces to $9^x = 27^4$. What is the value of x?

A) $\frac{4}{3}$

B) 4

C) 12

D) 6

Q16 — Medium

An equation from a rate problem is $\dfrac{12}{x} + 1 = \dfrac{8}{x} + 3$, with the unknown in two denominators. What is the solution?

F) 4

G) -2

H) 2

J) 20

Section 6 — Rearranging a Formula

"Solve for R in terms of the others" is a standard ACT question and it appeared on both official enhanced forms. There is no new mathematics in it: treat every other letter as if it were a number, and isolate the one you want.

The only real difficulty is that the answer is an expression rather than a number, so there is nothing to substitute back — which makes it worth being slow.

Q17 — Medium

The perimeter P of a rectangular field is related to its length l and its width w by $P = 2l + 2w$. A surveyor knows the perimeter and the length and needs a formula giving the width directly. Which of the following expresses w in terms of P and l?

A) $\frac{P}{2} - 2l$

B) $\frac{P}{2} - l$

C) $\frac{P}{2} - \frac{l}{2}$

D) $P - 2l$

Q18 — Hard

A physics formula is $F = \dfrac{mv}{t}$, and it must be rearranged to give t on its own, where t appears only in the denominator. Which of the following expresses t correctly?

F) Fmv

G) $\frac{mv}{F}$

H) $\frac{F}{mv}$

J) $\frac{Fm}{v}$

Q19 — Hard

A model requires the value of x satisfying $\sqrt{2x+7} = x - 4$, where the root sign denotes the non-negative root. What is the solution?

A) -1

B) 1

C) 10

D) 9

Q20 — Medium

A rate problem gives $\dfrac{20}{x} + 3 = \dfrac{8}{x} + 7$, with the unknown in two denominators. What is the solution?

F) 12

G) 4

H) 3

J) 28

Q21 — Medium

A colony of 500 bacteria grows by 20% each hour, compounding from one hour to the next. What is the population after 2 hours?

A) 600

B) 700

C) 720

D) 620

Q22 — Hard

A sample decays to 85% of its mass each hour, starting from 400 grams. Which expression gives its mass, in grams, after h hours?

F) $400(0.15)^h$

G) $400(0.85)^h$

H) $400 - 0.15h$

J) $400(1.15)^h$

Q23 — Medium

An equation from a mixing problem is $\dfrac{6}{x-2} = 3$, where the unknown sits inside a denominator. What is the solution?

A) 2

B) 4

C) 20

D) -4

Q24 — Medium

A rectangular plot's area, in square metres, is given by $x^2 - 3x - 40$, where x is a length in metres. The area falls to zero at the values satisfying $x^2 - 3x - 40 = 0$. What is that value of x?

F) -5

G) 8

H) 40

J) 3

Q25 — Hard

An equation from a model is $3x^2 - 5x - 2 = 0$. What is the **larger** of the two solutions?

A) $-\frac{1}{3}$

B) $\frac{5}{6}$

C) 2

D) 3

Q26 — Medium

A design constraint reduces to $4x^2 - 49 = 0$. What are the solutions?

F) $\left(-\frac{49}{4},\ \frac{49}{4}\right)$

G) $(-7,\ 7)$

H) $\left(-\frac{7}{2},\ \frac{7}{2}\right)$

J) $\left(-\frac{49}{2},\ \frac{49}{2}\right)$

Q27 — Hard

A model requires the value of x satisfying $\sqrt{x+5} = \sqrt{x} + 1$, where each root sign denotes the non-negative root. What is the solution?

A) 2

B) 8

C) 16

D) 4

Q28 — Medium

A rate problem gives $\frac{3}{x} = \frac{12}{x+6}$, with the unknown in both denominators. What is the solution?

F) 2

G) 6

H) -6

J) 18

Q29 — Medium

An equation reduces to $x + \frac{6}{x} = 5$, where the unknown appears both on its own and in a denominator. What is the **smaller** solution?

A) 6

B) 3

C) 5

D) 2

Q30 — Medium

An investment of $2000 grows by 6% each year, compounding annually. Which expression gives its value in dollars after n years?

F) $2000(0.06)^n$

G) $2000(1.06)^n$

H) $2000 + 0.06n$

J) $2000(6)^n$

Q31 — Hard

An equation from a growth model is $8^{x+1} = 32$. What is the value of x?

A) 4

B) $\frac{5}{3}$

C) $\frac{2}{3}$

D) 2

Q32 — Medium

A sample loses 20% of its mass each hour, starting at 500 grams. What is its mass, in grams, after 3 hours?

F) 200

G) 400

H) 256

J) 140

Q33 — Medium

The formula $A = \dfrac{bh}{2}$ gives the area of a triangle. A designer knows the area and the base and needs a formula for the height directly. Which of the following expresses h in terms of A and b?

A) $\frac{2A}{b}$

B) $\frac{A}{2b}$

C) $\frac{Ab}{2}$

D) $2Ab$

Q34 — Hard

The formula $T = 2\pi r^2 + 2\pi rh$ gives the surface area of a cylinder. An engineer knows T and r and needs h on its own. Which of the following gives h?

F) $\frac{T-2\pi r^2}{2\pi}$

G) $\frac{T-2\pi r^2}{2\pi r}$

H) $\frac{T}{2\pi r} - r^2$

J) $\frac{T}{2\pi r}$

Q35 — Medium

The formula $A = \pi r^2$ gives the area of a circle. A manufacturer knows the area and needs the radius. Which of the following gives r?

A) $\frac{\sqrt{A}}{\sqrt{\pi}}$

B) $\frac{A}{\pi}$

C) $\frac{\sqrt{A}}{\pi}$

D) $\frac{A}{2\pi}$

Q36 — Medium

A model requires the value of x satisfying $\sqrt{3x+1} = 4$, where the root sign denotes the non-negative root. What is the solution?

F) 1

G) 5

H) 16

J) $\frac{4}{3}$

Q37 — Hard

An equation from a rate problem is $\dfrac{x}{x-3} = \dfrac{5}{x-3} + 2$. What is the solution?

A) 1

B) 3

C) 5

D) 11

Q38 — Medium

A control rule requires $-3(x-4) > 2x+1$, where x is an adjustable input. Which of the following gives all values of x that satisfy the inequality?

F) $x < \frac{11}{5}$

G) $x > \frac{11}{5}$

H) $x < -\frac{13}{5}$

J) $x > -\frac{13}{5}$

Q39 — Medium

The system $\begin{cases} 3x + 2y = 7 \\ 6x + 4y = c \end{cases}$, where c is a constant, has **infinitely many** solutions. What is the value of c?

A) 7

B) 2

C) 14

D) 21

Q40 — Medium

Two conditions give $x + y = 9$ and $x - y = 3$. The question asks for $x^2 - y^2$. What is the value of $x^2 - y^2$?

F) 12

G) 6

H) 36

J) 27

Q41 — Medium

An expression in a design formula is $3x^2 - 27$. Which of the following is $3x^2 - 27$ fully factored?

A) $3(x - 3)(x + 3)$

B) $(3x - 9)(x + 3)$

C) $3(x - 3)^2$

D) $(x - 3)(x + 9)$

Q42 — Hard

The equation $x^2 + 6x - 7 = 0$ can be rewritten as $(x + a)^2 = b$, where a and b are constants. What is the value of b?

F) 7

G) 9

H) 16

J) 3

Q43 — Medium

An analyst needs the sum of the two solutions of $2x^2 - 10x + 8 = 0$. What is that sum?

A) 5

B) 10

C) 4

D) -5

Q44 — Medium

A growth model reduces to $2^x = 128$. What is the value of x?

F) 7

G) 64

H) 6

J) 8

Q45 — Hard

A delivery van weighs 1200 kg empty and each crate loaded into it weighs 45 kg. A bridge has a weight limit of 2100 kg. Which inequality gives all the numbers of crates n the van may carry across?

A) $0 \leq n \leq 46$

B) $n \geq 20$

C) $0 \leq n \leq 20$

D) $0 \leq n \leq 73$

Q46 — Medium

A cost model needs $(3x - 2)(2x + 5)$ written without brackets before its terms can be collected. Which expression is equivalent?

F) $6x^2 - 10$

G) $6x^2 + 19x + 10$

H) $6x^2 + 11x - 10$

J) $6x^2 - 11x - 10$

Q47 — Hard

An equation from a comparison is $\frac{x}{2} + 3 = \frac{x}{5} + 6$. What is the value of x?

A) 10

B) 3

C) 30

D) $\frac{10}{3}$

Q48 — Medium

An equation from a scaling problem is $\frac{x+1}{4} = \frac{x-2}{2}$. What is the value of x?

F) 3

G) 5

H) -5

J) 1

Answers and Explanations

Section 1 — Linear Equations and Inequalities

Q1 — Basic — Answer: D

Topic: A linear equation with brackets on both sides

Expand both sides first:

$$10x - 15 = 3x + 12 + 7 = 3x + 19$$

Gather the x terms on the left and the numbers on the right:

$$7x = 34 \qquad x = \frac{34}{7}$$

An awkward fraction is not a sign of an error. Check by substituting: the left gives $5\left(\frac{68}{7} - 3\right) = 5 \times \frac{47}{7} = \frac{235}{7}$, and the right gives $\frac{102}{7} + 19 = \frac{235}{7}$. Correct.

Why each wrong option is wrong:

- **A,** $\frac{4}{7}$ — multiplied the 5 by $2x$ but not by the -3.
- **B,** 2 — dropped the $+7$ on the right.
- **C,** $\frac{20}{7}$ — expanded $3(x + 4)$ as $3x + 4$, multiplying only the first term.

> **Takeaway:** the bracket multiplies **everything** inside it. Expand fully on both sides before you gather anything.

Q2 — Medium — Answer: F

Topic: An inequality that needs the sign reversed

Subtract 7 from both sides — this does not affect the direction:

$$-4x \geq 12$$

Now divide by -4. **Because -4 is negative, the sign turns round**:

$$x \leq -3$$

Check with a value. Take $x = -5$, which satisfies $x \leq -3$: $-4(-5) + 7 = 27$, and $27 \geq 19$. Correct. Now take $x = 0$, which does not: $7 \geq 19$ is false. Correct again.

Why each wrong option is wrong:

- **G,** $x \geq -3$ — divided correctly but left the sign pointing the same way.
- **H,** $x \leq 3$ — flipped the sign but lost the minus on the 3.
- **J,** $x \geq 3$ — did neither.

> **Takeaway:** dividing an inequality by a negative reverses it. If you are ever unsure, **test one number** from your answer — it settles the question in five seconds.

Q3 — Medium — Answer: B

Topic: An equation with no solution

Both sides are of the form $(\text{number})x + (\text{number})$.

"No solution" means the x terms cancel — leaving no x to solve for — while the constants disagree. The x terms cancel exactly when their coefficients match:

$$k = 4$$

Then the equation becomes $4x + 9 = 4x - 3$. Subtract $4x$ from both sides:

$$9 = -3$$

which is false whatever x is. So there is no solution.

Contrast with infinitely many solutions, which the ACT also asks: that needs the coefficients to match **and** the constants to match too. Same first condition, opposite conclusion — the constants decide which one you get.

Why each wrong option is wrong:

- **A,** -4 — right size, wrong sign; $k = -4$ gives $8x = -12$, which has one solution.
- **C,** 9 and **D,** -3 — matched a constant rather than the coefficient.

> **Takeaway:** no solution means the x terms cancel and the numbers do not. Infinitely many means both cancel. Check the constants to tell them apart.

Q4 — Medium — Answer: J

Topic: A three-part inequality

Treat it as one chain and do the same thing to all three parts.

Subtract 4 everywhere:

$$-9 < 3x \leq 9$$

Divide by 3 everywhere — 3 is positive, so nothing flips:

$$-3 < x \leq 3$$

The **kind** of each end matters as much as its value. The left was strict ($<$) and stays strict; the right allowed equality ($\leq$) and still does.

Why each wrong option is wrong:

- **F,** $-3 \leq x \leq 3$, **H,** $-3 \leq x < 3$ and **G,** $-3 < x < 3$ — all have the right numbers and the wrong ends. Copy each inequality sign down unchanged as you go; that is the whole difficulty.

> **Takeaway:** in a three-part inequality, operate on all three parts together and carry each sign down exactly as it was.

Section 2 — Systems of Equations

Q5 — Basic — Answer: B

Topic: A system solved by elimination

Add the two equations. The $+2y$ and $-2y$ cancel:

$$(3x + 5x) + (2y - 2y) = 19 + 13$$

$$8x = 32 \qquad x = 4$$

Check by finding y too: $3(4) + 2y = 19$ gives $2y = 7$, so $y = 3.5$. Put both into the second equation: $5(4) - 2(3.5) = 20 - 7 = 13$. Correct.

Why each wrong option is wrong:

- **A,** $\frac{7}{2}$ — solved correctly and gave y. Always check which letter the question asked for; this is the most common way to lose a mark you have earned.
- **C,** 32 — added the two right-hand sides to get 32 and never divided by 8.
- **D,** 8 — added the coefficients of x and stopped.

> **Takeaway:** when the coefficients of one variable are equal and opposite, add. When they are equal and the same, subtract. Then answer for the letter asked for.

Q6 — Medium — Answer: F

Topic: A system needing both equations scaled

To remove x, make its coefficients match. Multiply the first equation by 3 and the second by 2:

$$6x + 15y = 72$$
$$6x - 4y = -4$$

Now subtract the second from the first:

$$19y = 76 \qquad y = 4$$

Check: from the first equation $2x + 20 = 24$, so $x = 2$; and $3(2) - 2(4) = 6 - 8 = -2$. Correct.

Why each wrong option is wrong:

- **H,** 2 — gave x, which is 2.
- **G,** 76 — scaled the equations and then never combined them.
- **J,** -2 — copied the -2 from the right of the second equation.

> **Takeaway:** to eliminate a variable, multiply each equation by the *other's* coefficient. Then subtract if the signs match and add if they do not.

Q7 — Hard — Answer: B

Topic: A system with no solution

Parallel lines have the same slope. Rearrange each into $y = mx + c$ form.

From the first: $-2y = -6x + 10$, so $y = 3x - 5$, slope 3.

From the second: $ky = -9x + 3$, so $y = -\frac{9}{k}x + \frac{3}{k}$, slope $-\frac{9}{k}$.

Set the slopes equal:

$$-\frac{9}{k} = 3 \qquad \Longrightarrow \qquad k = -3$$

Check that the lines are not identical, which would give *infinitely many* solutions instead: with $k = -3$ the second equation is $9x - 3y = 3$, or $y = 3x - 1$. Same slope, different intercept (-1 against -5), so they are parallel and distinct. No solution. Correct.

Why each wrong option is wrong:

- **A,** 3 — got the size right and the sign wrong.
- **C,** $-\frac{4}{3}$ — divided the wrong pair of coefficients.

- **D,** -2 — took a coefficient from the first equation.

> **Takeaway:** no solution means equal slopes and different intercepts. Always check the intercept too — equal slopes *and* equal intercepts is the opposite answer, infinitely many.

Section 3 — Expanding and Factoring

Q8 — Basic — Answer: J

Topic: Squaring a binomial

$(2x-3)^2$ means $(2x-3)(2x-3)$. Multiply every term by every term:

$$2x \cdot 2x = 4x^2 \qquad 2x \cdot (-3) = -6x$$
$$-3 \cdot 2x = -6x \qquad -3 \cdot (-3) = +9$$

There are **two** middle terms, both $-6x$, so together they make $-12x$:

$$(2x-3)^2 = 4x^2 - 12x + 9$$

Check at $x = 1$: the original is $(-1)^2 = 1$, and $4 - 12 + 9 = 1$. Correct.

Why each wrong option is wrong:

- **F,** $4x^2 + 9$ — squared each term separately. There is always a middle term.
- **G,** $4x^2 - 9$ — used the difference-of-squares pattern, which is for $(a+b)(a-b)$, not for a square.
- **H,** $4x^2 - 6x + 9$ — found one middle term instead of two.

> **Takeaway:** $(a \pm b)^2 = a^2 \pm 2ab + b^2$. Square the **whole** first term, coefficient included, and double the middle.

Q9 — Medium — Answer: C

Topic: Factoring with a leading coefficient

With a leading coefficient, the two numbers must multiply to $2 \times (-3) = -6$ and add to $+5$. That pair is $+6$ and -1.

Split the middle term using them:

$$2x^2 + 6x - x - 3$$

Factor in pairs:

$$2x(x+3) - 1(x+3) = (x+3)(2x-1)$$

So the factors are $x+3$ and $2x-1$. Of the options, $2x-1$ is there.

Check by expanding: $(x+3)(2x-1) = 2x^2 - x + 6x - 3 = 2x^2 + 5x - 3$. Correct.

Why each wrong option is wrong:

- **A,** $x-3$ — used factors of 3 alone, ignoring the leading 2.
- **B,** $2x+1$ — right numbers with the signs swapped; $(x-3)(2x+1)$ expands to $2x^2 - 5x - 3$, with the middle sign wrong.
- **D,** $x+1$ — not a factor at all.

> **Takeaway:** with a leading coefficient a, look for two numbers multiplying to $a \times c$ and adding to b, then split the middle term and factor in pairs.

Q10 — Medium — Answer: F

Topic: Recognising the difference of two squares

$9x^2 - 49$ is a **difference of two squares**: $9x^2 = (3x)^2$ and $49 = 7^2$, with a minus between them and nothing else.

$$a^2 - b^2 = (a-b)(a+b)$$

with $a = 3x$ and $b = 7$:

$$9x^2 - 49 = (3x-7)(3x+7)$$

Check by expanding: $9x^2 + 21x - 21x - 49$, and the middle terms cancel — which is exactly why the pattern has no middle term.

Why each wrong option is wrong:

- **G,** $(3x-7)^2$ — $(3x-7)^2$ expands to $9x^2 - 42x + 49$, which has a middle term and the wrong constant sign.
- **H,** $(x+7)\,(9x-7)$ and **J,** $(3x-49)\,(3x+1)$ — took a square root of only one part.

> **Takeaway:** two perfect squares with a minus between them and no middle term factors as $(a-b)(a+b)$. Take the square root of **both** parts.

Section 4 — Quadratic Equations

Q11 — Medium — Answer: D

Topic: Solving a quadratic by factoring

Find two numbers multiplying to -14 and adding to -5. They are -7 and $+2$:

$$x^2 - 5x - 14 = (x - 7)(x + 2) = 0$$

A product is zero when one of its factors is zero, so $x = 7$ or $x = -2$.

Time cannot be negative, so the answer is $x = 7$.

Why each wrong option is wrong:

- **A,** -2 — gave the negative root, which the context rules out. This is why the question says the solution must be positive.
- **B,** 14 and **C,** 5 — read numbers straight off the equation.

> **Takeaway:** factor, set each bracket to zero, then **check the context**. A length, a time or a count cannot be negative, and the ACT relies on you noticing.

Q12 — Hard — Answer: J

Topic: The discriminant and exactly one solution

Exactly one real solution means the discriminant is zero:

$$b^2 - 4ac = 0$$

Here $a = 1$, $b = k$ and $c = 16$:

$$k^2 - 4(1)(16) = 0$$
$$k^2 = 64$$
$$k = \pm 8$$

The question says k is positive, so $k = 8$.

Check: $x^2 + 8x + 16 = (x + 4)^2$, which is zero only at $x = -4$. Exactly one solution. Correct.

Why each wrong option is wrong:

- **F,** 4 — took the square root of 16 rather than of $4ac$.
- **G,** 16 — copied the constant.
- **H,** 64 — computed $4ac = 64$ and stopped before the square root.

> **Takeaway:** one solution means $b^2 = 4ac$. And when a squared quantity is solved, there are two answers — read the question to see which sign it wants.

Q13 — Medium — Answer: B

Topic: A quadratic solved by taking roots

Take the square root of both sides, remembering **both** signs:

$$x - 3 = \pm 5$$

That is two separate equations:

$$x - 3 = 5 \Rightarrow x = 8 \qquad x - 3 = -5 \Rightarrow x = -2$$

Check both: $(8-3)^2 = 25$ and $(-2-3)^2 = (-5)^2 = 25$. Both correct.

Why each wrong option is wrong:

- **A,** $(-8,\ 2)$ — subtracted the 3 rather than adding it when isolating x.
- **C,** $(5,\ 5)$ — used only $+5$, giving one solution twice. A squared equation almost always has two.
- **D,** $(2,\ 8)$ — got $+8$ right and slipped the sign on the other case.

> **Takeaway:** taking a square root gives $\pm$. Write out both equations separately — trying to hold both in your head is where the sign errors come from.

Section 5 — Radical, Rational and Exponential Equations

Q14 — Hard — Answer: J

Topic: A radical equation with an extraneous solution

Square both sides:

$$x + 6 = x^2$$

Bring everything to one side and factor:

$$x^2 - x - 6 = 0 \quad \Longrightarrow \quad (x-3)(x+2) = 0$$

So the squared equation gives $x = 3$ and $x = -2$. **Both must now be tested in the original**, because squaring turns a false statement into a true one: $-2 \neq 2$, but $(-2)^2 = 2^2$.

At $x = 3$: $\sqrt{9} = 3$. ✓

At $x = -2$: $\sqrt{4} = 2$, and $2 \neq -2$. ✗

The square-root symbol means the non-negative root, so it can never equal a negative number. $x = -2$ is **extraneous** — manufactured by the squaring.

Why each wrong option is wrong:

- **F,** -2 — kept the extraneous root without testing it.
- **G,** 1 — added the two roots of the squared equation.
- **H,** 6 — doubled the right-hand side instead of squaring the left.

> **Takeaway:** after squaring, every solution is only a **candidate**. Substitute each back into the original equation. A square root is never negative.

Q15 — Medium — Answer: D

Topic: An exponential equation with matched bases

Both 9 and 27 are powers of 3:

$$9 = 3^2 \qquad 27 = 3^3$$

Rewrite both sides:

$$(3^2)^x = (3^3)^4$$
$$3^{2x} = 3^{12}$$

With the same base, the exponents must be equal:

$$2x = 12 \qquad x = 6$$

Check: $9^6 = 531441$ and $27^4 = 531441$. Correct.

Why each wrong option is wrong:

- **B,** 4 — copied the 4 from the right-hand side.
- **C,** 12 — reached $2x = 12$ and gave 12 instead of dividing.
- **A,** $\frac{4}{3}$ — divided the exponents without converting to a common base.

> **Takeaway:** rewrite both sides as powers of the **same base**, then set the exponents equal. Look for the small base hiding inside both numbers — usually 2 or 3.

Q16 — Medium — Answer: H

Topic: An equation with the variable in a denominator

Gather the fractions on one side and the numbers on the other:

$$\frac{12}{x} - \frac{8}{x} = 3 - 1$$
$$\frac{4}{x} = 2$$

Multiply both sides by x and divide by 2:

$$4 = 2x \qquad x = 2$$

Check in the original: $\frac{12}{2} + 1 = 7$ and $\frac{8}{2} + 3 = 7$. Correct. And $x = 2$ does not make any denominator zero, so it is allowed.

Why each wrong option is wrong:

- **F,** 4 — subtracted the numerators and stopped, ignoring the constants.
- **G,** -2 — sign slip while gathering terms.
- **J,** 20 — added the numerators.

> **Takeaway:** with the same denominator on both sides, combine the fractions first — it is quicker than multiplying everything out. Then check the answer does not make a denominator zero.

Section 6 — Rearranging a Formula

Q17 — Medium — Answer: B

Topic: Solving a formula for one of its variables

Isolate the term containing w:

$$P - 2l = 2w$$

Divide both sides by 2:

$$w = \frac{P - 2l}{2}$$

Test it with numbers, which is the only real check available. Take $l = 3$ and $w = 4$; then $P = 2(3) + 2(4) = 14$. Put $P = 14$ and $l = 3$ into the answer: $\frac{14-6}{2} = 4$. Correct.

Why each wrong option is wrong:

- **A,** $\frac{P}{2} - 2l$ — halved P before subtracting, so the $2l$ never got halved.
- **C,** $\frac{P}{2} - \frac{l}{2}$ — subtracted l rather than $2l$.
- **D,** $P - 2l$ — subtracted correctly and never divided.

> **Takeaway:** isolate the term with your letter in it, then divide by everything multiplying it. **Test with easy numbers** — it catches almost every rearranging error in ten seconds.

Q18 — Hard — Answer: G

Topic: Rearranging when the variable appears inside a fraction

Multiply both sides by t to bring it up out of the denominator:

$$Ft = mv$$

Now divide by F:

$$t = \frac{mv}{F}$$

Test with numbers: let $m = 2$, $v = 3$, $t = 6$. Then $F = \frac{6}{6} = 1$, and the answer gives $t = \frac{2 \times 3}{1} = 6$. Correct.

Why each wrong option is wrong:

- **F,** Fmv — multiplied by F instead of dividing.
- **H,** $\frac{F}{mv}$ — the reciprocal of the right answer, which is what you get by dividing in the wrong direction.
- **J,** $\frac{Fm}{v}$ — moved m but not v.

> **Takeaway:** to free a variable from a denominator, multiply both sides by it first. Then test with three small numbers; an inverted answer shows up immediately.

Q19 — Hard — Answer: D

Topic: A radical equation with a check that matters

Square both sides:

$$2x + 7 = (x - 4)^2 = x^2 - 8x + 16$$

$$x^2 - 10x + 9 = 0 \quad \Longrightarrow \quad (x - 9)(x - 1) = 0$$

Test both. At $x = 9$: $\sqrt{25} = 5$ and $9 - 4 = 5$. ✓ At $x = 1$: $\sqrt{9} = 3$ but $1 - 4 = -3$. ✗

A square root cannot be negative, so $x = 1$ is extraneous.

Why each wrong option is wrong:

- **B,** 1 — kept the extraneous root without testing it.
- **C,** 10 — added the two roots of the squared equation.
- **A,** -1 — a sign slip expanding $(x-4)^2$.

> **Takeaway:** every solution after squaring is a candidate until substituted back into the **original** equation.

Q20 — Medium — Answer: H

Topic: A rational equation

Gather fractions on one side and numbers on the other:

$$\frac{20}{x} - \frac{8}{x} = 7 - 3 \quad \Longrightarrow \quad \frac{12}{x} = 4$$

$$x = 3$$

Check: $\frac{20}{3} + 3 = \frac{29}{3}$ and $\frac{8}{3} + 7 = \frac{29}{3}$. ✓

Why each wrong option is wrong:

- **F,** 12 and **G,** 4 — did one half of the gathering and stopped.
- **J,** 28 — added the numerators.

> **Takeaway:** with a shared denominator, combine the fractions before doing anything else, and check the answer does not make the bottom zero.

Q21 — Medium — Answer: C

Topic: An exponential relationship

$$500 \times 1.2 \times 1.2 = 500 \times 1.44 = 720$$

The second hour's growth is calculated on 600, not on 500 — which is why the answer exceeds $500 + 100 + 100$.

Why each wrong option is wrong:

- **A,** 600 — stopped after one hour.
- **B,** 700 — added 20% of the **original** twice, which is simple growth, not compound.
- **D,** 620 — grew the wrong base.

> **Takeaway:** compound growth multiplies by $(1 + r)$ once per period. Each period grows from the **new** amount.

Q22 — Hard — Answer: G

Topic: Exponential decay to a target

"Decays to 85%" states the multiplier directly: 0.85.

$$M = 400(0.85)^h$$

Check one hour: $400 \times 0.85 = 340$, a loss of 60, which is 15% of 400.

Why each wrong option is wrong:

- **F,** $400(0.15)^h$ — used 0.15, which would leave only 15% each hour.
- **H,** $400 - 0.15h$ — linear decay, losing a fixed 0.15 grams per hour.
- **J,** $400(1.15)^h$ — a multiplier above 1 grows the sample.

> **Takeaway:** the multiplier is what **remains**. Decay is below 1, growth above.

Q23 — Medium — Answer: B

Topic: An equation with the variable in a denominator

Multiply both sides by $(x - 2)$:

$$6 = 3(x - 2) = 3x - 6$$

$$3x = 12 \qquad x = 4$$

Check: $\frac{6}{4-2} = \frac{6}{2} = 3$. ✓ And $x = 4$ does not make the denominator zero, so it is allowed.

Why each wrong option is wrong:

- **A,** $2 - x = 2$ is precisely the value that is **not** allowed: it makes the denominator zero and the expression undefined.
- **C,** 20 — multiplied 6 by 3 rather than dividing.
- **D,** -4 — a sign slip.

> **Takeaway:** clear the denominator by multiplying, solve, then confirm the answer does not make any denominator zero.

Q24 — Medium — Answer: G

Topic: A quadratic equation solved by factoring

Find two numbers multiplying to -40 and adding to -3. They are -8 and $+5$:

$$x^2 - 3x - 40 = (x - 8)(x + 5) = 0$$

A product is zero when one factor is zero, so $x = 8$ or $x = -5$. A length cannot be negative, so $x = 8$.

Why each wrong option is wrong:

- **F,** -5 — gave the root the context rules out.
- **H,** 40 and **J,** 3 — read numbers straight off the equation without factoring.

> **Takeaway:** factor, set each bracket to zero, then check which root the situation allows.

Q25 — Hard — Answer: C

Topic: A quadratic equation needing the formula

With $a = 3$, $b = -5$, $c = -2$:

$$x = \frac{5 \pm \sqrt{25 - 4(3)(-2)}}{6} = \frac{5 \pm \sqrt{49}}{6} = \frac{5 \pm 7}{6}$$

$$x = 2 \qquad \text{or} \qquad x = -\tfrac{1}{3}$$

Watch the $-4ac$ term: c is negative, so $-4(3)(-2) = +24$, not -24. That double negative is where the formula usually goes wrong.

Why each wrong option is wrong:

- **A,** $-\frac{1}{3}$ — the smaller root; the question asked for the larger.
- **B,** $\frac{5}{6}$ — computed $\frac{5}{6}$, forgetting the $\pm\sqrt{\ }$ part entirely.
- **D,** 3 — read the leading coefficient off the equation.

> **Takeaway:** substitute a, b and c **with their signs**, and expect $-4ac$ to be positive whenever c is negative.

Q26 — Medium — Answer: H

Topic: A quadratic equation with no middle term

Move the constant across and divide by 4:

$$4x^2 = 49 \qquad x^2 = \frac{49}{4}$$

Take the square root of both sides, remembering **both** signs:

$$x = \pm\frac{7}{2}$$

Check: $4 \times \frac{49}{4} = 49$. ✓

Why each wrong option is wrong:

- **F,** $\left(-\frac{49}{4},\ \frac{49}{4}\right)$ — divided by 4 and stopped before the square root.
- **G,** $(-7,\ 7)$ — rooted the 49 and ignored the 4.
- **J,** $\left(-\frac{49}{2},\ \frac{49}{2}\right)$ — halved 49 instead of rooting.

> **Takeaway:** no middle term means take square roots. Isolate x^2 first, and never lose the $\pm$.

Q27 — Hard — Answer: D

Topic: A radical equation squared twice over

One squaring is not enough here, because a root sign survives it:

$$x + 5 = \left(\sqrt{x} + 1\right)^2 = x + 2\sqrt{x} + 1$$

The x terms cancel, which leaves the remaining root on its own:

$$4 = 2\sqrt{x} \quad \Longrightarrow \quad \sqrt{x} = 2$$

Square a second time: $x = 4$.

Check in the **original**: $\sqrt{4+5} = 3$ and $\sqrt{4} + 1 = 3$. ✓

Why each wrong option is wrong:

- **A,** 2 — found $\sqrt{x} = 2$ and stopped before the second squaring.
- **B,** 8 — squared $2\sqrt{x}$ as $2x$ instead of $4x$, so $16 = 2x$.
- **C,** 16 — dropped the 2 and read $\sqrt{x} = 4$.

> **Takeaway:** $(\sqrt{x} + 1)^2$ is $x + 2\sqrt{x} + 1$, not $x + 1$. The middle term keeps a root alive, so isolate it and square again, then check the answer in the original equation.

Q28 — Medium — Answer: F

Topic: A rational equation with two denominators

Cross-multiply:

$$3(x+6) = 12x$$

$$3x + 18 = 12x$$

$$18 = 9x \qquad x = 2$$

Check: $\frac{3}{2} = 1.5$ and $\frac{12}{8} = 1.5$. ✓ Neither denominator is zero at $x = 2$, so it is allowed.

Why each wrong option is wrong:

- **G,** 6 — copied the 6 out of the denominator.
- **H,** -6 — $x = -6$ makes the second denominator zero, which is exactly the value the equation excludes.
- **J,** 18 — added the two numbers in the question.

> **Takeaway:** cross-multiply, solve, then confirm the answer does not make any denominator zero.

Q29 — Medium — Answer: D

Topic: A rational equation that becomes quadratic

Multiply every term by x:

$$x^2 + 6 = 5x \quad \Longrightarrow \quad x^2 - 5x + 6 = 0$$
$$(x-2)(x-3) = 0 \quad \Longrightarrow \quad x = 2 \text{ or } x = 3$$

The smaller is 2. Check: $2 + \frac{6}{2} = 5$. ✓

Why each wrong option is wrong:

- **B,** 3 — the larger of the two solutions.
- **C,** 5 and **A,** 6 — copied numbers from the equation.

> **Takeaway:** clearing a denominator that contains x usually produces a quadratic. Expect **two** answers and read which one is wanted.

Q30 — Medium — Answer: G

Topic: An exponential relationship written as a percentage

A 6% rise multiplies by 1.06 — the whole original, plus another 6% of it:

$$V = 2000(1.06)^n$$

Check one year: $2000 \times 1.06 = 2120$, which is $2000 + 120$, and 120 is 6% of 2000. ✓

Why each wrong option is wrong:

- **F,** $2000(0.06)^n$ — 0.06 would leave 6% of the money after a year, a 94% loss.
- **H,** $2000 + 0.06n$ — linear growth, adding 6 cents a year.
- **J,** $2000(6)^n$ — multiplies the money sixfold every year.

> **Takeaway:** a $p\%$ rise per period is a multiplier of $1 + \frac{p}{100}$. Test one period against plain arithmetic.

Q31 — Hard — Answer: C

Topic: An exponential equation with matched bases

Both 8 and 32 are powers of 2:

$$8 = 2^3 \qquad 32 = 2^5$$
$$(2^3)^{x+1} = 2^5 \quad \Longrightarrow \quad 2^{3(x+1)} = 2^5$$

Same base, so the exponents match:

$$3(x+1) = 5 \quad \Longrightarrow \quad 3x = 2 \quad \Longrightarrow \quad x = \tfrac{2}{3}$$

Why each wrong option is wrong:

- **A,** 4 — divided 32 by 8, which is not how exponents combine.
- **B,** $\frac{5}{3}$ — solved $3x = 5$, forgetting the $+1$ inside the bracket.
- **D,** 2 — a whole-number guess; $8^3 = 512$, not 32.

> **Takeaway:** find the small base hiding inside both numbers — usually 2 or 3 — then set the exponents equal. Do not forget what is inside the bracket.

Q32 — Medium — Answer: H

Topic: Exponential decay over several periods

Keeping 80% each hour means multiplying by 0.8 three times:

$$500 \times 0.8 \times 0.8 \times 0.8 = 500 \times 0.512 = 256$$

Note it is **not** $500 - 300$. Each hour's loss is 20% of what is left, which shrinks: 100, then 80, then 64.

Why each wrong option is wrong:

- **F,** 200 — took 20% of the **original** three times, which is simple decay, not compound.
- **G,** 400 — stopped after one hour.
- **J,** 140 — subtracted a fixed amount three times.

> **Takeaway:** decay multiplies by $1 - \frac{p}{100}$ once per period. Each period works from the **new** amount, so the loss gets smaller every time.

Q33 — Medium — Answer: A

Topic: Rearranging a formula with a fraction

Multiply both sides by 2 to clear the fraction:

$$2A = bh$$

Then divide by b:

$$h = \frac{2A}{b}$$

Test with numbers: $b = 4$, $h = 3$ gives $A = 6$; and $\frac{2(6)}{4} = 3$. ✓

Why each wrong option is wrong:

- **B,** $\frac{A}{2b}$ — divided by 2 rather than multiplying, so the fraction was never cleared.
- **C,** $\frac{Ab}{2}$ and **D,** $2Ab$ — multiplied by b instead of dividing.

> **Takeaway:** clear the fraction first, then divide by everything multiplying your letter. Test with three easy numbers — an inverted answer shows at once.

Q34 — Hard — Answer: G

Topic: Rearranging a formula where the variable appears twice

Only the second term contains h, so move the first one across:

$$T - 2\pi r^2 = 2\pi rh$$

Now divide by everything multiplying h, which is $2\pi r$:

$$h = \frac{S - 2\pi r^2}{2\pi r}$$

Why each wrong option is wrong:

- **F,** $\frac{T-2\pi r^2}{2\pi}$ — divided by 2π and lost the r.
- **H,** $\frac{T}{2\pi r} - r^2$ — divided before subtracting, so the first term was not divided too.
- **J,** $\frac{T}{2\pi r}$ — dropped the $2\pi r^2$ altogether.

> **Takeaway:** subtract **before** you divide, and divide the whole side. A term that does not contain your letter must be moved across first.

Q35 — Medium — Answer: A

Topic: Rearranging to make a squared variable the subject

Divide both sides by π:

$$\frac{A}{\pi} = r^2$$

Then take the square root of the **whole** side:

$$r = \sqrt{\frac{A}{\pi}}$$

A radius cannot be negative, so only the positive root is kept.

Why each wrong option is wrong:

- **B,** $\frac{A}{\pi}$ — isolated r^2 and stopped.
- **C,** $\frac{\sqrt{A}}{\pi}$ — rooted the A but not the π. The root applies to everything under it.
- **D,** $\frac{A}{2\pi}$ — rearranged the circumference formula instead.

> **Takeaway:** isolate the squared term, then root the **entire** side. Keep only the positive root when the letter is a physical length.

Q36 — Medium — Answer: G

Topic: A radical equation with a whole-number solution

Square both sides:

$$3x + 1 = 16 \quad \Longrightarrow \quad 3x = 15 \quad \Longrightarrow \quad x = 5$$

Check: $\sqrt{15+1} = \sqrt{16} = 4.$ ✓

Nothing extraneous can appear here, because the right-hand side is already positive — the trouble only starts when it contains a variable that might be negative.

Why each wrong option is wrong:

- **F,** 1 — subtracted the 1 before squaring, so the 4 was never squared.
- **H,** 16 — squared to 16 and stopped, ignoring the rest of the equation.
- **J,** $\frac{4}{3}$ — divided 4 by 3 without squaring at all.

> **Takeaway:** square first to clear the root, **then** solve. Substitute back whenever the other side contains a variable.

Q37 — Hard — Answer: A

Topic: A rational equation with an excluded value

Multiply every term by $(x - 3)$ to clear the fractions:

$$x = 5 + 2(x - 3)$$

Expand the bracket:

$$x = 5 + 2x - 6 = 2x - 1$$

Subtract x from both sides:

$$0 = x - 1 \implies x = 1$$

Check in the original: $\dfrac{1}{-2} = -\frac{1}{2}$, and $\dfrac{5}{-2} + 2 = -2.5 + 2 = -\frac{1}{2}$. ✓

And $x = 1$ does not make the denominator zero, so it is allowed.

Why each wrong option is wrong:

- **B,** 3 — $x = 3$ makes both denominators zero. It is the one value the equation rules out before any working begins.
- **C,** 5 — copied the numerator.
- **D,** 11 — a sign slip while gathering the x terms.

> **Takeaway:** combine fractions over a shared denominator, then solve — and note the excluded value **before** you start, so you can reject it on sight.

Q38 — Medium — Answer: F

Topic: An inequality with brackets

Expand, remembering the -3 multiplies **both** terms:

$$-3x + 12 > 2x + 1$$

Gather x on the side that keeps it positive — subtract $-3x$, that is, add $3x$:

$$12 > 5x + 1 \implies 11 > 5x \implies x < \tfrac{11}{5}$$

Gathering this way avoids dividing by a negative, so no sign flip is needed at all.

Why each wrong option is wrong:

- **G,** $x > \frac{11}{5}$ — right boundary, wrong direction.
- **H,** $x < -\frac{13}{5}$ — expanded $-3(x-4)$ as $-3x - 12$, keeping the sign of the 4.
- **J,** $x > -\frac{13}{5}$ — both errors together.

> **Takeaway:** expand carefully through a negative, then gather x on whichever side leaves it positive. That removes the sign-flip rule entirely.

Q39 — Medium — Answer: C

Topic: A system with infinitely many solutions

The second equation's left side is exactly twice the first's:

$$6x + 4y = 2(3x + 2y)$$

For the two to be the same line, the right side must be doubled too:

$$c = 2 \times 7 = 14$$

Check: $6x + 4y = 14$ divides by 2 to give $3x + 2y = 7$ — the same equation. Every solution of one is a solution of the other.

Contrast with no solution: any other value of c makes the lines parallel and distinct, and they never meet.

Why each wrong option is wrong:

- **A,** 7 — leaves the lines parallel and distinct: no solution at all.
- **B,** 2 — gave the scale factor rather than applying it.
- **D,** 21 — used a factor of 3.

> **Takeaway:** infinitely many solutions means one equation is a **multiple** of the other, right-hand side included. Find the factor from the coefficients and apply it.

Q40 — Medium — Answer: J

Topic: A system solved for an expression rather than a variable

$$x^2 - y^2 = (x + y)(x - y) = 9 \times 3 = 27$$

Solving for x and y works too — $x = 6$, $y = 3$, and $36 - 9 = 27$ — but the factorisation gets there in one line.

Why each wrong option is wrong:

- **F,** 12 and **G,** 6 — combined the two given values with the wrong operation.
- **H,** 36 — found $x = 6$ and squared it, forgetting the $-y^2$.

> **Takeaway:** when a question asks for an **expression** rather than a variable, look for a factorisation that uses what you were given directly. The difference of two squares is the one the ACT reaches for most.

Q41 — Medium — Answer: A

Topic: Factoring out a common factor first

Take out the 3 first:

$$3x^2 - 27 = 3(x^2 - 9)$$

What is left is a difference of two squares:

$$3(x^2 - 9) = 3(x - 3)(x + 3)$$

Check by expanding: $3(x^2 - 9) = 3x^2 - 27$. ✓

Why each wrong option is wrong:

- **B,** $(3x - 9)(x + 3)$ — expands to the right thing and is **not wrong as an expression**, but it is not **fully** factored: $3x - 9$ still contains a common factor of 3. That is the whole point of the word "fully".
- **C,** $3(x - 3)^2$ — a perfect square would give a middle term, and this expression has none.
- **D,** $(x - 3)(x + 9)$ — expands to $x^2 + 6x - 27$.

> **Takeaway:** always look for a common factor **first**. It makes the numbers smaller and often exposes a pattern that was hidden.

Q42 — Hard — Answer: H

Topic: Completing the square

Move the constant across:

$$x^2 + 6x = 7$$

Halve the coefficient of x and square it: half of 6 is 3, and $3^2 = 9$. Add 9 to **both** sides:

$$x^2 + 6x + 9 = 16 \quad \Longrightarrow \quad (x + 3)^2 = 16$$

So $a = 3$ and $b = 16$.

Why each wrong option is wrong:

- **F,** 7 — copied the original constant.
- **G,** 9 — gave the 9 that was added, not the total on the right.
- **J,** 3 — gave a rather than b.

> **Takeaway:** halve the x coefficient, square it, add to both sides. Then read which letter the question wants — a and b are easy to confuse.

Q43 — Medium — Answer: A

Topic: The sum of a quadratic's solutions

For $ax^2 + bx + c = 0$ the two solutions add to $-\frac{b}{a}$:

$$-\frac{-10}{2} = 5$$

Check by solving: $2x^2 - 10x + 8 = 2(x-1)(x-4)$, so the roots are 1 and 4, which add to 5. ✓

The companion fact is worth learning at the same time: the roots **multiply** to $\frac{c}{a} = \frac{8}{2} = 4$, and $1 \times 4 = 4$. ✓

Why each wrong option is wrong:

- **B,** 10 — used b without dividing by a.
- **C,** 4 — gave the **product** of the roots.
- **D,** -5 — dropped the minus in $-\frac{b}{a}$; b is already negative, so the answer is positive.

> **Takeaway:** roots sum to $-\frac{b}{a}$ and multiply to $\frac{c}{a}$. Both are faster than solving when the question asks only for the sum or product.

Q44 — Medium — Answer: F

Topic: An exponential equation solved by inspection

Count the doublings: 2, 4, 8, 16, 32, 64, 128 — seven of them.

$$2^7 = 128 \quad \Longrightarrow \quad x = 7$$

Why each wrong option is wrong:

- **G,** 64 — divided 128 by 2. An exponent is not a quotient.
- **H,** 6 — $2^6 = 64$, one doubling short.
- **J,** 8 — $2^8 = 256$, one doubling too many.

> **Takeaway:** with small powers, counting the multiplications is faster than any rule. Knowing the powers of 2 up to 1024 pays for itself repeatedly.

Q45 — Hard — Answer: C

Topic: An inequality from a real constraint

Total weight is the van plus the crates:

$$1200 + 45n \leq 2100$$

Remove the van's own weight first — it is there whether or not any crates are loaded:

$$45n \leq 900 \quad \Longrightarrow \quad n \leq 20$$

A count cannot be negative, so $0 \leq n \leq 20$.

Why each wrong option is wrong:

- **A,** $0 \leq n \leq 46$ — divided 2100 by 45 without removing the van's weight, allowing 46 crates and over 3000 kg.
- **B,** $n \geq 20$ — reversed the inequality, permitting exactly the loads the bridge forbids.
- **D,** $0 \leq n \leq 73$ — added the van's weight instead of subtracting it.

> **Takeaway:** a fixed weight is not part of the rate. Subtract it **before** dividing, and check the answer against the original limit.

Q46 — Medium — Answer: H

Topic: Multiplying two binomials with coefficients

Four products:

$$3x \cdot 2x = 6x^2 \qquad 3x \cdot 5 = 15x$$
$$-2 \cdot 2x = -4x \qquad -2 \cdot 5 = -10$$

Collect the middle: $15x - 4x = 11x$.

$$(3x-2)(2x+5) = 6x^2 + 11x - 10$$

Check at $x = 1$: the original is $(1)(7) = 7$, and $6 + 11 - 10 = 7$. ✓

Why each wrong option is wrong:

- **F,** $6x^2 - 10$ — multiplied only the first terms and the last terms, missing both middle products.
- **G,** $6x^2 + 19x + 10$ — treated the -2 as $+2$, so both of its products came out positive.
- **J,** $6x^2 - 11x - 10$ — sign wrong on the middle term; $15x - 4x$ is positive, because the larger product is the positive one.

> **Takeaway:** four products, then collect the two middle terms. Check at $x = 1$ — it takes five seconds and catches nearly every slip.

Q47 — Hard — Answer: A

Topic: An equation with a variable on both sides and fractions

Multiply every term by 10, the lowest common denominator of 2 and 5:

$$5x + 30 = 2x + 60$$

$$3x = 30 \quad \Longrightarrow \quad x = 10$$

Check: $\frac{10}{2} + 3 = 8$ and $\frac{10}{5} + 6 = 8$. ✓

Why each wrong option is wrong:

- **B,** 3 — subtracted the constants and ignored the x terms.
- **C,** 30 — reached $3x = 30$ and multiplied rather than divided.
- **D,** $\frac{10}{3}$ — divided 10 by 3 instead of 30 by 3.

> **Takeaway:** clear all fractions in one move by multiplying **every** term by the lowest common denominator. Then it is an ordinary linear equation.

Q48 — Medium — Answer: G

Topic: A rational equation solved by cross-multiplying

Cross-multiply:

$$2(x+1) = 4(x-2)$$
$$2x + 2 = 4x - 8$$
$$10 = 2x \quad \Longrightarrow \quad x = 5$$

Check: $\frac{6}{4} = 1.5$ and $\frac{3}{2} = 1.5$. ✓

Why each wrong option is wrong:

- **F,** 3 — multiplied out only one side.
- **H,** -5 — a sign slip while gathering the x terms.
- **J,** 1 — copied a constant from a numerator.

> **Takeaway:** cross-multiplying handles two fractions in one move. Expand both brackets fully before gathering.

Chapter 4 — Functions

Functions are 17 to 20% of the test — eight questions — and they are the topic where notation does most of the damage. $f(3)$ does not mean f times 3. Once that is genuinely clear, most of this chapter follows.

A **function** is a machine: you put a number in, it gives one number out. $f(x) = 2x + 1$ says "double it and add one". So $f(3) = 7$. The letter in the brackets is just a label for whatever goes in — $f(\text{banana}) = 2 \times \text{banana} + 1$ is silly but correct, and it is the right mental picture.

Logarithms are here too. They are on the ACT and not on the SAT, and they appear roughly once or twice per paper.

Topics covered: function notation · working backwards from an output · composite functions · graphs and key features · transformations · domain · piecewise functions · logarithms · modelling with functions

---

Section 1 — Notation and Evaluation

To evaluate $f(3)$, replace **every** x in the definition with 3 — including the ones inside powers — and work it out. Brackets around a negative input are not optional: $(-2)^2 = 4$, but $-2^2 = -4$.

Q1 — Basic

A profit model is defined by $f(x) = 3x^2 - 4x + 1$, where x is a quantity that may be negative. What is the value of $f(-2)$?

A) -19

B) 5

C) -3

D) 21

Q2 — Medium

A conversion is carried out by the linear function f, defined by $f(x) = 5x - 8$. On one occasion the output was recorded as 27, but the input that produced it was not written down. If $f(a) = 27$, what is the value of a?

F) 127

G) 7

H) $\frac{19}{5}$

J) 35

Q3 — Medium

Two functions are defined by $f(x) = x - 4$ and $g(x) = x^2$. A calculation applies g to an input first and then applies f to the result. What is the value of $f(g(3))$?

A) 1

B) 5

C) -1

D) 13

Q4 — Hard

Two functions are defined by $f(x) = 2x + 1$ and $g(x) = x^2$. An engineer needs the positive input a for which $f(g(a)) = 33$. What is the value of a?

F) 16

G) 4

H) 8

J) 2

Section 2 — Graphs, Key Features and Transformations

Reading a graph is mostly vocabulary:

- an x**-intercept** is where the graph crosses the x-axis, so $y = 0$. It is also called a **zero** or a **root**.
- a y**-intercept** is where it crosses the y-axis, so $x = 0$.
- a **vertex** is the turning point of a parabola.

Transformations follow one rule that catches everybody: changes **inside** the brackets act on x and go the **opposite way** to what they look like; changes **outside** act on y and behave normally.

what you write	what happens
$f(x) + c$	up by c
$f(x) - c$	down by c
$f(x + c)$	**left** by c
$f(x - c)$	**right** by c
$-f(x)$	reflected in the x-axis
$a\,f(x)$	stretched vertically by a

Q5 — Medium

The graph of $y = f(x)$ records a measurement over time and passes through the point $(2, 5)$. A colleague redraws the whole graph as $y = f(x - 3)$. Through which point must the redrawn graph pass?

A) $(-1,\ 5)$

B) $(5,\ 5)$

C) $(2,\ 8)$

D) $(2,\ 2)$

Q6 — Medium

The graph of $y = f(x)$ has a minimum at the point $(1, -4)$. The whole model is then raised by 6 units to give $y = f(x) + 6$. What are the coordinates of the minimum of the new graph?

F) $(1,\ -10)$

G) $(1,\ 2)$

H) $(7,\ -4)$

J) $(-5,\ -4)$

Q7 — Hard

The graph of $y = f(x)$ is shifted 4 units to the right and 2 units down to give a new model. Which equation describes the new graph?

A) $y = f(x+4) + 2$

B) $y = f(x+4) - 2$

C) $y = f(x-4) + 2$

D) $y = f(x-4) - 2$

Q8 — Medium

A function is defined by $f(x) = \dfrac{x+3}{x-5}$. For which value of x is the function undefined?

F) -3

G) 0

H) 3

J) 5

Section 3 — Logarithms

A logarithm asks: **what power do I need?**

$$\log_b(y) = x \quad \text{means exactly} \quad b^x = y$$

Read $\log_2(8) = 3$ as "2 to the power 3 is 8". Converting between those two forms answers most ACT logarithm questions on its own.

The three rules worth knowing:

$$\log(mn) = \log m + \log n$$
$$\log\left(\frac{m}{n}\right) = \log m - \log n$$
$$\log(m^k) = k \log m$$

Multiplying inside becomes adding outside — which is what logarithms were invented for.

Q9 — Medium

An equation from a decibel model is $\log_3(x) = 4$, where the base of the logarithm is 3. What is the value of x?

A) 12

B) 64

C) 81

D) $\frac{4}{3}$

Q10 — Hard

An expression reduces to $\log_2(8) + \log_2(4)$, where both terms share the same base. What is the value of the expression?

F) 12

G) 32

H) 6

J) 5

Section 4 — Modelling with Functions

The ACT writes growth and decay in function notation, and the shape to recognise is:

$$f(x) = a \cdot b^x$$

- a is the **starting value**, what you have when $x = 0$
- b is the **multiplier per step**: above 1 is growth, below 1 is decay

A 15% rise per year means $b = 1.15$. A 12% fall means $b = 0.88$ — and note it is 0.88, not -0.12.

Q11 — Medium

A laboratory colony starts with 400 bacteria and grows by 15% during each hour, compounding from one hour to the next. The technician wants one function giving the population at any whole number of hours. Which function gives the population after h hours?

A) $P = 400(15)^h$

B) $P = 400(0.15)^h$

C) $P = 400 + 15h$

D) $P = 400(1.15)^h$

Q12 — Medium

A car bought new is modelled by $V = 24000(0.88)^t$, where V is its value in dollars and t is the number of years since purchase. The owner is trying to explain to a friend what the 0.88 means. Which statement is best supported by the model?

F) The car loses 12% of its value each year

G) The car loses \$88 each year

H) The car loses 88% of its value each year

J) The car gains 88% of its value each year

Q13 — Hard

A population of 800 organisms doubles every 5 years. The study runs for 20 years altogether. What is the population after 20 years?

A) 3200

B) 838860800

C) 12800

D) 8000

Q14 — Hard

A colony of organisms triples in number every day, and after 4 days of growth the count stands at 810. The researcher needs the size of the colony at the moment observation began. How many were there at the start?

F) 10

G) 270

H) 2430

J) 202

Q15 — Medium

Two functions are defined by $f(x) = 2x - 1$ and $g(x) = x + 5$. A process applies g first and then f. What is the value of $f(g(2))$?

A) 8

B) 13

C) 7

D) 3

Q16 — Hard

Two functions are defined by $f(x) = x^2$ and $g(x) = x - 3$. An expression for $f(g(x))$ is required, valid for every x. Which of the following is $f(g(x))$?

F) $x^2 - 3$

G) $x^2 - 9$

H) $x^2 + 6x + 9$

J) $x^2 - 6x + 9$

Q17 — Medium

A conversion is carried out by $f(x) = 3x + 4$, and the reverse conversion is needed. Which of the following is $f^{-1}(x)$?

A) $\frac{1}{3x+4}$

B) $\frac{x}{3} + \frac{4}{3}$

C) $3x - 4$

D) $\frac{x}{3} - \frac{4}{3}$

Q18 — Medium

A function is defined by $f(x) = 2x - 1$, and a process applies it twice in succession, which is written $f(f(3))$. What is the value of $f(f(3))$?

F) 5

G) 11

H) 6

J) 9

Q19 — Hard

A function is defined by $f(x) = \sqrt{x-5}$. Which of the following gives all values of x in the domain of f?

A) $x > 5$

B) $x \geq 5$

C) $x \leq 5$

D) $x \geq -5$

Q20 — Medium

A charge is defined by a piecewise function:

$$f(x) = \begin{cases} 2x+1 & \text{if } x < 3 \\ x^2 - 4 & \text{if } x \geq 3 \end{cases}$$

What is the value of $f(5)$?

F) 11

G) 25

H) 21

J) 1

Q21 — Medium

A delivery charge is defined by a piecewise function:

$$c(x) = \begin{cases} 5 + 2x & \text{if } x \leq 10 \\ 30 & \text{if } x > 10 \end{cases}$$

What is $c(10)$?

A) 30

B) 35

C) 25

D) 15

Q22 — Hard

A function is defined by $f(x) = x^2 + 3$ for all real x. Which of the following gives all values the function can output?

F) $y \geq 3$

G) $y > 3$

H) $y \leq 3$

J) $y \geq 0$

Q23 — Medium

A tolerance is modelled by $f(x) = |x - 4| + 1$. What is the minimum value of f?

A) 4

B) 1

C) 5

D) 0

Q24 — Medium

A sound-level calculation needs the value of $\log_2(64)$. What is $\log_2(64)$?

F) 32

G) 8

H) 6

J) 128

Q25 — Hard

A model requires the value of $\log_9(3)$. What is $\log_9(3)$?

A) $\frac{1}{3}$

B) 3

C) 2

D) $\frac{1}{2}$

Q26 — Medium

An expression reduces to $\log_5(50) - \log_5(2)$, where both terms share a base. What is the value?

F) 48

G) 2

H) 25

J) 100

Q27 — Medium

A taxi firm's total charge is modelled by $C(m) = 3.5 + 1.8m$ dollars for a journey of m miles. A passenger is quoted a fare of $21.50 and wants to know how far that takes them. How many miles is the journey?

A) 10

B) $\frac{215}{18}$

C) 18

D) $\frac{43}{7}$

Q28 — Hard

A quantity starts at 200 and **halves** every 4 days. Which function gives its value after d days?

F) $200(0.5)^d$

G) $200 - 25d$

H) $200(0.5)^{d/4}$

J) $200(4)^{d/2}$

Q29 — Medium

A population is modelled by $P(t) = 1200(1.03)^t$, where t is years since the study began. A report needs the population at the moment the study started. What was it?

A) 1236

B) $\frac{103}{100}$

C) 1200

D) 0

Q30 — Basic

A model uses $f(x) = 8x - 3$, and the input is $x = \dfrac{1}{4}$. What is $f\left(\dfrac{1}{4}\right)$?

F) -1

G) 2

H) $\frac{5}{4}$

J) 5

Q31 — Medium

A function f is given by the table below.

x	**1**	**2**	**3**	**4**
$f(x)$	5	8	11	14

What is the value of $f(3) - f(1)$?

A) 2

B) 16

C) 6

D) 3

Q32 — Medium

Two functions are defined by $f(x) = x + 2$ and $g(x) = 3x$. A process applies f first and then g. What is $g(f(4))$?

F) 14

G) 6

H) 12

J) 18

Q33 — Hard

Two functions are defined by $f(x) = 2x + 6$ and $g(x) = \dfrac{x-6}{2}$. A process applies f and then g. What is $g(f(5))$?

A) 5

B) 16

C) 10

D) 8

Q34 — Medium

A cost model is $f(x) = x^2 - 4x + 7$. What is the y-intercept?

F) 0

G) -4

H) 7

J) 3

Q35 — Medium

A function is defined by $f(x) = (x - 2)(x + 5)$. What is the **sum** of the zeros?

A) 7

B) 10

C) -3

D) 3

Q36 — Medium

A parabola is given by $f(x) = x^2 - 6x + 5$. For which values of x is the function increasing?

F) $x \leq 3$

G) $x \geq 3$

H) $x \geq 5$

J) $x \leq 5$

Q37 — Hard

A projectile's height in metres is $h(t) = -2t^2 + 12t + 5$, where t is seconds. What is the greatest height reached?

A) 23

B) 3

C) 5

D) 17

Q38 — Medium

The graph of $y = f(x)$ passes through the point $(4, 3)$. The whole graph is then stretched vertically by a factor of 2 to give $y = 2f(x)$. Through which point must the new graph pass?

F) $(8,\ 3)$

G) $(2,\ 3)$

H) $(4,\ 6)$

J) $(8,\ 6)$

Q39 — Hard

The graph of $y = f(x)$ passes through $(-3, 7)$. The graph is then redrawn as $y = f(-x)$. Through which point must the reflected graph pass?

A) $(3,\ 7)$

B) $(-3,\ -7)$

C) $(3,\ -7)$

D) $(7,\ -3)$

Q40 — Medium

The graph of $y = f(x)$ passes through $(2, 5)$. It is then redrawn as $y = f(x + 1) - 4$. Through which point must the new graph pass?

F) $(1,\ 1)$

G) $(3,\ 1)$

H) $(1,\ 9)$

J) $(3,\ 9)$

Q41 — Medium

A tariff is defined by

$$g(x) = \begin{cases} 3x & \text{if } x < 2 \\ x + 8 & \text{if } x \geq 2 \end{cases}$$

What is $g(-1)$?

A) -3

B) 7

C) 3

D) -9

Q42 — Hard

An equation from a sound model is $\log_4(x-3)=2$. What is the value of x?

F) 11

G) 8

H) 19

J) 5

Q43 — Medium

A quantity is 100 at time 0, 150 at time 1 and 225 at time 2. Which model fits, and what is the value at time 3?

A) Exponential, and 337.5

B) Linear, and 300

C) Exponential, and 300

D) Linear, and 275

Answers and Explanations

Section 1 — Notation and Evaluation

Q1 — Basic — Answer: D

Topic: Evaluating a function at a negative input

Replace every x with -2, keeping the brackets:

$$f(-2) = 3(-2)^2 - 4(-2) + 1$$

Take the power first:

$$(-2)^2 = 4 \quad \text{so} \quad 3 \times 4 = 12$$

Then the middle term. Subtracting a negative adds:

$$-4 \times (-2) = +8$$
$$12 + 8 + 1 = 21$$

Why each wrong option is wrong:

- **A,** -19 — lost the brackets, treating $3(-2)^2$ as -12.
- **B,** 5 — handled the square correctly but took $-4(-2)$ as -8.
- **C,** -3 — a sign slip in the middle term.

> **Takeaway:** put brackets round a negative input and keep them. The two places signs go wrong are the squared term and the term with a minus in front.

Q2 — Medium — Answer: G

Topic: Working backwards from an output

$f(a) = 27$ means the machine produced 27, so set the rule equal to 27 and solve for the input:

$$5a - 8 = 27$$
$$5a = 35$$
$$a = 7$$

Check forwards: $f(7) = 35 - 8 = 27$. Correct.

Why each wrong option is wrong:

- **F,** 127 — put 27 **into** the function instead of solving for the input. Going forwards when the question goes backwards is the standard error here.
- **H,** $\frac{19}{5}$ — subtracted the 8 rather than adding it.
- **J,** 35 — added the 8 correctly and never divided by 5.

> **Takeaway:** $f(a) = 27$ gives you the **output**. Set the rule equal to it and solve. Then substitute your answer forwards to check.

Q3 — Medium — Answer: B

Topic: A composite function

Work from the **inside out**. The inner bracket is $g(3)$:

$$g(3) = 3^2 = 9$$

Now that 9 is the input to f:

$$f(9) = 9 - 4 = 5$$

Why each wrong option is wrong:

- **A,** 1 — applied f first and then g: $f(3) = -1$, and $(-1)^2 = 1$. Order matters, and $f(g(x))$ is generally not $g(f(x))$.
- **C,** -1 — worked out $f(3)$ and stopped.
- **D,** 13 — added 4 instead of subtracting.

> **Takeaway:** in $f(g(x))$, g **goes first** — it is the one wrapped in the inner brackets. Work outwards from the innermost bracket, exactly as you would with arithmetic.

Q4 — Hard — Answer: G

Topic: A composite function with an unknown input

Undo the outer function first. Whatever $g(a)$ is, f turned it into 33:

$$2 \cdot g(a) + 1 = 33 \quad \Longrightarrow \quad 2 \cdot g(a) = 32 \quad \Longrightarrow \quad g(a) = 16$$

Now undo the inner one:

$$a^2 = 16 \qquad a = \pm 4$$

The question asks for the positive input, so $a = 4$.

Check forwards, which is always worth doing on a composite: $g(4) = 16$, and $f(16) = 32 + 1 = 33$. Correct.

Why each wrong option is wrong:

- **F,** 16 — gave $g(a) = 16$, the value halfway through the chain, rather than a itself. Stopping one function early is the standard error here.
- **J,** 2 — took the square root twice, $\sqrt{\sqrt{16}} = 2$.
- **H,** 8 — halved 16 instead of taking its square root.

> **Takeaway:** peel a composite from the **outside in** — undo f, then undo g — which is the reverse of the order you would use to evaluate it. Then run your answer forwards through both functions to check.

Section 2 — Graphs, Key Features and Transformations

Q5 — Medium — Answer: B

Topic: A horizontal shift

The change is **inside** the brackets, so it moves the graph horizontally — and inside-the-bracket changes go the **opposite** way to their sign. A minus 3 shifts the graph 3 units to the **right**.

$$(2, 5) \longrightarrow (2 + 3,\ 5) = (5, 5)$$

If the direction feels backwards, reason it through: the new graph at $x = 5$ computes $f(5 - 3) = f(2) = 5$. So the height that used to be at $x = 2$ now appears at $x = 5$ — three units to the right.

Why each wrong option is wrong:

- **A,** $(-1,\ 5)$ — moved left, the direction the minus sign suggests.
- **C,** $(2,\ 8)$ and **D,** $(2,\ 2)$ — changed the y-coordinate. A change inside the brackets never moves a graph vertically.

> **Takeaway:** inside the brackets means horizontal and **opposite**; outside means vertical and normal. When in doubt, substitute one x and see which output comes back.

Q6 — Medium — Answer: G

Topic: A vertical shift of a minimum

The $+6$ is **outside** the brackets, so it moves the graph vertically, upwards, by 6. The x-coordinate of the minimum does not change — the graph slides straight up.

$$(1, -4) \longrightarrow (1,\ -4+6) = (1, 2)$$

Why each wrong option is wrong:

- **H,** $(7,\ -4)$ and **J,** $(-5,\ -4)$ — moved horizontally. Outside the brackets is always vertical.
- **F,** $(1,\ -10)$ — moved down.

> **Takeaway:** outside the brackets, the change acts on the output, so only the y-coordinate moves — and it moves the way the sign says.

Q7 — Hard — Answer: D

Topic: A transformation described in words

Take the two shifts one at a time.

Right by 4 is a horizontal shift, so it goes inside the brackets, and inside changes act opposite to their sign. Right means **minus**:

$$f(x-4)$$

Down by 2 is vertical, so it goes outside, and outside changes behave normally. Down means **minus**:

$$f(x-4)-2$$

If the horizontal rule still feels wrong, test it on $f(x) = x^2$, whose lowest point is at $x = 0$. The graph of $(x-4)^2$ has its lowest point where $x - 4 = 0$, that is at $x = 4$ — four units to the right. Confirmed.

Why each wrong option is wrong:

- **B,** $y = f(x+4) - 2$ — wrote the right-shift as $+4$.
- **C,** $y = f(x-4) + 2$ — wrote the downward shift as $+2$.
- **A,** $y = f(x+4) + 2$ — both signs the wrong way round.

> **Takeaway:** horizontal shifts are **inside and opposite**; vertical shifts are **outside and obvious**. Do them one at a time, and check with x^2 if unsure.

Q8 — Medium — Answer: J

Topic: The domain of a rational function

A fraction is undefined only when its **bottom** is zero:

$$x - 5 = 0 \qquad x = 5$$

At $x = 5$ the function reads $\frac{8}{0}$, which has no value.

Why each wrong option is wrong:

- **F,** -3 — set the **numerator** to zero. That gives a zero of the function, $f(-3) = 0$, which is a perfectly good value, not an excluded one.
- **G,** 0 — $f(0) = \frac{3}{-5}$, which is fine.
- **H,** 3 — took a number from the top.

> **Takeaway:** only the denominator can make a function undefined. Setting the numerator to zero answers a different question — where the graph crosses the x-axis.

Section 3 — Logarithms

Q9 — Medium — Answer: C

Topic: Converting a logarithm to exponential form

$\log_3(x) = 4$ says: **3 to the power 4 is** x.

$$x = 3^4 = 81$$

The pattern to hold on to: the **base** goes to the bottom of the power, and the **answer of the log** becomes the exponent.

Why each wrong option is wrong:

- **A,** 12 — multiplied 3 by 4.
- **B,** 64 — computed $4^3 = 64$, swapping base and exponent. The two are not interchangeable.
- **D,** $\frac{4}{3}$ — divided.

> **Takeaway:** $\log_b(y) = x$ means $b^x = y$. Write the exponential form out every time; it turns a log question into an ordinary powers question.

Q10 — Hard — Answer: J

Topic: Combining logarithms

Two routes, both quick.

Directly: $\log_2(8) = 3$ because $2^3 = 8$, and $\log_2(4) = 2$ because $2^2 = 4$. So the sum is $3 + 2 = 5$.

By the rule: adding logs multiplies their insides:

$$\log_2(8) + \log_2(4) = \log_2(32) = 5$$

because $2^5 = 32$. Same answer, which is a good check that the rule is being used correctly.

Why each wrong option is wrong:

- **F,** 12 — added the insides. Adding logs multiplies the insides, it does not add them.
- **G,** 32 — multiplied the insides correctly to get 32, then forgot to take the logarithm of it.
- **H,** 6 — multiplied the two logarithms, $3imes2$, instead of adding them.

> **Takeaway:** adding logs **multiplies** what is inside. When the numbers are small powers of the base, evaluating each log separately is faster than using the rule — and doing both is a free check.

Section 4 — Modelling with Functions

Q11 — Medium — Answer: D

Topic: Building an exponential model

Growth of 15% per hour multiplies by 1.15 each hour — the original whole plus another 15%:

$$P = 400(1.15)^h$$

Check one hour: $400 \times 1.15 = 460$, which is $400 + 60$, and 60 is 15% of 400. Correct.

Why each wrong option is wrong:

- **B,** $P = 400(0.15)^h$ — used 0.15 as the multiplier. That would leave 15% of the colony each hour, an 85% **collapse**, not growth.
- **C,** $P = 400 + 15h$ — modelled it as linear, adding 15 bacteria per hour. A percentage applies to the current amount, so the increase grows as the colony does.
- **A,** $P = 400(15)^h$ — used 15, multiplying the colony fifteenfold every hour.

> **Takeaway:** a $p\%$ rise per period is a multiplier of $1 + \frac{p}{100}$; a $p\%$ fall is $1 - \frac{p}{100}$. Always test one period against the plain arithmetic.

Q12 — Medium — Answer: F

Topic: Reading a decay factor from a model

A multiplier of 0.88 means the car **keeps** 88% of its value each year. What it loses is the rest:

$$100\% - 88\% = 12\%$$

Check with numbers. After one year:

$$24000 \times 0.88 = 21120$$

a fall of 2880, and $\frac{2880}{24000} = 0.12 = 12\%$. Correct.

Why each wrong option is wrong:

- **H, The car loses 88% of its value each year** — read 0.88 as the amount lost rather than the amount kept. Losing 88% a year would leave the car worth 2880 after twelve months.
- **G, The car loses $88 each year** — read it as a fixed number of dollars. Exponential models work in proportions, so the dollar loss shrinks each year.
- **J, The car gains 88% of its value each year** — a multiplier below 1 always shrinks a quantity.

> **Takeaway:** the multiplier is what is **kept**. Subtract it from 1 to find what is lost. Below 1 is decay; above 1 is growth.

Q13 — Hard — Answer: C

Topic: An exponential model with a period other than one

Count the doubling **periods**, not the years:

$$\frac{20 \text{ years}}{5 \text{ years per doubling}} = 4 \text{ doublings}$$

$$800 \times 2^4 = 800 \times 16 = 12800$$

Written as a formula, that is $P = 800 \cdot 2^{t/5}$ — the division by 5 is what converts years into periods.

Check by doubling four times: 800, 1600, 3200, 6400, 12800. Correct.

Why each wrong option is wrong:

- **A,** 3200 — used only two doublings.

- **B,** 838860800 — doubled every year, giving a number in the hundreds of millions.
- **D,** 8000 — treated the growth as linear.

> **Takeaway:** when the growth period is not 1, the exponent is $\frac{\text{time}}{\text{period}}$. Count the periods first, then apply the multiplier that many times.

Q14 — Hard — Answer: F

Topic: Working backwards to a starting value

Let p be the starting count. Four days of tripling multiplies it by 3^4:

$$p \times 3^4 = 810$$

$$81p = 810$$

$$p = 10$$

Check forwards: 10, 30, 90, 270, 810. Four triplings. Correct.

Why each wrong option is wrong:

- **G,** 270 — divided by 3 once, undoing a single day.
- **H,** 2430 — multiplied, growing the colony further into the future.
- **J,** 202 — divided by the number of days, which treats the growth as linear.

> **Takeaway:** to go backwards through exponential growth, **divide by the multiplier raised to the number of periods**. Then run it forwards to check — five seconds, and it catches every off-by-one.

Q15 — Medium — Answer: B

Topic: A composite function evaluated twice

$g(2) = 7$, and then $f(7) = 14 - 1 = 13$.

Why each wrong option is wrong:

- **A,** 8 — applied f first: $f(2) = 3$, then $g(3) = 8$. The order is not interchangeable.
- **C,** 7 and **D,** 3 — stopped after one function.

> **Takeaway:** the inner bracket goes first. $f(g(x))$ means g, then f.

Q16 — Hard — Answer: J

Topic: A composite function as an expression

Substitute the whole of $g(x)$ into f:

$$f(g(x)) = (x-3)^2 = x^2 - 6x + 9$$

Check at $x = 5$: $g(5) = 2$ and $f(2) = 4$; and $25 - 30 + 9 = 4$. ✓

Why each wrong option is wrong:

- **F,** $x^2 - 3$ — computed $g(f(x)) = x^2 - 3$, the other order.
- **G,** $x^2 - 9$ — squared each term, forgetting the middle term.
- **H,** $x^2 + 6x + 9$ — used $(x+3)^2$.

> **Takeaway:** substitute the inner function **whole**, brackets and all, then expand. A squared bracket always has a middle term.

Q17 — Medium — Answer: D

Topic: The inverse of a linear function

Write $y = 3x + 4$, swap the letters, and solve for the new y:

$$x = 3y + 4 \quad \Longrightarrow \quad y = \frac{x-4}{3}$$

Check: $f(2) = 10$, and $f^{-1}(10) = \frac{6}{3} = 2$. ✓

Why each wrong option is wrong:

- **B,** $\frac{x}{3} + \frac{4}{3}$ — added rather than subtracted.
- **C,** $3x - 4$ — reversed the signs without reversing the operations.
- **A,** $\frac{1}{3x+4}$ — an inverse **function** is not a reciprocal. The notation f^{-1} does not mean "one over".

> **Takeaway:** swap x and y, then solve. f^{-1} undoes f; it is not $\frac{1}{f}$.

Q18 — Medium — Answer: J

Topic: A composite function applied to itself

$$f(3) = 6 - 1 = 5 \qquad f(5) = 10 - 1 = 9$$

Why each wrong option is wrong:

- **F,** 5 — applied f once.
- **G,** 11 — used 6 rather than 5 as the second input, dropping the first subtraction.
- **H,** 6 — doubled and never subtracted.

> **Takeaway:** $f(f(x))$ means run the machine twice. Write the intermediate value down before feeding it back in.

Q19 — Hard — Answer: B

Topic: The domain of a function with a square root

The expression under a square root cannot be negative, so

$$x - 5 \geq 0 \quad \Longrightarrow \quad x \geq 5$$

The end **is** included: at $x = 5$ the root is $\sqrt{0} = 0$, a perfectly good value.

Why each wrong option is wrong:

- **A,** $x > 5$ — excluded $x = 5$. Zero under a root is fine; only negatives are not.
- **C,** $x \leq 5$ — reversed the inequality, which is the region where the inside is negative.
- **D,** $x \geq -5$ — a sign error on the boundary.

> **Takeaway:** under a square root, set the inside ≥ 0. Note it is **greater than or equal**, because $\sqrt{0}$ exists.

Q20 — Medium — Answer: H

Topic: A piecewise function

The input is 5, and $5 \geq 3$, so the **second** rule applies:

$$f(5) = 5^2 - 4 = 25 - 4 = 21$$

The first rule is only for inputs below 3, and it plays no part here.

Why each wrong option is wrong:

- **F,** 11 — used the rule for $x < 3$, which does not apply to 5.
- **G,** 25 — used the right rule and dropped the -4.
- **J,** 1 — substituted at the boundary rather than at the input given.

> **Takeaway:** with a piecewise function, **check the condition first** and pick the line, then substitute. The other line is irrelevant.

Q21 — Medium — Answer: C

Topic: A piecewise function at its boundary

Check the condition before choosing a line. The first says $x \leq 10$, and 10 satisfies it, so:

$$c(10) = 5 + 2(10) = 25$$

The second line applies only **above** 10. Note the two rules do not agree at the boundary — c jumps from 25 to 30 — which is why the $\leq$ matters.

Why each wrong option is wrong:

- **A,** 30 — used the rule for $x > 10$; 10 is not greater than 10.
- **B,** 35 — added both rules together.
- **D,** 15 — dropped the coefficient of x.

> **Takeaway:** read the condition first and note whether it uses $<$ or $\leq$. That single symbol decides which rule owns the boundary.

Q22 — Hard — Answer: F

Topic: The range of a function

x^2 is at its smallest when $x = 0$, where it equals 0. Everywhere else it is positive. So

$$f(x) = x^2 + 3 \geq 0 + 3 = 3$$

The value 3 **is** reached, at $x = 0$, so the end is included.

Why each wrong option is wrong:

- **G,** $y > 3$ — excluded 3, but $f(0) = 3$ exactly.

- **H,** $y \leq 3$ — reversed the inequality; the parabola opens upward, so 3 is a minimum, not a maximum.
- **J,** $y \geq 0$ — gave the range of x^2 alone, forgetting the $+3$ lifts every output.

> **Takeaway:** for range questions, find the extreme value the expression can reach and ask whether it is actually attained. A square's minimum is 0, at $x = 0$.

Q23 — Medium — Answer: B

Topic: An absolute-value function

$|x - 4|$ is smallest when the inside is zero, which happens at $x = 4$. There it equals 0, so

$$f(4) = 0 + 1 = 1$$

Everywhere else $|x - 4|$ is positive, so f is larger.

Why each wrong option is wrong:

- **A,** 4 — gave $x = 4$, the input **where** the minimum occurs, not the minimum itself.
- **C,** 5 — added the two constants.
- **D,** 0 — gave the minimum of $|x - 4|$ and forgot the $+1$.

> **Takeaway:** an absolute value bottoms out at zero where its inside is zero. Then read whether the question wants the input or the output.

Q24 — Medium — Answer: H

Topic: Evaluating a logarithm

Ask: 2 to what power gives 64?

$$2^6 = 64 \quad \Longrightarrow \quad \log_2(64) = 6$$

Counting the doublings is often quicker than any rule: 2, 4, 8, 16, 32, 64 — six of them.

Why each wrong option is wrong:

- **F,** 32 — divided 64 by 2. A logarithm is an exponent, not a quotient.
- **G,** 8 — took a square root.
- **J,** 128 — multiplied by the base.

> **Takeaway:** $\log_b(y)$ asks "b to what power gives y?" With small numbers, count the multiplications.

Q25 — Hard — Answer: D

Topic: A logarithm with a fractional result

Ask: 9 to what power gives 3?

$$9^{1/2} = \sqrt{9} = 3 \quad \Longrightarrow \quad \log_9(3) = \tfrac{1}{2}$$

The check that settles it instantly: whenever the number is **smaller** than the base, the logarithm is between 0 and 1.

Why each wrong option is wrong:

- **B,** 3 — divided 9 by 3.
- **C,** 2 — computed $\log_3(9) = 2$, the two arguments swapped. Note it is the reciprocal of the right answer, which is no coincidence.
- **A,** $\frac{1}{3}$ — a fraction, but the wrong one.

> **Takeaway:** a logarithm is between 0 and 1 exactly when the number is between 1 and the base. Use that to sanity-check before calculating.

Q26 — Medium — Answer: G

Topic: Splitting a logarithm

Subtracting logs of the same base divides their insides:

$$\log_5(50) - \log_5(2) = \log_5\left(\frac{50}{2}\right) = \log_5(25)$$

And $5^2 = 25$, so the answer is 2.

Why each wrong option is wrong:

- **F,** 48 — subtracted the insides. Subtracting logs divides; it does not subtract.
- **H,** 25 — divided correctly to reach 25 and then forgot to take the logarithm.
- **J,** 100 — multiplied, which is what **adding** logs does.

> **Takeaway:** adding logs multiplies inside; subtracting divides. Then finish by evaluating the log you are left with.

Q27 — Medium — Answer: A

Topic: Modelling with a linear function

The fixed charge is paid once, so remove it before dividing:

$$3.5 + 1.8m = 21.50$$

$$1.8m = 18 \qquad m = 10$$

Check: $1.8 \times 10 = 18$, and $18 + 3.5 = 21.50$. ✓

Why each wrong option is wrong:

- **B,** $\frac{215}{18}$ — divided the whole fare by the rate, as though there were no fixed charge. That is the trap this question is built around.
- **C,** 18 — copied the per-mile rate.
- **D,** $\frac{43}{7}$ — divided by the fixed charge instead of the rate.

> **Takeaway:** in $C = \text{fixed} + \text{rate} \times x$, subtract the fixed part **before** dividing by the rate.

Q28 — Hard — Answer: H

Topic: Choosing a model from a description

The halving period is 4 days, not 1, so the exponent counts **periods**:

$$\text{periods} = \frac{d}{4} \qquad \Longrightarrow \qquad V = 200(0.5)^{d/4}$$

Check 4 days: the exponent is 1, giving $200 \times 0.5 = 100$. ✓ Check 8 days: the exponent is 2, giving 50. ✓

Why each wrong option is wrong:

- **F,** $200(0.5)^d$ — halves every **day**, so by day 4 it would be 12.5.
- **G,** $200 - 25d$ — linear decay, which would reach zero on day 8 and go negative after.
- **J,** $200(4)^{d/2}$ — grows the quantity instead of shrinking it.

> **Takeaway:** when the period is not 1, the exponent is $\frac{\text{time}}{\text{period}}$. Test the model at exactly one period.

Q29 — Medium — Answer: C

Topic: Reading a starting value out of a model

The study began at $t = 0$, and anything to the power 0 is 1:

$$P(0) = 1200 \times (1.03)^0 = 1200 \times 1 = 1200$$

So the number in front of the bracket is always the **starting value**.

Why each wrong option is wrong:

- **A,** 1236 — gave $P(1)$, the population after one year.
- **B,** $\frac{103}{100}$ — gave the multiplier, which describes growth rather than size.
- **D,** 0 — a model does not have to start at zero; this one starts at 1200.

> **Takeaway:** in $a \cdot b^t$, a is where it starts and b is how it changes. Setting $t = 0$ recovers a every time, because $b^0 = 1$.

Q30 — Basic — Answer: F

Topic: Evaluating a function at a fraction

$$f\left(\tfrac{1}{4}\right) = 8\left(\tfrac{1}{4}\right) - 3 = 2 - 3 = -1$$

Only the x is replaced. The -3 is unaffected by the input.

Why each wrong option is wrong:

- **G,** 2 — multiplied correctly and stopped before subtracting.
- **H,** $\frac{5}{4}$ — divided the 3 by 4 as well, but the 3 is not multiplied by x.
- **J,** 5 — ignored the input and used $x = 1$.

> **Takeaway:** substitute for **every** x and nothing else. Terms without an x stay exactly as they are.

Q31 — Medium — Answer: C

Topic: Reading a function from a table

Read the two outputs off the table and subtract:

$$f(3) = 11 \qquad f(1) = 5 \qquad f(3) - f(1) = 6$$

Why each wrong option is wrong:

- **A,** 2 — subtracted the **inputs**, $3 - 1$, rather than the outputs.
- **B,** 16 — added the two outputs.
- **D,** 3 — gave the step between consecutive rows, which is the slope of this linear function but not what was asked.

> **Takeaway:** $f(3)$ means the output when the input is 3 — look it up in the bottom row. The top row holds inputs, never answers.

Q32 — Medium — Answer: J

Topic: A composite function in the other order

Work from the inner bracket outwards:

$$f(4) = 6 \qquad g(6) = 18$$

Why each wrong option is wrong:

- **F,** 14 — applied g first: $g(4) = 12$, then $f(12) = 14$. The order is not interchangeable, and this question is the mirror image of one where it is the other way round.
- **G,** 6 and **H,** 12 — stopped after one function.

> **Takeaway:** the function named **inside** the brackets goes first. In $g(f(x))$ that is f; in $f(g(x))$ it is g.

Q33 — Hard — Answer: A

Topic: A composite function that returns its input

$$f(5) = 2(5) + 6 = 16 \qquad g(16) = \frac{16 - 6}{2} = 5$$

The answer is the number we started with — and that is no accident. g undoes exactly what f does, so they are **inverse** functions, and $g(f(x)) = x$ for every x.

Why each wrong option is wrong:

- **B,** 16 — gave $f(5)$ and stopped halfway.
- **C,** 10 — subtracted the 6 and forgot to halve.
- **D,** 8 — halved 16 without subtracting the 6 first.

> **Takeaway:** if a composite hands back the input you started with, the two functions are inverses. Spotting that saves the arithmetic entirely.

Q34 — Medium — Answer: H

Topic: The y-intercept of a function's graph

Set $x = 0$:

$$f(0) = 0 - 0 + 7 = 7$$

So the constant term **is** the y-intercept, for any polynomial — every other term contains an x and vanishes at zero.

Why each wrong option is wrong:

- **F,** 0 — a graph need not pass through the origin, and this one does not.
- **G,** -4 — gave the coefficient of x.
- **J,** 3 — combined two coefficients that never meet.

> **Takeaway:** the y-intercept is the value at $x = 0$, which for a polynomial is just the constant term.

Q35 — Medium — Answer: C

Topic: The zeros of a factored function

Set each bracket to zero:

$$x - 2 = 0 \Rightarrow x = 2 \qquad x + 5 = 0 \Rightarrow x = -5$$

$$2 + (-5) = -3$$

The signs **flip**: a factor of $(x + 5)$ gives a zero at -5, not $+5$. That is the whole difficulty of the question.

Why each wrong option is wrong:

- **A,** 7 — added the numbers as printed, ignoring the sign flip.
- **B,** 10 — multiplied them.
- **D,** 3 — subtracted with the signs the wrong way round.

> **Takeaway:** a factor $(x - r)$ gives a zero at $x = r$, so the sign in the bracket is the **opposite** of the zero.

Q36 — Medium — Answer: G

Topic: Where a graph is increasing

The vertex sits at

$$x = -\frac{b}{2a} = -\frac{-6}{2} = 3$$

The parabola opens upward (the x^2 coefficient is positive), so it **falls** to the left of 3 and **rises** to the right.

Check: $f(1) = 0$, $f(2) = -3$ — falling. And $f(4) = -3$, $f(5) = 0$ — rising.

Why each wrong option is wrong:

- **F,** $x \leq 3$ — that is where it decreases.
- **H,** $x \geq 5$ and **J,** $x \leq 5$ — used 5, the constant term, as the vertex.

> **Takeaway:** an upward parabola decreases before its vertex and increases after. Find the vertex at $-\frac{b}{2a}$ first.

Q37 — Hard — Answer: A

Topic: The maximum of a downward parabola

The vertex is at

$$t = -\frac{b}{2a} = -\frac{12}{2(-2)} = 3 \text{ seconds}$$

That is **when** the maximum happens. The height itself needs substituting back:

$$h(3) = -2(9) + 36 + 5 = -18 + 41 = 23 \text{ metres}$$

Why each wrong option is wrong:

- **B,** 3 — gave $t = 3$, the time of the maximum rather than its height. This is the single most common error on vertex questions.
- **C,** 5 — gave $h(0) = 5$, the height at launch.
- **D,** 17 — a slip substituting back.

> **Takeaway:** $-\frac{b}{2a}$ gives the **input**. Substitute it back to get the output. Read the question to see which one it wants.

Q38 — Medium — Answer: H

Topic: A vertical stretch

The 2 is **outside** the function, so it multiplies the output:

$$(4, 3) \longrightarrow (4,\ 2 \times 3) = (4, 6)$$

The input is untouched. At $x = 4$ the new graph computes $2f(4) = 2 \times 3$.

Why each wrong option is wrong:

- **F,** $(8,\ 3)$ and **G,** $(2,\ 3)$ — changed the input. A factor outside the function never moves a point sideways.
- **J,** $(8,\ 6)$ — doubled both coordinates.

> **Takeaway:** outside the function acts on y; inside acts on x. A vertical stretch leaves every x-coordinate exactly where it was.

Q39 — Hard — Answer: A

Topic: A reflection in the y-axis

The minus is **inside** the function, so it acts on the input: the graph is flipped left to right.

$$(-3, 7) \longrightarrow (3,\ 7)$$

Reason it through: the new graph at $x = 3$ computes $f(-3) = 7$. So the height that was at $x = -3$ now appears at $x = 3$.

Why each wrong option is wrong:

- **B,** $(-3,\ -7)$ — that is $y = -f(x)$, a reflection in the **horizontal** axis, where the outputs change sign.
- **C,** $(3,\ -7)$ — reflected in both axes.
- **D,** $(7,\ -3)$ — swapped the coordinates, which is a reflection in the line $y = x$.

> **Takeaway:** $f(-x)$ flips left–right; $-f(x)$ flips up–down. Inside acts on x, outside acts on y — the same rule as every other transformation.

Q40 — Medium — Answer: F

Topic: Two transformations at once

Take the two separately.

Inside: $f(x+1)$ shifts the graph 1 unit **left**, because inside changes act opposite to their sign. The x-coordinate goes from 2 to 1.

Outside: the -4 lowers every output by 4. The y-coordinate goes from 5 to 1.

$$(2,5) \longrightarrow (1,1)$$

Why each wrong option is wrong:

- **G,** $(3,\ 1)$ — moved right; a $+1$ inside moves **left**.
- **H,** $(1,\ 9)$ — moved up.
- **J,** $(3,\ 9)$ — both directions wrong.

> **Takeaway:** do the two shifts one at a time and label which is which. Inside–opposite, outside–obvious.

Q41 — Medium — Answer: A

Topic: A piecewise function below its boundary

-1 is less than 2, so the **first** rule applies:

$$g(-1) = 3(-1) = -3$$

The second rule would give 7, but it applies only from 2 upward.

Why each wrong option is wrong:

- **B,** 7 — used the rule for $x \geq 2$ on an input below 2.
- **C,** 3 — gave the coefficient rather than the output.
- **D,** -9 — an arithmetic slip.

> **Takeaway:** check the condition **before** substituting. A piecewise function is several functions wearing one name, and only one of them applies at a time.

Q42 — Hard — Answer: H

Topic: Solving a logarithmic equation

$\log_4(x-3) = 2$ says: **4 to the power 2 gives $x - 3$.**

$$x - 3 = 4^2 = 16 \quad \Longrightarrow \quad x = 19$$

Check what the logarithm allows: its inside must be positive, and $19 - 3 = 16 > 0$. ✓

Why each wrong option is wrong:

- **F,** 11 — multiplied the base by the result.
- **G,** 8 — subtracted the 3 instead of adding it when isolating x.
- **J,** 5 — added 3 to the 2 without raising anything to a power.

> **Takeaway:** rewrite $\log_b(\text{something}) = c$ as something $= b^c$, then solve — and confirm the inside of the logarithm came out positive.

Q43 — Medium — Answer: A

Topic: Choosing between a linear and an exponential model

Test for linear first by subtracting:

$$150 - 100 = 50 \qquad 225 - 150 = 75$$

Not constant, so it is not linear. Now divide:

$$\frac{150}{100} = 1.5 \qquad \frac{225}{150} = 1.5$$

Constant, so it is **exponential** with a multiplier of 1.5.

$$225 \times 1.5 = 337.5$$

Why each wrong option is wrong:

- **B, Linear, and** 300 and **D, Linear, and** 275 — assumed a constant difference, which the data contradicts.
- **C, Exponential, and** 300 — identified the model correctly and then added 75 anyway, which is the linear step.

> **Takeaway:** subtract to test for linear; if that fails, divide to test for exponential. Then **use** the model you identified — spotting it and then doing linear arithmetic is a real and common way to lose the mark.

Chapter 5 — Geometry

Geometry is 17 to 20% of the test — eight questions — and on the ACT it is **larger and more varied than on the SAT**. Circles, solids, polygons and coordinate geometry all appear, and there is no formula sheet to lean on.

That last point governs this whole chapter. Every formula used here is in Chapter 8, with the reason it is true. Learn them there; use them here.

One habit worth building from the start: **draw the picture**. Almost every ACT geometry question describes a shape in words, and a sketch with the numbers written on it turns a confusing paragraph into an obvious calculation. It takes fifteen seconds and it is the single most reliable way to gain marks in this category.

Topics covered: angles in triangles and on lines · isosceles and equilateral triangles · polygons and quadrilaterals · similar figures · circles, arcs and sectors · the equation of a circle · solids and volume · coordinate geometry

---

Section 1 — Angles, Triangles and Polygons

The facts this section needs, all of them worth knowing by heart:

- angles on a **straight line** add to 180°
- angles **round a point** add to 360°
- the three angles of a **triangle** add to 180°
- an **isosceles** triangle has two equal sides and two equal angles, the equal angles being **opposite** the equal sides
- an **equilateral** triangle has three 60° angles
- the **exterior angle** of a triangle equals the sum of the two interior angles not next to it
- the interior angles of a polygon with n sides add to $(n-2) \times 180^\circ$

That last formula covers quadrilaterals (360°), pentagons (540°), hexagons (720°) and everything beyond, so it is one fact rather than five.

Q1 — Basic

A surveyor measures two of the three interior angles of a triangular plot of land and records them as 47° and 68°. What is the measure of the third angle?

A) 115

B) 45

C) 65

D) 113

Q2 — Medium

A roof truss is built as triangle ABC with $AB = AC$, making it isosceles, and the angle at the apex A measures 40°. What is the measure of angle B?

F) 40

G) 140

H) 100

J) 70

Q3 — Medium

In a triangle, the two interior angles not adjacent to a particular exterior angle measure 52° and 61°. What is the measure of that exterior angle?

A) 67

B) 113

C) 128

D) 119

Q4 — Medium

A regular hexagon is used as a tile pattern, and each of its six interior angles is the same size. What is the measure of each interior angle?

F) 120

G) 60

H) 720

J) 90

Q5 — Hard

A flagpole casts a shadow 9 metres long at the same moment that a 2-metre post casts a shadow 1.5 metres long on level ground. How tall is the flagpole, in metres?

A) 6

B) 12

C) $\frac{27}{4}$

D) 135

Section 2 — Circles

Four facts carry almost every ACT circle question:

$$\text{circumference} = 2\pi r \qquad \text{area} = \pi r^2$$

An **arc** is part of the circumference and a **sector** is a slice of the area. Both are the same fraction of the whole, and that fraction is $\dfrac{\text{central angle}}{360}$:

$$\text{arc length} = \frac{\theta}{360} \times 2\pi r \qquad \text{sector area} = \frac{\theta}{360} \times \pi r^2$$

And the **equation of a circle** with centre (h, k) and radius r:

$$(x - h)^2 + (y - k)^2 = r^2$$

Note the **minus** signs in the brackets and that the right-hand side is r^2, not r. Both are where the marks go.

Q6 — Medium

A circular flower bed of radius 6 metres is to be divided into wedge-shaped planting sections. One section is a sector with a central angle of 60° measured at the centre of the bed. What is the area of that sector?

F) 6π

G) 36π

H) 2π

J) 12π

Q7 — Medium

A radio transmitter's coverage area is described in the xy-plane by $(x-4)^2+(y+3)^2=25$. What are its centre and radius?

A) $(-4,\ 3,\ 25)$

B) $(-4,\ 3,\ 5)$

C) $(4,\ -3,\ 25)$

D) $(4,\ -3,\ 5)$

Q8 — Hard

A circular arena is drawn on a plan with a diameter running between the endpoints $(1, 2)$ and $(7, 10)$. What is the radius?

F) 10

G) 5

H) 14

J) $\sqrt{10}$

Section 3 — Solids and Volume

The volumes worth knowing, all of which are in Chapter 8 with their reasons:

solid	volume
box (rectangular prism)	lwh
cylinder	$\pi r^2 h$
any prism	(area of cross-section) $\times$ length
sphere	$\frac{4}{3}\pi r^3$
cone	$\frac{1}{3}\pi r^2 h$

The ACT also likes **scaling**: what happens to area and volume when a length changes. The rule is worth more than any single formula:

> If every length is multiplied by k, then every **area** is multiplied by k^2 and every **volume** by k^3.

Q9 — Medium

A cylindrical water tank has a base of radius 3 metres and stands 10 metres tall. What is its volume?

A) 60π

B) 30π

C) 90π

D) 900π

Q10 — Hard

A spherical balloon is inflated until its radius is three times what it was before. By what factor does its volume increase?

F) 3

G) 9

H) 12

J) 27

Q11 — Medium

A closed cardboard box measures 4 by 3 by 2 units, and every face has to be covered. What is its surface area?

A) 52

B) 24

C) 26

D) 18

Section 4 — Coordinate Geometry

Three formulas connect algebra to the picture, and all three come from Pythagoras or from counting:

$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

The midpoint is an **average**, so it adds; distance and slope both **subtract**. Mixing up which one adds is the commonest error here.

Q12 — Basic

In the standard (x, y) coordinate plane, a line segment has endpoints $(3, 8)$ and $(-1, -4)$. What is the midpoint?

F) $(2,\ 6)$

G) $(4,\ 12)$

H) $(1,\ 2)$

J) $(-2,\ -6)$

Q13 — Hard

In the xy-plane, a point must lie exactly 5 units from the origin and also exactly 3 units from the horizontal axis. How many such points are there?

A) 1

B) 2

C) 3

D) 4

Q14 — Medium

A survey records two markers at $(-2, 1)$ and $(4, 9)$ in the xy-plane, and the straight-line distance between them is needed. What is the distance?

F) 14

G) 10

H) $\sqrt{14}$

J) 100

Q15 — Medium

Two triangles are similar, and the sides of the smaller measure 4, 6 and 8 units. The longest side of the larger triangle measures 20 units. What is the perimeter of the larger triangle?

A) 45

B) 18

C) 30

D) 54

Q16 — Medium

Two triangles are similar. The smaller has sides of 3 and 5 units; the larger has a side of 12 corresponding to the 3. What is the length of the larger triangle's side corresponding to the 5?

F) 14

G) 15

H) 20

J) $\frac{5}{4}$

Q17 — Hard

Two similar rectangles have corresponding sides in the ratio $2 : 5$. What is the ratio of their areas?

A) $\frac{2}{5}$

B) $\frac{4}{5}$

C) $\frac{4}{25}$

D) $\frac{8}{125}$

Q18 — Medium

Two triangles have three pairs of equal sides. Which of the following must be true of them?

F) Their corresponding angles are equal and their areas are equal

G) Their corresponding angles are equal but their areas may differ

H) Their areas are equal but their angles may differ

J) Neither the angles nor the areas need match

Q19 — Medium

A regular octagon is used as a paving stone, and each of its eight interior angles is the same size. What is each interior angle?

A) 135

B) 45

C) 1080

D) 120

Q20 — Medium

A parallelogram-shaped panel has a base of 15 centimetres and a perpendicular height of 8 centimetres. What is the area of the panel, in square centimetres?

F) 150

G) 60

H) 120

J) 46

Answers and Explanations

Section 1 — Angles, Triangles and Polygons

Q1 — Basic — Answer: C

Topic: The third angle of a triangle

The three angles of a triangle add to 180°:

$$47 + 68 + x = 180$$
$$115 + x = 180$$
$$x = 65^\circ$$

Check: $47 + 68 + 65 = 180$. Correct.

Why each wrong option is wrong:

- **A,** 115 — added the two given angles and stopped. 115 is a step on the way, not the answer.
- **B,** 45 and **D,** 113 — arithmetic slips. Always add all three of your angles at the end; it takes two seconds and catches every one of these.

> **Takeaway:** the angles of a triangle add to 180°. Subtract the sum of the two you know, then add all three back to check.

Q2 — Medium — Answer: J

Topic: The base angles of an isosceles triangle

$AB = AC$, so the angles opposite those sides — angles C and B — are equal. Call each of them x.

$$40 + x + x = 180$$
$$2x = 140$$
$$x = 70^\circ$$

Check: $40 + 70 + 70 = 180$. Correct.

Which angles are equal? The ones **opposite** the equal sides. AB and AC both start at A, so the equal angles are the other two, B and C. Getting this backwards is the usual error, and drawing the triangle makes it obvious.

Why each wrong option is wrong:

- **F,** 40 — assumed the triangle is equilateral.
- **G,** 140 — found the 140° left over and forgot it is shared between two angles.
- **H,** 100 — treated angle C as 40° as well.

> **Takeaway:** equal sides face equal angles. Subtract the odd angle from 180°, then **halve** what is left.

Q3 — Medium — Answer: B

Topic: An exterior angle of a triangle

The exterior angle rule:

$$\text{exterior angle} = 52 + 61 = 113^\circ$$

Why it works: the third interior angle is $180 - 52 - 61 = 67^\circ$, and the exterior angle sits on a straight line with it, so it is $180 - 67 = 113^\circ$. The rule is just those two steps combined — which is why it saves time.

Why each wrong option is wrong:

- **A,** 67 — gave the third **interior** angle, the one the exterior angle sits next to.
- **C,** 128 and **D,** 119 — used only one of the two given angles.

> **Takeaway:** an exterior angle equals the sum of the two **remote** interior angles. If you forget the rule, find the third angle and subtract from 180° — same answer, one more step.

Q4 — Medium — Answer: F

Topic: The interior angles of a polygon

Total interior angles:

$$(6 - 2) \times 180 = 4 \times 180 = 720^\circ$$

The hexagon is regular, so all six angles are equal:

$$\frac{720}{6} = 120^\circ$$

Why each wrong option is wrong:

- **G,** 60 — gave the **exterior** angle. The exterior angles of any polygon add to 360°, so each is $\frac{360}{6} = 60^\circ$ — and indeed $120 + 60 = 180$, as they sit on a straight line together.
- **H,** 720 — gave the total for all six angles.
- **J,** 90 — divided by 8 rather than 6.

> **Takeaway:** $(n-2) \times 180$ is the **total**. Divide by n for one angle of a regular polygon. The exterior angles always total 360°, whatever n is.

Q5 — Hard — Answer: B

Topic: Similar triangles in a shadow problem

Draw both triangles. Each has a vertical object and a horizontal shadow, and the sun makes the same angle in both — so they are **similar**, and matching sides are in the same ratio.

Set up the proportion with height on top in both:

$$\frac{\text{flagpole height}}{\text{flagpole shadow}} = \frac{\text{post height}}{\text{post shadow}}$$
$$\frac{h}{9} = \frac{2}{1.5}$$
$$h = 9 \times \frac{2}{1.5} = 9 \times \frac{4}{3} = 12 \text{ m}$$

Sanity check: the post is taller than its shadow (2 against 1.5), so the flagpole must be taller than its 9-metre shadow. 12 is. Correct.

Why each wrong option is wrong:

- **A,** 6 — paired the wrong sides together.
- **C,** $\frac{27}{4}$ — set the proportion up upside down, which makes the flagpole *shorter* than its shadow and fails the sanity check.
- **D,** 135 — multiplied the numbers without forming a ratio.

> **Takeaway:** similar shapes give equal ratios. Put the **same** measurement on top of both fractions, then check the answer is on the right side of the number you started with.

Section 2 — Circles

Q6 — Medium — Answer: F

Topic: The area of a sector

A 60° sector is $\frac{60}{360} = \frac{1}{6}$ of the circle.

The whole circle's area:

$$\pi r^2 = \pi \times 36 = 36\pi$$

One sixth of it:

$$\frac{1}{6} \times 36\pi = 6\pi \text{ square metres}$$

Why each wrong option is wrong:

- **G,** 36π — gave the area of the whole circle without taking the fraction.
- **H,** 2π — used the arc-length formula, which gives a **length**, not an area. Check the units: an area needs r^2 in it.
- **J,** 12π — doubled the correct sector.

> **Takeaway:** a sector is $\frac{\theta}{360}$ of the circle. Use πr^2 for area and $2\pi r$ for arc length, and let the units tell you which you need.

Q7 — Medium — Answer: D

Topic: Reading the centre and radius from a circle's equation

The standard form is

$$(x-h)^2 + (y-k)^2 = r^2$$

with centre (h, k) and radius r.

Match it up carefully. $(x-4)^2$ gives $h = 4$. For the y term, rewrite $(y+3)^2$ as $(y-(-3))^2$, so $k = -3$.

The right-hand side is $r^2 = 25$, so $r = 5$ — **take the square root**.

Centre $(4, -3)$, radius 5.

Why each wrong option is wrong:

- **B,** $(-4,\ 3,\ 5)$ — copied the signs as they appear in the brackets. The formula has a minus built in, so a $+3$ inside means -3 in the centre.

- **C,** $(4,\ -3,\ 25)$ — gave 25 as the radius. The equation gives r^2.
- **A,** $(-4,\ 3,\ 25)$ — both errors together.

> **Takeaway:** the signs in the brackets **flip**, and the number on the right is r^2. Rewrite $(y+3)$ as $(y-(-3))$ if the sign flip is not automatic yet.

Q8 — Hard — Answer: G

Topic: Finding the radius from a diameter's endpoints

Find the distance between the two points first:

$$d = \sqrt{(7-1)^2 + (10-2)^2} = \sqrt{6^2 + 8^2} = \sqrt{36+64} = \sqrt{100} = 10$$

That distance is the **diameter**, so the radius is half of it:

$$r = \frac{10}{2} = 5$$

The 6, 8, 10 triangle here is a scaled 3, 4, 5 — worth recognising, because it saves the arithmetic entirely.

Why each wrong option is wrong:

- **F,** 10 — gave the diameter. Stopping one step early is the usual error in this question, which is exactly why it says "radius".
- **H,** 14 — added the horizontal and vertical differences without squaring. That is the distance you would walk along two sides, not the straight line.
- **J,** $\sqrt{10}$ — did not square the differences before adding.

> **Takeaway:** the distance formula is Pythagoras in disguise. And read the last word — diameter or radius decides whether you halve.

Section 3 — Solids and Volume

Q9 — Medium — Answer: C

Topic: The volume of a cylinder

$$V = \pi r^2 h = \pi \times 3^2 \times 10 = \pi \times 9 \times 10 = 90\pi$$

Square the radius **first**, then multiply by the height. Doing it in the other order is fine too, as long as only the radius gets squared.

Why each wrong option is wrong:

- **A,** 60π — used $2\pi rh$, which is the curved surface **area**, not a volume. A volume needs three lengths multiplied together, and this has only two.
- **B,** 30π — forgot to square the radius.
- **D,** 900π — squared 3×10 rather than just the 3.

> **Takeaway:** $\pi r^2 h$ — only the radius is squared. If your expression has fewer than three lengths multiplied together, it is not a volume.

Q10 — Hard — Answer: J

Topic: Scaling a solid

The volume of a sphere is $\frac{4}{3}\pi r^3$. Replacing r with $3r$:

$$\frac{4}{3}\pi(3r)^3 = \frac{4}{3}\pi \cdot 27r^3$$

so the volume is **27 times** what it was. The bracket matters: $(3r)^3$ cubes the 3 as well as the r.

The general rule: multiply every length by k and volume multiplies by k^3. Here $k = 3$, so $3^3 = 27$.

Why each wrong option is wrong:

- **F,** 3 — assumed volume grows in proportion to length.
- **G,** 9 — used k^2, which is the factor for **area**, not volume.
- **H,** 12 — brought the $\frac{4}{3}$ into the scaling. Constants in the formula do not affect how it scales; only the power of r does.

> **Takeaway:** lengths scale by k, areas by k^2, volumes by k^3. Cube the bracket, not just the letter inside it.

Q11 — Medium — Answer: A

Topic: The surface area of a box

The six faces come in three pairs, one pair for each combination of two dimensions:

$$4 \times 3 = 12 \quad \text{(twice)} \qquad 4 \times 2 = 8 \quad \text{(twice)} \qquad 3 \times 2 = 6 \quad \text{(twice)}$$
$$\text{surface area} = 2(12 + 8 + 6) = 2 \times 26 = 52$$

Why each wrong option is wrong:

- **B,** 24 — gave the **volume**, $4 \times 3 \times 2$. Check the units: a surface area is in square units, and multiplying three lengths gives cubic units.
- **C,** 26 — added one of each face and forgot the box has two of each.
- **D,** 18 — added the dimensions rather than multiplying them in pairs.

> **Takeaway:** a box has six faces in three matching pairs, so $2(lw + lh + wh)$. If you find yourself multiplying all three dimensions together, you are calculating a volume.

Section 4 — Coordinate Geometry

Q12 — Basic — Answer: H

Topic: The midpoint of a line segment

Average each coordinate separately:

$$x: \frac{3 + (-1)}{2} = \frac{2}{2} = 1$$
$$y: \frac{8 + (-4)}{2} = \frac{4}{2} = 2$$

Midpoint $(1, 2)$.

A good check: the midpoint must lie **between** the two endpoints in both coordinates. 1 is between -1 and 3; 2 is between -4 and 8. Correct.

Why each wrong option is wrong:

- **F,** $(2,\ 6)$ and **J,** $(-2,\ -6)$ — subtracted instead of adding. Both fail the "between" check.
- **G,** $(4,\ 12)$ — gave the differences without halving.

> **Takeaway:** the midpoint **adds and halves**; distance and slope **subtract**. Then check your answer lies between the two endpoints.

Q13 — Hard — Answer: D

Topic: Counting points at a fixed distance

"5 units from the origin" means

$$x^2 + y^2 = 25$$

"3 units from the horizontal axis" means the distance from the x-axis is 3, so

$$y = 3 \quad \text{or} \quad y = -3$$

Take $y = 3$:

$$x^2 + 9 = 25 \quad \Longrightarrow \quad x^2 = 16 \quad \Longrightarrow \quad x = \pm 4$$

That gives $(4, 3)$ and $(-4, 3)$. Now $y = -3$ gives the same two x values, so $(4, -3)$ and $(-4, -3)$.

Four points in total.

Why each wrong option is wrong:

- **A,** 1, **B,** 2 and **C,** 3 — each misses some of the sign combinations. There are two independent choices, $\pm$ for x and $\pm$ for y, so $2 \times 2 = 4$.

> **Takeaway:** "a distance from an axis" allows **both** sides of it, and a squared equation gives **both** signs. Sketch the circle and the two lines — the four crossings are then impossible to miscount.

Q14 — Medium — Answer: G

Topic: The distance between two points

Find the two gaps first:

$$\text{across: } 4 - (-2) = 6 \qquad \text{up: } 9 - 1 = 8$$

Then Pythagoras:

$$\sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$$

Another 6, 8, 10 triangle. The 3-4-5 family turns up constantly on the ACT.

Why each wrong option is wrong:

- **F,** 14 — added the gaps, which is the distance walked along two sides of the right triangle rather than across it. It is always larger than the true distance.
- **H,** $\sqrt{14}$ — added before squaring.
- **J,** 100 — did everything right and forgot the square root.

> **Takeaway:** find both gaps, square, add, **then** root. Watch the double negative in $4 - (-2)$; that is where the arithmetic usually goes wrong.

Q15 — Medium — Answer: A

Topic: Corresponding sides of similar triangles

The scale factor comes from the pair of corresponding sides you know — the longest of each:

$$k = \frac{20}{8} = 2.5$$

Perimeter scales by the same factor as any length:

$$(4 + 6 + 8) \times 2.5 = 18 \times 2.5 = 45$$

Why each wrong option is wrong:

- **B,** 18 — gave the smaller triangle's perimeter.
- **C,** 30 — replaced the longest side and left the other two unscaled.
- **D,** 54 — used a scale factor of 3.

> **Takeaway:** find the scale factor from a matching pair, then apply it to **every** length. Perimeter scales by k; area would scale by k^2.

Q16 — Medium — Answer: H

Topic: A missing side in similar triangles

Find the scale factor from the pair you know:

$$k = \frac{12}{3} = 4$$

Apply it to the other side:

$$5 \times 4 = 20$$

Why each wrong option is wrong:

- **F,** 14 — added the **difference** (9) rather than multiplying by the ratio. Similar figures scale by multiplication, not addition — adding a fixed amount would change the shape.
- **G,** 15 — used 3 as the scale factor.
- **J,** $\frac{5}{4}$ — divided instead of multiplying, shrinking the larger triangle.

> **Takeaway:** similar figures scale by a **ratio**, never by a difference. Find the factor from a known pair, then apply it.

Q17 — Hard — Answer: C

Topic: Areas of similar figures

Lengths are in the ratio $2 : 5$, so **both** dimensions scale by $\frac{5}{2}$ going from the smaller to the larger. Area is length × width, so the factor applies twice:

$$\text{area ratio} = 2^2 : 5^2 = 4 : 25$$

Why each wrong option is wrong:

- **A,** $\frac{2}{5}$ — used the length ratio unchanged.
- **B,** $\frac{4}{5}$ — squared only the 2.
- **D,** $\frac{8}{125}$ — cubed, which is the factor for **volumes**.

> **Takeaway:** lengths scale by k, areas by k^2, volumes by k^3. Square **both** parts of the ratio.

Q18 — Medium — Answer: F

Topic: Congruent triangles

Three pairs of equal sides is the **SSS** condition, and it makes the triangles **congruent** — identical in every respect, not merely the same shape.

So the corresponding angles are equal **and** the areas are equal. A triangle's three sides fix it completely: there is no way to build two different triangles from the same three lengths.

Why each wrong option is wrong:

- **G, Their corresponding angles are equal but their areas may differ** — describes **similar** triangles, which share angles but may differ in size. Congruent is stronger.
- **H, Their areas are equal but their angles may differ** — equal areas do not on their own force equal angles, but here the angles are equal too.

- **J, Neither the angles nor the areas need match** — both do match.

> **Takeaway: similar** means same shape, possibly different size. **Congruent** means identical. Three equal sides give congruence, and congruence gives you everything.

Q19 — Medium — Answer: A

Topic: An angle in a regular polygon

Total interior angles:

$$(8-2)\times 180 = 6\times 180 = 1080^\circ$$

Shared equally between eight angles:

$$\frac{1080}{8} = 135^\circ$$

Why each wrong option is wrong:

- **B,** 45 — gave the **exterior** angle, $\frac{360}{8} = 45^\circ$. Note $135 + 45 = 180$, as they sit on a straight line together.
- **C,** 1080 — gave the total rather than one angle.
- **D,** 120 — used a hexagon's answer.

> **Takeaway:** $(n-2)\times 180$ is the **total**; divide by n for one angle. Exterior angles always total 360°, so each is $\frac{360}{n}$.

Q20 — Medium — Answer: H

Topic: The area of a parallelogram

The area of a parallelogram is base times **perpendicular** height:

$$15 \times 8 = 120 \text{ square centimetres}$$

The 10 cm slanted side is there to be ignored. Height always means the perpendicular distance between the two parallel sides, never the sloping edge — and on the ACT that extra length is offered precisely because it is tempting.

Why each wrong option is wrong:

- **F,** 150 — used the slanted side, which overstates the area. The perpendicular height is always the **shorter** of the two.

- **G,** 60 — halved it, which is the **triangle** formula. A parallelogram is two triangles, so it does not carry the half.
- **J,** 46 — added the two lengths, giving something in the wrong units entirely.

> **Takeaway:** a parallelogram is base × perpendicular height, with no half. A slanted side given alongside the height is there to be left alone.

Chapter 6 — Trigonometry

ACT counts trigonometry inside Geometry, but it deserves its own chapter because **this is where the ACT goes furthest beyond the SAT**. The SAT tests right triangles and almost nothing else. The ACT adds the law of sines, the law of cosines, radians, identities, and the graphs of sine and cosine.

There are usually three or four trigonometry questions on a paper. They are completely learnable — the whole topic rests on one mnemonic and three formulas, and the ACT prints none of them.

Topics covered: SOHCAHTOA · finding a side · finding an angle · the special right triangles · trigonometric identities · the law of sines · the law of cosines · degrees and radians · the graphs of sine and cosine

---

Section 1 — Right-Triangle Trigonometry

Label the sides **relative to the angle you are using**:

- the **hypotenuse** is always opposite the right angle — the longest side
- the **opposite** side is across the triangle from your angle
- the **adjacent** side is the other one touching your angle

Then:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

SOH-CAH-TOA. Sine is Opposite over Hypotenuse, Cosine is Adjacent over Hypotenuse, Tangent is Opposite over Adjacent.

The word "opposite" is the trap: opposite *the angle you are working with*, which changes when you switch angles. Always mark your angle on the sketch first.

Q1 — Basic

A ramp is modelled by right triangle ABC with the right angle at C. The sloping edge AB measures 13 units and the side BC, which lies opposite angle A, measures 5 units. What is the value of sin A?

A) $\frac{12}{13}$

B) $\frac{5}{12}$

C) $\frac{13}{5}$

D) $\frac{5}{13}$

Q2 — Medium

A builder is installing a wheelchair ramp that rises at an angle of 12° to the horizontal ground, and the sloping surface measures 9 metres from end to end. Building regulations require the vertical rise to be recorded. Which expression gives that height, in metres?

F) $9\cos 12^\circ$

G) $9\sin 12^\circ$

H) $\dfrac{9}{\sin 12^\circ}$

J) $9\tan 12^\circ$

Q3 — Medium

A support bracket is cut as a 30°–60°–90° triangle. The side opposite the 30° angle has length 7. What is the length of the hypotenuse?

A) $7\sqrt{3}$

B) 14

C) $\frac{7}{2}$

D) $7\sqrt{2}$

Q4 — Hard

A calculation gives $\tan\alpha = \dfrac{5}{12}$ and $\sin\alpha = \dfrac{5}{13}$ for an acute angle α, and $\cos\alpha$ is needed. What is $\cos\alpha$?

F) $\frac{5}{13}$

G) $\frac{13}{12}$

H) $\frac{5}{12}$

J) $\frac{12}{13}$

Section 2 — Beyond Right Triangles

When a triangle has **no right angle**, SOHCAHTOA does not apply. Two formulas cover every such case, and the ACT expects both from memory.

The law of sines — use it when you have an angle and the side opposite it:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The law of cosines — use it when you have two sides and the angle between them, or all three sides:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Notice that if $C = 90^\circ$ then $\cos C = 0$ and the law of cosines collapses to $c^2 = a^2 + b^2$ — Pythagoras. It is the general version of a rule you already know.

Which to use: if you can pair a side with the angle opposite it, use sines. Otherwise use cosines.

Q5 — Hard

A surveyor needs the distance across a lake. Two sides of a triangular plot measure 7 and 9 metres, and the angle between those two sides is 60°. What is the length of the third side?

A) 16

B) $\sqrt{67}$

C) $\sqrt{130}$

D) 4

Q6 — Hard

In triangle PQR, angle P measures 30°, angle Q measures 45°, and the side q opposite angle Q measures 10 units. The side p opposite angle P is required. What is the length of p?

F) 15

G) $10\sqrt{2}$

H) 5

J) $5\sqrt{2}$

Section 3 — Radians and Graphs

A **radian** is another unit for angle, like measuring in metres instead of feet. The one conversion to know:

$$\pi \text{ radians} = 180^\circ$$

So to go from degrees to radians, multiply by $\dfrac{\pi}{180}$; the other way, multiply by $\dfrac{180}{\pi}$. If the answer has a π in it, it is in radians.

For **graphs** of $y = a\sin(bx)$:

- the **amplitude** is $|a|$ — the height from the middle to a peak
- the **period** is $\dfrac{2\pi}{b}$ — how far along before the wave repeats

---

Q7 — Medium

An angle of 40° has to be written in radians for a calculation that expects that unit. What is 40° in radians?

A) 40π

B) $\frac{9}{2\pi}$

C) $\frac{2\pi}{9}$

D) $\frac{2}{9}$

Q8 — Medium

A tide is modelled by $y = 3\sin(2x)$. What are the amplitude and the period of this graph?

F) $(3,\ \pi)$

G) $(3,\ 2\pi)$

H) $(2,\ 3)$

J) $(6,\ \pi)$

Q9 — Medium

For an acute angle θ, $\sin\theta = \dfrac{3}{5}$, and $\cos\theta$ is required. What is $\cos\theta$?

A) $\frac{5}{4}$

B) $\frac{3}{4}$

C) $\frac{2}{5}$

D) $\frac{4}{5}$

Q10 — Hard

For an acute angle β, $\cos\beta = \dfrac{8}{17}$ and $\sin\beta = \dfrac{15}{17}$. What is $\tan\beta$?

F) $\frac{8}{15}$

G) $\frac{15}{17}$

H) $\frac{15}{8}$

J) $\frac{23}{17}$

Q11 — Medium

A surveyor stands 40 metres from the base of a tower on level ground and measures the angle to its top as 35°. Which expression gives the tower's height in metres?

A) $40\sin 35^\circ$

B) $40\tan 35^\circ$

C) $40\cos 35^\circ$

D) $\dfrac{40}{\tan 35^\circ}$

Q12 — Hard

All three sides of a triangle are known: 6, 7 and 9 units. What is the value of $\cos C$ for that angle?

F) $\frac{9}{13}$

G) $-\frac{1}{21}$

H) $\frac{1}{21}$

J) 0

Q13 — Medium

A ladder 10 metres long leans against a wall at 65° to the horizontal ground. Which expression gives that distance, in metres?

A) $10\cos 65^\circ$

B) $10\sin 65^\circ$

C) $10\tan 65^\circ$

D) $\dfrac{10}{\cos 65^\circ}$

Q14 — Hard

A right triangle has a side of 7 opposite an angle θ and a hypotenuse of 25. Which expression gives θ?

F) $\sin\left(\frac{7}{25}\right)$

G) $\sin^{-1}\left(\frac{25}{7}\right)$

H) $\sin^{-1}\left(\frac{7}{25}\right)$

J) $\cos^{-1}\left(\frac{7}{25}\right)$

Q15 — Medium

A square gate is braced by a diagonal, forming a 45°–45°–90° triangle whose two equal sides each measure 6 units. What is the length of the hypotenuse?

A) 12

B) $6\sqrt{3}$

C) $6\sqrt{2}$

D) $3\sqrt{2}$

Q16 — Medium

An angle is given as $\dfrac{5\pi}{6}$ radians and must be reported in degrees. What is the angle in degrees?

F) 30

G) 150

H) 300

J) 216

Q17 — Hard

A tide is modelled by $y = 5\cos(3x)$. What is the period of this graph?

A) $\frac{2\pi}{3}$

B) 2π

C) 6π

D) 3

Q18 — Medium

A temperature model is $y = 4\sin(x) + 20$. What are the maximum and minimum values of y?

F) $(24,\ 16)$

G) $(20,\ 4)$

H) $(4,\ -4)$

J) $(80,\ 20)$

Q19 — Medium

For an acute angle θ, $\cos\theta = \dfrac{7}{25}$, and $\sin\theta$ is required. What is $\sin\theta$?

A) $\frac{24}{25}$

B) $\frac{18}{25}$

C) $\frac{25}{7}$

D) $\frac{7}{24}$

Q20 — Hard

An expression in a model reduces to $\frac{\sin\theta}{\cos\theta} \times \cos\theta$, where $\cos\theta \neq 0$. Which of the following is the expression equal to?

F) $\sin\theta$

G) $\cos\theta$

H) $\tan\theta$

J) 1

Q21 — Hard

An expression reduces to $\sin^2\theta + \cos^2\theta + 3$. What is the value of the expression?

A) 3

B) 5

C) 4

D) 6

Q22 — Medium

A cable runs from the top of a 12-metre mast to a point on the ground, making an angle of 40° with the ground. Which expression gives the cable's length, in metres?

F) $\frac{12}{\sin 40^\circ}$

G) $12\sin 40^\circ$

H) $\frac{12}{\cos 40^\circ}$

J) $12\tan 40^\circ$

Answers and Explanations

Section 1 — Right-Triangle Trigonometry

Q1 — Basic — Answer: D

Topic: Finding a trigonometric ratio in a right triangle

The right angle is at C, so the hypotenuse is $AB = 13$ — the side opposite the right angle.

For angle A, the **opposite** side is $BC = 5$ (the question says so).

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{5}{13}$$

The third side is $AC = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$, which is where the other options come from. 5, 12, 13 is a Pythagorean triple worth memorising.

Why each wrong option is wrong:

- **A,** $\frac{12}{13}$ — $\frac{12}{13}$ is $\cos A$: adjacent over hypotenuse.
- **B,** $\frac{5}{12}$ — $\frac{5}{12}$ is $\tan A$: opposite over adjacent.
- **C,** $\frac{13}{5}$ — inverted the ratio. Sine and cosine are always **at most 1**, because the hypotenuse is the longest side, so any answer bigger than 1 is wrong immediately.

> **Takeaway:** mark the angle, label opposite / adjacent / hypotenuse relative to **it**, then apply SOHCAHTOA. Sine and cosine can never exceed 1.

Q2 — Medium — Answer: G

Topic: Using a ratio to find a side

Sketch it: the ramp is the **hypotenuse** (9 m), the vertical rise is **opposite** the 12° angle, and the ground is adjacent.

Opposite and hypotenuse means **sine**:

$$\sin 12^\circ = \frac{\text{height}}{9}$$

Multiply both sides by 9:

$$\text{height} = 9 \sin 12^\circ$$

Sanity check without a calculator: $\sin 12^\circ$ is a small number well below 1, so the height is a small fraction of 9 metres. That is what a gentle ramp should be.

Why each wrong option is wrong:

- **F,** $9\cos 12^\circ$ — cosine gives the horizontal distance along the ground, not the rise.
- **H,** $\dfrac{9}{\sin 12^\circ}$ — dividing by $\sin 12^\circ$ gives about 43 metres, which is far longer than the ramp itself. Impossible: a leg cannot exceed the hypotenuse.
- **J,** $9\tan 12^\circ$ — tangent uses the adjacent side, which is not given here.

> **Takeaway:** pick the ratio that uses the side you **have** and the side you **want**. Then check the size: a leg is always shorter than the hypotenuse.

Q3 — Medium — Answer: B

Topic: A special right triangle

In a 30°–60°–90° triangle the sides are always in the ratio

$$1 : \sqrt{3} : 2$$

with the **1 opposite the** 30°, the $\sqrt{3}$ opposite the 60°, and the **2 as the hypotenuse**.

Here the side opposite 30° is 7, so that is the "1" of the ratio and everything scales by 7:

$$\text{hypotenuse} = 2 \times 7 = 14$$

The shortcut worth remembering: **the hypotenuse is twice the shortest side**, and the shortest side is always the one facing the smallest angle.

Why each wrong option is wrong:

- **A,** $7\sqrt{3}$ — gave the side opposite 60°, which is $7\sqrt{3}$.
- **C,** $\frac{7}{2}$ — halved rather than doubled. The hypotenuse is the longest side, so it cannot be smaller than a leg.
- **D,** $7\sqrt{2}$ — used the 45°–45°–90° ratio $1 : 1 : \sqrt{2}$.

> **Takeaway:** 30–60–90 gives $1 : \sqrt{3} : 2$, and 45–45–90 gives $1 : 1 : \sqrt{2}$. Match the side you are given to its place in the ratio, then scale.

Q4 — Hard — Answer: J

Topic: A trigonometric identity

The identity that links all three:

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

Rearrange it to make $\cos\alpha$ the subject:

$$\cos\alpha = \frac{\sin\alpha}{\tan\alpha} = \frac{5/13}{5/12} = \frac{5}{13} \times \frac{12}{5} = \frac{12}{13}$$

Check with the other identity, $\sin^2 + \cos^2 = 1$:

$$\left(\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2 = \frac{25+144}{169} = \frac{169}{169} = 1 \checkmark$$

Why each wrong option is wrong:

- **F,** $\frac{5}{13}$ and **H,** $\frac{5}{12}$ — repeated a quantity already given.
- **G,** $\frac{13}{12}$ — divided tangent by sine instead of sine by tangent, giving $\frac{13}{12}$. A cosine can never exceed 1, so this is wrong on sight.

> **Takeaway:** $\tan = \frac{\sin}{\cos}$ and $\sin^2 + \cos^2 = 1$. Between them they connect any two ratios. And sine and cosine are never above 1 — use that to reject an answer instantly.

Section 2 — Beyond Right Triangles

Q5 — Hard — Answer: B

Topic: The law of cosines

Two sides and the angle **between** them: that is the law of cosines.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

with $a = 7$, $b = 9$ and $C = 60^\circ$. Since $\cos 60^\circ = \frac{1}{2}$:

$$c^2 = 49 + 81 - 2(7)(9)\left(\tfrac{1}{2}\right) = 130 - 63 = 67$$

$$c = \sqrt{67} \approx 8.19$$

Sanity check: the third side of a triangle must be less than the sum of the other two (16) and more than their difference (2). 8.19 sits comfortably between. Correct.

Why each wrong option is wrong:

- **A,** 16 — added the two sides, which is the absolute maximum a third side could approach and never reach.
- **C,** $\sqrt{130}$ — used Pythagoras, which assumes a right angle. The angle here is 60°, so the $-2ab\cos C$ term cannot be dropped.
- **D,** 4 — subtracted the sides.

> **Takeaway:** two sides and the angle between them means the law of cosines. The $-2ab\cos C$ term is the correction that turns Pythagoras into a rule for any triangle.

Q6 — Hard — Answer: J

Topic: The law of sines

Angles paired with their opposite sides: use the law of sines.

$$\frac{p}{\sin P} = \frac{q}{\sin Q}$$

$$\frac{p}{\sin 30^\circ} = \frac{10}{\sin 45^\circ}$$

$$p = \frac{10\sin 30^\circ}{\sin 45^\circ} = \frac{10 \times \frac{1}{2}}{\frac{\sqrt{2}}{2}} = \frac{5}{\frac{\sqrt{2}}{2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2} \approx 7.07$$

Sanity check: angle P (30°) is smaller than angle Q (45°), so the side opposite P must be **shorter** than the side opposite Q. $7.07 < 10$. Correct.

Why each wrong option is wrong:

- **G,** $10\sqrt{2}$ — inverted the ratio, giving about 14.1. That would make the side opposite the smaller angle the longer one, which the sanity check rejects.
- **H,** 5 — halved 10 because 30 is half of 60. Sines are not proportional to angles.
- **F,** 15 — combined the angle numbers arithmetically.

> **Takeaway:** the law of sines needs a side paired with its opposite angle. Then check the ordering: the **bigger angle always faces the longer side**.

Section 3 — Radians and Graphs

Q7 — Medium — Answer: C

Topic: Converting degrees to radians

Multiply by $\dfrac{\pi}{180}$:

$$40 \times \frac{\pi}{180} = \frac{40\pi}{180} = \frac{2\pi}{9}$$

Cancel carefully: $\frac{40}{180}$ divides top and bottom by 20 to give $\frac{2}{9}$.

Sanity check: $\frac{2\pi}{9}$ is about 0.70 radians. A right angle is $\frac{\pi}{2} \approx 1.57$ radians, and 40° is less than 90°, so an answer below 1.57 is right.

Why each wrong option is wrong:

- **A,** 40π — multiplied by π without dividing by 180.
- **B,** $\frac{9}{2\pi}$ — used the conversion the other way round.
- **D,** $\frac{2}{9}$ — divided by 180 and lost the π, leaving a number in no unit at all.

> **Takeaway:** degrees to radians multiplies by $\frac{\pi}{180}$. The π stays in the answer — if it has vanished, the conversion went the wrong way.

Q8 — Medium — Answer: F

Topic: The amplitude and period of a sine graph

For $y = a\sin(bx)$:

$$\text{amplitude} = |a| = 3$$
$$\text{period} = \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$$

The amplitude is the coefficient **outside**, and it stretches the wave vertically. The number **inside**, multiplying x, squeezes it horizontally — and it squeezes rather than stretches, so a larger b gives a **shorter** period.

Why each wrong option is wrong:

- **H,** $(2,\ 3)$ — swapped the roles of the two coefficients.
- **G,** $(3,\ 2\pi)$ — gave 2π, the period of $\sin x$ with no coefficient inside.
- **J,** $(6,\ \pi)$ — doubled the amplitude, applying the inside coefficient outside.

> **Takeaway:** outside sets the height, inside sets the width, and the period is $\frac{2\pi}{b}$ — so the bigger the number inside, the **faster** the wave repeats.

Q9 — Medium — Answer: D

Topic: The Pythagorean identity

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \left(\tfrac{3}{5}\right)^2 = 1 - \tfrac{9}{25} = \tfrac{16}{25}$$

$$\cos\theta = \tfrac{4}{5}$$

The angle is acute, so the cosine is positive. This is the 3-4-5 triangle in disguise.

Why each wrong option is wrong:

- **B,** $\frac{3}{4}$ — gave $\tan\theta = \frac{3}{4}$.
- **C,** $\frac{2}{5}$ — subtracted the ratios without squaring them.
- **A,** $\frac{5}{4}$ — inverted the answer, giving a cosine above 1, which is impossible.

> **Takeaway:** $\sin^2 + \cos^2 = 1$ links the two directly. Square, subtract, root — and expect an answer no greater than 1.

Q10 — Hard — Answer: H

Topic: A reciprocal trigonometric identity

$$\tan\beta = \frac{\sin\beta}{\cos\beta} = \frac{15/17}{8/17} = \frac{15}{17} \times \frac{17}{8} = \frac{15}{8}$$

The 17s cancel, which is why this is quicker than rebuilding the triangle.

Check with the other identity: $\left(\frac{15}{17}\right)^2 + \left(\frac{8}{17}\right)^2 = \frac{225+64}{289} = 1.$ ✓

Why each wrong option is wrong:

- **F,** $\frac{8}{15}$ — divided cosine by sine, giving $\cot\beta$.
- **G,** $\frac{15}{17}$ — repeated the sine.
- **J,** $\frac{23}{17}$ — added the two ratios.

> **Takeaway:** $\tan = \frac{\sin}{\cos}$, in that order. Unlike sine and cosine, a tangent **may** exceed 1.

Q11 — Medium — Answer: B

Topic: A right triangle and the tangent ratio

Opposite and adjacent means **tangent**:

$$\tan 35^\circ = \frac{\text{height}}{40} \quad\Longrightarrow\quad \text{height} = 40\tan 35^\circ$$

Sanity check: 35° is less than 45°, so the tangent is below 1 and the tower is shorter than the 40 metres of ground. That rules out **D** immediately.

Why each wrong option is wrong:

- **A,** $40\sin 35^\circ$ and **C,** $40\cos 35^\circ$ — sine and cosine both need the hypotenuse, which is not given here.
- **D,** $\dfrac{40}{\tan 35^\circ}$ — dividing by the tangent gives about 57 metres, taller than the distance, which the 35° check rejects.

> **Takeaway:** two legs and no hypotenuse means tangent. Below 45° the tangent is under 1; above it, over 1 — a free size check.

Q12 — Hard — Answer: H

Topic: The law of cosines to find an angle

Start from the law of cosines with $c = 9$ the side opposite C:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$81 = 36 + 49 - 84\cos C$$

$$84\cos C = 85 - 81 = 4 \quad\Longrightarrow\quad \cos C = \frac{4}{84} = \frac{1}{21}$$

The cosine is small and **positive**, so C is a little under 90° — which makes sense, since $9^2 = 81$ is only just less than $6^2 + 7^2 = 85$.

Why each wrong option is wrong:

- **F,** $\frac{9}{13}$ — invented a ratio of sides with no formula behind it.
- **G,** $-\frac{1}{21}$ — subtracted in the wrong order, giving a negative cosine and so an obtuse angle. The check above shows the angle is acute.
- **J,** 0 — a cosine of 0 would mean exactly 90°, which needs $9^2 = 6^2 + 7^2$. It is not: 81 against 85.

> **Takeaway:** the law of cosines works in both directions. Rearranged it gives $\cos C = \frac{a^2+b^2-c^2}{2ab}$, and the sign of the answer tells you whether the angle is acute or obtuse.

Q13 — Medium — Answer: A

Topic: Finding a leg with the cosine ratio

Sketch it. The ladder is the **hypotenuse** (10 m), the ground distance is **adjacent** to the 65° angle, and the wall is opposite.

Adjacent and hypotenuse means **cosine**:

$$\cos 65^\circ = \frac{\text{distance}}{10} \quad \Longrightarrow \quad \text{distance} = 10\cos 65^\circ$$

Since 65° is a steep angle, the cosine is small, so the foot of the ladder sits close to the wall — about 4.2 metres. That is what a steep ladder should do.

Why each wrong option is wrong:

- **B,** $10\sin 65^\circ$ — sine gives the height reached **up the wall**, not the distance along the ground.
- **C,** $10\tan 65^\circ$ — tangent needs the opposite and adjacent sides, and the hypotenuse is what is given.
- **D,** $\dfrac{10}{\cos 65^\circ}$ — dividing gives about 24 metres, longer than the ladder. A leg cannot exceed the hypotenuse.

> **Takeaway:** pick the ratio joining the side you **have** to the side you **want**. Then check the size against the hypotenuse.

Q14 — Hard — Answer: H

Topic: Finding an angle from two sides

Opposite over hypotenuse is the **sine**:

$$\sin\theta = \frac{7}{25}$$

To get θ itself, apply the inverse:

$$\theta = \sin^{-1}\left(\frac{7}{25}\right)$$

Why each wrong option is wrong:

- **F,** $\sin\left(\frac{7}{25}\right)$ — no inverse, so it computes the sine **of** a number rather than the angle whose sine is that number.
- **G,** $\sin^{-1}\left(\frac{25}{7}\right)$ — $\frac{25}{7}$ is greater than 1, and no angle has a sine above 1, so this expression has no value at all.
- **J,** $\cos^{-1}\left(\frac{7}{25}\right)$ — cosine uses the adjacent side, which here is 24, not 7.

> **Takeaway:** to find a **side** use sin, cos or tan; to find an **angle** use their inverses. A sine or cosine above 1 is a signal you have inverted the ratio.

Q15 — Medium — Answer: C

Topic: A 45-45-90 triangle

A 45–45–90 triangle has sides in the ratio

$$1 : 1 : \sqrt{2}$$

with the $\sqrt{2}$ on the hypotenuse. Both legs are 6, so everything scales by 6:

$$\text{hypotenuse} = 6\sqrt{2} \approx 8.49$$

Check with Pythagoras: $6^2 + 6^2 = 72$, and $\sqrt{72} = 6\sqrt{2}$. ✓

Why each wrong option is wrong:

- **A,** 12 — doubled the leg, which is the 30–60–90 rule.
- **B,** $6\sqrt{3}$ — used $\sqrt{3}$, also from the other special triangle.
- **D,** $3\sqrt{2}$ — gave a hypotenuse shorter than the legs, which is impossible.

> **Takeaway:** 45–45–90 is $1 : 1 : \sqrt{2}$; 30–60–90 is $1 : \sqrt{3} : 2$. Keep the two apart by remembering which has two equal sides.

Q16 — Medium — Answer: G

Topic: Converting radians to degrees

Multiply by $\dfrac{180}{\pi}$:

$$\frac{5\pi}{6} \times \frac{180}{\pi} = \frac{5 \times 180}{6} = 150^\circ$$

The π cancels, which is the sign the conversion is going the right way — an answer in degrees should have no π left in it.

Sanity check: π is 180°, so $\frac{5\pi}{6}$ is a little less than that. 150 fits.

Why each wrong option is wrong:

- **F,** 30 — divided 180 by 6 and forgot the 5.
- **H,** 300 — used 360° instead of 180°.
- **J,** 216 — inverted the fraction.

> **Takeaway:** radians to degrees multiplies by $\frac{180}{\pi}$, and the π must cancel. If a π survives, the conversion went the wrong way.

Q17 — Hard — Answer: A

Topic: The period of a trigonometric graph

For $y = a\cos(bx)$ the period is

$$\frac{2\pi}{b} = \frac{2\pi}{3}$$

The 5 outside sets the **amplitude** and has no effect on the period at all.

Note the direction: a **bigger** number inside gives a **shorter** period, because it squeezes the wave. Three full cycles now fit where one used to.

Why each wrong option is wrong:

- **B,** 2π — the period of $\cos x$, ignoring the coefficient.
- **C,** 6π — multiplied by 3 rather than dividing, stretching the wave when it should be squeezed.
- **D,** 3 — gave the coefficient.

> **Takeaway:** period is $\frac{2\pi}{b}$, where b is the number multiplying x. Outside sets height, inside sets width — and inside works in reverse.

Q18 — Medium — Answer: F

Topic: The amplitude and midline of a trigonometric graph

$\sin x$ runs between -1 and 1. Multiplying by 4 makes it run between -4 and 4, and adding 20 lifts the whole thing:

$$\text{maximum} = 20 + 4 = 24 \qquad \text{minimum} = 20 - 4 = 16$$

The 20 is the **midline** — the level the wave oscillates about — and the 4 is the **amplitude**, the swing either side of it.

Why each wrong option is wrong:

- **G,** $(20,\ 4)$ — gave the two coefficients rather than the values they produce.
- **H,** $(4,\ -4)$ — ignored the $+20$, so the wave sits about zero.
- **J,** $(80,\ 20)$ — multiplied the two numbers.

> **Takeaway:** for $y = a\sin(x) + c$, the graph runs from $c - a$ to $c + a$. The constant is the midline; the coefficient is the swing.

Q19 — Medium — Answer: A

Topic: The Pythagorean identity rearranged

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta = 1 - \left(\tfrac{7}{25}\right)^2 = 1 - \tfrac{49}{625} = \tfrac{576}{625}$$
$$\sin\theta = \tfrac{24}{25}$$

The angle is acute, so the sine is positive. This is the 7-24-25 triangle.

Why each wrong option is wrong:

- **B,** $\frac{18}{25}$ — subtracted the ratio without squaring it.
- **C,** $\frac{25}{7}$ — inverted the cosine, giving a value above 1, which no sine can take.
- **D,** $\frac{7}{24}$ — gave the tangent, $\frac{24}{7}$ inverted.

> **Takeaway:** square, subtract from 1, root. And use "no more than 1" to reject an answer before doing any arithmetic.

Q20 — Hard — Answer: F

Topic: Simplifying a trigonometric expression

The fraction is the definition of the tangent:

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

So the expression is $\tan\theta \times \cos\theta$. Writing the tangent back out shows the cancellation:

$$\frac{\sin\theta}{\cos\theta} \times \cos\theta = \sin\theta$$

Check at $\theta = 30^\circ$: $\tan 30^\circ \approx 0.577$ and $\cos 30^\circ \approx 0.866$, and their product is $0.5 = \sin 30^\circ$. ✓

Why each wrong option is wrong:

- **G,** $\cos\theta$ — cancelled the sine instead of the cosine.
- **H,** $\tan\theta$ — stopped at the tangent without multiplying.
- **J,** 1 — cancelled everything, which would need the numerator and denominator to be identical.

> **Takeaway:** $\tan = \frac{\sin}{\cos}$ works in both directions. When an expression mixes the three ratios, rewrite the tangent as a fraction and look for what cancels.

Q21 — Hard — Answer: C

Topic: An identity used to simplify a sum

$$\sin^2\theta + \cos^2\theta = 1 \qquad \text{for every } \theta$$

So the expression is

$$1 + 3 = 4$$

no matter what θ is. That is what makes it an **identity** rather than an equation: it holds everywhere, so no value of θ needs to be found.

Why each wrong option is wrong:

- **A,** 3 — treated the two squared terms as zero.
- **B,** 5 — counted each term as 1, giving $1 + 1 + 3$. The identity says their **sum** is 1, not each of them.
- **D,** 6 — doubled the constant.

> **Takeaway:** spot $\sin^2 + \cos^2$ and replace the pair with **1** immediately. It is the most useful identity on the paper.

Q22 — Medium — Answer: F

Topic: Finding the hypotenuse with a trigonometric ratio

Sketch it. The mast (12 m) is opposite the 40° angle, and the cable is the hypotenuse. Opposite over hypotenuse is the **sine**:

$$\sin 40^\circ = \frac{12}{\text{cable}} \quad \Longrightarrow \quad \text{cable} = \frac{12}{\sin 40^\circ}$$

Here the unknown is on the **bottom**, so the equation is rearranged by dividing rather than multiplying — the opposite of the usual case.

Sanity check: a hypotenuse is always the longest side, so the answer must exceed 12. Dividing by $\sin 40^\circ \approx 0.64$ gives about 18.7 metres. ✓

Why each wrong option is wrong:

- **G,** $12 \sin 40^\circ$ — multiplying gives about 7.7 metres, shorter than the mast it is attached to. Impossible for a hypotenuse.
- **H,** $\dfrac{12}{\cos 40^\circ}$ — cosine uses the adjacent side, which is the ground distance and is not given.
- **J,** $12 \tan 40^\circ$ — tangent does not involve the hypotenuse at all.

> **Takeaway:** when the unknown is the **hypotenuse**, it ends up on the bottom of the ratio, so you divide. Check the answer is the longest side.

Chapter 7 — Statistics and Probability

Statistics and Probability is 12 to 15% of the test — six questions. Very little of it is calculation. Most of it is **reading**: a table, a chart, or a sentence describing how data was collected.

That makes it the category where careless reading costs most. A two-way table question is arithmetic a ten-year-old could do; the difficulty is entirely in working out which row, which column, and which total the question wants.

Topics covered: mean, median and mode · range and standard deviation · outliers · reading two-way and frequency tables · Venn diagrams · scatterplots and lines of best fit · probability · complements · mutually exclusive events · counting · samples and study design

---

Section 1 — Centre and Spread

Four words, and the ACT expects all four:

- **mean** — the total divided by how many. Uses every value's size.
- **median** — the middle value **once sorted**. Uses only position.
- **mode** — the most common value.
- **range** — largest minus smallest. A crude measure of spread.

Standard deviation measures how far the values sit from their own mean, on average. You will almost never have to calculate one on the ACT — you will have to **compare** two, and for that a single idea is enough:

> More spread out means a bigger standard deviation. Values bunched near the mean means a smaller one.

Q1 — Medium

Two laboratories each recorded five measurements of the same quantity. Set P is $20,\ 20,\ 20,\ 20,\ 20$ and set Q is $10,\ 15,\ 20,\ 25,\ 30$. Both sets have the same mean. Which statement is true?

A) Set P has the greater standard deviation

B) Set Q has the greater standard deviation

C) The two standard deviations are equal

D) Standard deviation cannot be compared without calculating it

---

Q2 — Medium

A set of readings is $12,\ 14,\ 15,\ 13,\ 16$. A sixth reading of 60 is then recorded. Which statement about the effect of that value is true?

F) Neither the mean nor the median changes

G) The median rises far more than the mean does

H) Both rise by the same amount

J) The mean rises far more than the median does

Section 2 — Reading Tables

Most ACT statistics questions are a table and a careful sentence. The method is always the same three steps:

1. Find the **group** the question restricts you to — that is your denominator.
2. Find the **count** it asks about — that is your numerator.
3. Check the totals add up before you trust the table.

The word to watch is **"of"**. "Of the students who cycle" restricts you to the cyclists' row; the grand total is then irrelevant.

Q3 — Medium

A school recorded which of its 120 students study Spanish and which study history, and set the counts out in the table below.

	Studies history	Does not study history	Total
Studies Spanish	45	25	70
Does not study Spanish	20	30	50
Total	65	55	120

A student who studies Spanish is chosen at random. What is the probability that the student also studies history?

A) $\frac{3}{8}$

B) $\frac{7}{12}$

C) $\frac{5}{14}$

D) $\frac{9}{14}$

Q4 — Medium

A tourist survey recorded how many cities each visitor went to during their trip, and the results are summarised below.

Cities visited	1	2	3	4
Number of tourists	30	40	50	20

What is the mean number of cities visited per tourist?

F) $\frac{5}{2}$

G) 35

H) $\frac{17}{7}$

J) 3

Q5 — Hard

On a farm survey, 34 farms raise cows, 28 raise sheep, and 12 raise both. Every farm surveyed raises at least one of the two. How many farms were surveyed?

A) 62

B) 50

C) 74

D) 38

Section 3 — Probability

$$P(\text{event}) = \frac{\text{outcomes you want}}{\text{outcomes possible}}$$

Every probability is between 0 and 1. An answer outside that range is wrong without further thought.

Three ideas the ACT tests:

- **the complement** — $P(\text{not } A) = 1 - P(A)$. Often far quicker than counting what you want directly.
- **mutually exclusive** events cannot both happen, so $P(A \text{ or } B) = P(A) + P(B)$.
- **independent** events do not affect each other, so $P(A \text{ and } B) = P(A) \times P(B)$.

"Or" tends to add; "and" tends to multiply.

Q6 — Basic

A bag used in a demonstration holds counters identical apart from colour, and the counts are recorded below.

Colour	Red	Blue	Green	Total
Number of counters	5	7	8	20

One counter is drawn at random. What is the probability that it is **not** green?

F) $\frac{2}{5}$

G) $\frac{3}{5}$

H) $\frac{2}{3}$

J) $\frac{1}{3}$

Q7 — Medium

Two events A and B are mutually exclusive. The probability of A is 0.3 and the probability of B is 0.45. What is the probability that A or B occurs?

A) $\frac{27}{200}$

B) $\frac{3}{20}$

C) $\frac{3}{4}$

D) 1

Q8 — Hard

A bag holds 3 red counters and 5 blue counters. One counter is drawn and **not** replaced, then a second is drawn. What is the probability that both are red?

F) $\frac{9}{64}$

G) $\frac{3}{4}$

H) $\frac{3}{28}$

J) $\frac{3}{8}$

Section 4 — Samples and Studies

A short but reliable topic. Two rules cover almost every question:

- A conclusion may only be drawn about the **population that was sampled**, and only if the sample was **random**.
- An **observational study** can show that two things go together. Only a **controlled experiment**, with random assignment, can show that one **causes** the other.

Q9 — Hard

Researchers recorded the sleep and the test scores of 500 randomly selected students at one university, and found that students who slept longer tended to score higher. No one was told how much to sleep; the researchers only recorded what was already happening. Which conclusion is best supported?

A) Higher scores cause students to sleep longer

B) Sleeping longer causes students to score higher

C) More sleep is associated with higher scores among students at this university

D) More sleep is associated with higher scores among all adults

Q10 — Medium

A café offers 4 kinds of bread, 3 fillings and 2 sauces, and a sandwich is made by choosing exactly one of each. Each choice is made independently of the others. How many different sandwiches are possible?

F) 14

G) 9

H) 12

J) 24

Q11 — Medium

Two sets of five readings are recorded. Set X is $40,\ 41,\ 42,\ 43,\ 44$ and set Y is $10,\ 25,\ 42,\ 60,\ 73$. Both have the same median. Which statement about their spread is true?

A) Neither range nor standard deviation can be compared here

B) Set X has the greater range but set Y the greater standard deviation

C) The two sets have the same range, because they share a median

D) Set Y has both the greater range and the greater standard deviation

---

Q12 — Hard

A researcher surveys 300 randomly selected students at one university and finds that 42% support a proposed change. The sample was random, but it was drawn from a single university. Which conclusion is best supported?

F) About 42% of students at that university support the change

G) About 42% of students nationwide support the change

H) Exactly 42% of students at that university support the change

J) The survey shows the change will improve student results

Q13 — Medium

A researcher wants to establish whether a new revision method **causes** better test results. Which study design would support that conclusion?

A) Survey teachers about which method they believe works better

B) Ask students which method they used and compare their results

C) Compare results at a school that adopted the method with one that did not

D) Randomly assign students to use the new method or the old one, then compare results

---

Q14 — Medium

A clinic recorded the age and height of a group of children and drew a line of best fit through the resulting scatterplot. The line has equation $y = 6x + 78$, where x is age in years and y is height in centimetres. Which is the best interpretation of the number 6?

F) The height of a child at birth, in centimetres

G) The average increase in height, in centimetres, per year of age

H) The age at which a child reaches 6 centimetres

J) The number of children in the study

Q15 — Medium

A line of best fit through a scatterplot is $y = 2x + 5$. One of the data points used to draw it is $(4, 15)$. How does the actual value compare with the predicted one?

A) -2

B) 2

C) 13

D) 0

Q16 — Hard

A scatterplot of 40 towns shows that towns with more bookshops tend to have higher average incomes, and a line of best fit slopes upward. Which conclusion is best supported by the scatterplot alone?

F) Among these towns, more bookshops is associated with higher average income

G) Opening more bookshops would raise a town's average income

H) Higher average income causes towns to open more bookshops

J) Bookshop numbers and income are unrelated in these towns

Q17 — Hard

Four runners finish a race and no two finish together. In how many different orders can they finish?

A) 24

B) 16

C) 10

D) 4

Q18 — Medium

A cinema recorded which of 300 customers bought a snack, split by ticket type.

	Bought a snack	No snack	Total
Adult	84	96	180
Child	78	42	120
Total	162	138	300

What fraction of **all** customers bought a snack?

F) $\frac{7}{15}$

G) $\frac{13}{20}$

H) $\frac{27}{50}$

J) $\frac{27}{23}$

Q19 — Medium

A shop recorded how many items each customer bought during one hour.

Items bought	1	2	3	4
Number of customers	6	9	4	1

What is the median number of items bought?

A) $\frac{5}{2}$

B) 2

C) 9

D) $\frac{3}{2}$

Q20 — Hard

A survey of 50 households recorded the number of cars owned. Twelve households owned no car, twenty owned one, fourteen owned two, and four owned three. What is the mean number of cars per household?

F) 1

G) $\frac{6}{5}$

H) $\frac{3}{2}$

J) 50

Q21 — Medium

A scatterplot of temperature against ice-cream sales has the line of best fit $y = 12x - 40$, where x is the temperature in degrees and y is the number of sales. What does the line predict for a day at 15 degrees?

A) 180

B) -40

C) 140

D) 12

Q22 — Medium

A scatterplot of a car's age against its value has a line of best fit with a **negative** slope. Which statement does that support?

F) Older cars tend to be worth less

G) Older cars tend to be worth more

H) Age and value are unrelated

J) Every older car is worth less than every newer one

Q23 — Hard

A line of best fit is $y = 4x + 2$. One observed data point is $(3, 11)$. What is the residual for that point?

A) -3

B) 3

C) 14

D) 11

Q24 — Medium

A four-digit code is made from the digits 1 to 9, and no digit may be repeated. How many different codes are possible?

F) 6561

G) 36

H) 24

J) 3024

Q25 — Medium

A meal deal offers one of 5 mains, one of 4 sides and one of 3 drinks. Each choice is made independently of the others. How many different meals are possible?

A) 12

B) 23

C) 60

D) 15

Q26 — Hard

Five books are placed in a row on a shelf, and one particular book must go on the far left. The remaining four can go anywhere among the other positions. In how many orders can the books be arranged?

F) 120

G) 20

H) 24

J) 5

Q27 — Medium

A school wants to estimate how much time its 900 students spend on homework. Four methods of choosing 100 students are proposed. Which is most likely to give a representative sample?

A) Pick 100 students at random from the full list of all 900

B) Ask the first 100 students who arrive at school one morning

C) Ask 100 students in the library after school

D) Ask for 100 volunteers to fill in a form

Q28 — Hard

A researcher randomly samples 200 adults **in one city** and finds that 38% cycle to work. Which conclusion is best supported?

F) About 38% of adults in that city cycle to work

G) About 38% of adults in the country cycle to work

H) Exactly 38% of adults in that city cycle to work

J) Cycling to work makes adults healthier

Q29 — Medium

A company tests a new training course by measuring employees' scores before and after it, and finds the scores went up. To claim the course caused the rise, what does the study most need?

A) A comparison group of similar employees who did not take the course

B) A larger number of employees taking the course

C) A second test of the same employees after the course

D) A survey asking employees whether they enjoyed the course

Answers and Explanations

Section 1 — Centre and Spread

Q1 — Medium — Answer: B

Topic: Comparing standard deviations

Both sets have a mean of 20.

Set P is **every value identical**. Nothing deviates from the mean at all, so its standard deviation is exactly **zero** — the only time that happens.

Set Q spreads from 10 to 30, so its values sit well away from 20.

Q therefore has the greater standard deviation.

Why each wrong option is wrong:

- **A, Set P has the greater standard deviation** — has it backwards. A set of identical values cannot be the more spread out.
- **C, The two standard deviations are equal** — equal means do not imply equal spreads. That is exactly what standard deviation exists to distinguish.
- **D, Standard deviation cannot be compared without calculating it** — no calculation is needed. One set has zero spread, and no set can have less than that.

> **Takeaway:** standard deviation is about **spread**, not centre. Identical values give zero. Two sets can share a mean and be nothing alike.

Q2 — Medium — Answer: J

Topic: What an outlier does to the mean and the median

Work out both, before and after.

Before: the five values total 70, so the mean is 14. Sorted they are $12, 13, 14, 15, 16$, so the median is 14 as well.

After: the six values total 130, so the mean is $130 \div 6 = 21.67$. Sorted they are $12, 13, 14, 15, 16, 60$, so the median is $\frac{14+15}{2} = 14.5$.

The mean moved nearly 8; the median moved a half.

The reason is structural. The mean uses the **size** of every value, so one huge number drags it a long way. The median only cares about **position**, and 60 counts for no more than 17 would.

Why each wrong option is wrong:

- **G, The median rises far more than the mean does** — exactly backwards.
- **H, Both rise by the same amount** — they move by very different amounts, which is the point.
- **F, Neither the mean nor the median changes** — both change.

> **Takeaway:** an outlier drags the **mean** and barely moves the **median**. That is why incomes and house prices are reported as medians.

Section 2 — Reading Tables

Q3 — Medium — Answer: D

Topic: Conditional probability from a two-way table

"A student **who studies Spanish** is chosen" restricts the pool to the Spanish row. That row has 70 students, so 70 is the denominator — not 120.

Of those 70, the number who also study history is 45.

$$P = \frac{45}{70} = \frac{9}{14}$$

Why each wrong option is wrong:

- **A,** $\frac{3}{8}$ — used 120, the whole school. That would be the answer to "a student is chosen at random from the school; what is the probability they study both?" — a different question.
- **B,** $\frac{7}{12}$ — gave the probability that a random student studies Spanish.
- **C,** $\frac{5}{14}$ — gave the probability of **not** studying history within the same row. Note it and the answer add to 1, as they must.

> **Takeaway:** the group the question restricts you to is the **denominator**. Underline the "of the ..." phrase before you look at the table.

Q4 — Medium — Answer: H

Topic: Reading a frequency table

A frequency table is a shorthand: 30 tourists each visited 1 city, 40 each visited 2, and so on. To find the mean, add up the **total cities visited** and divide by the **total tourists**.

$$\text{total cities} = 1(30) + 2(40) + 3(50) + 4(20) = 30 + 80 + 150 + 80 = 340$$

$$\text{total tourists} = 30 + 40 + 50 + 20 = 140$$

$$\text{mean} = \frac{340}{140} = \frac{17}{7} \approx 2.43$$

Sanity check: the answer must lie between 1 and 4, and nearer the middle where most tourists are. It does.

Why each wrong option is wrong:

- **F,** $\frac{5}{2}$ — averaged $1, 2, 3, 4$, ignoring how many tourists gave each answer. That treats one tourist and fifty tourists as equally important.
- **G,** 35 — averaged the frequencies instead, which answers nothing.
- **J,** 3 — gave 3, the **mode** — the most common answer, not the mean.

> **Takeaway:** in a frequency table, multiply each value by its frequency, add, then divide by the total frequency. Never average the values on the top row by themselves.

Q5 — Hard — Answer: B

Topic: A Venn diagram in words

Draw two overlapping circles. The 12 farms raising both sit in the overlap, and they have been counted **twice** — once in the 34 and once in the 28. Subtract them once to put that right:

$$34 + 28 - 12 = 50$$

Check by parts: cows only is $34 - 12 = 22$; sheep only is $28 - 12 = 16$; both is 12. And $22 + 16 + 12 = 50$. Correct.

Why each wrong option is wrong:

- **A,** 62 — added without removing the double count.
- **C,** 74 — added the overlap a third time.
- **D,** 38 — subtracted it twice, which removes the "both" farms from the survey altogether.

> **Takeaway:** for two overlapping groups, total $= A + B -$ both. Subtract the overlap **once**, because it was counted twice. Then check by adding the three separate regions.

Section 3 — Probability

Q6 — Basic — Answer: G

Topic: Probability using the complement

Two routes, and they agree.

Count directly: not green means red or blue, which is $5 + 7 = 12$ counters out of 20:

$$\frac{12}{20} = \frac{3}{5}$$

Use the complement: green is $\frac{8}{20}$, so not green is

$$1 - \frac{8}{20} = \frac{12}{20} = \frac{3}{5}$$

Why each wrong option is wrong:

- **F,** $\frac{2}{5}$ — answered the opposite question. Note it and the correct answer add to 1.
- **H,** $\frac{2}{3}$ — compared green with the other two colours rather than with the whole bag. The denominator of a probability is everything possible.
- **J,** $\frac{1}{3}$ — assumed the three colours are equally likely because there are three of them. They are not; the counts differ.

> **Takeaway:** the denominator is **everything possible**. And when a question says "not", the complement is usually the faster route: $1 - P(A)$.

Q7 — Medium — Answer: C

Topic: Mutually exclusive events

Mutually exclusive means the two events never overlap, so nothing gets counted twice and the probabilities simply add:

$$P(A \text{ or } B) = 0.3 + 0.45 = 0.75$$

Note the answer is below 1, as it must be. And it does **not** have to equal 1 — there is a 0.25 chance that neither happens.

Why each wrong option is wrong:

- **A,** $\frac{27}{200}$ — multiplied. Multiplying is for "and" with independent events, and here "and" is impossible: mutually exclusive events have $P(A \text{ and } B) = 0$.
- **B,** $\frac{3}{20}$ — subtracted.
- **D,** 1 — assumed the two events cover every possibility.

> **Takeaway:** mutually exclusive means **add** for "or". Independent means **multiply** for "and". Mutually exclusive is not the same as covering everything.

Q8 — Hard — Answer: H

Topic: Two draws without replacement

The first draw: 3 red out of 8 counters.

$$P(\text{first red}) = \frac{3}{8}$$

Now **both totals have changed**. One red has gone, so there are 2 reds left out of 7 counters.

$$P(\text{second red}) = \frac{2}{7}$$

"And" with these two steps means multiply:

$$\frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$$

Why each wrong option is wrong:

- **F,** $\frac{9}{64}$ — used $\frac{3}{8}$ twice, which is the answer for drawing **with** replacement.
- **G,** $\frac{3}{4}$ — added the counts.
- **J,** $\frac{3}{8}$ — gave the first draw only.

> **Takeaway:** without replacement, **both** the numerator and the denominator drop by one for the second draw. Write the two fractions side by side before multiplying.

Section 4 — Samples and Studies

Q9 — Hard — Answer: C

Topic: What an observational study can conclude

Two limits apply, and both matter.

No cause. Nobody was assigned an amount of sleep — the researchers only watched. So this is an **observational study**, and it can establish that two things move together, not that one produces the other. Something else could explain both: a student who is organised may sleep more *and* study more.

No wider population. The sample was random, but only *within one university*. The conclusion can extend to that university's students and no further.

Option **A** claims association, and only for students at this university. Both limits respected.

Why each wrong option is wrong:

- **B, Sleeping longer causes students to score higher** and **A, Higher scores cause students to sleep longer** — claim causation in one direction or the other. Neither is available from an observational study, and the fact that the two opposite claims are equally consistent with the data is itself the argument.
- **D, More sleep is associated with higher scores among all adults** — extends the finding to all adults. The sample was random within one university, so that is as far as it reaches.

> **Takeaway:** observational studies give **association**; only a randomised experiment gives **causation**. And a conclusion never travels beyond the population that was actually sampled.

Q10 — Medium — Answer: J

Topic: Counting arrangements

Each bread can go with each filling, and each of those pairs with each sauce, so the choices multiply:

$$4 \times 3 \times 2 = 24$$

Why it multiplies rather than adds: for **each** of the 4 breads there are 3 fillings, giving 12 combinations; for each of those 12 there are 2 sauces, giving 24. Adding would answer "how many ingredients are there altogether", which is a different question.

Why each wrong option is wrong:

- **G,** 9 — added the three counts.
- **H,** 12 — multiplied two of the three and forgot the sauces.
- **F,** 14 — mixed multiplying and adding.

> **Takeaway:** independent choices **multiply**. If the question says "one of each", multiply the counts together.

Q11 — Medium — Answer: D

Topic: The range and the spread of a data set

Both medians are 42, which tells you nothing about spread.

Set X runs from 40 to 44, a range of 4. Set Y runs from 10 to 73, a range of 63. Y's values sit far further from their mean, so its standard deviation is greater too.

Why each wrong option is wrong:

- **B, Set X has the greater range but set Y the greater standard deviation** — has the range the wrong way round.
- **C, The two sets have the same range, because they share a median** — a shared median says nothing about range. That is precisely why both statistics are reported.
- **A, Neither range nor standard deviation can be compared here** — both are perfectly comparable from the values given.

> **Takeaway:** median describes the **centre**; range and standard deviation describe the **spread**. Two sets can share one and differ wildly in the other.

Q12 — Hard — Answer: F

Topic: What a random sample supports

Three limits, and each rules out one option.

The population. The sample was random within *one university*, so the conclusion reaches that university's students and no further.

Estimate, not certainty. A sample gives an estimate with a margin of error around it. "About 42%" is right; "exactly 42%" is not.

Opinion is not outcome. The survey asked what students *support*. It says nothing about whether the change would *work*.

Why each wrong option is wrong:

- **G, About 42% of students nationwide support the change** — extends beyond the population sampled.
- **H, Exactly 42% of students at that university support the change** — claims exactness a sample cannot give.
- **J, The survey shows the change will improve student results** — turns a question about opinion into a claim about results.

> **Takeaway:** a random sample supports an **estimate** about the **population it was drawn from**, and nothing else.

Q13 — Medium — Answer: D

Topic: An experiment versus an observational study

Causation requires a **controlled experiment**: the researcher assigns the treatment, and assigns it **at random**.

Random assignment is what makes the two groups alike in every other respect — motivation, prior attainment, hours available — so that any difference in results can be attributed to the method itself.

Why each wrong option is wrong:

- **B, Ask students which method they used and compare their results** — students chose their own method, so the groups may differ in ways that also affect results. That is an observational study: it shows association, not cause.
- **C, Compare results at a school that adopted the method with one that did not** — the schools differ in many ways besides the method.
- **A, Survey teachers about which method they believe works better** — measures belief, not effect.

> **Takeaway: assigned at random** means an experiment, and only an experiment supports a claim about cause. Anything self-selected gives association only.

Q14 — Medium — Answer: G

Topic: Reading a line of best fit

In $y = mx + c$ the coefficient of x is the **slope**: how much y changes for each one-unit increase in x.

Here one extra year of age corresponds to 6 more centimetres of height, on average.

Why each wrong option is wrong:

- **F, The height of a child at birth, in centimetres** — that is the 78, the value where the line meets the vertical axis.
- **H, The age at which a child reaches 6 centimetres** and **J, The number of children in the study** — neither quantity appears in the model.

> **Takeaway:** in a line of best fit, the slope is a **rate of change** and the intercept is the value at zero. Say the units out loud: "6 centimetres per year".

Q15 — Medium — Answer: B

Topic: Comparing an actual value with a predicted one

The line predicts

$$y = 2(4) + 5 = 13$$

The actual value is 15, so the point sits **2 above** the line:

$$15 - 13 = 2$$

That difference is called the **residual**. A positive residual means the line under-predicted.

Why each wrong option is wrong:

- **A,** -2 — subtracted actual from predicted, giving the residual's negative.
- **C,** 13 — gave the prediction rather than the difference.
- **D,** 0 — assumed the point lies exactly on the line. A line of best fit rarely passes through any particular point.

> **Takeaway:** residual $=$ actual $-$ predicted. Positive means the point is above the line.

Q16 — Hard — Answer: F

Topic: What a scatterplot's association does and does not show

An upward-sloping line of best fit shows **association**: the two quantities tend to rise together across these towns.

It cannot show **cause**, in either direction. Nobody opened bookshops experimentally to see what happened; the data was only observed. A third factor — population size, say — could easily drive both.

Why each wrong option is wrong:

- **G, Opening more bookshops would raise a town's average income** and **H, Higher average income causes towns to open more bookshops** — claim causation, in opposite directions. That both are equally consistent with the same scatterplot is the argument against either.
- **J, Bookshop numbers and income are unrelated in these towns** — contradicts the upward slope.

> **Takeaway:** a scatterplot shows association. Only a randomised experiment shows cause, and "these two rise together" never says which one moves the other.

Q17 — Hard — Answer: A

Topic: Counting arrangements in order

Four choices for first place, then three left for second, two for third, and the last runner takes fourth:

$$4 \times 3 \times 2 \times 1 = 24$$

The counts **shrink** by one each time because a runner cannot finish in two places at once — unlike the sandwich question, where each choice was independent and the counts stayed the same.

Why each wrong option is wrong:

- **B,** 16 — used 4^2, which would allow repeats.
- **C,** 10 — added the choices.
- **D,** 4 — gave the number of runners.

> **Takeaway:** arrangements of n distinct things in order come to $n \times (n-1) \times \cdots \times 1$. When the same item cannot be reused, the count drops by one at every step.

Q18 — Medium — Answer: H

Topic: Reading a two-way table for a total

"All customers" means the grand total, 300, goes underneath. The snack column totals 162.

$$\frac{162}{300} = \frac{27}{50}$$

Why each wrong option is wrong:

- **F,** $\frac{7}{15}$ and **G,** $\frac{13}{20}$ — restricted the question to one row. Neither was asked for.
- **J,** $\frac{27}{23}$ — compared the two column totals with each other rather than with the whole.

> **Takeaway:** the denominator is whatever group the question names. "All" means the grand total; "of the adults" would mean 180.

Q19 — Medium — Answer: B

Topic: Reading a frequency table for a median

There are $6 + 9 + 4 + 1 = 20$ customers, so the median sits between the 10th and the 11th when they are lined up in order.

Counting along: customers 1–6 bought 1 item, customers 7–15 bought 2. Both the 10th and the 11th fall in that second group, so both bought 2 items, and the median is 2.

Why each wrong option is wrong:

- **A,** $\frac{5}{2}$ — averaged the numbers on the top row, ignoring how many customers gave each.
- **C,** 9 — gave the largest **frequency**, which counts customers, not items.
- **D,** $\frac{3}{2}$ — took a middle value of the wrong list.

> **Takeaway:** in a frequency table, the median is found by counting **along** the frequencies to the middle position, not by looking at the top row.

Q20 — Hard — Answer: G

Topic: Reading a bar chart described in words

Total cars owned:

$$0(12) + 1(20) + 2(14) + 3(4) = 0 + 20 + 28 + 12 = 60$$

Households: 50.

$$\text{mean} = \frac{60}{50} = 1.2$$

Why each wrong option is wrong:

- **F,** 1 — gave the **mode**, the most common answer, which is 1.
- **H,** $\frac{3}{2}$ — averaged $0, 1, 2, 3$ without weighting by how many households gave each.
- **J,** 50 — gave the number of households.

> **Takeaway:** multiply each value by its frequency, add, then divide by the total frequency. A mean need not be a whole number even when every value is.

Q21 — Medium — Answer: C

Topic: Predicting from a line of best fit

Substitute $x = 15$ into the line:

$$y = 12(15) - 40 = 180 - 40 = 140$$

Why each wrong option is wrong:

- **A,** 180 — multiplied and forgot to subtract.
- **B,** -40 — gave the intercept, which is the prediction at **zero** degrees — and note it is negative, which is why a line of best fit should not be trusted far outside the data it came from.
- **D,** 12 — gave the slope.

> **Takeaway:** a line of best fit predicts by substitution. The intercept is the prediction at zero, which is often physically meaningless.

Q22 — Medium — Answer: F

Topic: Interpreting the sign of a slope in a scatterplot

A negative slope means that as x increases, y decreases. Here x is age and y is value, so **older cars tend to be worth less**.

The word "tend" is doing real work. A line of best fit describes a **trend** across the whole scatterplot; individual points sit above and below it.

Why each wrong option is wrong:

- **G, Older cars tend to be worth more** — describes a positive slope.

- **H, Age and value are unrelated** — a slope of zero would mean no relationship; this one is negative.
- **J, Every older car is worth less than every newer one** — far too strong. A trend is not a rule about every pair of cars, and a scatterplot with a clear negative slope will still contain an old car worth more than some newer one.

> **Takeaway:** the sign of the slope gives the direction of the trend. "Tends to" is right; "always" is not.

Q23 — Hard — Answer: A

Topic: The residual of a data point

The line predicts

$$y = 4(3) + 2 = 14$$

The actual value is 11, so

$$\text{residual} = 11 - 14 = -3$$

A **negative** residual means the point lies below the line — the model over-predicted.

Why each wrong option is wrong:

- **B,** 3 — subtracted predicted from actual the wrong way round. The sign carries the meaning, so the order matters.
- **C,** 14 and **D,** 11 — gave one of the two values rather than their difference.

> **Takeaway:** residual = actual − predicted. Negative means the point sits below the line.

Q24 — Medium — Answer: J

Topic: Counting with a restriction

Nine digits for the first position; once one is used, eight remain for the second, then seven, then six:

$$9 \times 8 \times 7 \times 6 = 3024$$

Why each wrong option is wrong:

- **F,** 6561 — 9^4 allows repeats, which the question forbids.
- **G,** 36 — multiplied the digit count by the number of positions.

- **H,** 24 — counted the orderings of four fixed items, ignoring the nine digits to choose from.

> **Takeaway:** when items cannot repeat, the count **drops by one** at each step. When they can, it stays the same.

Q25 — Medium — Answer: C

Topic: Counting choices that combine

Every main can go with every side, and each of those pairs with every drink, so the counts multiply:

$$5 \times 4 \times 3 = 60$$

The counts stay the same at each step here, because choosing a main does not use up any side.

Why each wrong option is wrong:

- **A,** 12 — added, which answers "how many items are on the menu".
- **B,** 23 — mixed multiplying and adding.
- **D,** 15 — left out one of the three choices.

> **Takeaway:** independent choices multiply. Compare with the previous question: there the counts shrank, because a digit once used was gone.

Q26 — Hard — Answer: H

Topic: Counting arrangements with a fixed position

The far-left position is fixed, so there is only **one** choice for it. That leaves four books for four positions:

$$1 \times 4 \times 3 \times 2 \times 1 = 24$$

Why each wrong option is wrong:

- **F,** 120 — $5! = 120$ counts every arrangement, including the ones with the wrong book on the left.
- **G,** 20 — counted only the first two positions.
- **J,** 5 — gave the number of books.

> **Takeaway:** handle the restricted position **first**, then count the free ones. A fixed position contributes a factor of 1.

Q27 — Medium — Answer: A

Topic: What makes a sample representative

A sample is representative when **every** member of the population has the same chance of being chosen. Only a random draw from the full list of 900 does that.

The other three are **self-selected**, and each in a way that is plausibly linked to the very thing being measured — which is what makes the bias serious rather than merely untidy.

Why each wrong option is wrong:

- **B, Ask the first 100 students who arrive at school one morning** — students who arrive early may be more organised, and so may do more homework.
- **C, Ask 100 students in the library after school** — students in the library after school are almost certainly doing homework.
- **D, Ask for 100 volunteers to fill in a form** — volunteers choose themselves, and people with strong feelings volunteer more.

> **Takeaway:** representative means **randomly selected from the whole population**. Ask whether the way people ended up in the sample is connected to what is being measured.

Q28 — Hard — Answer: F

Topic: How far a conclusion may travel

Three limits, one per wrong option.

The population. The sample was random *within one city*, so the conclusion reaches that city and no further.

Estimate, not certainty. A sample gives an estimate surrounded by a margin of error. "About 38%" is supportable; "exactly 38%" is not.

Behaviour is not effect. The survey recorded what people do, not what it does to them.

Why each wrong option is wrong:

- **G, About 38% of adults in the country cycle to work** — extends beyond the population sampled.
- **H, Exactly 38% of adults in that city cycle to work** — claims a precision a sample cannot give.

- **J, Cycling to work makes adults healthier** — a claim about health from a survey about travel.

> **Takeaway:** a random sample supports an **estimate** about **the population it was drawn from**. Check both of those before accepting a conclusion.

Q29 — Medium — Answer: A

Topic: Why a control group is needed

Scores rising after a course is consistent with the course working — and equally consistent with employees simply getting better at their jobs over the same period, or getting better at the test itself.

A **comparison group** of similar employees who did not take the course separates those explanations: if the course group improved more, the course is the plausible reason.

Why each wrong option is wrong:

- **B, A larger number of employees taking the course** — a bigger group makes the same flawed comparison more precisely. Size fixes noise, not bias.
- **C, A second test of the same employees after the course** — retesting the same people adds no comparison.
- **D, A survey asking employees whether they enjoyed the course** — enjoyment is not attainment.

> **Takeaway:** to claim cause, you need something to compare against. "Before and after" on its own cannot rule out everything else that happened in between.

Chapter 8 — Every Formula You Have to Know

The SAT prints a page of formulas at the start of every maths module. **The ACT prints none.** Everything you need has to be in your head when you sit down.

This chapter is that list, with the reason each formula is true beside it. Reasons matter: a formula you understand survives a nervous morning, and one you have only memorised often does not. Where a formula can be rebuilt from a simpler one in five seconds, that is said too — it is one fewer thing to carry.

Work through it a section at a time rather than in one sitting, and come back to it between the other chapters. These are the cheapest marks in the book.

The list

Lines and coordinates

formula	why
slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$	rise over run — how far up for each step across
$y = mx + b$	m is the slope, b the height where $x = 0$
distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Pythagoras on the horizontal and vertical gaps
midpoint $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$	the average of each coordinate
parallel lines	equal slopes
perpendicular lines	slopes multiply to -1 (flip it and change the sign)

Do not confuse them: midpoint **adds**; distance and slope **subtract**.

Triangles and angles

formula	why
angles in a triangle $= 180^\circ$	tear off the three corners and they form a straight line
angles in an n-sided polygon $= (n-2) \times 180^\circ$	it splits into $n - 2$ triangles
exterior angles of any polygon $= 360^\circ$	one full turn walking round it
exterior angle of a triangle = the two remote interior angles	both equal 180° minus the third angle
area of a triangle $= \frac{1}{2}bh$	half of the rectangle that encloses it
Pythagoras: $a^2 + b^2 = c^2$	right triangles only, c the hypotenuse

Pythagorean triples worth recognising on sight, because they save all the arithmetic: **3-4-5**, **5-12-13**, **8-15-17**, and any multiple of them (6-8-10, 9-12-15, 10-24-26).

Special right triangles:

$$30^\circ\text{–}60^\circ\text{–}90^\circ \quad \longrightarrow \quad 1 : \sqrt{3} : 2$$
$$45^\circ\text{–}45^\circ\text{–}90^\circ \quad \longrightarrow \quad 1 : 1 : \sqrt{2}$$

In both, the smallest side faces the smallest angle.

Circles

formula	why
circumference $= 2\pi r$	π is defined as circumference over diameter
area $= \pi r^2$	
arc length $= \dfrac{\theta}{360} \times 2\pi r$	the same fraction of the way round
sector area $= \dfrac{\theta}{360} \times \pi r^2$	the same fraction of the whole slice
$(x-h)^2 + (y-k)^2 = r^2$	centre (h, k), radius r — it is the distance formula

The circle equation is worth seeing that way: it says "every point is a distance r from the centre", which is what a circle *is*.

Solids

formula	why
box: $V = lwh$	
any prism or cylinder: $V = (\text{cross-section}) \times \text{length}$	stacked copies of the end face
cylinder: $V = \pi r^2 h$	the prism rule with a circular end
sphere: $V = \frac{4}{3}\pi r^3$	
cone: $V = \frac{1}{3}\pi r^2 h$	exactly a third of the cylinder around it
box surface area $= 2(lw + lh + wh)$	six faces in three matching pairs

Scaling, which is worth more than any single volume formula: multiply every length by k and **areas** multiply by k^2, **volumes** by k^3.

Algebra

formula	why
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	solves any quadratic
discriminant $b^2 - 4ac$	positive → 2 roots, zero → 1, negative → none
$(a+b)^2 = a^2 + 2ab + b^2$	there is **always** a middle term
$(a-b)^2 = a^2 - 2ab + b^2$	
$a^2 - b^2 = (a-b)(a+b)$	the middle terms cancel
vertex of $y = ax^2 + bx + c$ at $x = -\dfrac{b}{2a}$	halfway between the two roots

Exponents, logs and sequences

formula	why
$x^a x^b = x^{a+b}$, $\dfrac{x^a}{x^b} = x^{a-b}$, $(x^a)^b = x^{ab}$	count the copies
$x^0 = 1$, $x^{-a} = \dfrac{1}{x^a}$, $x^{m/n} = \sqrt[n]{x^m}$	
$\log_b y = x$ means $b^x = y$	a log is an exponent
$\log(mn) = \log m + \log n$	multiplying inside adds outside
arithmetic: $a + (n-1)d$	$n - 1$ steps to reach the nth term
geometric: $a \cdot r^{n-1}$	same reason, multiplying instead

Trigonometry

formula	why
SOH-CAH-TOA	sin = opp/hyp, cos = adj/hyp, tan = opp/adj
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	divide the first two
$\sin^2\theta + \cos^2\theta = 1$	Pythagoras, divided through by c^2
$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$	law of sines
$c^2 = a^2 + b^2 - 2ab\cos C$	law of cosines — Pythagoras plus a correction
π radians $= 180^\circ$	
$y = a\sin(bx)$: amplitude $\lvert a\rvert$, period $\dfrac{2\pi}{b}$	outside stretches, inside squeezes

Statistics

formula	why
mean $= \dfrac{\text{total}}{\text{count}}$, so total $=$ mean $\times$ count	the rearranged form is the useful one
median	the middle value **after sorting**
range	largest minus smallest
$P(\text{not } A) = 1 - P(A)$	everything either happens or does not

---

Section 1 — Using Them From Memory

The questions below give you nothing. Each one needs a formula from the list above, applied once. If you have to look one up, that is the formula to keep practising.

Q1 — Basic

A triangular sail has a base measuring 14 units along its foot and a perpendicular height of 9 units from that base to the opposite corner. What is its area?

A) 126

B) 63

C) 23

D) 46

Q2 — Medium

An equation from a model is $2x^2 + 3x - 2 = 0$, and it must be solved. What are the solutions?

F) $\left(-\frac{3}{2},\ 0\right)$

G) $(-1,\ 2)$

H) $\left(-2,\ \frac{1}{2}\right)$

J) $(-4,\ 1)$

Q3 — Medium

A conical container has a base radius of 3 units and a height of 8 units. What is its volume?

A) 72π

B) 24π

C) 8π

D) 96π

Q4 — Medium

A road is to be built perpendicular to an existing one whose equation is $y = \dfrac{2}{5}x + 7$. What is the slope of the new road?

F) $\frac{2}{5}$

G) $\frac{5}{2}$

H) $-\frac{2}{5}$

J) $-\frac{5}{2}$

Q5 — Hard

A manufacturer's cost, in thousands of dollars, is modelled by $y = x^2 - 8x + 3$, and the production level giving the lowest cost is wanted. At what value of x does the minimum occur?

A) -8

B) -4

C) 4

D) 8

Q6 — Hard

Two sides of a triangular plot measure 5 and 8 metres and the angle between them is 60°. What is the length of the third side?

F) 7

G) $\sqrt{89}$

H) 13

J) 3

Q7 — Medium

A savings scheme pays out amounts forming the sequence $7,\ 21,\ 35,\ 49, \ldots$, and the 20th payment is wanted. What is the 20th term?

A) 287

B) 140

C) 133

D) 273

Q8 — Medium

A circular region in the xy-plane has its centre at $(-2, 5)$ and a radius of 4 units. Which equation describes this circle?

F) $(x-2)^2 + (y+5)^2 = 16$

G) $(x+2)^2 + (y-5)^2 = 16$

H) $(x+2)^2 + (y-5)^2 = 4$

J) $(x-2)^2 + (y-5)^2 = 16$

Q9 — Medium

What is the distance between the points $(2, -3)$ and $(7, 9)$ in the xy-plane?

A) 13

B) 17

C) $\sqrt{17}$

D) 169

Q10 — Medium

A spherical tank has a radius of 3 units. What is its volume?

F) 9π

G) 12π

H) 36π

J) 27π

Q11 — Medium

A line segment has endpoints $(-3, 8)$ and $(7, -2)$. What is the midpoint?

A) $(2,\ 3)$

B) $(5,\ -5)$

C) $(4,\ 6)$

D) $(-5,\ 5)$

Answers and Explanations

Section 1 — Using Them From Memory

Q1 — Basic — Answer: B

Topic: A formula you must know — the area of a triangle

$$\text{area} = \tfrac{1}{2} \times \text{base} \times \text{height} = \tfrac{1}{2} \times 14 \times 9 = 63$$

The reason for the half: a triangle is exactly half of the rectangle that encloses it, with the same base and height. Picturing that rectangle is a reliable way to remember which formula carries the $\frac{1}{2}$.

Why each wrong option is wrong:

- **A,** 126 — used bh, the **rectangle** formula, forgetting the half.
- **C,** 23 and **D,** 46 — added lengths rather than multiplying them. An area needs two lengths multiplied; adding gives something in the wrong units.

> **Takeaway:** a triangle is half its enclosing rectangle. And the height must be **perpendicular** to the base — not a slanted side.

Q2 — Medium — Answer: H

Topic: A formula you must know — the quadratic formula

With $a = 2$, $b = 3$, $c = -2$:

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-2)}}{2(2)} = \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm 5}{4}$$

$$x = \frac{2}{4} = \frac{1}{2} \qquad \text{or} \qquad x = \frac{-8}{4} = -2$$

Watch the $-4ac$ term: c is negative, so $-4(2)(-2)$ is $+16$, not -16. That double negative is where the formula usually goes wrong.

Check by factoring, which also works here: $(2x-1)(x+2) = 2x^2 + 3x - 2$. ✓

Why each wrong option is wrong:

- **F,** $\left(-\frac{3}{2},\ 0\right)$ — dropped the $-4ac$ term, using $\sqrt{9}$ instead of $\sqrt{25}$. Forgetting it entirely, or getting its sign wrong, is the single commonest quadratic-formula error.

- **G,** $(-1,\ 2)$ — factored as if the leading coefficient were 1, giving $(x+1)(x-2)$, which expands to $x^2 - x - 2$ rather than $2x^2 + 3x - 2$.
- **J,** $(-4,\ 1)$ — divided by 2 rather than by $2a = 4$.

> **Takeaway:** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Substitute a, b and c **with their signs** before doing any arithmetic.

Q3 — Medium — Answer: B

Topic: A formula you must know — the volume of a cone

$$V = \tfrac{1}{3}\pi r^2 h = \tfrac{1}{3} \times \pi \times 9 \times 8 = 24\pi$$

The reason for the third: a cone holds exactly **one third** of the cylinder with the same base and height. That is worth knowing as a fact in its own right — it means you only really have to remember the cylinder.

Why each wrong option is wrong:

- **A,** 72π — gave the cylinder's volume, three times too big.
- **C,** 8π — forgot to square the radius.
- **D,** 96π — used $\frac{4}{3}$, which belongs to the sphere.

> **Takeaway:** cone $= \frac{1}{3} \times$ cylinder. Sphere is $\frac{4}{3}\pi r^3$ and has **no** height in it — if your sphere formula contains an h, it is wrong.

Q4 — Medium — Answer: J

Topic: A formula you must know — perpendicular slopes

Perpendicular slopes are **negative reciprocals**: flip the fraction *and* change its sign. Both steps, not one.

$$\text{flip: } \frac{5}{2} \qquad \text{change sign: } -\frac{5}{2}$$

Check with the multiplication test — two perpendicular slopes multiply to -1:

$$\frac{2}{5} \times \left(-\frac{5}{2}\right) = -1 \checkmark$$

The $+7$ is the intercept and plays no part; only the slope matters for perpendicularity.

Why each wrong option is wrong:

- **F,** $\frac{2}{5}$ — that is the **parallel** slope.
- **G,** $\frac{5}{2}$ and **H,** $-\frac{2}{5}$ — did one of the two steps.

> **Takeaway:** perpendicular means flip **and** negate. Multiply your answer by the original slope; if you do not get -1, you have done only half of it.

Q5 — Hard — Answer: C

Topic: A formula you must know — the vertex of a parabola

For $y = ax^2 + bx + c$ the vertex sits at

$$x = -\frac{b}{2a}$$

Here $a = 1$ and $b = -8$:

$$x = -\frac{-8}{2(1)} = \frac{8}{2} = 4$$

The two minus signs cancel, which is exactly where option **B** goes wrong.

Why the formula is true: a parabola is symmetric, so its vertex lies halfway between its two roots — and the quadratic formula puts those roots at $\frac{-b}{2a}$ plus and minus the same amount. The midpoint is $\frac{-b}{2a}$.

Check by testing either side: at $x = 3$, $y = -12$; at $x = 4$, $y = -13$; at $x = 5$, $y = -12$. Lowest at 4. ✓

Why each wrong option is wrong:

- **A,** -8 — read b off the equation.
- **B,** -4 — dropped the minus sign in front of the formula.
- **D,** 8 — used b without dividing by $2a$.

> **Takeaway:** $x = -\frac{b}{2a}$, and b carries its own sign. Substituting a negative b into a formula that already has a minus gives a **positive** answer.

Q6 — Hard — Answer: F

Topic: A formula you must know — the law of cosines

$$c^2 = a^2 + b^2 - 2ab\cos C$$

With $a = 5$, $b = 8$, $C = 60^\circ$ and $\cos 60^\circ = \frac{1}{2}$:

$$c^2 = 25 + 64 - 2(5)(8)\left(\tfrac{1}{2}\right) = 89 - 40 = 49$$
$$c = 7$$

Sanity check: the third side must be between $8 - 5 = 3$ and $8 + 5 = 13$. It is.

Notice what the formula is doing. Without the last term it would be Pythagoras, giving $\sqrt{89} \approx 9.4$. The $-2ab\cos C$ term is the correction for the angle not being 90° — and when $C = 90^\circ$, $\cos C = 0$ and the correction disappears.

Why each wrong option is wrong:

- **G,** $\sqrt{89}$ — used Pythagoras on a triangle that has no right angle.
- **H,** 13 and **J,** 3 — added or subtracted the sides, which give the absolute limits the third side must lie strictly between.

> **Takeaway:** law of cosines is Pythagoras plus a correction term. Two sides and the angle **between** them is the signal to use it.

Q7 — Medium — Answer: D

Topic: A formula you must know — the nth term of a sequence

Check which kind of sequence it is by subtracting:

$$21 - 7 = 14 \qquad 35 - 21 = 14 \qquad 49 - 35 = 14$$

Constant, so it is arithmetic with $a = 7$ and $d = 14$.

$$a + (n-1)d = 7 + 19 \times 14 = 7 + 266 = 273$$

Why $n - 1$: you are already standing on the first term, so getting to the 20th takes 19 steps, not 20.

Why each wrong option is wrong:

- **A,** 287 — used $a + nd$, one difference too many. This is the error the $n - 1$ exists to prevent.
- **B,** 140 — multiplied the first term by 20.
- **C,** 133 — used 7 as the common difference.

> **Takeaway:** $a + (n-1)d$ for arithmetic, $a \cdot r^{n-1}$ for geometric. Both have $n - 1$, for the same reason: count the gaps, not the terms.

Q8 — Medium — Answer: G

Topic: A formula you must know — the equation of a circle

The standard form is

$$(x-h)^2+(y-k)^2=r^2$$

with centre (h,k). The formula already contains minus signs, so a **negative** coordinate produces a **plus** in the bracket:

$$h=-2 \implies (x-(-2))^2=(x+2)^2$$
$$k=5 \implies (y-5)^2$$

And the right-hand side is $r^2=4^2=16$.

$$(x+2)^2+(y-5)^2=16$$

Check: the point $(2,5)$ is 4 units right of the centre and should be on the circle. $(2+2)^2+(5-5)^2=16$. ✓

Why each wrong option is wrong:

- **F,** $(x-2)^2+(y+5)^2=16$ and **J,** $(x-2)^2+(y-5)^2=16$ — copied a coordinate's sign straight into the bracket instead of flipping it.
- **H,** $(x+2)^2+(y-5)^2=4$ — put the radius on the right rather than its square.

> **Takeaway:** the signs **flip** and the right side is r^2. Write $(x-(-2))$ out in full first if the flip is not automatic yet, then test one point.

Q9 — Medium — Answer: A

Topic: A formula you must know — the distance formula

The gaps are $7-2=5$ across and $9-(-3)=12$ up:

$$\sqrt{5^2+12^2}=\sqrt{25+144}=\sqrt{169}=13$$

A 5-12-13 triangle — worth recognising, since it removes the arithmetic.

Why each wrong option is wrong:

- **B,** 17 — added the gaps, which is the distance along two sides rather than across.
- **C,** $\sqrt{17}$ — added before squaring.
- **D,** 169 — stopped before the square root.

> **Takeaway:** the distance formula is Pythagoras. Watch the double negative when subtracting a negative coordinate.

Q10 — Medium — Answer: H

Topic: A formula you must know — the volume of a sphere

$$V = \tfrac{4}{3}\pi r^3 = \tfrac{4}{3} \times \pi \times 27 = 36\pi$$

The $\frac{4}{3}$ and the **cube** are both essential, and there is no height in the formula — a sphere has only one measurement.

Why each wrong option is wrong:

- **F,** 9π — used πr^2, the area of a circle.
- **G,** 12π — squared the radius instead of cubing it.
- **J,** 27π — omitted the $\frac{4}{3}$.

> **Takeaway:** sphere is $\frac{4}{3}\pi r^3$. If your sphere formula has an h in it, it is the wrong formula.

Q11 — Medium — Answer: A

Topic: A formula you must know — the midpoint

Average each coordinate separately:

$$x: \frac{-3+7}{2} = 2 \qquad y: \frac{8+(-2)}{2} = 3$$

Check that the answer lies **between** the endpoints in both coordinates: 2 is between -3 and 7, and 3 is between -2 and 8. ✓

Why each wrong option is wrong:

- **B,** $(5,\ -5)$ — subtracted instead of adding, which is the distance formula's first step, not the midpoint's.
- **C,** $(4,\ 6)$ — added correctly and forgot to halve.
- **D,** $(-5,\ 5)$ — sign errors in both coordinates.

> **Takeaway:** midpoint **adds and halves**; distance and slope **subtract**. Then check the answer sits between the two endpoints.

Chapter 9 — Two Full Practice Papers

Two complete ACT Mathematics papers, each built to the real specification:

- **45 questions in 50 minutes**, one undivided section
- **four answer choices**, lettered A–D on odd questions and F–J on even ones
- **a calculator permitted throughout**
- **no formula sheet** — nothing is given to you
- questions running from easier to harder, as the real paper does
- the mix built to ACT's published blueprint: 20% Algebra, 18% Functions, 18% Geometry, 13% Statistics & Probability, 11% Number & Quantity, and **20% Integrating Essential Skills**

Time them properly. 50 minutes, one sitting, no interruptions, no looking anything up. An untimed paper measures your mathematics; a timed one measures your ACT score, and they are not the same number.

Do the second paper only once you have worked through everything the first one showed you. A practice paper you have not learned from is a practice paper wasted.

Scoring

Count how many you got right, then read the scale score off the table below. It is ACT's own conversion table for the official enhanced form, with the raw score scaled to 45 questions.

right	scale	right	scale	right	scale
45	36	30	28	15	17
43	35	28	27	13	16
41	34	26	25	11	15
39	33	24	23	9	14
37	32	22	21	7	13
35	31	20	20	5	12
33	30	18	18	3	9
31	29	16	17	1	5

Read the middle of that table: around the centre of the range **one extra right answer is worth about one scale point**. Five more is roughly five points.

Paper 1

45 questions · 50 minutes · calculator permitted · no formula sheet

Q1 — Basic

A regional council surveyed 240 households about recycling and found that 30% of them separate glass from other waste. How many households separate glass?

A) 30

B) 80

C) 168

D) 72

Q2 — Basic

If $3x + 5 = 20$, what is the value of x?

F) $\frac{20}{3}$

G) 5

H) $\frac{25}{3}$

J) 15

Q3 — Basic

A machine fills 45 bottles every 3 minutes at a steady rate. How many bottles does it fill in 8 minutes?

A) 15

B) 45

C) 96

D) 120

Q4 — Basic

A surveyor measures two interior angles of a triangular plot as 35° and 95°. What is the measure of the third angle?

F) 50

G) 130

H) 60

J) 85

Q5 — Basic

Which expression is equivalent to $4(2x - 5)$?

A) $8x - 5$

B) $8x - 20$

C) $6x - 20$

D) $2x - 20$

Q6 — Basic

Which expression is equivalent to $x^5 \cdot x^3$?

F) x^{15}

G) x^8

H) x^2

J) $2x^8$

Q7 — Basic

The function f is defined by $f(x) = x^2 - 3x$, where $f(x)$ is the profit in hundreds of dollars from selling x hundred units. What is $f(5)$?

A) 16

B) 25

C) -10

D) 10

Q8 — Basic

Two friends share 45 sweets in the ratio $4 : 5$. How many sweets does the friend with the larger share receive?

F) 20

G) 5

H) 9

J) 25

Q9 — Basic

A delivery service charges a fixed handling fee plus an amount for each kilometre, so the total charge in dollars is $f(x) = 4x + 7$ for x kilometres. What is the value of $f(0)$?

A) 0

B) 4

C) 11

D) 7

Q10 — Basic

Five readings are $8,\ 11,\ 6,\ 14$ and 11. What is their mean?

F) 10

G) 11

H) 50

J) 8

Q11 — Medium

If $y = 2x - 1$ and $3x + y = 14$, what is the value of x?

A) 2

B) 3

C) 5

D) $\frac{13}{5}$

Q12 — Medium

A jacket priced at $80 has its price raised by 15%. What is the new price?

F) 12

G) 95

H) 92

J) 68

Q13 — Medium

Two functions are defined by $f(x) = x + 3$ and $g(x) = 2x$. An engineer applies f to an input and then applies g to the result. What is $g(f(4))$?

A) 11

B) 7

C) 8

D) 14

Q14 — Medium

A circular pond has a radius of 7 metres. What is its area, in square metres?

F) 14π

G) 49π

H) 7π

J) 28π

Q15 — Medium

Which of the following is 0.00036 written in scientific notation?

A) 3.6×10^{-3}

B) 36×10^{-5}

C) 3.6×10^{-4}

D) 3.6×10^{4}

Q16 — Medium

The integers 1 to 30 are each written on a separate slip and one slip is drawn at random. What is the probability that the number drawn is a multiple of 6?

F) $\frac{1}{5}$

G) $\frac{1}{30}$

H) $\frac{1}{6}$

J) $\frac{1}{3}$

Q17 — Medium

What is the least integer value of x that satisfies $9 - 2x < 4x - 15$?

A) -4

B) 4

C) 6

D) 5

Q18 — Medium

A boat travels at 30 miles per hour. Given that 1 mile is 5280 feet and 1 hour is 3600 seconds, what is its speed in feet per second?

F) 88

G) $\frac{225}{11}$

H) 2640

J) 44

Q19 — Medium

A rectangular gate is braced by a diagonal strut. The gate is 9 units wide and 12 units tall. How long is the strut?

A) 21

B) 15

C) $\sqrt{21}$

D) 11

Q20 — Medium

The graph of $y = f(x)$ passes through the point $(3, 8)$. Through which point must the graph of $y = f(x) - 5$ pass?

F) $(-2,\ 8)$

G) $(8,\ 3)$

H) $(3,\ 3)$

J) $(3,\ 13)$

Q21 — Medium

A sequence begins $7,\ 21,\ 35,\ 49, \ldots$ What is its 10th term?

A) 147

B) 70

C) 126

D) 133

Q22 — Medium

The mean of four numbers is 21. Three of them are 18, 24 and 19. What is the fourth?

F) 61

G) 23

H) 84

J) 21

Q23 — Medium

The height of a ball is zero at the times satisfying $x^2 + 4x - 21 = 0$, where x is the number of seconds since it was thrown. What is the positive solution?

A) 3

B) -7

C) 21

D) 7

Q24 — Medium

A cylindrical tank has radius 3 metres and height 10 metres. What is its volume, in cubic metres?

F) 30π

G) 60π

H) 900π

J) 90π

Q25 — Medium

A survey of 200 people recorded whether they cycle and whether they own a car.

	Owns a car	No car	Total
Cycles	30	50	80
Does not cycle	90	30	120
Total	120	80	200

One person is chosen at random from those who cycle. What is the probability that they own a car?

A) $\frac{3}{20}$

B) $\frac{3}{8}$

C) $\frac{1}{4}$

D) $\frac{2}{5}$

Q26 — Medium

A recipe uses 3 cups of flour for every 8 biscuits. How many cups are needed for 24 biscuits?

F) 9

G) 64

H) 19

J) 72

Q27 — Medium

For which value of x is $f(x) = \dfrac{x+2}{x-7}$ undefined?

A) -2

B) 0

C) 7

D) 2

Q28 — Medium

What is the slope of the line through $(-2, 7)$ and $(3, -3)$?

F) 2

G) $-\frac{1}{2}$

H) $\frac{4}{5}$

J) -2

Q29 — Medium

The formula $P = 2l + 2w$ gives the perimeter of a rectangle. Which of the following expresses w in terms of P and l?

A) $\frac{P}{2} - l$

B) $\frac{P}{2} - 2l$

C) $\frac{P}{2} - \frac{l}{2}$

D) $P - 2l$

Q30 — Medium

Six readings are $4,\ 9,\ 2,\ 11,\ 6$ and 8. What is their median?

F) $\frac{20}{3}$

G) 6

H) 7

J) 9

Q31 — Hard

In the equation $6x + 5 = kx - 4$, k is a constant. The equation has no solution. What is the value of k?

A) -6

B) 6

C) 5

D) -4

Q32 — Hard

A 2-metre post casts a shadow 1.5 metres long at the same moment a flagpole casts a shadow 9 metres long. How tall is the flagpole, in metres?

F) 6

G) 12

H) $\frac{27}{4}$

J) 18

Q33 — Hard

A culture starts with 60 cells and doubles every 3 hours. How many cells are there after 12 hours?

A) 240

B) 245760

C) 960

D) 720

Q34 — Hard

Given that $i^2 = -1$, what is the value of $\sqrt{-9} + \sqrt{-16}$?

F) $5i$

G) -7

H) 5

J) $7i$

Q35 — Hard

Two classes both average 70 on a test. Their results are summarised below.

Class	Mean	Lowest	Highest
A	70	68	72
B	70	45	95

Which statement is true?

A) Class B has the greater standard deviation

B) Class A has the greater standard deviation

C) The standard deviations are equal, because the means are equal

D) Standard deviation cannot be judged from a range

Q36 — Hard

A price rises by 25% one year and falls by 20% the next. Over the two years together, the price has:

F) returned to exactly its original value

G) risen by 5%

H) fallen by 5%

J) risen by 45%

Q37 — Hard

A ramp rises at 12° to the horizontal and its sloping surface is 9 metres long. Which expression gives its vertical height, in metres?

A) $9\cos 12^\circ$

B) $\dfrac{9}{\sin 12^\circ}$

C) $9\sin 12^\circ$

D) $9\tan 12^\circ$

Q38 — Hard

What is the solution to $\sqrt{x+6} = x$, where the root sign denotes the non-negative root?

F) -2

G) 1

H) 3

J) 6

Q39 — Hard

If $\log_3(x) = 4$, what is the value of x?

A) 12

B) 64

C) $\frac{4}{3}$

D) 81

Q40 — Hard

Two sides of a triangle measure 5 and 8, and the angle between them is 60°. What is the length of the third side?

F) $\sqrt{89}$

G) 7

H) 13

J) 3

Q41 — Hard

What is $5\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$?

A) $\begin{bmatrix}5 & 10\\ 15 & 20\end{bmatrix}$

B) $\begin{bmatrix}5 & 2\\ 3 & 4\end{bmatrix}$

C) $\begin{bmatrix}6 & 7\\ 8 & 9\end{bmatrix}$

D) $\begin{bmatrix}5 & 10\\ 3 & 4\end{bmatrix}$

Q42 — Hard

One pump empties a tank in 6 hours and another empties the same tank in 12 hours. Working together, how many hours do they take?

F) 4

G) 18

H) 9

J) 72

Q43 — Hard

A café offers 4 breads, 3 fillings and 2 sauces, and a sandwich uses one of each. How many different sandwiches are possible?

A) 9

B) 24

C) 12

D) 14

Q44 — Hard

A circle has radius 6. What is the area of a sector with a central angle of 60°?

F) 36π

G) 2π

H) 6π

J) 12π

Q45 — Hard

In the equation $x^2 + 10x + c = 0$, c is a constant, and the equation has exactly one real solution. What is the value of c?

A) 10

B) 5

C) 25

D) 100

Paper 2

45 questions · 50 minutes · calculator permitted · no formula sheet

Do this one only after you have worked through what Paper 1 showed you.

Q46 — Basic

What percent of 250 is 40?

F) 40

G) 16

H) 625

J) 84

Q47 — Basic

If $2(x + 6) = 26$, what is the value of x?

A) 13

B) 19

C) 10

D) 7

Q48 — Basic

A rectangular room measures 6 metres by 4 metres and carpet costs $18 per square metre. What is the total cost?

F) 24

G) 360

H) 42

J) 432

Q49 — Basic

Eight readings are $4,\ 7,\ 4,\ 9,\ 4,\ 7,\ 2$ and 5. What is the mode?

A) 7

B) 4

C) $\frac{21}{4}$

D) 3

Q50 — Basic

Which expression is equivalent to $5(2x - 3) - 2(x + 4)$?

F) $8x - 7$

G) $12x - 7$

H) $8x - 23$

J) $12x - 23$

Q51 — Basic

A fair six-sided die is rolled once. What is the probability of rolling a number greater than 4?

A) $\frac{2}{3}$

B) $\frac{1}{6}$

C) $\frac{1}{2}$

D) $\frac{1}{3}$

Q52 — Basic

If $g(x) = \dfrac{x+6}{2}$, what is $g(-4)$?

F) 1

G) -1

H) 2

J) 5

Q53 — Basic

A tap runs at 3 litres per minute. How many litres does it deliver in 2 hours?

A) 6

B) 180

C) 360

D) 120

Q54 — Basic

Six values are $5,\ 9,\ 5,\ 12,\ 7$ and 5. What is the range?

F) 5

G) $\frac{43}{6}$

H) 7

J) 12

Q55 — Basic

A recipe mixes cement and sand in the ratio $2 : 7$. If 63 kilograms of sand are used, how many kilograms of cement are needed?

A) 18

B) 9

C) 126

D) 14

Q56 — Medium

If $2x + 3y = 16$ and $2x - y = 4$, what is the value of y?

F) 20

G) $\frac{7}{2}$

H) 6

J) 3

Q57 — Medium

After a 20% discount, a coat costs $96. What was the original price?

A) 116

B) $\frac{576}{5}$

C) 120

D) 76

Q58 — Medium

The graph of $y = x^2$ is shifted 3 units right. Which equation describes the new graph?

F) $y = (x - 3)^2$

G) $y = (x + 3)^2$

H) $y = x^2 - 3$

J) $y = x^2 + 3$

Q59 — Medium

A circle has centre $(3, -2)$ and radius 5. Which equation describes it?

A) $(x+3)^2 + (y-2)^2 = 25$

B) $(x-3)^2 + (y+2)^2 = 25$

C) $(x-3)^2 + (y+2)^2 = 5$

D) $(x+3)^2 + (y+2)^2 = 25$

Q60 — Medium

A class of 20 students averages 68 and a class of 30 students averages 78. What is the mean across both classes?

F) 73

G) 146

H) 74

J) 50

Q61 — Medium

A sequence begins $3,\ 12,\ 48,\ 192, \ldots$ What is its 6th term?

A) 12288

B) 23

C) 768

D) 3072

Q62 — Medium

A shop recorded how many drinks each customer bought.

Drinks bought	**1**	**2**	**3**
Number of customers	5	10	5

What is the mean number of drinks per customer?

F) 20

G) 2

H) $\frac{40}{3}$

J) 10

Q63 — Medium

A machine's value is modelled by $V = 18000(0.85)^t$ dollars, where t is years since purchase. Which statement is best supported?

A) The machine loses 15% of its value each year

B) The machine loses 85% of its value each year

C) The machine loses $85 each year

D) The machine gains 85% of its value each year

Q64 — Medium

A box holds 4 red and 6 blue pens. Two are taken out one after another without replacement. What is the probability that both are red?

F) $\frac{4}{25}$

G) $\frac{2}{5}$

H) $\frac{4}{5}$

J) $\frac{2}{15}$

Q65 — Medium

Which of the following gives all values of x for which $-3x + 4 > 19$?

A) $x < -5$

B) $x > -5$

C) $x < 5$

D) $x > 5$

Q66 — Medium

A bag holds 3 green, 5 yellow and 12 red counters. One counter is drawn at random. What is the probability that it is **not** red?

F) $\frac{2}{5}$

G) $\frac{3}{5}$

H) $\frac{3}{2}$

J) $\frac{1}{3}$

Q67 — Medium

The graph of $y = x^2 - 6x + 5$ has a minimum. At what value of x does it occur?

A) -6

B) -3

C) 3

D) 6

Q68 — Medium

What is 135° expressed in radians?

F) 135π

G) $\frac{4\pi}{3}$

H) $\frac{3\pi}{4}$

J) $\frac{3}{4}$

Q69 — Medium

What is the determinant of $\begin{pmatrix} 4 & 3 \\ 2 & 5 \end{pmatrix}$?

A) 26

B) 14

C) -14

D) 20

Q70 — Medium

In a club, 34 members play chess, 28 play draughts, and 12 play both. Every member plays at least one. How many members are there?

F) 62

G) 50

H) 74

J) 38

Q71 — Medium

A linear function's values are recorded below.

x	**1**	**2**	**3**	**4**
$f(x)$	9	13	17	21

What is the slope of the function?

A) 12

B) 3

C) 4

D) $\frac{1}{4}$

Q72 — Medium

If $x^2 + kx + 36$ is a perfect square for some positive constant k, what is the value of k?

F) 6

G) 36

H) 12

J) 18

Q73 — Medium

If $-7 < 2x + 1 \leq 9$, which of the following gives all possible values of x?

A) $-4 < x \leq 4$

B) $-4 \leq x \leq 4$

C) $-4 \leq x < 4$

D) $-4 < x < 4$

Q74 — Medium

A cinema sells adult tickets at $14 and child tickets at $9. One evening it sold 60 tickets and took $710. How many child tickets were sold?

F) 34

G) 60

H) 30

J) 26

Q75 — Medium

One gym charges $25 a month plus $4 per class; another charges a flat $65 a month. At how many classes a month do they cost the same?

A) 10

B) $\frac{65}{4}$

C) 40

D) 90

Q76 — Hard

The system $\begin{cases} 4x - 2y = 8 \\ 6x + ky = 5 \end{cases}$ has no solution. What is the value of k?

F) -3

G) 3

H) $-\frac{3}{2}$

J) -2

Q77 — Hard

What is the distance between $(-3, 2)$ and $(1, -1)$ in the xy-plane?

A) 7

B) 5

C) $\sqrt{7}$

D) 25

Q78 — Hard

A population is modelled by $f(x) = 5470(0.64)^x$, where x is years since the study began. Which statement is true?

F) The population grows by 64% each year

G) The population falls by 64% each year

H) The population falls by 36% each year

J) The population falls by 5470 each year

Q79 — Hard

Given $i^2 = -1$, what is i^{27}?

A) 1

B) i

C) $-i$

D) -1

Q80 — Hard

In triangle PQR, angle P is 30°, angle Q is 45°, and the side opposite Q is 10. What is the length of the side opposite P?

F) $5\sqrt{2}$

G) $10\sqrt{2}$

H) 5

J) 15

Q81 — Hard

A rectangle's length and width are both tripled. By what factor does its area increase?

A) 3

B) 27

C) 6

D) 9

Q82 — Hard

What is the solution to $\frac{15}{x} - 2 = \frac{9}{x}$?

F) 6

G) 3

H) 24

J) 12

Q83 — Hard

What is the value of $\log_2(8) + \log_2(4)$?

A) 5

B) 12

C) 32

D) 6

Q84 — Hard

What is the magnitude of the vector $\langle 5, -12 \rangle$?

F) -7

G) 7

H) 17

J) 13

Q85 — Hard

A line is perpendicular to $y = -2x + 1$ and passes through $(4, 3)$. What is its y-intercept?

A) 1

B) 11

C) $\frac{1}{2}$

D) -2

Q86 — Hard

A taxi fare in dollars is $C = 4 + 2.5m$ for a journey of m miles. Which is the best interpretation of the 4?

F) A fixed charge applied before any distance is travelled

G) The cost of each mile travelled

H) The number of miles included in the base fare

J) The total cost of a four-mile journey

Q87 — Hard

A quadratic has roots $x = -4$ and $x = 6$. Which equation could it be?

A) $x^2 + 2x - 24$

B) $x^2 - 10x - 24$

C) $x^2 - 2x - 24$

D) $x^2 - 2x + 24$

Q88 — Hard

Two machines were tested, and their output is summarised below. Both produced the same mean.

Machine	Mean	Lowest	Highest
A	50	48	52
B	50	20	80

Which statement is true?

F) Machine B's output has the greater standard deviation

G) Machine A's output has the greater standard deviation

H) The two standard deviations are equal, because the means are equal

J) Standard deviation cannot be compared from a range

Q89 — Hard

In a town, 60% of residents own a bicycle, and 25% of those owners ride daily. What percent of all residents ride daily?

A) 85

B) 15

C) 35

D) 25

Q90 — Hard

A rectangular garden's length is twice its width, and its perimeter is 54 metres. What is its area, in square metres?

F) 27

G) 81

H) 9

J) 162

Answers and Explanations

Q1 — Basic — Answer: D

Topic: A percentage of a number

$$240 \times 0.30 = 72$$

C gives the households that do not separate glass; **B** takes a third rather than 30%.

> **Takeaway:** "of" means multiply. Check whether the question wants the group described or the rest.

Q2 — Basic — Answer: G

Topic: Solving a two-step linear equation

Undo the operations in the reverse of the order they were applied: the $+5$ first, then the multiplication by 3.

Subtract 5, then divide by 3:

$$3x = 15 \qquad x = 5$$

> **Takeaway:** undo in reverse order — the addition first, then the multiplication.

Q3 — Basic — Answer: D

Topic: A unit rate

Find what happens in **one** unit of time first, then scale up. Going straight from 3 minutes to 8 tempts you into multiplying by 8 directly.

$45 \div 3 = 15$ bottles per minute, so $15 \times 8 = 120$.

> **Takeaway:** find the rate for one unit first, then multiply.

Q4 — Basic — Answer: F

Topic: Angles in a triangle

The three angles of any triangle total 180°, so the third is whatever is left after the other two.

$$180 - 35 - 95 = 50^\circ$$

Check: $35 + 95 + 50 = 180$.

> **Takeaway:** subtract both known angles from 180°, then add all three back.

Q5 — Basic — Answer: B

Topic: Expanding a bracket

The number outside the bracket multiplies **both** terms inside it, signs included.

$$4 \times 2x = 8x \qquad 4 \times (-5) = -20$$

> **Takeaway:** the number outside multiplies **both** terms inside.

Q6 — Basic — Answer: G

Topic: An exponent rule

Multiplying powers of the same base **adds** the exponents — you are just counting how many copies of x are being multiplied together.

Multiplying powers of the same base **adds** the exponents: $5 + 3 = 8$.

> **Takeaway:** multiply → add exponents; divide → subtract; power of a power → multiply.

Q7 — Basic — Answer: D

Topic: Evaluating a function

Substitute 5 for every x, taking the power first:

$$f(5) = 5^2 - 3(5) = 25 - 15 = 10$$

> **Takeaway:** apply the power before the subtraction.

Q8 — Basic — Answer: J

Topic: A part-to-whole ratio

$4 + 5 = 9$ parts, so one part is $45 \div 9 = 5$. The larger share is $5 \times 5 = 25$.

> **Takeaway:** add the parts first, then multiply by the part asked for.

Q9 — Basic — Answer: D

Topic: Reading a linear function

$$f(0) = 4(0) + 7 = 7$$

It is the handling fee — what you pay before travelling anywhere.

> **Takeaway:** the constant term is the value at zero: the fixed part of a linear model.

Q10 — Basic — Answer: F

Topic: The mean of a small data set

The mean is the total shared equally, so add everything and divide by how many there are.

$$\frac{8 + 11 + 6 + 14 + 11}{5} = \frac{50}{5} = 10$$

> **Takeaway:** mean is total ÷ count. The mode (11) and the range (8) are different things.

Q11 — Medium — Answer: B

Topic: A system solved by substitution

Substitute the first into the second:

$$3x + (2x - 1) = 14 \quad \Longrightarrow \quad 5x = 15 \quad \Longrightarrow \quad x = 3$$

C is y, which is 5.

> **Takeaway:** when one equation already gives a variable alone, substitute. Then check which letter was asked for.

Q12 — Medium — Answer: H

Topic: A percentage increase

A 15% rise multiplies by 1.15 — the original whole, plus another 15% of it:

$$80 \times 1.15 = 92$$

> **Takeaway:** increase by $p\%$ in one step, multiplying by $1 + \frac{p}{100}$.

Q13 — Medium — Answer: D

Topic: A composite function

Work outwards from the inner bracket. $f(4) = 4 + 3 = 7$, and that 7 is then the input to g, giving $g(7) = 14$.

> **Takeaway:** work from the inside bracket outwards.

Q14 — Medium — Answer: G

Topic: The area of a circle

Area uses the **square** of the radius; circumference does not. Check which the question wants before choosing the formula.

$$A = \pi r^2 = 49\pi$$

F is the circumference, $2\pi r$.

> **Takeaway:** area squares the radius; circumference does not.

Q15 — Medium — Answer: C

Topic: Scientific notation

Move the decimal point until exactly one non-zero digit stands in front of it, then count how many places it moved. A number below 1 gives a negative power.

The point moves 4 places right to give 3.6, so the power is -4.

> **Takeaway:** a number below 1 gets a **negative** power. Expand your answer to check.

Q16 — Medium — Answer: H

Topic: Probability from a set of outcomes

The multiples of 6 up to 30 are 6, 12, 18, 24, 30 — five of them.

$$P = \frac{5}{30} = \frac{1}{6}$$

> **Takeaway:** count the outcomes that work, then divide by how many there are altogether.

Q17 — Medium — Answer: D

Topic: A linear inequality

Add $2x$ to both sides and add 15:

$$24 < 6x \quad \implies \quad 4 < x$$

Strictly greater than 4, so the least **integer** is 5.

> **Takeaway:** a strict inequality excludes its boundary. Gather x on the side that keeps it positive and no sign flip is needed.

Q18 — Medium — Answer: J

Topic: Converting a rate between units

$$\frac{30 \times 5280}{3600} = 44$$

Remember 60 mph = 88 ft/s; half the speed is half the answer.

> **Takeaway:** write both conversions as fractions and let the units cancel.

Q19 — Medium — Answer: B

Topic: Pythagoras

The strut runs corner to corner, so it is the hypotenuse of a right triangle whose legs are the two sides of the gate.

$$\sqrt{81 + 144} = \sqrt{225} = 15$$

A 9-12-15 triangle — three times 3-4-5.

> **Takeaway:** square, add, then root. Recognising the triple saves the arithmetic.

Q20 — Medium — Answer: H

Topic: A function transformation

The -5 is outside the brackets, so it lowers every output by 5. The x-coordinate is unchanged.

> **Takeaway:** outside the brackets moves the graph vertically, in the direction the sign says.

Q21 — Medium — Answer: D

Topic: An arithmetic sequence

Check first that the sequence is arithmetic by subtracting consecutive terms. They differ by 14 every time.

The common difference is 14, so

$$7 + 9 \times 14 = 133$$

> **Takeaway:** $a + (n - 1)d$ — nine steps to reach the tenth term.

Q22 — Medium — Answer: G

Topic: A missing value from a mean

The four total $4 \times 21 = 84$; the three given total 61; so the fourth is 23.

> **Takeaway:** mean × count = total. Get the total first.

Q23 — Medium — Answer: A

Topic: Solving a quadratic by factoring

Two numbers multiplying to -21 and adding to $+4$: $+7$ and -3.

$$(x+7)(x-3)=0 \implies x=-7 \text{ or } x=3$$

Time cannot be negative, so $x=3$.

> **Takeaway:** factor, set each bracket to zero, then reject any root the context forbids.

Q24 — Medium — Answer: J

Topic: The volume of a cylinder

A cylinder's volume is the area of its circular base multiplied by its height, so **only the radius** gets squared.

$$V=\pi r^2 h=\pi \times 9 \times 10=90\pi$$

> **Takeaway:** only the radius is squared.

Q25 — Medium — Answer: B

Topic: Conditional probability from a two-way table

Restricted to cyclists, so the denominator is 80:

$$\frac{30}{80}=\frac{3}{8}$$

> **Takeaway:** the group named after "from those who" is the denominator.

Q26 — Medium — Answer: F

Topic: A proportion

Set the two ratios equal with the same quantity on top of both, then scale. More biscuits must mean more flour, which rules out any smaller answer.

24 biscuits is three times 8, so the flour is three times 3, which is 9.

> **Takeaway:** keep the same quantity on top of both fractions, and check the direction — more biscuits means more flour.

Q27 — Medium — Answer: C

Topic: The domain of a function

A fraction has no value only when its **denominator** is zero. Setting the numerator to zero answers a different question — where the graph crosses the x-axis.

A fraction is undefined only when its denominator is zero: $x = 7$.

> **Takeaway:** the bottom decides the domain; the top decides the zeros.

Q28 — Medium — Answer: J

Topic: The slope of a line through two points

$$m = \frac{-3-7}{3-(-2)} = \frac{-10}{5} = -2$$

The line falls, so a negative slope is right.

> **Takeaway:** subtract in the same order top and bottom, and watch the double negative.

Q29 — Medium — Answer: A

Topic: Rearranging a formula

Isolate the term containing w first, then divide by everything multiplying it. Test the result with easy numbers.

$$P - 2l = 2w \quad \Longrightarrow \quad w = \frac{P - 2l}{2}$$

> **Takeaway:** isolate the term, then divide by everything multiplying it. Test with small numbers.

Q30 — Medium — Answer: H

Topic: The median of an even number of values

Sort the values first — this is the step people skip. With an even count there is no single middle value, so average the middle two.

Sorted: 2, 4, 6, 8, 9, 11. The middle two are 6 and 8, so the median is 7.

> **Takeaway:** sort first. With an even count, average the middle two.

Q31 — Hard — Answer: B

Topic: An equation with no solution

No solution means the x terms cancel and the constants disagree, so $k = 6$ and the equation becomes $5 = -4$, which is false.

> **Takeaway:** no solution needs matching coefficients and differing constants; infinitely many needs both to match.

Q32 — Hard — Answer: G

Topic: Similar triangles

$$\frac{h}{9} = \frac{2}{1.5} \quad \Longrightarrow \quad h = 9 \times \frac{4}{3} = 12$$

> **Takeaway:** put the same measurement on top of both fractions, then sanity-check the size.

Q33 — Hard — Answer: C

Topic: An exponential model

Count the doubling **periods**, not the hours: 12 hours at one doubling every 3 hours is four doublings, not twelve.

$12 \div 3 = 4$ doublings, so $60 \times 2^4 = 960$.

> **Takeaway:** divide the time by the period to count the doublings.

Q34 — Hard — Answer: J

Topic: Complex numbers

Rewrite each square root of a negative as a multiple of i **before** combining them. The rule $\sqrt{a}\sqrt{b} = \sqrt{ab}$ does not survive a negative under the sign.

$$\sqrt{-9} = 3i \qquad \sqrt{-16} = 4i \qquad 3i + 4i = 7i$$

> **Takeaway:** convert each root to a multiple of i **before** combining. The rule $\sqrt{a}\sqrt{b} = \sqrt{ab}$ fails for negatives.

Q35 — Hard — Answer: A

Topic: Interpreting standard deviation

Class A's scores sit within 2 marks of the mean; class B's spread 25 either side. B is far more spread out.

> **Takeaway:** equal means say nothing about spread. That is what standard deviation measures.

Q36 — Hard — Answer: F

Topic: Two successive percentage changes

Percentage changes multiply rather than add, so take a convenient starting value of 100 and apply each multiplier in turn.

$$1.25 \times 0.80 = 1.00$$

A multiplier of exactly 1 means no overall change.

> **Takeaway:** successive percentage changes multiply. Start from 100.

Q37 — Hard — Answer: C

Topic: Trigonometry in context

The ramp is the hypotenuse and the height is opposite the angle, so use sine: height $= 9 \sin 12^\circ$.

> **Takeaway:** pick the ratio linking the side you have to the side you want. A leg is always shorter than the hypotenuse.

Q38 — Hard — Answer: H

Topic: A radical equation

Squaring gives $x^2 - x - 6 = 0$, so $x = 3$ or $x = -2$. Testing back, $\sqrt{4} = 2 \neq -2$, so -2 is extraneous.

> **Takeaway:** after squaring, every root is only a candidate. Substitute back.

Q39 — Hard — Answer: D

Topic: A logarithm

A logarithm states which power the base must be raised to, so rewrite it as an ordinary power immediately.

$\log_3(x) = 4$ means $3^4 = x$, so $x = 81$.

> **Takeaway:** a logarithm is an exponent. Rewrite it as a power immediately.

Q40 — Hard — Answer: G

Topic: The law of cosines

Two sides with the known angle **between** them is the signal for the law of cosines. Pythagoras would assume a right angle, and this one is 60°.

$$c^2 = 25 + 64 - 2(5)(8)(0.5) = 49 \quad \Longrightarrow \quad c = 7$$

> **Takeaway:** two sides and the angle between them means the law of cosines.

Q41 — Hard — Answer: A

Topic: Scalar multiplication of a matrix

A scalar multiplies **every** entry, not just the first:

$$5\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 5 & 10 \\ 15 & 20 \end{pmatrix}$$

> **Takeaway:** a scalar reaches all four entries. Count them.

Q42 — Hard — Answer: F

Topic: Two rates working together

Times do not add — two pumps together must be faster than either alone, so the answer has to be under 6 hours. What adds is the **rates**.

Rates add: $\frac{1}{6} + \frac{1}{12} = \frac{1}{4}$, so 4 hours.

> **Takeaway:** add the rates, then invert. The answer must be less than the faster worker alone.

Q43 — Hard — Answer: B

Topic: Counting combinations

Each bread can go with each filling, and each of those pairs with each sauce, so the three counts multiply:

$$4 \times 3 \times 2 = 24$$

> **Takeaway:** independent choices multiply.

Q44 — Hard — Answer: H

Topic: The area of a sector

A sector is the same fraction of the circle's area as its angle is of a full turn. Area uses πr^2; arc length would use $2\pi r$.

A 60° sector is one sixth of the circle: $\frac{1}{6} \times 36\pi = 6\pi$.

> **Takeaway:** a sector is $\frac{\theta}{360}$ of the whole. Area uses r^2; arc length does not.

Q45 — Hard — Answer: C

Topic: The discriminant

One solution means $b^2 - 4ac = 0$:

$$100 - 4c = 0 \quad \Longrightarrow \quad c = 25$$

Then $x^2 + 10x + 25 = (x+5)^2$, zero only at $x = -5$.

> **Takeaway:** exactly one solution means the discriminant is zero.

Q46 — Basic — Answer: G

Topic: Finding a percentage

"What percent **of** 250" makes 250 the whole, so it goes underneath:

$$\frac{40}{250} \times 100 = 16\%$$

Check the other way: 16% of 250 is 40.

> **Takeaway:** the number after "percent **of**" goes underneath. Over 100% means you divided upside down.

Q47 — Basic — Answer: D

Topic: Solving a linear equation with brackets

Either divide by the 2 first or expand the bracket fully. Doing half of each is what produces the wrong answers here.

Divide by 2 first: $x + 6 = 13$, so $x = 7$.

> **Takeaway:** either divide by the bracket's multiplier first, or expand fully. Do not do half of each.

Q48 — Basic — Answer: J

Topic: The area of a rectangle in context

Carpet covers a surface, so this is an **area**, not a perimeter. Then finish the question — the area is not the answer.

Area is 24 m^2, so the cost is $24 \times 18 = 432$.

> **Takeaway:** covering a floor is an area. Finish the question — the area is rarely the answer.

Q49 — Basic — Answer: B

Topic: The mode of a data set

The mode is the value that appears most often. Counting: 4 appears three times, 7 twice, and 9, 2 and 5 once each.

$$\text{mode} = 4$$

> **Takeaway:** mode is the most **frequent** value, not the middle one and not the average. Tally before answering.

Q50 — Basic — Answer: H

Topic: Simplifying an expression

$$10x - 15 - 2x - 8 = 8x - 23$$

The -2 multiplies **both** terms of the second bracket.

> **Takeaway:** a minus in front of a bracket changes every sign inside it.

Q51 — Basic — Answer: D

Topic: Probability from equally likely outcomes

"Greater than 4" means 5 or 6 — two outcomes out of six.

$$P = \frac{2}{6} = \frac{1}{3}$$

Note that 4 itself is **not** included; "greater than" excludes the number named.

> **Takeaway:** count the outcomes that qualify, then divide by the total. Read whether the boundary value counts.

Q52 — Basic — Answer: F

Topic: Function notation

The whole numerator is divided by 2, not just part of it, so work out the top before dividing.

$$g(-4) = \frac{-4 + 6}{2} = \frac{2}{2} = 1$$

> **Takeaway:** the whole numerator is divided, not just part of it.

Q53 — Basic — Answer: C

Topic: Converting a rate

Convert the time into the unit the rate uses before multiplying. The rate is per minute, so the 2 hours must become minutes.

2 hours is 120 minutes, so $3 \times 120 = 360$ litres.

> **Takeaway:** convert the time to the rate's unit before multiplying.

Q54 — Basic — Answer: H

Topic: The mode and the range

The range is the largest value minus the smallest, and nothing else:

$$12 - 5 = 7$$

Here the mode is 5 and the mean is about 7.2 — three different numbers from the same six values.

> **Takeaway:** range is largest minus smallest. The mode here is 5 and the mean is about 7.2 — three different numbers.

Q55 — Basic — Answer: A

Topic: A ratio in two parts

Here 63 kg is one **part** of the ratio, not the whole. Find what a single part is worth, then multiply by the number of parts wanted.

One part is $63 \div 7 = 9$ kg, so cement is $2 \times 9 = 18$ kg.

> **Takeaway:** here the 63 is one **part**, not the whole. Read which quantity the number belongs to.

Q56 — Medium — Answer: J

Topic: A system by elimination

Subtracting the second equation from the first removes x:

$$4y = 12 \quad \Longrightarrow \quad y = 3$$

> **Takeaway:** matching coefficients means subtract. Then check which letter was asked for.

Q57 — Medium — Answer: C

Topic: Working backwards from a discount

After a 20% reduction the customer pays 80% of the original, so undo it by **dividing**. Adding 20% back on does not return you to the start.

96 is 80% of the original, so the original is $96 \div 0.8 = 120$.

> **Takeaway:** to undo a percentage change, **divide** by the multiplier. Adding 20% back does not return you to the start.

Q58 — Medium — Answer: F

Topic: A transformation of a parabola

Horizontal shifts go **inside** the brackets and act **opposite** to their sign. Right by 3 means $(x-3)^2$, whose lowest point is at $x = 3$.

> **Takeaway:** inside and opposite for horizontal; outside and obvious for vertical.

Q59 — Medium — Answer: B

Topic: The equation of a circle

$(x-h)^2 + (y-k)^2 = r^2$ with $h = 3$, $k = -2$, $r = 5$. The -2 becomes $(y+2)$ and the right side is 25.

> **Takeaway:** the signs flip and the right-hand side is r^2.

Q60 — Medium — Answer: H

Topic: A weighted average

Averages of unequal groups cannot themselves be averaged. Convert each class to a total, add the totals, and divide by the combined count.

$$\frac{20(68) + 30(78)}{50} = \frac{3700}{50} = 74$$

> **Takeaway:** averages of unequal groups cannot be averaged. Go back to totals.

Q61 — Medium — Answer: D

Topic: A geometric sequence

Subtracting consecutive terms gives no constant, so try dividing: each term is 4 times the one before. That makes it geometric.

The ratio is 4, so the 6th term is $3 \times 4^5 = 3072$.

> **Takeaway:** $a \cdot r^{n-1}$ — the exponent is one less than the term number.

Q62 — Medium — Answer: G

Topic: The mean from a frequency table

Total drinks:

$$1(5) + 2(10) + 3(5) = 5 + 20 + 15 = 40$$

Customers: $5 + 10 + 5 = 20$.

$$\text{mean} = \frac{40}{20} = 2$$

> **Takeaway:** multiply each value by its frequency, add, then divide by the total frequency. Averaging the top row alone ignores how many people gave each answer.

Q63 — Medium — Answer: A

Topic: Reading a decay factor

The multiplier is what the machine **keeps**, so what it loses is the rest. Subtract from 1 rather than reading 0.85 as the loss.

A multiplier of 0.85 keeps 85%, so 15% is lost each year.

> **Takeaway:** the multiplier is what is **kept**. Subtract from 1 for what is lost.

Q64 — Medium — Answer: J

Topic: Probability without replacement

Without replacement, both the count of reds and the total drop by one before the second draw.

$$\frac{4}{10} \times \frac{3}{9} = \frac{12}{90} = \frac{2}{15}$$

> **Takeaway:** without replacement, both the top and the bottom drop by one on the second draw.

Q65 — Medium — Answer: A

Topic: An inequality needing a sign flip

Dividing an inequality by a negative number reverses its direction. Test one value from your answer to confirm.

$-3x > 15$, and dividing by -3 reverses the sign: $x < -5$.

> **Takeaway:** dividing by a negative flips the inequality. Test one value to confirm.

Q66 — Medium — Answer: F

Topic: Probability of an event not happening

Not red means green or yellow: $3 + 5 = 8$ counters out of 20.

$$\frac{8}{20} = \frac{2}{5}$$

Or use the complement: red is $\frac{12}{20} = \frac{3}{5}$, so not red is $1 - \frac{3}{5} = \frac{2}{5}$.

> **Takeaway:** the denominator is **everything possible**. For a "not" question, $1 - P(A)$ is usually the quicker route.

Q67 — Medium — Answer: C

Topic: The vertex of a parabola

The vertex of $y = ax^2 + bx + c$ sits at $x = -\frac{b}{2a}$, and b carries its own sign — here two minus signs cancel.

$$x = -\frac{b}{2a} = -\frac{-6}{2} = 3$$

> **Takeaway:** b carries its own sign, and the formula has a minus of its own. Two minuses make a plus.

Q68 — Medium — Answer: H

Topic: Degrees and radians

Multiply by $\frac{\pi}{180}$. The π stays in the answer; if it has vanished, the conversion went the wrong way.

$$135 \times \frac{\pi}{180} = \frac{3\pi}{4}$$

> **Takeaway:** multiply by $\frac{\pi}{180}$. The π stays in the answer.

Q69 — Medium — Answer: B

Topic: The determinant of a matrix

The determinant is the product along the main diagonal minus the product along the other one:

$$ad - bc = (4)(5) - (3)(2) = 20 - 6 = 14$$

> **Takeaway:** main diagonal first, then subtract the other.

Q70 — Medium — Answer: G

Topic: A Venn diagram

The 12 who play both have been counted twice, once in each total, so subtract them once:

$$34 + 28 - 12 = 50$$

Check by parts: 22 chess only, 16 draughts only, 12 both.

> **Takeaway:** subtract the overlap **once** — it was counted twice.

Q71 — Medium — Answer: C

Topic: Slope from a table

Slope is the change in output divided by the change in input, and on a linear table it is the same between any two rows.

$$\frac{21 - 9}{4 - 1} = \frac{12}{3} = 4$$

> **Takeaway:** slope is change in output over change in input, and on a linear table it is the same between any two rows.

Q72 — Medium — Answer: H

Topic: A perfect-square trinomial

A perfect square is $(x + a)^2 = x^2 + 2ax + a^2$. Here $a^2 = 36$ so $a = 6$, and the middle term is $2a = 12$.

> **Takeaway:** in a perfect square the middle coefficient is **twice** the square root of the constant.

Q73 — Medium — Answer: A

Topic: A three-part inequality

Do the same thing to all three parts at once, and copy each inequality sign down exactly as it was — the ends decide between the options here.

Subtract 1 from all three parts and divide by 2:

$$-4 < x \leq 4$$

> **Takeaway:** operate on all three parts, and carry each inequality sign down unchanged.

Q74 — Medium — Answer: J

Topic: A word problem with two unknowns

With $a + c = 60$ and $14a + 9c = 710$, substituting $a = 60 - c$ gives $840 - 5c = 710$, so $c = 26$ and $a = 34$.

> **Takeaway:** name both unknowns, write one equation for the count and one for the money.

Q75 — Medium — Answer: A

Topic: Comparing two linear models

$$25 + 4n = 65 \quad \Longrightarrow \quad 4n = 40 \quad \Longrightarrow \quad n = 10$$

> **Takeaway:** set the two totals equal. Remove the fixed part before dividing.

Q76 — Hard — Answer: F

Topic: A system with no solution

The first line has slope 2. The second has slope $-\frac{6}{k}$, so $-\frac{6}{k} = 2$ gives $k = -3$. The intercepts then differ, so the lines are parallel and distinct.

> **Takeaway:** no solution means equal slopes and different intercepts. Check the intercept too.

Q77 — Hard — Answer: B

Topic: The distance between two points

The distance formula is Pythagoras on the horizontal and vertical gaps. Watch the double negative in $1-(-3)$.

Gaps of 4 and 3, so $\sqrt{16+9}=5$. A 3-4-5 triangle.

> **Takeaway:** square, add, then root. Watch the double negatives in the subtraction.

Q78 — Hard — Answer: H

Topic: An exponential model in function notation

A multiplier below 1 shrinks the population, and what is lost is $1-b$, not b itself.

A multiplier of 0.64 keeps 64%, so 36% is lost each year.

> **Takeaway:** below 1 is decay. What is lost is $1-b$, not b.

Q79 — Hard — Answer: C

Topic: Powers of the imaginary unit

The powers of i repeat every four, so divide the exponent by 4 and use the **remainder**.

$27 \div 4$ leaves remainder 3, and $i^3=-i$.

> **Takeaway:** the powers of i cycle with period 4. Divide the exponent by 4 and use the remainder.

Q80 — Hard — Answer: F

Topic: The law of sines

$$\frac{p}{\sin 30^\circ}=\frac{10}{\sin 45^\circ} \implies p=\frac{10\times 0.5}{\frac{\sqrt{2}}{2}}=5\sqrt{2}\approx 7.07$$

Smaller angle, shorter side — as it must be.

> **Takeaway:** the law of sines pairs a side with its opposite angle. The bigger angle always faces the longer side.

Q81 — Hard — Answer: D

Topic: Scaling an area

Both dimensions triple, and area is a product of two lengths, so the factor applies twice.

Both dimensions triple, so the area is multiplied by $3 \times 3 = 9$.

> **Takeaway:** lengths by k, areas by k^2, volumes by k^3.

Q82 — Hard — Answer: G

Topic: A rational equation

$$\frac{15}{x} - \frac{9}{x} = 2 \implies \frac{6}{x} = 2 \implies x = 3$$

> **Takeaway:** combine fractions over the shared denominator first. Then check the answer does not make it zero.

Q83 — Hard — Answer: A

Topic: Combining logarithms

$\log_2 8 = 3$ and $\log_2 4 = 2$, so the sum is 5. Equivalently $\log_2(32) = 5$.

> **Takeaway:** adding logs multiplies what is inside. With small powers, evaluate each separately and use the rule as a check.

Q84 — Hard — Answer: J

Topic: The magnitude of a vector

Magnitude is Pythagoras on the two components, and squaring removes the minus sign:

$$\sqrt{5^2 + (-12)^2} = \sqrt{169} = 13$$

> **Takeaway:** magnitude is Pythagoras on the components, and is never negative.

Q85 — Hard — Answer: A

Topic: Perpendicular lines

The perpendicular slope is $\frac{1}{2}$. Then $3 = \frac{1}{2}(4) + b$ gives $b = 1$.

> **Takeaway:** flip **and** negate for perpendicular, then substitute the point.

Q86 — Hard — Answer: F

Topic: Interpreting a linear model

Setting $m = 0$ gives $C = 4$, so the 4 is what you pay before moving. The 2.5 is the cost per mile.

> **Takeaway:** the constant is the value at zero; the coefficient is the rate.

Q87 — Hard — Answer: C

Topic: A quadratic from its roots

Roots -4 and 6 give factors $(x + 4)$ and $(x - 6)$:

$$(x + 4)(x - 6) = x^2 - 2x - 24$$

> **Takeaway:** a root of r gives a factor of $(x - r)$, so the signs flip.

Q88 — Hard — Answer: F

Topic: Comparing spread from a summary table

Both means are 50, which says nothing about spread. Machine A's output stays within 2 of the mean; machine B's ranges 30 either side.

B's values sit much further from the mean, so its standard deviation is larger.

> **Takeaway:** equal means do not imply equal spread. That is precisely what standard deviation measures, and a range is enough to compare two sets roughly.

Q89 — Hard — Answer: B

Topic: A percentage of a percentage

"Of those owners" means the 25% applies to the 60%, not to the town, and "of" means multiply:

$$0.60 \times 0.25 = 0.15 = 15\%$$

> **Takeaway:** "of" means multiply. A percentage of a percentage is smaller than either.

Q90 — Hard — Answer: J

Topic: Several ideas together

With width w and length $2w$, the perimeter is $6w = 54$, so $w = 9$ and the length is 18. The area is $9 \times 18 = 162$.

> **Takeaway:** name the smaller quantity, write the other in terms of it, then answer the question actually asked — here area, not a side.

Chapter 10 — The Challenge Set

Twenty questions, all of them harder than anything the ACT is likely to ask you.

That is deliberate. Every other chapter in this book is calibrated to the real test. This one is calibrated to the *edge* of what the test's own content can be made to do. Work through these and the real thing will feel narrow.

What makes these different. A normal ACT question tests one idea and gives you sixty seconds to do it. These need two or three ideas joined together, and none of them tells you which. Several have a first answer that is wrong for a reason you will only see if you check — a root the logarithm forbids, a quadrant that flips a sign, a probability whose denominator changed while you were not looking.

Nothing here is off-syllabus. Logarithms, matrices, complex numbers, sequences, vectors and trigonometric identities are all ACT content and all appear in the earlier chapters. The difficulty is in the combination and in the traps, not in unfamiliar material.

The letters still alternate. Odd-numbered questions are lettered A B C D and even-numbered ones F G H J, exactly as on the real test. If you find yourself looking for option E, you have lost your place.

A warning about the wrong options. Every wrong option here is a value you can actually reach by making one specific, reasonable mistake. None of them are padding, and arriving at a tidy number is not evidence that you are right.

Topics covered: logarithmic equations with a forbidden root · determinants and scaling · complex division · arithmetic series · quadrants and sign · vectors · circles from general form · dependent probability · inverse functions · mixtures · absolute value inequalities · similar solids · change of base · arrangements with repeats · nonlinear systems · trigonometric equations · exponential decay · endpoints from a midpoint · means after a removal · the law of cosines

---

Section 1 — When the First Answer Is Not the Answer

Every question in this section has a step after the step you think is last: a root the logarithm forbids, a sign the quadrant decides, a square root nobody took, a denominator that changed while you were not looking. Reaching a tidy number is not evidence that you are finished.

Q1 — Brutal

What is the value of x that satisfies $\log_2(x) + \log_2(x-2) = 3$?

A) 4

B) -2

C) 2

D) -8

Q2 — Brutal

For the 2×2 matrix $A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$, what is the determinant of $2A$?

F) 5

G) 10

H) 20

J) 40

Q3 — Brutal

What is $\dfrac{2+3i}{1-i}$ in the form $a + bi$?

A) $-1 + 5i$

B) $-\frac{1}{2} - \frac{5i}{2}$

C) $\frac{5}{2} + \frac{i}{2}$

D) $-\frac{1}{2} + \frac{5i}{2}$

Q4 — Brutal

What is the sum of all the multiples of 7 between 100 and 300?

F) 5586

G) 28

H) 399

J) 8232

Q5 — Brutal

If $\sin\theta = \dfrac{3}{5}$ and $90^\circ < \theta < 180^\circ$, what is the value of $\tan\theta$?

A) $\frac{3}{4}$

B) $-\frac{3}{4}$

C) $-\frac{4}{3}$

D) $\frac{4}{3}$

Q6 — Brutal

Let $\mathbf{u} = \langle 5, -12 \rangle$ and $\mathbf{v} = \langle -1, 5 \rangle$. What is $\|\mathbf{u} + \mathbf{v}\|$?

F) 11

G) $5\sqrt{13}$

H) 13

J) $\sqrt{65}$

Q7 — Brutal

In the standard coordinate plane, a circle has equation $x^2 + y^2 + 4x - 6y - 12 = 0$. What is its radius?

A) 5

B) $2\sqrt{3}$

C) 12

D) 25

Q8 — Brutal

A bag contains 5 red marbles and 3 blue marbles. Two marbles are drawn at random, one after the other, without replacement. What is the probability that both are red?

F) $\frac{25}{64}$

G) $\frac{5}{14}$

H) $\frac{5}{16}$

J) $\frac{15}{56}$

Q9 — Brutal

For $f(x) = \dfrac{2x+1}{x-3}$, what is the value of $f^{-1}(5)$?

A) 3

B) $\frac{11}{2}$

C) $\frac{16}{3}$

D) $-\frac{16}{3}$

Q10 — Brutal

A container holds 20 litres of a solution that is 30% acid. How many litres of **pure acid** must be added so that the result is 50% acid?

F) 8

G) 4

H) 20

J) 10

Section 2 — Two Ideas at Once

The second ten join two pieces of content that the earlier chapters teach apart. A ratio question that is really about cubes. A logarithm that is really about which base you are standing in. A counting question where the objects are not all different. None of the individual ideas is new; the joining is the difficulty, and no line of the question tells you which two are being joined.

Q11 — Brutal

How many integers x satisfy $|2x - 7| < 5$?

A) 5

B) 2

C) 6

D) 4

Q12 — Brutal

Two solid containers are similar in shape. Their volumes are 216 cm^3 and 729 cm^3. The surface area of the smaller container is 240 cm^2. What is the surface area of the larger container, in square centimetres?

F) 360

G) 720

H) 810

J) 540

Q13 — Brutal

What is the value of $\log_4 8$?

A) $\frac{3}{2}$

B) 3

C) 2

D) $\frac{2}{3}$

Q14 — Brutal

How many distinguishable arrangements are there of the six letters in the word BANANA?

F) 720

G) 120

H) 60

J) 360

Q15 — Brutal

In the standard coordinate plane, the graphs of $y = x^2 - 3x + 2$ and $y = 2x - 4$ intersect at exactly two points. What is the sum of the y-coordinates of those two points?

A) 5

B) 2

C) 0

D) 6

Q16 — Brutal

How many values of θ with $0^\circ \leq \theta < 360^\circ$ satisfy $2\sin^2\theta = \sin\theta$?

F) 4

G) 3

H) 1

J) 2

Q17 — Brutal

A radioactive material has a half-life of 6 years. A sample with a mass of 480 grams is stored. After how many years will 15 grams of it remain?

A) 192

B) 24

C) 30

D) 5

Q18 — Brutal

In the standard coordinate plane, $M(3, -1)$ is the midpoint of $\overline{AB}$. If A is the point $(-2, 4)$, what are the coordinates of B?

F) $\left(\frac{1}{2},\ \frac{3}{2}\right)$

G) $(8,\ -6)$

H) $(5,\ -5)$

J) $(-7,\ 9)$

Q19 — Brutal

The mean of a list of 12 numbers is 45. One number is removed from the list, and the mean of the remaining 11 numbers is 43. What number was removed?

A) 2

B) 22

C) 24

D) 67

Q20 — Brutal

In triangle ABC, $AB = 5$, $BC = 7$, and $AC = 8$. What is the area of triangle ABC, in square units?

F) $\frac{35}{2}$

G) 20

H) 300

J) $10\sqrt{3}$

Answers and Explanations

Section 1 — When the First Answer Is Not the Answer

Q1 — Brutal — Answer: A

Topic: A logarithmic equation with an inadmissible root

Two logarithms with the same base add by multiplying what is inside them:

$$\log_2(x) + \log_2(x-2) = \log_2\big(x(x-2)\big) = 3$$

Undo the logarithm by writing it as a power of the base:

$$x(x-2) = 2^3 = 8 \quad\Longrightarrow\quad x^2 - 2x - 8 = 0 \quad\Longrightarrow\quad (x-4)(x+2) = 0$$

So $x = 4$ or $x = -2$ — **from the equation you rewrote**, which is not quite the equation you were given.

Go back and test both. At $x = -2$ the original contains $\log_2(-2)$, and a logarithm of a negative number does not exist. That root is inadmissible. At $x = 4$: $\log_2 4 + \log_2 2 = 2 + 1 = 3$. ✓

So $x = 4$.

Why each wrong option is wrong:

- **B,** -2 — the forbidden root. It satisfies $x^2 - 2x - 8 = 0$ perfectly and cannot be substituted into the question. Combining two logarithms into one widens the domain, and everything gained that way has to be tested.
- **C,** 2 — added the two roots. There is only one solution.
- **D,** -8 — multiplied them, giving the constant term of the quadratic.

> **Takeaway:** Whenever you combine logarithms, you have changed the domain. Every root must go back into the **original** expression, and any that asks for the log of zero or a negative is discarded.

Q2 — Brutal — Answer: H

Topic: What scaling a matrix does to its determinant

There are two ways through, and they agree.

Directly. $2A = \begin{pmatrix} 4 & 2 \\ 6 & 8 \end{pmatrix}$, so

$$\det(2A) = 4 \times 8 - 2 \times 6 = 32 - 12 = 20$$

By the rule. Multiplying a matrix by a scalar multiplies **every** entry, and the determinant of a 2×2 is a product of two entries. So each term picks up two factors of the scalar:

$$\det(kA) = k^2 \det(A) \quad \text{for a } 2 \times 2$$

Here $\det(A) = 2(4) - 1(3) = 5$, so $\det(2A) = 2^2 \times 5 = 20$.

Why each wrong option is wrong:

- **F,** 5 — gave $\det(A)$ and ignored the scaling entirely.
- **G,** 10 — multiplied the determinant by 2 once. The scalar reaches every entry, not the determinant as a whole.
- **J,** 40 — used k^3, which is the rule for a 3×3. The exponent is the size of the matrix.

> **Takeaway:** $\det(kA) = k^n \det(A)$ for an $n \times n$ matrix. For the 2×2 matrices the ACT uses, that is k^2 — and if you cannot recall it, multiplying the four entries out takes ten seconds and settles it.

Q3 — Brutal — Answer: D

Topic: Dividing complex numbers

A complex number cannot be left in a denominator. Clear it by multiplying top and bottom by the **conjugate** of the denominator — same real part, opposite imaginary part. For $1 - i$ that is $1 + i$.

$$\frac{2+3i}{1-i} \times \frac{1+i}{1+i}$$

Top: $(2+3i)(1+i) = 2 + 2i + 3i + 3i^2 = 2 + 5i - 3 = -1 + 5i$, using $i^2 = -1$.

Bottom: $(1-i)(1+i) = 1 - i^2 = 1 + 1 = 2$. The conjugate is chosen precisely so that this comes out real.

$$\frac{-1+5i}{2} = -\frac{1}{2} + \frac{5}{2}i$$

Why each wrong option is wrong:

- **A,** $-1 + 5i$ — multiplied out the top correctly and never divided by the 2 underneath. The last step is the one that gets dropped.
- **B,** $-\frac{1}{2} - \frac{5i}{2}$ — right size, wrong sign on the imaginary part, from mishandling $3i^2$.

- **C,** $\frac{5}{2} + \frac{i}{2}$ — multiplied top and bottom by $1 - i$, the denominator itself rather than its conjugate. That leaves $(1-i)^2 = -2i$ underneath, still complex, which defeats the purpose.

> **Takeaway:** Multiply by the conjugate, remember $i^2 = -1$, and split the result into real and imaginary halves at the end. If your denominator still has an i in it, you used the wrong multiplier.

Q4 — Brutal — Answer: F

Topic: The sum of an arithmetic series you have to build first

Nothing is handed to you here — the first term, the last term and the count all have to be found before any formula can be used.

First multiple of 7 above 100: $7 \times 15 = 105$. **Last multiple of 7 below 300:** $7 \times 42 = 294$.

How many terms? They step by 7, so

$$n = \frac{294 - 105}{7} + 1 = 27 + 1 = 28$$

The $+1$ is what counts the first term itself, and leaving it off is the classic fencepost error.

Now sum. An arithmetic series is the number of terms times the average of the first and last:

$$S = 28 \times \frac{105 + 294}{2} = 28 \times \frac{399}{2} = 14 \times 399 = 5586$$

Why each wrong option is wrong:

- **G,** 28 — stopped at the number of terms.
- **H,** 399 — gave $105 + 294$, the first plus the last, without averaging or multiplying.
- **J,** 8232 — multiplied the count by the **last** term instead of by the average. That overcounts every term except the last.

> **Takeaway:** For "the sum of all multiples of k between two bounds", find the first and last multiples inside the range, count with $\frac{\text{last}-\text{first}}{k} + 1$, then use $S = n \times \frac{\text{first}+\text{last}}{2}$.

Q5 — Brutal — Answer: B

Topic: A trigonometric ratio in the second quadrant

Find the cosine first, then divide. The identity is

$$\sin^2\theta + \cos^2\theta = 1 \implies \cos^2\theta = 1 - \left(\tfrac{3}{5}\right)^2 = \tfrac{16}{25} \implies \cos\theta = \pm\tfrac{4}{5}$$

The sign is the question. In the second quadrant sine is positive and cosine is negative, so $\cos\theta = -\frac{4}{5}$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3/5}{-4/5} = -\frac{3}{4}$$

A quick sanity check: in the second quadrant only sine is positive, so tangent must be negative. Any positive option can be discarded before any arithmetic.

Why each wrong option is wrong:

- **A,** $\frac{3}{4}$ — took $\cos\theta = +\frac{4}{5}$ and ignored the range given. The range is the only reason it was stated.
- **C,** $-\frac{4}{3}$ — computed $\frac{\cos}{\sin}$, which is $\cot\theta$.
- **D,** $\frac{4}{3}$ — inverted *and* lost the sign.

> **Takeaway:** The Pythagorean identity gives a magnitude; the quadrant gives the sign. Decide the sign before dividing, and check it against "all, sine, tan, cos" at the end.

Q6 — Brutal — Answer: J

Topic: The magnitude of a resultant vector

Add the vectors component by component first, then take one magnitude at the end. Magnitudes cannot be added — that is the whole difficulty.

$$\mathbf{u} + \mathbf{v} = \langle 5 + (-1),\ -12 + 5\rangle = \langle 4, -7\rangle$$
$$\|\langle 4, -7\rangle\| = \sqrt{4^2 + (-7)^2} = \sqrt{16 + 49} = \sqrt{65}$$

That is about 8.06.

Note that $\|\mathbf{u}\| = 13$ and $\|\mathbf{v}\| = \sqrt{26} \approx 5.1$, and the resultant is shorter than **either** sum you might have guessed — because the two vectors point in nearly opposite directions and partly cancel.

Why each wrong option is wrong:

- **F,** 11 — added 4 and 7. That is the distance walked along the axes, not the straight-line length.
- **G,** $5\sqrt{13}$ — subtracted the vectors instead of adding, giving $\langle 6, -17\rangle$ and $\sqrt{325}$.
- **H,** 13 — gave $\|\mathbf{u}\| = 13$ and never used $\mathbf{v}$.

> **Takeaway:** Combine components, then apply Pythagoras once. $\|\mathbf{u} + \mathbf{v}\|$ is almost never $\|\mathbf{u}\| + \|\mathbf{v}\|$ — they are equal only when the vectors point the same way.

Q7 — Brutal — Answer: A

Topic: The radius of a circle given in general form

A general-form equation tells you nothing until the squares are completed. Group the variables and move the constant across:

$$x^2 + 4x + y^2 - 6y = 12$$
$$(x+2)^2 - 4 + (y-3)^2 - 9 = 12$$
$$(x+2)^2 + (y-3)^2 = 12 + 4 + 9 = 25$$

The centre is $(-2, 3)$ and the radius is $\sqrt{25} = 5$.

The two corrections both **add** to the right-hand side, because both were subtracted on the left. Getting one of those signs wrong is what the other options are made of.

Why each wrong option is wrong:

- **B,** $2\sqrt{3}$ — took the square root of the bare constant 12, without adding what completing the squares contributed.
- **C,** 12 — read the 12 as r^2 and stopped.
- **D,** 25 — reported $r^2 = 25$ rather than r. The last square root is the step most often skipped on this question.

> **Takeaway:** Complete the square in both variables before reading anything off. Then $r = \sqrt{\text{right-hand side}}$ — and confirm you have taken the root, not left the square.

Q8 — Brutal — Answer: G

Topic: Two draws without replacement

"Without replacement" means the bag is different for the second draw, and both numbers in the fraction change.

First draw. 5 red out of 8 marbles: $\frac{5}{8}$.

Second draw. One red has gone, so 4 red remain out of 7 marbles: $\frac{4}{7}$.

The two must both happen, so multiply:

$$\frac{5}{8} \times \frac{4}{7} = \frac{20}{56} = \frac{5}{14}$$

That is about 0.357.

Why each wrong option is wrong:

- **F,** $\frac{25}{64}$ — used $\frac{5}{8}$ twice, which is the answer *with* replacement.
- **H,** $\frac{5}{16}$ — reduced the number of reds to 4 but left the total at 8. Both counts drop, because the marble that left the numerator also left the bag.
- **J,** $\frac{15}{56}$ — computed red then blue.

> **Takeaway:** Without replacement, decrease the top **and** the bottom for the second draw. Write the two fractions side by side before multiplying, and check that each denominator is one smaller than the last.

Q9 — Brutal — Answer: C

Topic: The value of an inverse function

You never have to build the inverse function. $f^{-1}(5)$ is, by definition, the input whose output is 5 — so solve $f(x) = 5$.

$$\frac{2x+1}{x-3} = 5 \quad \Longrightarrow \quad 2x + 1 = 5(x-3) = 5x - 15$$

$$16 = 3x \quad \Longrightarrow \quad x = \frac{16}{3}$$

Check: $f\left(\frac{16}{3}\right) = \frac{32/3+1}{16/3-3} = \frac{35/3}{7/3} = 5$. ✓

Why each wrong option is wrong:

- **A,** 3 — gave $x = 3$, the value excluded from the domain because it makes the denominator zero.
- **B,** $\frac{11}{2}$ — computed $f(5) = \frac{11}{2}$. That is the function, not its inverse, and the two are only equal by coincidence.
- **D,** $-\frac{16}{3}$ — lost a sign moving the -15 across.

> **Takeaway:** $f^{-1}(k)$ means "solve $f(x) = k$". Rearranging the whole function into an inverse formula is extra work and an extra chance to slip.

Q10 — Brutal — Answer: F

Topic: Strengthening a mixture

Track the **acid**, and remember that adding pure acid increases the total volume as well. That second point is the whole question.

Acid at the start: 30% of $20 = 6$ litres.

Add L litres of pure acid. Acid becomes $6 + L$; total volume becomes $20 + L$.

The result must be 50% acid:

$$6 + L = 0.5(20 + L) = 10 + 0.5L$$

$$0.5L = 4 \implies L = 8$$

Check: $6 + 8 = 14$ litres of acid in 28 litres of solution, and $\frac{14}{28} = 50\%$. ✓

Why each wrong option is wrong:

- **G,** 4 — set the final amount against 50% of the **original** 20 litres, holding the total fixed. Pouring in 4 litres would give 10 litres of acid in 24 litres of solution, about 42%.
- **H,** 20 — assumed you must double the container to double the concentration. The concentration is not doubling; it is going from 30% to 50%.
- **J,** 10 — computed 50% of 20 and offered it as the amount to add.

> **Takeaway:** In a mixture question, write "amount of the substance" on one side and "fraction × new total" on the other. Whatever you add goes into the total as well as the substance, unless the question says it evaporates.

Section 2 — Two Ideas at Once

Q11 — Brutal — Answer: D

Topic: Counting the integers that satisfy an absolute value inequality

An absolute value less than something is a **band**, and it opens into a double inequality:

$$|2x - 7| < 5 \iff -5 < 2x - 7 < 5$$

Add 7 throughout, then halve throughout:

$$2 < 2x < 12 \implies 1 < x < 6$$

Now stop and read what that says. Both ends are **strict**, so 1 and 6 are out. The integers strictly between them are 2, 3, 4, 5 — four of them.

Counting integers in an interval is where this goes wrong. Do not subtract the endpoints; list them if the range is short, and if it is long, count as "largest minus smallest, plus one" **on the integers that are actually included** — here $5 - 2 + 1 = 4$.

Why each wrong option is wrong:

- **A,** 5 — computed $6-1$. That is the width of the interval, not the number of integers in it, and the two are never equal.
- **B,** 2 — solved only $2x-7<5$ with $2x-7$ positive, giving $3.5 \le x < 6$ and just $\{4,5\}$. An absolute value has two branches and both produce solutions.
- **C,** 6 — read the sign as $\le$, which admits $x=1$ and $x=6$ and gives six integers. Check: $|2(1)-7|=5$, and 5 is not less than 5.

> **Takeaway:** $|A|<b$ becomes $-b<A<b$; $|A|>b$ becomes two separate one-sided pieces. Solve to an interval, decide whether each end is included, then count the integers by listing or by "last minus first, plus one".

Q12 — Brutal — Answer: J

Topic: Surface area of a solid similar to another

Similar solids have **one** scale factor between them, and area and volume are different powers of it. Get the scale factor first; never scale one quantity by another quantity's ratio.

Length. The volumes are in the ratio $\frac{729}{216}=\frac{27}{8}$, and volume goes as the cube of length, so

$$\left(\frac{\text{length}_2}{\text{length}_1}\right)^3=\frac{27}{8} \quad\Longrightarrow\quad \frac{\text{length}_2}{\text{length}_1}=\sqrt[3]{\frac{27}{8}}=\frac{3}{2}$$

Area. Area goes as the **square** of length:

$$240\times\left(\frac{3}{2}\right)^2=240\times\frac{9}{4}=540$$

The whole family is worth carrying as one line: **length k, area k^2, volume k^3.** From any one of the three you can get to the other two.

Why each wrong option is wrong:

- **F,** 360 — multiplied by $\frac{3}{2}$, the **length** ratio. Area is the square of it.
- **G,** 720 — used 3 as the scale factor, because $729=3^6$ and the cube root of 729 is 9... the reasoning varies, but the mistake is the same one: the ratio is 3 to 2, not 3 to 1. The smaller solid is not a unit.
- **H,** 810 — multiplied by $\frac{27}{8}$, the **volume** ratio, which is the cube.

> **Takeaway:** Find the **length** scale factor first, from whichever ratio you are given, then square it for area and cube it for volume. Scaling an area by a volume ratio is the single most common error on this question type.

Q13 — Brutal — Answer: A

Topic: Evaluating a logarithm whose base is not the obvious one

$\log_4 8$ asks: **4 to what power is 8?** Neither 1 nor 2 works — $4^1 = 4$ and $4^2 = 16$ — so the answer is between them and is not a whole number. That alone rules out two options.

Write both numbers as powers of 2 and let y be the answer:

$$4^y = 8 \quad \Longrightarrow \quad (2^2)^y = 2^3 \quad \Longrightarrow \quad 2^{2y} = 2^3$$

Same base, so the exponents match: $2y = 3$ and $y = \frac{3}{2}$.

Change of base gets you there too, and is worth knowing because it works when the two numbers share no convenient base:

$$\log_4 8 = \frac{\log 8}{\log 4} = \frac{3 \log 2}{2 \log 2} = \frac{3}{2}$$

Why each wrong option is wrong:

- **B,** 3 — computed $\log_2 8 = 3$, using base 2 instead of base 4. The small number is the base, and changing it changes the answer.
- **C,** 2 — divided 8 by 4. A logarithm is an exponent, not a quotient.
- **D,** $\frac{2}{3}$ — computed $\log_8 4 = \frac{2}{3}$, with the base and the argument swapped. Notice that this is the reciprocal of the right answer, which is exactly what swapping them always produces.

> **Takeaway:** Read $\log_b a$ as "b to what power gives a", bracket the answer between two whole powers to sanity-check it, and use $\log_b a = \frac{\log a}{\log b}$ when the powers do not come out cleanly.

Q14 — Brutal — Answer: H

Topic: Arranging letters when some of them repeat

Six letters in a row would give $6! = 720$ orders **if every letter were different**. They are not: BANANA is B, three A's and two N's.

Swapping two of the A's produces the same word, so the 720 counts each real arrangement several times over — once for each way of shuffling the identical letters among themselves. There are 3! ways to permute the A's and 2! ways to permute the N's, so every distinguishable word has been counted $3! \times 2! = 6 \times 2 = 12$ times.

$$\frac{6!}{3!\,2!} = \frac{720}{12} = 60$$

Why each wrong option is wrong:

- **F,** 720 — $6! = 720$, treating all six letters as different.
- **G,** 120 — divided by 3! for the A's and forgot the N's.
- **J,** 360 — divided by 2! for the N's and forgot the A's.

> **Takeaway:** For arrangements with repeats, divide the factorial of the total by the factorial of **each** repeated count. Every repeated letter needs its own factor in the denominator — count the letters, then count the divisions, and make sure the two lists agree.

Q15 — Brutal — Answer: B

Topic: Where a parabola meets a line

At an intersection the two graphs have the same y for the same x, so set the right-hand sides equal:

$$x^2 - 3x + 2 = 2x - 4 \implies x^2 - 5x + 6 = 0 \implies (x-2)(x-3) = 0$$

So $x = 2$ and $x = 3$ — **and those are x-coordinates.** The question asked for y. Feed each one back into either equation; the line is easier:

$$y = 2(2) - 4 = 0 \qquad y = 2(3) - 4 = 2$$

The two points are $(2, 0)$ and $(3, 2)$, and the sum of the y-coordinates is $0 + 2 = 2$.

Why each wrong option is wrong:

- **A,** 5 — $2 + 3 = 5$, the sum of the x-coordinates. It is also what the quadratic hands you directly as $-\frac{b}{a}$, which makes it very easy to reach and stop.
- **C,** 0 — found $y = 0$ at the first point and stopped. Two intersections means two y-values.
- **D,** 6 — $2 \times 3 = 6$, the product of the roots, and the constant term of $x^2 - 5x + 6$.

> **Takeaway:** Solving a system gives you the x-values; substituting gives you the points. Before writing anything down, reread which coordinate — or which sum, or which difference — the question actually wants.

Q16 — Brutal — Answer: F

Topic: How many solutions a trigonometric equation has in one turn

Never divide both sides by $\sin\theta$. That is the whole question. Dividing by something that can be zero throws away every solution where it *is* zero, and here that is half of them.

Bring everything to one side and factor instead:

$$2\sin^2\theta - \sin\theta = 0 \quad \Longrightarrow \quad \sin\theta\,(2\sin\theta - 1) = 0$$

A product is zero when either factor is, so there are two equations to solve:

- $\sin\theta = 0$ gives $\theta = 0^\circ$ and $\theta = 180^\circ$ (not 360°, which the range excludes)
- $\sin\theta = \frac{1}{2}$ gives $\theta = 30^\circ$ and, because sine is positive in the second quadrant too, $\theta = 150^\circ$

That is $0^\circ, 30^\circ, 150^\circ, 180^\circ$ — four values.

Why each wrong option is wrong:

- **G,** 3 — kept both factors but took only the first-quadrant solution of $\sin\theta = \frac{1}{2}$, giving three.
- **H,** 1 — did both: divided by $\sin\theta$ and then gave the single principal value 30°.
- **J,** 2 — divided through by $\sin\theta$, kept only $\sin\theta = \frac{1}{2}$, and found two.

> **Takeaway:** Factor, never divide, when a trigonometric function appears on both sides. Then for each factor ask how many angles in one full turn give that value — usually two, and the CAST diagram says which two.

Q17 — Brutal — Answer: C

Topic: Reading a half-life backwards

Halve repeatedly and count the steps, then convert steps to years. Doing it by hand is fastest and least error-prone:

$$480 \to 240 \to 120 \to 60 \to 30 \to 15$$

That is **five** halvings. Each one takes 6 years, so

$$5 \times 6 = 30 \text{ years}$$

Algebraically, with n half-lives: $480 \times \left(\frac{1}{2}\right)^n = 15$, so $\left(\frac{1}{2}\right)^n = \frac{15}{480} = \frac{1}{32}$ and $n = 5$, because $2^5 = 32$.

Why each wrong option is wrong:

- **A,** 192 — the mass fell by a factor of $\frac{480}{15} = 32$, and this treats 32 as the number of half-lives: $32 \times 6 = 192$. The **ratio** is 32; the number of halvings is the power of 2 that makes 32, which is 5.
- **B,** 24 — counted four halvings instead of five. The arrows are easy to miscount; the check is that the last number you write must be the target, 15, not one step above it.
- **D,** 5 — gave 5, the number of half-lives, and never multiplied by the 6 years each one takes.

> **Takeaway:** Halving problems have two different numbers in them — how many times you halve, and how long each halving takes. Write the chain of masses out, count the arrows, then multiply.

Q18 — Brutal — Answer: G

Topic: Finding the other endpoint from a midpoint

The midpoint formula averages the endpoints. Run it in reverse: if M is the average of A and B, then B is **twice** M **minus** A, coordinate by coordinate.

$$\frac{-2+B_x}{2}=3 \quad\Longrightarrow\quad B_x=2(3)-(-2)=8$$
$$\frac{4+B_y}{2}=-1 \quad\Longrightarrow\quad B_y=2(-1)-4=-6$$

So $B=(8,-6)$.

The step from A to M is $(+5,-5)$; the step from M to B must be the same again, landing on $(3+5,-1-5)=(8,-6)$. ✓ Walking the step twice is often quicker than the formula and self-checks as you go.

Why each wrong option is wrong:

- **F,** $\left(\frac{1}{2},\ \frac{3}{2}\right)$ — averaged M with A, giving the midpoint of $\overline{AM}$ instead of the far end of $\overline{AB}$.
- **H,** $(5,\ -5)$ — gave $(5,-5)$, which is the **step** from A to M, not a point. It is a useful quantity; it just needs adding to M.
- **J,** $(-7,\ 9)$ — computed $2A-M$, running the segment out from A in the wrong direction. Check any candidate by averaging it with A: it must give M, and $(-7,9)$ averages with $(-2,4)$ to $(-4.5,6.5)$.

> **Takeaway:** $B=2M-A$. Double the midpoint, subtract the known end, and verify by averaging your answer back with the known end.

Q19 — Brutal — Answer: D

Topic: Recovering a value that was removed from a data set

Means cannot be added or subtracted, but **totals** can. Convert both means to totals first — that is the only move this question needs.

Total of all 12: $12\times45=540$. Total of the remaining 11: $11\times43=473$.

The number removed is the difference between the two totals:

$$540 - 473 = 67$$

The size of it is worth noticing. The mean fell by only 2, and the value removed was 67 — well above the old mean of 45. That is the point of the question: removing one number moves the mean of eleven others only a little, so the removed value has to be a long way out to do it.

Why each wrong option is wrong:

- **A,** 2 — gave $45 - 43 = 2$, the change in the **mean**. Removing a 2 from a list averaging 45 would push the mean up, not down.
- **B,** 22 — $11 \times 2 = 22$, multiplying the drop by the new count. Close in shape to a correct method, but the drop applies to eleven numbers while the removed value belongs to the original twelve.
- **C,** 24 — $12 \times 2 = 24$, the same idea with the old count.

> **Takeaway:** Turn every mean into a total the moment you see it: total $=$ mean $\times$ count. Once both totals are on the page, the answer is a subtraction, and a sanity check is free — the removed value must sit on the side of the old mean that explains which way the mean moved.

Q20 — Brutal — Answer: J

Topic: The area of a triangle from its three sides

Three sides and no angle, and no height either. Nothing you know about area applies until an angle exists, so **make one** with the law of cosines.

Step 1 — find $\cos B$. Angle B sits between AB and BC, and the side opposite it is $AC = 8$:

$$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos B$$

$$64 = 25 + 49 - 2(5)(7)\cos B = 74 - 70\cos B$$

$$70\cos B = 10 \quad \Longrightarrow \quad \cos B = \frac{1}{7}$$

Step 2 — find $\sin B$. From $\sin^2 B + \cos^2 B = 1$, and taking the positive root because an angle in a triangle is between 0° and 180°, where sine is always positive:

$$\sin B = \sqrt{1 - \tfrac{1}{49}} = \sqrt{\tfrac{48}{49}} = \frac{4\sqrt{3}}{7}$$

Step 3 — area. With two sides and the angle between them:

$$\text{Area} = \tfrac{1}{2}(AB)(BC)\sin B = \tfrac{1}{2}(5)(7)\cdot\frac{4\sqrt{3}}{7} = \tfrac{1}{2}(5)(4\sqrt{3}) = 10\sqrt{3}$$

That is about 17.3 square units.

If you happen to know Heron's formula, it confirms this in one line: with $s = \frac{5+7+8}{2} = 10$,

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{10 \cdot 5 \cdot 3 \cdot 2} = \sqrt{300} = 10\sqrt{3}$$

Why each wrong option is wrong:

- **F,** $\frac{35}{2}$ — $\frac{1}{2}(5)(7) = 17.5$, treating angle B as a right angle. It is not: $5^2 + 7^2 = 74 \neq 64$. Notice how close 17.5 is to the correct 17.3 — the triangle is *nearly* right-angled, which is exactly why this option cannot be ruled out by estimating.
- **G,** 20 — $\frac{1}{2}(5)(8) = 20$, using two sides as though one were a base and the other its height. A side is only a height when it is perpendicular to the base.
- **H,** 300 — computed $s(s-a)(s-b)(s-c) = 10 \cdot 5 \cdot 3 \cdot 2 = 300$ under Heron's formula and never took the square root. A triangle with sides 5, 7 and 8 cannot have an area of 300; the largest area those sides could enclose is under 20.

> **Takeaway:** Three sides and no angle means the law of cosines. Get cos of the angle between the two sides you want to use, convert it to sin with the Pythagorean identity, then apply $\text{Area} = \frac{1}{2}ab\sin C$. Heron's formula is a shortcut worth knowing, and its square root is the step people drop.