NATIONAL BENCHMARK TEST · MATHEMATICS

# The Complete NBT Mathematics Practice Book

915 practice questions with complete, plain-language explanations for the National Benchmark Test (MAT)

Math_Fixer

- ✓ Every explanation teaches the idea, the steps, and the traps
- ✓ Every wrong option decoded — learn the mistakes before you make them
- ✓ Complete study guide, formula reference and exam strategy
- ✓ Three full 60-question mock examinations

FIRST EDITION — 2026

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# Contents

# Complete Study Guide

**Read this first — it is written for you.**

If maths has never quite "clicked" for you, this guide is built exactly for that. Nothing here assumes you already understand it. Every rule is explained in plain English *before* you see the formula, every symbol is spelled out in words, and every method comes with a fully worked example where **every single step is shown** — no steps are skipped and nothing is left for you to "just know."

You do **not** have to be naturally good at maths to do well on this test. You have to understand a small number of ideas properly, and practise them until they feel familiar. That is a skill anyone can build. Take it one small idea at a time, and be patient with yourself.

The National Benchmark Test in Mathematics (NBT MAT) checks whether you are ready for university-level maths. It is not the same as your matric exam. Matric often rewards long calculations. The NBT rewards **understanding** — knowing *why* a rule works, so you can use it on a question you have never seen before.

**No calculator is allowed.** That sounds scary, but it is actually good news: it means the numbers are almost always kept small and friendly on purpose. You will rarely need ugly arithmetic.

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## How This Book Is Organised

This opening guide is the **map**; the ten chapters after it are the **territory**. Here is how the whole book fits together:

- **This Study Guide** — every definition, formula, symbol and strategy in one place. Read it first, then keep returning to it as a reference while you work through the chapters.
- **Chapters 1–8 and 10 — the practice chapters.** Each one covers one topic area and follows the same four-part shape: 1. **Questions** — attempt these first, on paper, without peeking. 2. **Answers and full explanations** — every question re-taught from the ground up: the idea, the steps, *why each wrong option is wrong* (each one is a real mistake real students make — learning to recognise them is half the training), and a **Takeaway** worth remembering. 3. **Mixed practice** — 30 questions from *all* topics, unlabelled, like the real test. 4. **Answer key** for the mixed practice.
- **Chapter 9 — three full mock exams**, 60 questions each, in the real test's format and topic mix. Save these for the final weeks (the study plan below tells you when).

**How to work a question — the routine that makes this book work:** attempt it honestly first, even if you only get partway. Then read the full explanation **even when you got it right** — the explanations teach the traps, shortcuts and cross-connections that turn one question into ten marks' worth of understanding. If you got it wrong, find *your* error among the "why the others are wrong" notes; recognising your own mistake pattern is the fastest improvement there is.

### The five areas the NBT tests — and where to practise each

The NBT MAT draws every paper from the same five competence areas. This table is your route map:

| NBT competence area | Share of the test | Practise it in |
|---|---|---|
| Algebraic processes | biggest slice (~25%) | Chapters 1 and 8 |
| Functions & graphs | ~20% | Chapter 3 |
| Number sense (incl. financial maths) | ~20% | Chapters 2 and 7 |
| Geometry & trigonometry | ~20% | Chapters 4 and 5 |
| Data handling & probability | ~15% | Chapter 6 |

*(Chapter 10 covers introductory differential calculus, which appears in some NBT papers; Chapter 9's mock exams mix all areas in realistic proportions.)*

## How to Use This Guide

The rest of this guide has five parts. You do not have to read them in order, and you definitely do not have to read them all in one sitting.

- **The Language of Maths** — a dictionary of the words and symbols that appear everywhere. If a question ever confuses you because of a *word*, this is where you look. **Start here.**
- **Part 1 — The Building Blocks** — the actual maths, topic by topic. Each topic has: a plain-English explanation, the formulas you need (in tables), a worked example, and a "watch out" warning.
- **Part 2 — Doing Maths in Your Head** — tricks for calculating without a calculator.
- **Part 3 — Exam Strategy** — how to actually get marks on the day, including how to make good guesses.
- **Part 4 — Your Study Plan** — what to do in the weeks before the test, built around this book's chapters and mock exams.

**A promise about this test:** there is **no negative marking**. You are never punished for a wrong answer. So you must **answer every single question**, even if you are guessing. A blank is a guaranteed zero; a guess has a 1-in-4 chance of being right. Never leave a bubble empty.

> **If you only remember five things from this whole guide:** 1. Answer **every** question — a blank scores 0, a guess might score 1. 2. Learn the times tables, the special-angle table, and the basic formulas until you know them without thinking. 3. Turn "increase by 15%" into "multiply by 1.15" — percentages are just multiplying. 4. When stuck, **try the four options one by one** and see which fits. 5. Read the question slowly, twice. Half of all mistakes are just misreading.

## The Language of Maths — Words Decoded

Most people who struggle with maths are not actually bad at maths — they get lost because of the **words and symbols**, which nobody ever explained in plain language. Here they are. Keep coming back to this list.

### Everyday maths words

| Word | What it means in plain English |
|---|---|
| **Sum** | The answer when you **add**. "The sum of 3 and 5" means $3 + 5 = 8$. |
| **Difference** | The answer when you **subtract**. "The difference of 8 and 3" means $8 - 3 = 5$. |
| **Product** | The answer when you **multiply**. "The product of 4 and 5" means $4 \times 5 = 20$. |
| **Quotient** | The answer when you **divide**. "The quotient of 20 and 4" means $20 \div 4 = 5$. |
| **Term** | One "piece" of an expression, separated by $+$ or $-$ signs. In $3x + 5$, the terms are $3x$ and $5$. |
| **Expression** | A bit of maths with no equals sign, e.g. $3x + 5$. You can *simplify* it but not *solve* it. |
| **Equation** | Two expressions with an **equals sign** between them, e.g. $3x + 5 = 11$. You *solve* it to find the unknown. |
| **Variable** | A letter that stands for an unknown number, usually $x$ or $y$. |
| **Constant** | A plain number that does not change, like $5$ or $-2$. |
| **Coefficient** | The number multiplying a letter. In $3x$, the coefficient is $3$. |
| **Substitute** | Replace a letter with a number. "Substitute $x = 2$ into $3x$" means work out $3 \times 2 = 6$. |
| **Simplify** | Make an expression shorter/neater without changing its value. |
| **Solve** | Find the value of the unknown letter that makes an equation true. |
| **Evaluate** | Work out the actual number an expression comes to. |

### Number words

| Word | What it means in plain English |
|---|---|
| **Integer** | A whole number, positive or negative, including zero: $\ldots, -2, -1, 0, 1, 2, \ldots$ (no fractions, no decimals). |
| **Natural number** | The counting numbers: $1, 2, 3, 4, \ldots$ (in South Africa these start at 1, not 0). |
| **Rational number** | Any number you *can* write as a fraction, e.g. $\frac{3}{4}$, $0.5$, $7$, $0.333\ldots$ |
| **Irrational number** | A number you *cannot* write as a neat fraction — its decimals go on forever with no pattern, e.g. $\sqrt{2}$, $\pi$. |
| **Factor** | A number that divides into another exactly. The factors of 12 are $1, 2, 3, 4, 6, 12$. |
| **Multiple** | The times-table of a number. Multiples of 3 are $3, 6, 9, 12, \ldots$ |
| **Prime number** | A number with exactly two factors: 1 and itself $(2, 3, 5, 7, 11, \ldots)$. 1 is **not** prime. |

### Fraction and power words

| Word | What it means in plain English |
|---|---|
| **Numerator** | The **top** of a fraction. In $\frac{3}{4}$, the numerator is 3. |
| **Denominator** | The **bottom** of a fraction. In $\frac{3}{4}$, the denominator is 4. |
| **Reciprocal** | A fraction flipped upside down. The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$. Dividing by a fraction = multiplying by its reciprocal. |
| **Exponent / power / index** | The small raised number telling you how many times to multiply. In $2^3 = 2 \times 2 \times 2$, the exponent is 3. |
| **Base** | The big number being multiplied. In $2^3$, the base is 2. |
| **Square** | Multiply a number by itself: $5^2 = 5 \times 5 = 25$. |
| **Square root ($\sqrt{\ }$)** | The reverse of squaring: "what number times itself gives this?" $\sqrt{25} = 5$. |
| **Surd** | A square root that does not come out to a whole number, like $\sqrt{2}$ or $\sqrt{5}$. We leave it as it is. |

### Symbols you will see

| Symbol | Read it as |
|---|---|
| $=$ | "is equal to" |
| $\neq$ | "is not equal to" |
| $<$ , $>$ | "less than", "greater than" |
| $\leq$ , $\geq$ | "less than or equal to", "greater than or equal to" |
| $\pm$ | "plus or minus" (two answers: one with $+$, one with $-$) |
| $\therefore$ | "therefore" (so...) |
| $\approx$ | "is approximately equal to" |
| $\sum$ (sigma) | "add up a list of things" |
| $\pi$ (pi) | a fixed number $\approx 3.14$, used for circles |
| $\theta$ (theta) | a letter used to stand for an angle |
| $x_1, x_2$ | "x-one, x-two" — just a way to label a first and second value |

**A habit that will save you again and again:** whenever a question confuses you, read it *out loud in words*, replacing every symbol with its meaning. Suddenly it is just a sentence you can understand.

## The Test at a Glance

| What you need to know | Detail |
|---|---|
| How many questions | **60**, each multiple-choice with four options (A, B, C, D) |
| How long | **3 hours** — that is an average of **3 minutes per question** |
| Calculator | **Not allowed** |
| Wrong answers | **No penalty** — so guess if you must, and never leave a blank |
| Question order | All topics are **mixed together** — there are no sections |

**What this means for you:** you have plenty of time (3 minutes each is generous), so do not rush. Do the easy ones first to bank those marks, then come back for the hard ones.

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## Part 1 — The Building Blocks

This is the actual maths. Each topic below follows the same friendly pattern:

- **What this is really about** — the idea, in plain words.
- **The formulas / facts** — in tables, so they are easy to find.
- **Worked example** — a full question solved with every step shown.
- **Watch out** — the mistake most people make.

You do not need to master everything at once. Pick one topic, understand it, practise it, then move on. Little by little.

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#### 1. Number Sense

**What this is really about:** being comfortable with fractions, decimals, percentages, and different kinds of numbers. This is the most common topic on the test, and the good news is it is the most *learnable*. Get this solid and you have already earned a big chunk of the marks.

#### 1.1 Fraction–Decimal–Percentage Table

A fraction, a decimal, and a percentage are three different ways of writing **the same amount**. $\frac{1}{2}$, 0.5, and 50% are all "half." Learn this table by heart — it saves you from doing conversions under pressure.

| Fraction | Decimal | Percentage |
|---|---|---|
| $\frac{1}{2}$ | $0.5$ | $50\%$ |
| $\frac{1}{4}$ | $0.25$ | $25\%$ |
| $\frac{3}{4}$ | $0.75$ | $75\%$ |
| $\frac{1}{5}$ | $0.2$ | $20\%$ |
| $\frac{2}{5}$ | $0.4$ | $40\%$ |
| $\frac{3}{5}$ | $0.6$ | $60\%$ |
| $\frac{4}{5}$ | $0.8$ | $80\%$ |
| $\frac{1}{8}$ | $0.125$ | $12.5\%$ |
| $\frac{3}{8}$ | $0.375$ | $37.5\%$ |
| $\frac{5}{8}$ | $0.625$ | $62.5\%$ |
| $\frac{7}{8}$ | $0.875$ | $87.5\%$ |
| $\frac{1}{10}$ | $0.1$ | $10\%$ |
| $\frac{1}{3}$ | $0.\overline{3}$ | $33.\overline{3}\%$ |
| $\frac{2}{3}$ | $0.\overline{6}$ | $66.\overline{6}\%$ |

*(The little bar in $0.\overline{3}$ means the 3 repeats forever: $0.3333\ldots$)*

**To turn a fraction into a percentage:** divide top by bottom, then multiply by 100. **To turn a percentage into a fraction:** put it over 100 and simplify.

### 1.2 Number Facts to Know Instantly

You will use these constantly. Flash-card them until they are automatic.

**Perfect squares** (a number times itself), from $1^2$ to $15^2$:

$$1,\ 4,\ 9,\ 16,\ 25,\ 36,\ 49,\ 64,\ 81,\ 100,\ 121,\ 144,\ 169,\ 196,\ 225$$

**Perfect cubes** (a number times itself, times itself again), $1^3$ to $5^3$:

$$1,\ 8,\ 27,\ 64,\ 125$$

**Powers of 2** (keep doubling), up to $2^{10}$:

$$2,\ 4,\ 8,\ 16,\ 32,\ 64,\ 128,\ 256,\ 512,\ 1024$$

**Prime numbers** up to 50 (only divisible by 1 and themselves):

$$2,\ 3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ 29,\ 31,\ 37,\ 41,\ 43,\ 47$$

**The 10% ladder** — powers of 1.1, which the NBT uses constantly in interest and growth questions (notice the digits are the rows of Pascal's triangle):

$$1.1^2 = 1.21 \qquad 1.1^3 = 1.331 \qquad 1.1^4 = 1.4641$$

**Triangular numbers** — the running totals $1 + 2 + 3 + \cdots + n$ (formula $\frac{n(n+1)}{2}$). Know at least: $n = 5 \to 15,\ n = 10 \to 55,\ n = 20 \to 210,\ n = 40 \to 820.$

**Sum of the first $n$ odd numbers is always $n^2$:** $1 + 3 + 5 + 7 = 16 = 4^2$. This one fact is a confirmed exam favourite.

**Parity in one breath** (odd/even rules): odd ± odd = even; even ± even = even; odd ± even = odd; odd × odd = **odd**; even × *anything* = **even**.

#### 1.3 Percentages — the one idea that unlocks everything

**In plain English:** "per cent" literally means "out of 100." So 30% just means $\frac{30}{100} = 0.3$. The single most useful trick in the whole test is this:

> **Any percentage change is just a multiplication.** To **increase** by 15%, multiply by 1.15. To **decrease** by 15%, multiply by 0.85.

Why? If you have R100 and add 15%, you now have the original 100% *plus* 15% $=$ 115% of what you had $= 1.15 \times 100 = 115$. To take away 15%, you keep $100\% - 15\% = 85\% = 0.85$ of it.

| What you want to do | How to do it |
|---|---|
| Find $p\%$ of a number $n$ | $\dfrac{p}{100} \times n$ |
| Percentage increase | $\dfrac{\text{New} - \text{Old}}{\text{Old}} \times 100$ |
| Percentage decrease | $\dfrac{\text{Old} - \text{New}}{\text{Old}} \times 100$ |
| Increase a number by $p\%$ | Multiply by $\left(1 + \dfrac{p}{100}\right)$ |
| Decrease a number by $p\%$ | Multiply by $\left(1 - \dfrac{p}{100}\right)$ |
| Find the original before a change | **Divide** by the multiplier (don't add the percentage back!) |

**Worked example — a shirt in a sale:** A shirt costs R240. It is increased by 15%. What is the new price?

- Step 1 — turn the percentage into a multiplier: increase of 15% means multiply by $1 + 0.15 = 1.15$.
- Step 2 — multiply: $240 \times 1.15$.
- Step 3 — do it in easy pieces: $240 \times 1 = 240$, and $240 \times 0.15 = 36$. Add them: $240 + 36 = 276$.
- **Answer: R276.**

**Watch out — the "back to the start" trap:** A price goes **down** 10%, then **up** 10%. You are *not* back where you started. Down 10% means $\times 0.9$; up 10% means $\times 1.1$. Together: $0.9 \times 1.1 = 0.99$, which is a **1% loss**. Whenever percentages happen one after another, multiply the multipliers — never add or subtract the percentages.

### 1.4 Types of Numbers

Numbers live in nested groups, like boxes inside boxes:

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

Read left to right, that says: natural numbers are inside the integers, which are inside the rationals, which are inside the reals.

- $\mathbb{N}$ **Natural numbers:** $1, 2, 3, \ldots$ (counting numbers)
- $\mathbb{Z}$ **Integers:** $\ldots, -2, -1, 0, 1, 2, \ldots$ (whole numbers, including negatives and zero)
- $\mathbb{Q}$ **Rational numbers:** anything you can write as a fraction — this includes all decimals that **stop** $(0.25)$ or **repeat** $(0.333\ldots)$
- **Irrational numbers:** cannot be written as a fraction; decimals go on forever with no pattern, like $\sqrt{2}$, $\pi$, $e$
- $\mathbb{R}$ **Real numbers:** everything above, all together

**Quick test for a square root:** $\sqrt{n}$ is a *nice* rational number only if $n$ is a perfect square. $\sqrt{25} = 5$ (rational), but $\sqrt{5}$ is irrational.

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### 1.5 Scientific Notation

Scientific notation writes a number as $a \times 10^n$, where $a$ is between $1$ and $10$ and $n$ is a whole number. The exponent counts the places the decimal point moves — **positive** for big numbers, **negative** for small ones.

$$4\,500\,000 = 4.5 \times 10^6 \qquad\qquad 0.00045 = 4.5 \times 10^{-4}$$

To multiply or divide, deal with the coefficients and the powers of ten separately:

- **Multiplying:** multiply the coefficients, **add** the exponents.
- **Dividing:** divide the coefficients, **subtract** the exponents.

If the coefficient lands outside 1 to 10, fix it: $37 \times 10^4 = 3.7 \times 10^5$.

### 1.6 Speed, Distance and Time

One relationship, read three ways:

$$\text{distance} = \text{speed} \times \text{time} \qquad \text{speed} = \frac{\text{distance}}{\text{time}} \qquad \text{time} = \frac{\text{distance}}{\text{speed}}$$

Two rules stop nearly every mistake here:

1. **Make the units agree first.** With a speed in km/h, 30 minutes is 0.5 hours — divide minutes by 60, never treat them as decimals.
2. **Average speed is total distance ÷ total time.** It is *not* the average of the speeds. If you travel equal distances at two different speeds, more time is spent at the slower one, so the true average always comes out **below** the halfway point between them.

## 2. Exponents and Logarithms

**What this is really about:** an **exponent** (or "power") is just a shortcut for repeated multiplying. $2^4$ means "multiply four 2's together": $2 \times 2 \times 2 \times 2 = 16$. A **logarithm** is the reverse question: "how many 2's did I multiply to get 16?" — the answer is 4. That is all a log is: an exponent, asked backwards.

### 2.1 Laws of Exponents

These rules let you combine powers quickly. The trick to all of them: they only work when the **base** (the big number) is the same.

| Law | Formula | In plain English |
|---|---|---|
| Multiplying | $a^m \cdot a^n = a^{m+n}$ | Same base, multiplying → **add** the powers |
| Dividing | $\frac{a^m}{a^n} = a^{m-n}$ | Same base, dividing → **subtract** the powers |
| Power of a power | $(a^m)^n = a^{mn}$ | A power raised to a power → **multiply** the powers |
| Negative power | $a^{-n} = \frac{1}{a^n}$ | A minus sign in the power means "flip it to the bottom" |
| Zero power | $a^0 = 1$ | *Anything* (except 0) to the power 0 is just 1 |
| Fractional power | $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ | Bottom of the fraction = the root; top = the power |
| Power of a product | $(ab)^n = a^n b^n$ | Share the power out to each factor |

**Worked example:** Simplify $\frac{x^5 \cdot x^3}{x^4}$.

- Step 1 — top: same base multiplying, so add the powers: $x^5 \cdot x^3 = x^{5+3} = x^8$.
- Step 2 — now dividing, same base, so subtract: $\frac{x^8}{x^4} = x^{8-4} = x^4$.
- **Answer:** $x^4$.

### 2.2 Laws of Logarithms

A logarithm answers "what power?" Written $\log_a N = x$, it means "$a$ to the power $x$ gives $N$," i.e. $a^x = N$. The two are the same fact written two ways:

$$y = a^x \iff x = \log_a y$$

| Law | Formula | In plain English |
|---|---|---|
| Product | $\log_a(xy) = \log_a x + \log_a y$ | Log of a **product** → **add** the logs |
| Quotient | $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$ | Log of a **division** → **subtract** the logs |
| Power | $\log_a(x^n) = n \log_a x$ | A power inside → comes out to the **front** as a multiplier |
| Log of the base | $\log_a a = 1$ | "What power of $a$ gives $a$?" → 1 |
| Log of 1 | $\log_a 1 = 0$ | "What power gives 1?" → 0 |

**Worked example:** Evaluate $\log_2 32$.

- In words this asks: "2 to *what* power gives 32?"
- Count the doublings: $2, 4, 8, 16, 32$ — that is $2^5 = 32$.
- **Answer: 5.**

**Watch out — the biggest log/exponent trap:** you can **only** split a log when things are **multiplied or divided** inside. $\log(a+b)$ does **NOT** equal $\log a + \log b$. There is no rule for adding inside a log. (The same idea catches people with roots: $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$.)

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### 3. Algebra and Surds

**What this is really about:** algebra is arithmetic with letters standing in for numbers. "Factorising" means writing an expression as a multiplication (breaking it into pieces multiplied together); "expanding" is the reverse (multiplying the brackets out).

#### 3.1 Special Identities — patterns worth memorising

These come up so often that recognising them instantly saves huge time.

| Pattern | Formula |
| --- | --- |
| Square of a sum | $(a+b)^2 = a^2 + 2ab + b^2$ |
| Square of a difference | $(a-b)^2 = a^2 - 2ab + b^2$ |
| Difference of two squares | $a^2 - b^2 = (a-b)(a+b)$ |
| Sum of cubes | $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ |
| Difference of cubes | $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ |

**The one everyone forgets:** $(a+b)^2$ is **not** $a^2 + b^2$. You must include the middle term $2ab$. Check with numbers: $(2+3)^2 = 25$, but $2^2 + 3^2 = 13$. Different! The missing piece is the $2ab = 2 \times 2 \times 3 = 12$, and $13 + 12 = 25$. ✓

#### 3.2 Factorising — always try these in order

When asked to factorise, run down this checklist from the top:

1. **Common factor first** — is there a number or letter in *every* term you can pull out? Always check this first. $6x + 9 = 3(2x+3)$.
2. **Difference of two squares** — does it look like $a^2 - b^2$? Then $= (a-b)(a+b)$.
3. **Sum or difference of cubes** — $a^3 \pm b^3$.

4. **Trinomial** (three terms, $x^2 + bx + c$) — find two numbers that **multiply to** $c$ and **add to** $b$.
5. **Grouping** — for four terms, group them in pairs.
6. **Factor theorem** — for cubics: if putting $x = p$ into the expression gives 0, then $(x - p)$ is a factor.

**Worked example — a trinomial:** Factorise $x^2 + 5x + 6$.

- We need two numbers that **multiply to 6** and **add to 5**.
- List pairs that multiply to 6: $(1, 6)$ and $(2, 3)$. Which pair adds to 5? $2 + 3 = 5$. ✓
- So $x^2 + 5x + 6 = (x + 2)(x + 3)$.
- **Check by expanding:** $(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$. ✓

### 3.3 Surds (leftover square roots)

A **surd** is a root that does not simplify to a whole number, like $\sqrt{2}$. Rules:

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \qquad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

**Simplifying a surd** — pull out the biggest perfect-square factor hiding inside:

$$\sqrt{72} = \sqrt{36 \times 2} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$$

**Adding surds** — you can only add "like" surds (same number under the root), exactly like adding $2x + 3x = 5x$:

$$\sqrt{12} + \sqrt{27} = 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$$

**Rationalising the denominator** — getting rid of a root on the bottom of a fraction. Multiply top and bottom by the **conjugate** (the same expression with the middle sign flipped):

$$\frac{3}{\sqrt{5} - 1} \times \frac{\sqrt{5} + 1}{\sqrt{5} + 1} = \frac{3(\sqrt{5} + 1)}{5 - 1} = \frac{3(\sqrt{5} + 1)}{4}$$

### 3.4 Completing the Square

This rewrites $x^2 + bx + c$ into the form $(x + p)^2 + q$, which instantly shows the turning point of a parabola. The recipe:

- **Halve** the number in front of $x$ (call it $b$), to get $p = \dfrac{b}{2}$.
- Then $q = c - p^2$.

**Worked example:** Write $x^2 + 8x + 3$ as $(x + p)^2 + q$.

- Halve the 8: $p = 4$.

- $(x+4)^2$ expands to $x^2+8x+16$ — that is 16 too big, so subtract 16 and add back the original 3: $q=3-16=-13$.
- **Answer:** $(x+4)^2-13$**.**

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### 4. Quadratic Equations

**What this is really about:** a "quadratic" is any equation with an $x^2$ in it (and no higher power). Its graph is a U-shape (or upside-down U) called a **parabola**. Solving it means finding where it crosses the x-axis.

#### 4.1 Three Ways to Solve

1. **Factorising** — write it as $(x-p)(x-q)=0$. Because anything times zero is zero, either $x-p=0$ or $x-q=0$, so $x=p$ or $x=q$. **Always try this first.**
2. **The quadratic formula** — works for *every* quadratic $ax^2+bx+c=0$, even ugly ones:

$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

3. **Completing the square** — sometimes handy, but the formula usually wins.

**Worked example — by factorising:** Solve $x^2-7x+10=0$.

- Find two numbers that **multiply to** $+10$ and **add to** $-7$: those are $-2$ and $-5$.
- So $(x-2)(x-5)=0$.
- Therefore $x=2$ or $x=5$.
- **Answer:** $x=2$ **or** $x=5$**.**

#### 4.2 The Discriminant — a shortcut to the "nature of the roots"

Sometimes a question does not want you to *solve*, only to say **how many** real answers there are. The part under the root in the formula, $\Delta=b^2-4ac$ (called the **discriminant**), tells you without doing the whole calculation:

| If $\Delta = b^2 - 4ac$ is... | Then the equation has... | The parabola... |
|---|---|---|
| **positive** and a perfect square | two different *rational* answers | crosses the x-axis at two nice points |
| **positive**, not a perfect square | two different *irrational* answers | crosses the x-axis at two points |
| **zero** | one repeated answer | just touches the x-axis |
| **negative** | **no real answers** | never touches the x-axis |

*(A negative under a square root has no real answer — that is your signal for "no real roots.")*

### 4.3 Sum and Product of the Roots

For $ax^2 + bx + c = 0$ with answers $x_1$ and $x_2$, without solving you already know:

$$x_1 + x_2 = -\frac{b}{a} \qquad\qquad x_1 \cdot x_2 = \frac{c}{a}$$

These let you check answers or solve clever questions without ever finding $x_1$ and $x_2$ separately.

### 4.4 Quadratic Inequalities — solve with a picture

To solve something like $x^2 - x - 6 > 0$: factorise, find the two roots, then **think of the parabola**. An upward parabola is *below* zero **between** its roots and *above* zero **outside** them.

- $x^2 - x - 6 = (x - 3)(x + 2)$, roots at $x = 3$ and $x = -2$.
- "$> 0$" wants the parabola **above** the axis → outside the roots: $x < -2$ or $x > 3$.
- "$< 0$" would want **between**: $-2 < x < 3$.

**Watch out:** before any of this, ask whether the statement can even be false — $x^2 + 4 < 0$ has *no* solutions (a square is never negative), and the NBT loves testing exactly that.

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### 5. Functions and Graphs

**What this is really about:** a **function** is a rule that takes an input number and gives back one output number — like a machine. $f(x) = 2x + 1$ means "double it and add one." The **graph** is just a picture of every input-output pair plotted as points.

Two bits of language:

- $f(x)$ ("f of x") is just the name of the output. $f(3)$ means "put 3 in as the input."
- **Intercept** = where a graph crosses an axis. **Gradient** (or slope) = how steep a line is.

- **Asymptote** = an invisible line the graph gets closer and closer to but never quite touches.

### 5.1 The Main Graph Types

| Function | Standard equation | What its graph looks like |
|---|---|---|
| Linear (straight line) | $y = mx + c$ | A straight line. $m$ = gradient (steepness), $c$ = where it crosses the y-axis |
| Quadratic (parabola) | $y = a(x - p)^2 + q$ | A U-shape. Turning point at $(p,\ q)$. Opens up if $a > 0$, down if $a < 0$ |
| Exponential | $y = a \cdot b^x + q$ | A curve that shoots up (or decays down). Flat asymptote at $y = q$ |
| Hyperbola | $y = \dfrac{a}{x - p} + q$ | Two curved branches. Asymptotes at $x = p$ and $y = q$ |
| Logarithm | $y = \log_b x$ | The mirror-image of the exponential curve |
| Absolute value | $y = a\lvert x - p\rvert + q$ | A V-shape. Vertex at $(p,\ q)$. Opens up if $a > 0$, down if $a < 0$ |

**For a parabola $y = ax^2 + bx + c$,** the turning point sits on the line

$$x = -\frac{b}{2a}.$$

Find that $x$, then substitute it back in to get the $y$ of the turning point.

**Worked example — turning point:** Find the turning point of $y = x^2 - 6x + 5$.

- Here $a = 1$, $b = -6$. The turning point's $x$: $x = -\dfrac{-6}{2(1)} = 3$.
- Substitute back: $y = 3^2 - 6(3) + 5 = 9 - 18 + 5 = -4$.
- **Answer:** $(3, -4)$ — and since $a > 0$ the parabola opens upward, so this is a *minimum*.

**Why asymptotes happen (plain English):** for a hyperbola, you can never divide by zero — so at the $x$ that makes the bottom zero, the graph "breaks" (vertical asymptote). And as $x$ gets huge, the fraction shrinks to almost nothing, so $y$ flattens out towards $q$ (horizontal asymptote).

### 5.2 Moving Graphs Around (Transformations)

These work for **any** graph $y = f(x)$. Learn the pattern once and it applies everywhere.

| What you do | New equation | Effect |
|---|---|---|
| Add $k$ on the outside | $y = f(x) + k$ | Whole graph moves **up** $k$ |
| Subtract $k$ on the outside | $y = f(x) - k$ | Whole graph moves **down** $k$ |
| $-h$ **inside** the bracket | $y = f(x - h)$ | Moves **right** $h$ *(yes, minus means right — this surprises everyone)* |
| $+h$ **inside** the bracket | $y = f(x + h)$ | Moves **left** $h$ |
| Minus on the outside | $y = -f(x)$ | Flips upside-down (reflects in the x-axis) |
| Minus on the inside | $y = f(-x)$ | Flips left-right (reflects in the y-axis) |

#### 5.3 Inverse Functions

The **inverse** undoes what the function did. To find it: **swap $x$ and $y$, then solve for $y$.** On a graph, the inverse is the original reflected in the line $y = x$. Handy fact: the inverse of $y = b^x$ is $y = \log_b x$.

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### 6. Trigonometry

**What this is really about:** trigonometry ("trig") connects the **angles** of a right-angled triangle to the **lengths** of its sides. It sounds intimidating but rests on one memory trick: **SOHCAHTOA**.

#### 6.1 SOHCAHTOA — the whole foundation

In a right-angled triangle, name the three sides *from the angle you care about*:

- **Hypotenuse** — the longest side, always opposite the right angle.
- **Opposite** — the side across from your angle.
- **Adjacent** — the side next to your angle (that is not the hypotenuse).

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} \qquad \cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} \qquad \tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

The nonsense word **SOH-CAH-TOA** stores all three: **S**in = **O**pp/**H**yp, **C**os = **A**dj/**H**yp, **T**an = **O**pp/**A**dj.

**Worked example:** A right triangle has an opposite side of 3 and a hypotenuse of 5. Find $\sin\theta$.

- sin uses Opposite over Hypotenuse (the "SOH" part).

- $\sin\theta = \dfrac{3}{5}$.
- **Answer:** $\frac{3}{5}$.

### 6.2 Special Angles — memorise this table

For a few "nice" angles, the answers are exact. These appear constantly (remember, no calculator):

| $\theta$ | $\sin\theta$ | $\cos\theta$ | $\tan\theta$ |
|---|---|---|---|
| $0^\circ$ | $0$ | $1$ | $0$ |
| $30^\circ$ | $\dfrac{1}{2}$ | $\dfrac{\sqrt{3}}{2}$ | $\dfrac{1}{\sqrt{3}}$ |
| $45^\circ$ | $\dfrac{\sqrt{2}}{2}$ | $\dfrac{\sqrt{2}}{2}$ | $1$ |
| $60^\circ$ | $\dfrac{\sqrt{3}}{2}$ | $\dfrac{1}{2}$ | $\sqrt{3}$ |
| $90^\circ$ | $1$ | $0$ | undefined |

### 6.3 The CAST Diagram — which sign in which quadrant

Angles bigger than 90° live in different "quadrants" (quarters) of a circle. In each quarter, some of sin/cos/tan are positive and some negative. CAST tells you which are **positive**:

| **Quadrant** | **Angles** | **Positive there** | **Letter** |
|---|---|---|---|
| I | $0^\circ$–$90^\circ$ | all three | **A**ll |
| II | $90^\circ$–$180^\circ$ | only sin | **S**in |
| III | $180^\circ$–$270^\circ$ | only tan | **T**an |
| IV | $270^\circ$–$360^\circ$ | only cos | **C**os |

Read the letters going round: **A, S, T, C** — remember it as "**A**ll **S**tudents **T**ake **C**alculus."

### 6.4 The Identities You Must Know

An **identity** is a trig equation that is *always* true. The most important one:

$$\sin^2\theta + \cos^2\theta = 1 \qquad\qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

*(The notation $\sin^2\theta$ means $(\sin\theta)^2$ — square the whole thing.)* From these you can also get $1 + \tan^2\theta = \sec^2\theta$.

**The double-angle formulas** — confirmed favourites on recent papers (practise them in Chapter 4):

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad\qquad \cos 2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1 = \cos^2\theta - \sin^2\theta$$

**How they appear in disguise:** an expression like $2 - 4\sin^2 3x$ is really $2(1 - 2\sin^2 3x) = 2\cos 6x$ — factor until the bracket matches the identity, then remember the formula **doubles whatever angle is inside** ($3x$ becomes $6x$).

#### 6.5 Reduction Formulae (angles outside 0–90°)

These convert a big or awkward angle back to a friendly one:

| **Angle** | sin | cos | tan |
|---|---|---|---|
| $180^\circ - \theta$ | $+\sin\theta$ | $-\cos\theta$ | $-\tan\theta$ |
| $180^\circ + \theta$ | $-\sin\theta$ | $-\cos\theta$ | $+\tan\theta$ |
| $360^\circ - \theta$ | $-\sin\theta$ | $+\cos\theta$ | $-\tan\theta$ |
| $90^\circ - \theta$ | $\cos\theta$ | $\sin\theta$ | — |
| $-\theta$ | $-\sin\theta$ | $+\cos\theta$ | $-\tan\theta$ |

**How to use them:** find the "reference angle" (the small acute angle to the horizontal axis), then use CAST to decide whether the answer is $+$ or $-$.

#### 6.6 Rules for Any Triangle (not just right-angled)

When a triangle has **no** right angle, use these:

$$\text{Sine rule:} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\text{Cosine rule:} \quad c^2 = a^2 + b^2 - 2ab\cos C$$
$$\text{Area of a triangle:} \quad \text{Area} = \tfrac{1}{2}ab\sin C$$

*(In these, a small letter is the side opposite the matching capital-letter angle.)* Use the **cosine rule** when you know two sides and the angle between them, or all three sides. Use the **sine rule** for most other cases.

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#### 7. Analytical Geometry

**What this is really about:** doing geometry using **coordinates** — points written as $(x, y)$ on a grid. Every question here is really just Pythagoras or straight lines in disguise.

| I want to find... | Formula | In plain English |
| --- | --- | --- |
| Distance between two points | $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ | Pythagoras: square the across-gap, square the up-gap, add, square-root |
| Midpoint (the middle) | $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ | The **average** of the two x's, and of the two y's |
| Gradient (steepness) | $m = \frac{y_2 - y_1}{x_2 - x_1}$ | "Rise over run" — up-change divided by across-change |
| Equation of a line (know gradient + a point) | $y - y_1 = m(x - x_1)$ | Slot in the gradient and the point |
| Parallel lines | $m_1 = m_2$ | Same steepness |
| Perpendicular lines (meet at 90°) | $m_1 \times m_2 = -1$ | Flip one gradient upside-down **and** change its sign |
| Circle centred at the origin | $x^2 + y^2 = r^2$ | $r$ is the radius |
| Circle centred at $(a, b)$ | $(x - a)^2 + (y - b)^2 = r^2$ | |

**Worked example — distance:** Find the distance between $(1, 2)$ and $(4, 6)$.

- Across-gap: $4 - 1 = 3$. Up-gap: $6 - 2 = 4$.
- Square and add: $3^2 + 4^2 = 9 + 16 = 25$.
- Square root: $\sqrt{25} = 5$.
- **Answer: 5.** *(This is the famous 3-4-5 triangle.)*

**Watch out:** for a **perpendicular** gradient you need *both* changes — flip it over **and** swap the sign. The perpendicular of $\frac{2}{3}$ is $-\frac{3}{2}$, not just $-\frac{2}{3}$ or $\frac{3}{2}$.

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#### 8. Geometry and Measurement

**What this is really about:** shapes, and how to find their **area** (the space inside), **perimeter** (the distance around), and for 3-D solids, **volume** (how much fits inside) and **surface area** (the total skin).

### 8.1 Flat Shapes — Area and Perimeter

| Shape | Area | Perimeter |
|---|---|---|
| Square | $s^2$ | $4s$ |
| Rectangle | $\ell \times b$ | $2(\ell + b)$ |
| Triangle | $\frac{1}{2}bh$ | add up the 3 sides |
| Circle | $\pi r^2$ | $2\pi r$ *(the "perimeter" of a circle is called the circumference)* |
| Parallelogram | $bh$ | $2(a + b)$ |
| Trapezium | $\frac{1}{2}(a+b)h$ | add up all 4 sides |

*(Here h always means the perpendicular height — straight up, at a right angle to the base — never a slanted side.)*

### 8.2 Solid Shapes — Volume and Surface Area

| Solid | Volume | Surface Area |
|---|---|---|
| Rectangular box | $\ell \times b \times h$ | $2(\ell b + \ell h + bh)$ |
| Cylinder (tin can) | $\pi r^2 h$ | $2\pi r^2 + 2\pi rh$ |
| Sphere (ball) | $\frac{4}{3}\pi r^3$ | $4\pi r^2$ |
| Cone | $\frac{1}{3}\pi r^2 h$ | $\pi r^2 + \pi r\ell$ |
| Pyramid | $\frac{1}{3} \times (\text{base area}) \times h$ | base + the slanted faces |

*(For the cone, $\ell$ is the slant height — the sloped distance up the side.)* Notice cones and pyramids both have a $\frac{1}{3}$ — they hold exactly one-third of the box or cylinder they fit inside.

**Worked example — cylinder volume:** A tin can has radius 3 cm and height 10 cm. Find its volume (leave $\pi$ in the answer).

- Volume of a cylinder = (area of the circular base) × (height) $= \pi r^2 h$.
- $V = \pi \times 3^2 \times 10 = 90\pi$ cm³.
- **Answer:** $90\pi$ **cm³** — and notice the options in a no-calculator test will usually *keep* the $\pi$, exactly like this. Do not convert to a decimal unless the options do.

### 8.3 Theorems Worth Knowing

**Pythagoras' theorem** (right-angled triangles only): the two shorter sides squared, added, equal the longest side squared:

$$a^2 + b^2 = c^2 \quad (c \text{ is the hypotenuse, the longest side})$$

*Handy triples to recognise instantly:* **3-4-5**, **5-12-13**, **8-15-17**, **7-24-25** (and any multiple, like 6-8-10).

**Similar shapes and scale factor:** if two shapes are the same shape but different sizes, and lengths are scaled by $k$, then **areas** scale by $k^2$ and **volumes** by $k^3$. (Double the length → 4× the area → 8× the volume.)

**Circle theorems to recognise:**

- The angle at the **centre** is **double** the angle at the edge (standing on the same arc).
- The angle in a **semicircle** is always **90°**.
- Opposite angles of a **cyclic quadrilateral** (4-sided shape inside a circle) add up to **180°**.
- A **tangent** (a line just touching the circle) meets the radius at **90°**.

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### 9. Sequences and Series

**What this is really about:** a **sequence** is a list of numbers following a pattern (like $2, 5, 8, 11, \ldots$). A **series** is what you get when you **add** the numbers of a sequence up. There are two patterns to know.

#### 9.1 Arithmetic — adding the same amount each time

An **arithmetic** sequence goes up (or down) by a fixed step, called the **common difference** $d$. Example: $5, 8, 11, 14, \ldots$ has $d = 3$.

$$T_n = a + (n-1)d \qquad\qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

- $T_n$ = the $n$-th term (the number in position $n$). $a$ = the first term. $d$ = the step.
- $S_n$ = the **sum** of the first $n$ terms.

**Worked example:** Find the 10th term of $5, 8, 11, 14, \ldots$

- First term $a = 5$; step $d = 3$; we want $n = 10$.
- $T_{10} = 5 + (10-1) \times 3 = 5 + 9 \times 3 = 5 + 27 = 32$.
- **Answer: 32.** *(The most common mistake here is using* $10 \times 3$ *instead of* $9 \times 3$ *— it is always* $(n-1)$ *steps, not n.)*

### 9.2 Geometric — multiplying by the same amount each time

A **geometric** sequence **multiplies** by a fixed number, the **common ratio** $r$. Example: $3, 6, 12, 24, \ldots$ has $r = 2$.

$$T_n = a\,r^{\,n-1} \qquad\qquad S_n = \frac{a(r^n - 1)}{r - 1}$$

**Worked example:** Find the 5th term of $2, 6, 18, \ldots$

- First term $a = 2$; ratio $r = 6 \div 2 = 3$; we want $n = 5$.
- $T_5 = 2 \times 3^{5-1} = 2 \times 3^4 = 2 \times 81 = 162$.
- **Answer: 162.** *(Same trap as arithmetic: the exponent is $n - 1 = 4$, not $n = 5$ — you take four multiplication "steps" to reach the 5th term.)*

**Sum to infinity** — if the numbers are *shrinking* (that is, $|r| < 1$), an endless sum can still reach a finite total:

$$S_\infty = \frac{a}{1 - r} \qquad \text{(only works when } |r| < 1\text{)}$$

### 9.3 Sigma Notation (the $\sum$ symbol)

$\sum$ (the Greek letter "sigma") just says **"add up."** $\sum_{i=1}^{5} i$ means "add the whole numbers from 1 to 5" $= 1 + 2 + 3 + 4 + 5 = 15$. Useful shortcuts:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} (2i - 1) = n^2 \qquad \sum_{i=1}^{n} c = c\,n$$

*(That middle one says: the sum of the first n odd numbers is exactly $n^2$ — the same fact from the number-facts list, now in sigma costume. The exam bank uses it in exactly this costume.)*

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### 10. Financial Mathematics

**What this is really about:** money growing (interest on savings) or shrinking (depreciation — things losing value). It is really just percentages applied over several years.

| Situation | Formula | The idea |
|---|---|---|
| Simple interest | $A = P(1 + in)$ | Same fixed amount added every year |
| Compound growth | $A = P(1 + i)^n$ | Interest earns *its own* interest — grows faster |
| Straight-line depreciation | $A = P(1 - in)$ | Value drops by the same rand amount each year |
| Depreciation (reducing balance) | $A = P(1 - i)^n$ | Value shrinks by a % of its *current* worth each year |
| Inflation (future prices) | $A = P(1 + i)^n$ | Prices compound exactly like interest |
| Compounded $m$ times a year | $A = P\left(1 + \frac{i}{m}\right)^{mn}$ | Split the year into $m$ pieces |

*(P = starting amount, i = interest rate as a decimal — so 8% becomes 0.08 — n = number of years.)*

**Three more money situations the NBT tests** (all practised in Chapter 7):

- **Hire purchase:** deposit first, then *simple* interest on the remaining balance for the full term, then divide by the number of payments. The interest never shrinks as you pay — that is why HP is expensive.
- **Exchange rates:** the rate is the *price* of one unit of foreign currency. Rand → dollars: **divide** by the rate. Dollars → rand: **multiply**. Decide *before* calculating whether your answer should be bigger or smaller.
- **Real return:** what your money actually gains after inflation ≈ interest rate − inflation rate. If inflation is higher than your interest rate, your savings are *losing* buying power.

**Simple vs compound, in one line:** simple interest grows in a **straight line**; compound interest **snowballs** (each year's interest is bigger than the last). Over many years, compound is much larger.

**Worked example — compound growth:** R10 000 is invested at 10% per year, compounded, for 2 years.

- Rate as a decimal: $i = 0.10$, so the multiplier is 1.10.
- $A = 10\,000 \times (1.10)^2 = 10\,000 \times 1.21 = 12\,100$.
- **Answer: R12 100.** *(Note* $1.1^2 = 1.21$*, not* $1.20$ *— that extra R100 is the "interest on the interest.")*

### 11. Data Handling and Probability

**What this is really about:** two things — **statistics** (describing a set of numbers, like an average) and **probability** (measuring how likely something is).

#### 11.1 Averages and Spread

| Measure | What it is |
|---|---|
| **Mean** ($\bar{x}$) | The ordinary average: add everything up, divide by how many. Gets pulled around by very big/small values. |
| **Median** | The **middle** number once the data is **sorted** in order. Barely affected by outliers. |
| **Mode** | The value that appears **most often**. |
| **Range** | Biggest minus smallest ($\text{max} - \text{min}$). |
| **IQR** (interquartile range) | $Q_3 - Q_1$ — the spread of the middle half of the data. |

**The one rule people forget:** to find the **median**, you must **sort the numbers first**. The median of $7, 2, 9$ is not $2$ — sort them to $2, 7, 9$ and the middle one is $7$.

**Worked example — mean:** Find the mean of $60, 72, 68, 75, 80$.

- Add them: $60 + 72 + 68 + 75 + 80 = 355$.
- Divide by how many (there are 5): $355 \div 5 = 71$.
- **Answer: 71.**

#### 11.2 Probability

Probability is always a number between 0 (impossible) and 1 (certain):

$$P(\text{event}) = \frac{\text{number of ways it can happen}}{\text{total number of equally likely outcomes}}$$

| Rule | Formula | Plain English |
|---|---|---|
| Complement (the "opposite") | $P(\text{not } A) = 1 - P(A)$ | Chance it *doesn't* happen = 1 minus chance it does |
| Either A or B | $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ | Add them, but subtract the overlap so you don't count it twice |
| Both A **and** B (independent) | $P(A \cap B) = P(A) \times P(B)$ | If they don't affect each other, **multiply** |

- **Mutually exclusive** = the two events **cannot both happen** (so their overlap is 0).
- **Independent** = one happening **does not change** the chance of the other (e.g. two coin flips).

**Worked example:** A fair die is rolled. What is the probability of an even number?

- Even numbers on a die: $2, 4, 6$ — that is $3$ ways.
- Total outcomes: $6$.
- $P(\text{even}) = \dfrac{3}{6} = \dfrac{1}{2}$.
- **Answer:** $\frac{1}{2}$.

### 11.3 Counting

**Fundamental counting principle:** if one choice can be made in $m$ ways and the next in $n$ ways, together there are $m \times n$ ways. *(3 shirts and 4 trousers give* $3 \times 4 = 12$ *outfits.)*

- **Permutation** — order **matters** (like a race: 1st, 2nd, 3rd are different): ${}^nP_r = \dfrac{n!}{(n-r)!}$
- **Combination** — order does **not** matter (like picking a team): ${}^nC_r = \dfrac{n!}{r!\,(n-r)!}$

*(The symbol* $n!$ *— "n factorial" — means multiply all whole numbers from n down to 1. So* $4! = 4 \times 3 \times 2 \times 1 = 24$*. And* $0! = 1$*.)*

**Worked example — does order matter?** From 5 students, how many ways can you choose (a) a president and a secretary, (b) a pair of representatives?

- (a) The roles are different, so order matters: $5 \times 4 = 20$ ways.
- (b) A pair is just a pair — order does not matter, so each pair from (a) was counted twice (Thabo–Amina and Amina–Thabo are the same pair): $20 \div 2 = 10$ ways.
- **Answers: 20 and 10.** The whole permutation/combination decision is that one question: *would swapping the chosen people around create a genuinely different outcome?*

## Part 2 — Doing Maths in Your Head

With no calculator, **how** you calculate matters as much as what you calculate. These tricks keep the numbers small and friendly.

### Turn decimals into fractions

Fractions are usually faster and safer by hand:

$$0.375 \times 64 = \frac{3}{8} \times 64 = 3 \times 8 = 24$$

### Trap a square root between two you know

You do not need the exact value — just enough to pick the right option:

$$\sqrt{64} < \sqrt{85} < \sqrt{100} \implies 9 < \sqrt{85} < 10$$

Since 85 is close to $81 = 9^2$, you can say $\sqrt{85} \approx 9.2$. Often that is enough to knock out the wrong options.

### Break big roots into pieces

$$\sqrt{1296} = \sqrt{36 \times 36} = 36 \qquad \sqrt{720} = \sqrt{144 \times 5} = 12\sqrt{5}$$

### Cancel *before* you multiply

Cancelling first keeps the numbers tiny:

$$\frac{3}{8} \times \frac{16}{9} = \frac{3 \times 16}{8 \times 9} = \frac{1 \times 2}{1 \times 3} = \frac{2}{3}$$

*(The 3 cancels into the 9, and the 8 cancels into the 16, before you multiply.)*

### Percentages are just multipliers

Increase by 15% → $\times 1.15$. Decrease by 15% → $\times 0.85$. That one habit turns most "money" questions into a single multiplication.

## Part 3 — Exam Strategy

Knowing the maths is only half the battle. These are the habits that actually turn knowledge into marks on the day.

### Strategy 1 — When stuck, test the four options

The answer is *always one of the four*. If the algebra looks scary, try each option in the original question and see which one fits. Start with the simplest-looking one.

**Example:** Solve $\log_2(x+2) + \log_2(x-4) = 4$. Options: $-4,\ 2,\ 6,\ 12$.

- $x = -4$ and $x = 2$ both make something inside a log negative — impossible, cross them out.
- Try $x = 6$: $\log_2 8 + \log_2 2 = 3 + 1 = 4$. ✓ **The answer is 6** — no hard algebra needed.

### Strategy 2 — Put in easy numbers for the letters

If a question is all abstract letters, replace them with small numbers and see what happens.

**Example:** "Which expression is always even when $n$ is odd?" — just set $n = 1$ and test each option. If two still survive, try $n = 3$.

### Strategy 3 — The 3-Pass System

- **Pass 1 — the easy marks.** Go through all 60. Answer everything you can do quickly. If a question is slow or scary, **mark it and skip**. Do not get stuck.
- **Pass 2 — the middle ones.** Go back to the ones you skipped that you *can* do with a little more time.
- **Pass 3 — the hard ones and the guesses.** Attempt what is left. In the final two minutes, **fill in a guess for every remaining blank.**

### Strategy 4 — Read the options first

The options themselves are clues:

- Options far apart $\rightarrow$ a rough estimate is enough.
- Options with different signs $(+/-)$ $\rightarrow$ figure out the sign first.
- Options very close together $\rightarrow$ you need careful, exact working.
- Options with $\sqrt{\ }$ or $\pi$ $\rightarrow$ keep your answer in that exact form, do not turn it into a decimal.

#### Strategy 5 — The classic traps (know them, dodge them)

| The tempting-but-wrong move | Why it is wrong | Do this instead |
|---|---|---|
| $\sqrt{a^2+b^2}=a+b$ | You cannot split a root across a + | Leave it as $\sqrt{a^2+b^2}$ unless it simplifies exactly |
| Down 10% then up 10% = back to start | It is actually a **1% loss** | Multiply the multipliers: $0.9\times1.1=0.99$ |
| $\log(a+b)=\log a+\log b$ | Only **products** split | $\log(ab)=\log a+\log b$ |
| $\frac{x+a}{x+b}=\frac{a}{b}$ | You cannot cancel across a + | Only cancel things that are **multiplied** |
| $(-x)^2=-x^2$ | Squaring always gives a **positive** | $(-x)^2=+x^2$ |

#### Strategy 6 — Smart guessing

If you genuinely have no idea:

- Eliminate any option you *know* is impossible (wrong sign, way too big, negative where it can't be). Every option you remove improves your odds.
- Then pick one and **move on**. Never leave it blank — a blank is a guaranteed zero.

---

## Part 4 — Your Study Plan

You do not need to study everything. You need to study the **right** things, consistently. Little and often beats one long panic. This plan is built around this book — every step names exactly what to do with it.

#### Six to four weeks before

- Turn Part 1 of this guide into flashcards — one formula or fact per card. Test yourself a little **every day** until recall is instant.
- **Put your calculator away** for all homework from now on. Every sum by hand builds the mental muscle you need.

- Work through the practice chapters in the Priority List order (next page). For each chapter: do a section of questions, then read **all** the explanations — including for the ones you got right. Aim for one topic section per sitting, not one chapter per sitting.

### Three weeks before

- Finish the practice chapters you started, and do each chapter's **mixed practice** set in one timed sitting (30 questions in 90 minutes — the real test's pace).
- Keep a **mistake notebook**: for every question you miss, copy out the Takeaway from its explanation. This notebook becomes your personal revision list.

### Two weeks before

- Sit **Mock Exam 1** (Chapter 9) under real conditions: one quiet 3-hour block, no calculator, no phone, a clock on the wall, answers on paper.
- Mark it honestly with the key. For each miss, find the matching topic in the practice chapters and re-read those explanations.
- A few days later, sit **Mock Exam 2** the same way and check whether your weak areas improved.

### One week before

- Go through your mistake notebook and sort every entry:
- **Didn't know the rule** → learn that rule now (it is in Part 1 of this guide).
- **Misread the question** → practise reading slowly, twice.
- **Silly arithmetic slip** → slow down and double-check your working.
- Sit **Mock Exam 3** as your final dress rehearsal.
- Spend the remaining days making your **strong** topics rock-solid to lock in those marks — don't try to learn brand-new topics from scratch now.

### The night before

- Stop studying by early evening. A rested brain scores better than a crammed one.
- Pack your bag: ID / passport, your registration letter, several HB pencils, an eraser, a ruler, a protractor, a compass, and a clear water bottle.

---

## Priority List — If You Are Short on Time

If you cannot cover everything, study in **this order** — the topics near the top appear most often and are the most learnable:

1. Fractions, decimals, and percentages → **Chapter 2**
2. Basic algebra (factorising, simplifying, solving equations) → **Chapter 1**
3. Functions and graphs (especially parabolas and straight lines) → **Chapter 3**
4. Exponents and logarithms → **Chapter 1** (Sections 1.5–1.6)
5. Trigonometry (special angles, SOHCAHTOA, CAST) → **Chapter 4**
6. Analytical geometry (distance, midpoint, gradient) → **Chapter 5** (Section 5.8)
7. Probability and counting → **Chapter 6**
8. Sequences and series → **Chapter 8**
9. Financial maths → **Chapter 7**
10. Geometry and measurement → **Chapter 5**
11. Data and statistics → **Chapter 6**

However short your time is, protect **one full mock exam** (Chapter 9) — pacing a real 3-hour paper is a skill of its own, and the first time you practise it must not be on test day.

---

## A Final Word

When you sit down to write the NBT MAT, you **will** hit questions that look unlike anything you have practised. That is done on purpose — the examiners want to see how you think, not just what you have memorised.

So when a question looks strange, do not panic. Take a slow breath. Read it again, in plain words. Look at the four options for clues. Break the problem into smaller pieces. Ask yourself: *what simple idea is hiding underneath all this wording?* Almost always, the scary-looking question is just a familiar idea wearing a costume.

You have prepared. You understand the ideas, not just the tricks. Trust that.

**Work calmly. Show your steps. Answer every single question — even the ones you guess.**

You can do this.

# Chapter 1 — Algebraic Processes

Algebraic processes form the backbone of the NBT MAT — roughly a third of the questions you face will lean on the skills in this chapter, and weak algebra will quietly cost you marks in *every other* chapter too (trig, functions, geometry — they all eventually become algebra).

This chapter is organised so that each topic moves from "can you do the mechanical step" (Basic) to "can you do it under exam conditions without a calculator" (Intermediate) to "can you spot the shortcut that turns a five-minute problem into a thirty-second one" (Proficient — this is where the NBT really separates students).

## Section 1.1 — Factorisation

**Before you start, lock in this 3-step habit for every factorisation question:** 1. **Take out the common factor first** — always, even if you think there isn't one. 2. **Look for a pattern** in what's left: difference of squares ($a^2 - b^2$), sum/difference of cubes ($a^3 \pm b^3$), a trinomial ($x^2 + bx + c$ or $ax^2 + bx + c$), or four terms that suggest grouping. 3. **Check you're "fully" factorised** — can anything inside *any* bracket still be broken down further?

### Q1 — Basic

Factorise fully: $3x^2 - 12$

A) $(3x - 6)(x + 2)$

B) $3(x - 2)(x + 2)$

C) $3(x - 4)(x + 1)$

D) $(x - 2)(3x + 6)$

### Q2 — Basic

Factorise: $x^2 + 5x + 6$

A) $(x+1)(x+6)$

B) $(x+6)(x-1)$

C) $(x-2)(x-3)$

D) $(x+2)(x+3)$

---

### Q3 — Basic

Factorise fully: $9x^2 - 16y^2$

A) $(3x-4y)(3x+4y)$

B) $(9x-16y)(x+y)$

C) $(3x-4y)^2$

D) $(9x+4y)(x-4y)$

---

### Q4 — Basic

Factorise fully: $ax + ay + bx + by$

A) $abxy$

B) $(a+x)(b+y)$

C) $(a+b)(x+y)$

D) $(a-b)(x-y)$

---

### Q5 — Intermediate

Factorise fully: $x^3 + 27$

A) $(x+3)^3$

B) $(x+3)(x^2+3x+9)$

C) $(x+3)(x^2-3x+9)$

D) $(x+3)(x^2-6x+9)$

---

### Q6 — Intermediate

Factorise: $2x^2 + 5x - 3$

A) $(2x - 1)(x + 3)$

B) $(2x + 1)(x - 3)$

C) $(2x - 3)(x + 1)$

D) $(2x + 3)(x - 1)$

---

### Q7 — Intermediate

Factorise fully: $x^3 - 3x^2 - 4x + 12$

A) $(x - 3)(x - 4)(x + 1)$

B) $(x + 3)(x - 2)(x + 2)$

C) $(x - 3)(x^2 - 4)$

D) $(x - 3)(x - 2)(x + 2)$

---

### Q8 — Proficient

Factorise fully: $x^4 - 13x^2 + 36$

A) $(x - 4)(x + 4)(x - 9)(x + 9)$

B) $(x - 2)(x + 2)(x - 3)(x + 3)$

C) $(x - 2)(x + 2)(x^2 - 9)$

D) $(x^2 - 4)(x^2 - 9)$

---

## Section 1.2 — Linear & Quadratic Equations

**Two habits that prevent almost every careless error in this section:** 1. **Always rearrange to standard form before factorising** ($ax^2 + bx + c = 0$, everything on one side, zero on the other). 2. **Always check your final answer by substituting it back into the *original* equation** — not your rearranged version, the original. This catches sign errors *and* extraneous roots.

---

### Q9 — Basic

Solve for $x$: $\dfrac{3x-2}{4} = \dfrac{x+6}{2}$

A) $x = 14$

B) $x = 10$

C) $x = 2$

D) $x = -14$

---

### Q10 — Basic

Solve for $x$: $x^2 - 7x + 10 = 0$

A) $x = -2$ or $x = -5$

B) $x = 2$ or $x = -5$

C) $x = 2$ or $x = 5$

D) $x = -2$ or $x = 5$

---

### Q11 — Basic

Solve for $x$ (leave your answer in simplest surd form): $x^2 - 6x + 4 = 0$

A) $x = 6 \pm \sqrt{5}$

B) $x = 3 \pm \sqrt{5}$

C) $x = 3 \pm \sqrt{20}$

D) $x = 3 \pm 2\sqrt{5}$

---

### Q12 — Intermediate

Write $x^2 + 8x + 3$ in the form $(x+p)^2 + q$.

A) $(x+4)^2 - 19$

B) $(x+4)^2 + 3$

C) $(x+8)^2 - 13$

D) $(x+4)^2 - 13$

---

### Q13 — Intermediate

Solve for $x$: $x^2 = 5x - 6$

A) $x = 2$ or $x = 3$

B) $x = -6$ or $x = -1$

C) $x = -2$ or $x = -3$

D) $x = -2$ or $x = 3$

---

### Q14 — Intermediate

Solve for $x$: $\dfrac{3}{x-2} = \dfrac{1}{x+1}$

A) $x = \dfrac{5}{2}$

B) No solution, because $x \neq 2$

C) $x = \dfrac{7}{2}$

D) $x = -\dfrac{5}{2}$

---

### Q15 — Proficient

Solve for $x$: $x - 4\sqrt{x} - 5 = 0$

A) $x = 1$

B) $x = 25$ or $x = 1$

C) $x = 25$

D) $x = -1$

---

### Q16 — Proficient

Solve simultaneously for $x$ and $y$: $\quad y = x + 1 \quad \text{and} \quad y = x^2 - 5$

A) $(3; 2)$ and $(-2; -3)$

B) $(3; 4)$ and $(-2; -1)$

C) $(3; 4)$ only

D) $(-3; -2)$ and $(2; 3)$

---

## Section 1.3 — The Factor Theorem

**The one idea to lock in:**

- **Factor theorem:** $(x-a)$ is a factor of $f(x)$ **if and only if** $f(a)=0$.

**"If and only if"** means the substitution is complete proof, both ways. If $f(a)=0$ then $(x-a)$ **is** a factor — no extra checking needed. And if $(x-a)$ is a factor, then $f(a)$ **must** be 0.

**The detail people get wrong:** for the factor $(x-a)$ you substitute $x=a$ — the value that makes the bracket zero. So for $(x+2)$, substitute $x=-2$, not $x=+2$.

---

### Q17 — Basic

Given $f(x)=x^3-2x^2-5x+6$, evaluate $f(1)$ and hence state whether $(x-1)$ is a factor of $f(x)$.

A) $f(1)=0$, but more information is needed before concluding $(x-1)$ is a factor

B) $f(1)=0$; no, $(x-1)$ is not a factor of $f(x)$

C) $f(1)=4$; no, $(x-1)$ is not a factor of $f(x)$

D) $f(1)=0$; yes, $(x-1)$ is a factor of $f(x)$

---

### Q18 — Intermediate

Use the factor theorem to determine whether $(x+2)$ is a factor of $f(x)=x^3-7x-6$.

A) $f(-2)=-28$; no, $(x+2)$ is not a factor of $f(x)$

B) $f(-2)=0$; yes, $(x+2)$ is a factor of $f(x)$

C) $f(2)=-12$; no, $(x+2)$ is not a factor of $f(x)$

D) $f(-2)=0$, but polynomial long division is still needed to be certain

---

### Q19 — Intermediate

Given that $(x-2)$ is a factor of $f(x)=x^3+kx^2+2x-24$, find the value of $k$.

A) $k=3$

B) $k=9$

C) $k=12$

D) $k=-3$

---

### Q20 — Intermediate

Given that $(x-1)$ is a factor of $f(x)=x^3-2x^2-5x+6$, find **all** the roots of $f(x)=0$.

A) $x=1$ only

B) $x=1,\ x=-3,\ x=2$

C) $x=1,\ x=3,\ x=-2$

D) $x=-1,\ x=3,\ x=-2$

---

### Q21 — Proficient

The cubic $f(x)=x^3+ax^2+bx-6$ has $(x-1)$ and $(x+2)$ as factors. Find the values of $a$ and $b$.

A) $a=1,\ b=4$

B) $a=4,\ b=1$

C) $a=4,\ b=-1$

D) $a=4$, and $b$ cannot be determined from this information

---

### Q22 — Proficient

$f(x)$ is a cubic polynomial. You are told that $f(2)=0$, $f(-1)=0$, and $f(3)=0$. Which of the following statements about $f(x)$ **must** be true?

A) $f(x)=(x-2)(x+1)(x-3)$

B) $f(x)$ could have a fourth root that we haven't been told about

C) $f(x)$ has exactly one real root

D) $f(x)=k(x-2)(x+1)(x-3)$ for some non-zero constant $k$

---

## Section 1.4 — Inequalities

**The one rule that matters more than any other in this section:**

> **Multiplying or dividing both sides of an inequality by a *negative* number reverses the inequality sign** ($<$ becomes $>$, $\leq$ becomes $\geq$, and vice versa).

Why does this happen? Think of a simple true statement: $2 < 5$. Multiply both sides by $-1$: you'd get $-2 < -5$, which is **false** ($-2$ is actually *bigger* than $-5$ on the number line). To keep the statement true, the sign *must* flip: $-2 > -5$ ✓. This isn't an arbitrary rule to memorise — it's the only way to keep the inequality telling the truth.

---

**Q23 — Basic**

Solve for $x$: $3x - 5 < 7$

A) $x < 4$

B) $x > 4$

C) $x < \frac{2}{3}$

D) $x < -4$

---

**Q24 — Basic**

Solve for $x$: $-2x + 3 \geq 11$

A) $x \geq 4$

B) $x \geq -4$

C) $x \leq -4$

D) $x \leq 4$

---

**Q25 — Intermediate**

Solve for $x$: $x^2 - x - 6 < 0$

A) $-3 < x < 2$

B) $x < -2$ or $x > 3$

C) $-2 < x < 3$

D) $2 < x < 3$

---

### Q26 — Intermediate

Solve for $x$: $x^2 - 5x + 6 \geq 0$

A) $x \leq 2$ or $x \geq 3$

B) $2 \leq x \leq 3$

C) $x < 2$ or $x > 3$

D) $x \leq -2$ or $x \geq -3$

---

### Q27 — Intermediate

Solve for $x$: $\dfrac{x-2}{x+1} > 0$

A) $x < -1$ or $x \geq 2$

B) $-1 < x < 2$

C) $x \leq -1$ or $x > 2$

D) $x < -1$ or $x > 2$

---

### Q28 — Proficient

Solve for $x$: $x^2 + 4 < 0$

A) Every real value of $x$ satisfies this inequality

B) No real value of $x$ satisfies this inequality — the solution set is empty

C) $-2 < x < 2$

D) $x < -2$ or $x > 2$

---

## Section 1.5 — Exponents & Surds

**Two non-negotiable habits before you do anything else:** 1. **Same base, always.** Whenever you see different-looking numbers in an exponent question ($4, 8, 16$ or $9, 27$ or $6, 2, 3$), your first move is to ask *"can I rewrite all of these as powers of the same smaller base?"* Almost every exponent question on the NBT collapses once you do this. 2. **A fractional exponent is two instructions stacked on top of each other.** $a^{m/n}$ means *"take the $n$-th root, then raise to the power $m$"* (or the other order — both give the same answer). The denominator is the **root**; the numerator is the **power**. Mixing these up is the single most common exponent error students make.

---

### Q29 — Basic

Simplify: $\frac{x^5 \cdot x^3}{x^4}$

A) $x^4$

B) $x^{12}$

C) $x^2$

D) $x^{60}$

---

### Q30 — Basic

Write $\frac{1}{x^{-3}}$ using a **positive** exponent.

A) $x^{-3}$

B) $-x^3$

C) $x^3$

D) $\frac{1}{x^3}$

---

### Q31 — Basic

Evaluate: $8^{2/3}$

A) $\frac{16}{3}$

B) 4

C) 64

D) 8

---

### Q32 — Basic

Simplify: $\sqrt{12} + \sqrt{27}$

A) $\sqrt{12} + \sqrt{27}$ (cannot be simplified further)

B) $\sqrt{39}$

C) $6\sqrt{3}$

D) $5\sqrt{3}$

### Q33 — Intermediate

Rationalise the denominator: $\dfrac{2}{\sqrt{5}-1}$

A) $\dfrac{2\sqrt{5}+2}{4}$

B) $\dfrac{\sqrt{5}+1}{2}$

C) $\dfrac{\sqrt{5}+1}{3}$

D) $\dfrac{2}{\sqrt{5}+1}$

### Q34 — Intermediate

Simplify: $\dfrac{6^{x+1}}{2^x \cdot 3^{x-1}}$

A) 18

B) 12

C) 9

D) 36

### Q35 — Intermediate

Solve for $x$: $\sqrt{x+3} = x-3$

A) $x = 1$

B) $x = 1$ or $x = 6$

C) $x = 6$

D) No real solution

### Q36 — Proficient

Simplify: $\dfrac{2^{n+2} - 2^{n+1}}{2^n}$

A) $\dfrac{1}{2}$

B) 1

C) $2^{n+1}$

D) 2

---

### Q37 — Proficient

Simplify: $\sqrt{7 + 4\sqrt{3}}$

A) Cannot be simplified into the form $\sqrt{a} + \sqrt{b}$

B) $1 + \sqrt{6}$

C) $4 + \sqrt{3}$

D) $2 + \sqrt{3}$

---

## Section 1.6 — Logarithms

**The one conversion that unlocks almost everything:**

$$\log_a N = x \quad \Longleftrightarrow \quad a^x = N$$

Whenever a log expression confuses you, convert it to this exponential form — the fog usually clears immediately.

**The three log laws — and the one habit that protects you from inventing fake ones:**

$$\log_a(xy) = \log_a x + \log_a y \qquad \log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y \qquad \log_a(x^n) = n\log_a x$$

Notice the pattern: a *product* inside the log becomes a *sum* outside it; a *quotient* becomes a *difference*; a *power* becomes a *multiplying coefficient*. The laws only ever convert **multiplicative** structure (inside) into **additive** structure (outside) — never the other way round, and there is **no law at all** for $\log(x + y)$ or $\log(x - y)$. If you ever find yourself "simplifying" an addition or subtraction *inside* a logarithm, stop — that's the single most common way students invent rules that don't exist.

---

### Q38 — Basic

Evaluate: $\log_2 32$

A) 4

B) 16

C) 5

D) 6

---

### Q39 — Basic

Evaluate: $\log_5 \frac{1}{25}$

A) $-2$

B) 2

C) $-\frac{1}{2}$

D) $\frac{1}{2}$

---

### Q40 — Intermediate

Simplify: $\log_2 40 - \log_2 5$

A) $\log_2 200$

B) 3

C) 8

D) $\log_2 35$

---

### Q41 — Intermediate

Solve for $x$: $\log_2(x+3) = 4$

A) $x = 1$

B) $x = 5$

C) $x = 13$

D) $x = 19$

---

### Q42 — Intermediate

Evaluate: $\log_4 8$

A) $2$

B) $\frac{3}{2}$

C) $\frac{2}{3}$

D) $\log_4 8$ cannot be written as a simple fraction — it is irrational

---

### Q43 — Proficient

If $\log_a 2 = p$ and $\log_a 3 = q$, express $\log_a 12$ in terms of $p$ and $q$.

A) $2pq$

B) $p^2 + q$

C) $4p + q$

D) $2p + q$

---

### Q44 — Proficient

For all $x > 0$ and $y > 0$, which of the following statements is **always** true?

A) $\log(xy) = \log x + \log y$

B) $(\log x)(\log y) = \log(x + y)$

C) $\log(x - y) = \log x - \log y$

D) $\log(x + y) = \log x + \log y$

---

## Section 1.7 — Sequences & Series

**The one test that tells them apart:**

- **Arithmetic (AP):** consecutive terms differ by a **constant amount** — $T_{n+1} - T_n = d$ (the *common difference*) is the same every time. Growth is **additive**: $5, 8, 11, 14, \ldots$
- **Geometric (GP):** consecutive terms are related by a **constant multiplier** — $\frac{T_{n+1}}{T_n} = r$ (the *common ratio*) is the same every time. Growth is **multiplicative**: $5, 10, 20, 40, \ldots$

**The four formulas worth knowing cold:**

$$\text{AP: } T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\text{GP: } T_n = a \cdot r^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \quad (r \neq 1)$$

And **sigma notation** $\sum_{k=1}^{n}(\cdots)$ is nothing more than shorthand for "add up the expression in the brackets, substituting $k = 1$, then $k = 2$, ..., all the way up to $k = n$." It looks formal, but it's just a compact instruction to write out a sum term by term.

---

#### Q45 — Basic

Find the 10th term of the arithmetic sequence $5, 8, 11, 14, \ldots$

A) 27

B) 35

C) 30

D) 32

---

#### Q46 — Basic

Find the 6th term of the geometric sequence $3, 6, 12, 24, \ldots$

A) 96

B) 192

C) 48

D) 36

---

### Q47 — Basic

What is the common ratio of the geometric sequence $81, 27, 9, 3, \ldots$?

A) $-54$

B) 3

C) $\frac{1}{3}$

D) $\frac{1}{27}$

---

### Q48 — Intermediate

Find the sum of the first 20 terms of the arithmetic sequence $4, 7, 10, \ldots$

A) 610

B) 650

C) 680

D) 1300

---

### Q49 — Intermediate

Find the sum of the first 5 terms of the geometric sequence $2, 6, 18, \ldots$

A) 242

B) 243

C) 14

D) 50

---

### Q50 — Intermediate

Evaluate: $\sum_{k=1}^{5}(2k + 1)$

A) 30

B) 24

C) 31

D) 35

---

### Q51 — Intermediate

How many terms of the arithmetic sequence $3, 7, 11, \ldots$ must be added together for the sum to equal 210?

A) $n = 11$

B) $n = 10$

C) $n = 10$ or $n = -10.5$

D) No whole number of terms gives a sum of 210

---

### Q52 — Proficient

Find the sum to infinity of the geometric series $8 + 4 + 2 + 1 + \ldots$

A) $\dfrac{1}{16}$

B) No finite sum exists — the series goes on forever

C) 16

D) 4

---

### Q53 — Proficient

The 3rd term of an arithmetic sequence is 11, and the 7th term is 23. Find the first term $a$ and the common difference $d$.

A) $a = 3,\ d = 4$

B) $a = 11,\ d = 3$

C) $a = 5,\ d = 3$

D) $a = 17,\ d = 3$

---

## Section 1.8 — Financial Mathematics

**The crucial distinction — and a secret connection to Section 1.7:**

$$\text{Simple interest (linear growth): } A = P(1 + in)$$

$$\text{Compound interest (multiplicative growth): } A = P(1 + i)^n$$

Look closely at the compound formula — $P, P(1+i), P(1+i)^2, P(1+i)^3, \ldots$ — and you'll recognise it instantly: **it's a geometric sequence**, with first term $P$ and common ratio $(1+i)$. Everything you learned about GPs in Section 1.7 — including the "off-by-one in the exponent" trap — applies directly here. Simple interest, by contrast, adds the *same fixed amount* every year — it's an arithmetic sequence in disguise.

**Depreciation is just "negative growth":**

$$\text{Straight-line (simple) depreciation: } A = P(1 - in)$$

$$\text{Reducing-balance (compound) depreciation: } A = P(1 - i)^n$$

Same two formula families, same GP/AP structure underneath — only the sign in the bracket flips, because the value is *shrinking* rather than *growing*. The single biggest skill this section tests is matching the **right formula family** to the **right scenario description**: "compounded annually" / "reducing balance" → exponential; "simple interest" / "straight-line" → linear.

---

### Q54 — Basic

Tumi invests R4 000 at a simple interest rate of $6\%$ per annum. Which expression correctly represents the **value of her investment** after 5 years?

A) $4\,000(1.06)^5$

B) $4\,000(1 + 0.06 \times 5)$

C) $4\,000 \times 0.06 \times 5$

D) $4\,000(1 + 0.06) \times 5$

---

### Q55 — Basic

Sipho invests R8 000 at a compound interest rate of $9\%$ per annum, compounded annually. Which expression represents the value of his investment after 4 years?

A) $8\,000(0.09)^4$

B) $8\,000(1 + 0.09 \times 4)$

C) $8\,000 \times 1.09 \times 4$

D) $8\,000(1.09)^4$

---

### Q56 — Intermediate

A delivery van bought for R240 000 depreciates at a rate of $15\%$ per annum on the **reducing-balance** method. Which expression gives its value after 3 years?

A) $240\,000(1 - 0.15)^3$

B) $240\,000(1 - 0.15 \times 3)$

C) $240\,000(0.15)^3$

D) $240\,000(1 + 0.15)^3$

---

### Q57 — Intermediate

An investment of R10 000 grows to R12 100 after 2 years under annual compound interest. Find the annual interest rate.

A) $11\%$

B) $21\%$

C) $10.5\%$

D) $10\%$

---

### Q58 — Intermediate

An investment grows from a principal $P$ to an amount of R16 000 after 4 years, at a compound interest rate of $i$ per annum, compounded annually. Which expression correctly represents $P$ in terms of $i$?

A) $P = \dfrac{16\,000}{1+4i}$

B) $P = 16\,000(1+i)^4$

C) $P = \dfrac{16\,000}{(1+i)^4}$

D) $P = 16\,000 - 4i$

---

### Q59 — Proficient

A machine valued at $V$ depreciates on the reducing-balance method at $r\%$ per annum (where $0 < r < 100$). At the same time, the cost of a brand-new equivalent machine increases (inflates) at the *same* rate, $r\%$ per annum. After $n$ years, the old machine's depreciated value is $V(1-r)^n$ and the new machine's cost is $V(1+r)^n$ (writing $r$ here as a decimal). Which statement about the ratio $\dfrac{\text{old machine's value}}{\text{new machine's cost}}$ **must** be true?

A) The ratio equals $\left(\dfrac{1-r}{1+r}\right)^n$, and it becomes **smaller** with each passing year B) The two effects cancel exactly: the ratio always equals 1, no matter what $n$ is

C) The ratio equals $(1-r^2)^n$, formed by multiplying the two growth factors together

D) The ratio equals $\left(\dfrac{1+r}{1-r}\right)^n$, and it becomes **larger** with each passing year

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## Section 1.9 — Algebraic Insight, Vieta's Formulas & Manipulation Tricks

This section has no new procedures. It rewards you for **seeing the structure** instead of grinding through the algebra.

**The master tool: Vieta's formulas.** For a quadratic $ax^2 + bx + c = 0$ with roots $\alpha$ and $\beta$:

$$\alpha + \beta = -\frac{b}{a} \qquad\qquad \alpha\beta = \frac{c}{a}$$

These say something remarkable: **you can know the sum and product of a quadratic's roots without ever finding the roots themselves** — just by reading off its coefficients. Combined with the identity you met in the sample question at the start of this chapter,

$$a^2 + b^2 = (a+b)^2 - 2ab,$$

Vieta's formulas let you compute all sorts of expressions *built from* the roots — $\alpha^2 + \beta^2$, $\frac{1}{\alpha} + \frac{1}{\beta}$, $(\alpha + 1)(\beta + 1)$, and more — directly from the coefficients, without the slow, error-prone detour of solving the equation first.

**The recurring false belief worth naming, one last time:** several sections in this chapter have each, in their own way, warned you against the same imaginary rule — that an operation "distributes" across addition just because it feels like it should: $\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$ (Q32), $\log(x+y) \neq \log x + \log y$ (Q44), and — as you're about to see — $\frac{1}{\alpha} + \frac{1}{\beta} \neq \frac{1}{\alpha + \beta}$ too. Recognising this *one* recurring pattern protects you across an enormous range of NBT questions.

---

### Q60 — Intermediate

If $\alpha$ and $\beta$ are the roots of $2x^2 - 7x + 3 = 0$, what is the value of $\alpha + \beta$?

A) 7

B) $\frac{7}{2}$

C) $\frac{3}{2}$

D) $-\frac{7}{2}$

---

### Q61 — Intermediate

If $x - \frac{1}{x} = 3$, what is the value of $x^2 + \frac{1}{x^2}$?

A) 9

B) 11

C) 7

D) 5

---

### Q62 — Proficient

If $\alpha$ and $\beta$ are the roots of $x^2 - 5x + 6 = 0$, form a quadratic equation (with integer coefficients) whose roots are $\alpha + 1$ and $\beta + 1$.

A) $x^2 - 7x + 12 = 0$

B) $x^2 - 5x + 6 = 0$

C) $x^2 - 3x + 2 = 0$

D) $x^2 - 7x + 7 = 0$

---

### Q63 — Proficient

If $a + b + c = 6$, $ab + bc + ca = 11$, and $abc = 6$, find the value of $a^2 + b^2 + c^2$.

A) 58

B) 36

C) 25

D) 14

---

### Q64 — Proficient

For the equation $x^2 + bx + c = 0$, where $b$ and $c$ are real numbers, which statement **must** be true?

A) If $b = 0$, the equation has no real roots

B) If $b^2 - 4c > 0$, the equation has two real roots of the same sign

C) If $c < 0$, the equation has two real roots of opposite sign

D) If $c > 0$, the equation has two real roots of the same sign

---

#### Q65 — Proficient

Without solving the equation $3x^2 - 8x + 5 = 0$ for $x$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$, where $\alpha$ and $\beta$ are its roots.

A) $\frac{8}{5}$
B) $\frac{5}{8}$
C) $\frac{8}{3}$
D) $\frac{3}{8}$

---

## Exam-Bank Extras — Question Types Confirmed in Recent Papers

The NBT MAT draws on a stable question bank, and test-writers consistently report seeing the same question types — often near-identical questions — year after year. The six questions below are modelled directly on types repeatedly confirmed in recent papers that were not yet represented in this chapter. Every answer has been independently machine-verified.

*A note on Q71:* the remainder theorem sits slightly outside the current CAPS core (which is why the rest of this book works with the factor theorem only), but it has appeared on real NBT papers — so one authentic example is included here, taught from scratch.

---

#### Q66 — Basic

What is the sum of the solutions of $x^2 = 4$?

A) 2

B) 0

C) 4

D) $-2$

---

### Q67 — Intermediate

For real numbers $a$ and $b$, the operation $\diamond$ is defined by

$$a \diamond b = a^2 - 3b$$

Find the value of $(2 \diamond 1) \diamond 3$.

A) 10

B) $-2$

C) $-8$

D) 8

---

### Q68 — Proficient

Two numbers have a sum of $m$ and a product of $n$. Which expression gives the **sum of the squares** of the two numbers?

A) $m^2 + 2n$

B) $m^2 - 2n$

C) $m^2 - n$

D) $m - 2n$

---

### Q69 — Proficient

Both $(x + 1)$ and $(x - 2)$ are factors of

$$f(x) = 2x^4 - kx^2 + bx - 4$$

Find the value of $k$.

A) 4

B) $-6$

C) 2

D) $-4$

---

### Q70 — Proficient

The integers $x$ and $y$ satisfy

$$2^{x+2} + 2^x = 5^{y+1} - 5^y$$

Find the value of $x - y$.

A) 0

B) 3

C) 1

D) 2

---

### Q71 — Proficient

When $f(x) = 2x^4 - kx^2 + bx$ is divided by $(x+1)$, the remainder is $-3$. When $f(x)$ is divided by $(x-1)$, the remainder is 5. Find the value of $k$.

A) $-1$

B) 4

C) 2

D) 1

# Answers and Explanations

## Section 1.1 — Factorisation

### Q1 — Basic — Answer: B

**Topic:** Common factor + difference of squares

**Step 1 — Take out the common factor.** Both terms divide by 3.

$$3x^2 - 12 = 3(x^2 - 4)$$

**Step 2 — Factorise what is left.** $x^2 - 4$ is a difference of two squares, $a^2 - b^2 = (a - b)(a + b)$.

$$3x^2 - 12 = 3(x - 2)(x + 2)$$

*Why the others are wrong:*

- **A** and **D** still hide a 3 inside a bracket: $(3x - 6) = 3(x - 2)$. Not *fully* factorised.
- **C** — expand it: $(x - 4)(x + 1) = x^2 - 3x - 4$. That is not $x^2 - 4$.

> **Takeaway:** "Factorise fully" means check every bracket again. Can anything still be divided down?

---

### Q2 — Basic — Answer: D

**Topic:** Trinomial factorisation

For $x^2 + bx + c$, find **two numbers that multiply to** $c$ **and add to** $b$. Here $c = 6, b = 5$.

Factor pairs of 6: $(1, 6)$ and $(2, 3)$. Which adds to 5? $2 + 3 = 5$ ✓

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

Check: $(x + 2)(x + 3) = x^2 + 5x + 6$ ✓

*Why the others are wrong:*

- **A** uses $(1, 6)$ — multiplies to 6, but adds to 7, not 5.
- **C** uses $(-2, -3)$ — multiplies to $+6$, but adds to $-5$. That gives $x^2 - 5x + 6$.
- **B** uses $(6, -1)$ — multiplies to $-6$, not $+6$.

> **Takeaway:** If $b$ and $c$ are both positive, both factors are positive. Checking the signs first cuts your options in half.

---

### Q3 — Basic — Answer: A

**Topic:** Difference of squares with coefficients

Find the squares: $9x^2 = (3x)^2$ and $16y^2 = (4y)^2$. So $a = 3x$ and $b = 4y$, and $a^2 - b^2 = (a-b)(a+b)$:

$$9x^2 - 16y^2 = (3x - 4y)(3x + 4y)$$

Check: $(3x - 4y)(3x + 4y) = 9x^2 + 12xy - 12xy - 16y^2 = 9x^2 - 16y^2$ ✓ The middle terms cancel — that always happens with a difference of squares.

*Why the others are wrong:*

- **B** and **D** — expanding either leaves an $xy$ term. The original has no $xy$ term.
- **C** is a perfect square, not a difference of squares: $(3x - 4y)^2 = 9x^2 - 24xy + 16y^2$.

> **Takeaway:** Difference of squares needs two terms, a **minus** between them, and both terms perfect squares. A **plus** between two squares ($9x^2 + 16y^2$) cannot be factorised at all.

---

### Q4 — Basic — Answer: C

**Topic:** Factorising by grouping (four terms)

**Four terms, no overall common factor** — so group them in pairs that *do* share a factor.

$$ax + ay + bx + by = a(x + y) + b(x + y)$$

Now $(x + y)$ is common to both halves. Take it out:

$$= (a + b)(x + y)$$

Check: $(a + b)(x + y) = ax + ay + bx + by$ ✓

*Why the others are wrong:*

- **B** pairs the wrong terms. Expand it: $ab + ay + bx + xy$ — nothing like the original.
- **D** has the right shape but wrong signs. It answers $ax - ay - bx + by$.
- **A** just multiplies everything together. That is not factorising.

> **Takeaway:** With four terms, split them into two pairs, factorise each pair, then take out the bracket they share.

---

### Q5 — Intermediate — Answer: C

**Topic:** Sum of cubes

$27 = 3^3$, so this is a **sum of cubes** with $a = x$ and $b = 3$:

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$x^3 + 27 = (x+3)(x^2 - 3x + 9)$$

Watch the signs: the bracket $(x+3)$ keeps the **original** sign, but the middle term of the trinomial flips to the **opposite** sign, $-3x$.

*Why the others are wrong:*

- **B** forgot the middle sign flips. It expands to $x^3 + 6x^2 + 18x + 27$.
- **D** uses $-6x$, which belongs to $(x-3)^2$, a different expansion.
- **A** confuses "cube of a sum" with "sum of cubes". $(x+3)^3 = x^3 + 9x^2 + 27x + 27$.

> **Takeaway:** Learn both cube identities together:
> $$a^3 + b^3 = (a+b)(a^2 - ab + b^2) \qquad a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
> The bracket matches the original sign; the trinomial's middle term is always the opposite sign.

---

### Q6 — Intermediate — Answer: A

**Topic:** Trinomial with leading coefficient ≠ 1

When the leading coefficient is not 1, find two numbers that **multiply to** $a \times c$ and **add to** $b$.

Here $a \times c = 2 \times (-3) = -6$ and $b = 5$. The pair is 6 and $-1$.

Split the middle term and group:

$$2x^2 + 6x - x - 3 = 2x(x+3) - 1(x+3) = (2x-1)(x+3)$$

Check: $(2x-1)(x+3) = 2x^2 + 5x - 3$ ✓

*Why the others are wrong:* all three give the wrong middle term.

- **B**: $(2x+1)(x-3) = 2x^2 - 5x - 3$
- **C**: $(2x-3)(x+1) = 2x^2 - x - 3$
- **D**: $(2x+3)(x-1) = 2x^2 + x - 3$

> **Takeaway:** Always expand your answer to check. With $a \neq 1$ several bracket arrangements look right, and only one is.

---

### Q7 — Intermediate — Answer: D

**Topic:** Factorising a cubic by grouping

Four terms, no overall common factor — so group in pairs:

$$x^3 - 3x^2 - 4x + 12 = x^2(x-3) - 4(x-3) = (x-3)(x^2-4)$$

Now $x^2 - 4$ is a difference of squares, so keep going:

$$(x-3)(x-2)(x+2)$$

*Why the others are wrong:*

- **C** is a correct *intermediate* step, but $x^2 - 4$ still factorises. The question says **fully**.
- **B** has a sign error: $(x+3)(x^2-4) = x^3 + 3x^2 - 4x - 12$.
- **A** expands to $x^3 - 6x^2 + 5x + 12$ — not the original.

> **Takeaway:** After grouping, check each bracket again for another pattern. Difference of squares is the one people miss.

---

### Q8 — Proficient — Answer: B

**Topic:** Quadratic-in-disguise (substitution insight)

Only **even powers** of $x$ appear, so this is a quadratic in disguise. Let $y = x^2$:

$$y^2 - 13y + 36 = (y-4)(y-9)$$

Substitute back $y = x^2$:

$$(x^2-4)(x^2-9)$$

Both brackets are differences of squares, so keep going:

$$(x-2)(x+2)(x-3)(x+3)$$

*Why the others are wrong:*

- **D** stops right after substituting back. Correct, but not **fully** factorised.
- **C** factorises the first bracket and forgets the second.
- **A** takes $y = 4$ and $y = 9$ and writes $x = 4$ and $x = 9$. But $y = x^2$, so $x^2 = 4$ gives $x = \pm 2$ and $x^2 = 9$ gives $x = \pm 3$.

> **Takeaway:** Only even powers? Substitute $y = x^2$. When you swap back, remember you must take a **square root**, not copy the number across.

---

## Section 1.2 — Linear & Quadratic Equations

### Q9 — Basic — Answer: A

**Topic:** Linear equation with fractions

**Step 1 — Clear the fractions.** Cross-multiply:

$$2(3x - 2) = 4(x + 6)$$

**Step 2 — Expand.**

$$6x - 4 = 4x + 24$$

**Step 3 — Collect terms.** Anything crossing the equals sign flips its sign.

$$6x - 4x = 24 + 4 \quad \Rightarrow \quad 2x = 28 \quad \Rightarrow \quad x = 14$$

Check in the original: LHS $= \frac{40}{4} = 10$, RHS $= \frac{20}{2} = 10$ ✓

*Why the others are wrong:*

- **B** ($x = 10$) forgot a sign flip: $6x - 4x = 24 - 4$ instead of $24 + 4$.
- **C** ($x = 2$) cross-multiplied the wrong way: $4(3x - 2) = 2(x + 6)$.
- **D** ($x = -14$) flipped the signs on both constants.

> **Takeaway:** When a term crosses the equals sign, say the sign change out loud: "minus four becomes plus four." Sign slips are the most common algebra error there is.

---

### Q10 — Basic — Answer: C

**Topic:** Solving a quadratic by factorisation

Already in standard form, so factorise. Two numbers that **multiply to** $+10$ and **add to** $-7$: that is $-2$ and $-5$.

$$x^2 - 7x + 10 = (x-2)(x-5) = 0$$

**Zero-product rule:** if two things multiply to zero, one of them must be zero.

$$x = 2 \quad \text{or} \quad x = 5$$

*Why the others are wrong:* each solves a **different** equation.

- **A** solves $x^2 + 7x + 10 = 0$
- **B** solves $x^2 + 3x - 10 = 0$
- **D** solves $x^2 - 3x - 10 = 0$

They all use the numbers 2 and 5 with the wrong signs.

> **Takeaway:** Check signs before you write anything. In $x^2 + bx + c = 0$: if $c$ is **positive**, both factors share the sign of $b$. If $c$ is **negative**, the factors have opposite signs. Here $c = +10, b = -7$, so both factors are negative.

---

### Q11 — Basic — Answer: B

**Topic:** Quadratic formula with a surd answer

This will not factorise, so use the quadratic formula with $a = 1, b = -6, c = 4$:

$$x = \frac{6 \pm \sqrt{36-16}}{2} = \frac{6 \pm \sqrt{20}}{2}$$

**Simplify the surd:** $\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$.

$$x = \frac{6 \pm 2\sqrt{5}}{2}$$

**Divide *every* term by 2** — this is the step people rush:

$$x = 3 \pm \sqrt{5}$$

*Why the others are wrong:*

- **C** never simplified $\sqrt{20}$ to $2\sqrt{5}$.
- **D** divided the 6 by 2 but not the surd.
- **A** divided the surd but not the 6.

> **Takeaway:** The fraction bar applies to **every** term above it. After the quadratic formula, ask: did I divide both the whole number **and** the surd by $2a$?

---

### Q12 — Intermediate — Answer: D

**Topic:** Completing the square

You want to rewrite $x^2 + 8x$ as a perfect square plus a correction.

**Step 1 — Halve the coefficient of $x$.** Half of 8 is 4, so try $(x+4)^2$. Expand it:

$$(x+4)^2 = x^2 + 8x + 16$$

**Step 2 — Correct the constant.** You have $+16$ but want $+3$, so subtract 16 and add 3:

$$x^2 + 8x + 3 = (x+4)^2 - 16 + 3 = (x+4)^2 - 13$$

*Why the others are wrong:*

- **C** uses $p = 8$ — the original coefficient instead of **half** of it.
- **B** just tacks the $+3$ on, ignoring the extra $+16$ that $(x+4)^2$ brings in.
- **A** has the right method but slips on the arithmetic: $3 - 16 = -13$, not $-19$.

> **Takeaway:** Two moves every time: **(1)** halve $b$ to get $p$; **(2)** $q = c - p^2$.

---

### Q13 — Intermediate — Answer: A

**Topic:** Rearranging before solving

**The key step comes before factorising:** this is not in standard form. Move everything to one side.

$$x^2 - 5x + 6 = 0$$

Now factorise. Two numbers multiplying to $+6$ and adding to $-5$: that is $-2$ and $-3$.

$$(x-2)(x-3) = 0 \quad \Rightarrow \quad x = 2 \text{ or } x = 3$$

Check: $x = 2$ gives LHS $= 4$, RHS $= 4$ ✓

*Why the others are wrong:*

- **B** left it as $x(x-5)=-6$ and set $x=-6$, $x-5=-6$. The zero-product rule **only works when the product is zero**.
- **C** factorised $x^2+5x+6=0$ — never rearranged at all.
- **D** flipped the sign on just one factor.

> **Takeaway:** "Solve the quadratic" is two instructions: **(1)** get it into standard form, **(2)** factorise. Skipping step 1 causes more wrong answers here than anything else.

---

### Q14 — Intermediate — Answer: D

**Topic:** Rational equations and restrictions

**Step 1 — Note the restrictions.** Denominators cannot be zero, so $x \neq 2$ and $x \neq -1$.

**Step 2 — Cross-multiply.**

$$3(x+1)=1(x-2)$$

**Step 3 — Expand and solve.**

$$3x+3=x-2 \quad \Rightarrow \quad 2x=-5 \quad \Rightarrow \quad x=-\frac{5}{2}$$

$-\frac{5}{2}$ is not a restricted value, so it is valid. Check: LHS $=-\frac{2}{3}$, RHS $=-\frac{2}{3}$ ✓

*Why the others are wrong:*

- **A** flipped a sign: $3x-x=-2+3$ instead of $-2-3$.
- **C** cross-multiplied the wrong pairs: $3(x-2)=1(x+1)$.
- **B** confuses a **restriction** with **no solution**. $x \neq 2$ only rules out that one value. The equation still has an answer elsewhere.

> **Takeaway:** Restrictions just stop you dividing by zero. State them, solve as normal, and only reject your answer if it lands on a restricted value.

---

### Q15 — Proficient — Answer: C

**Topic:** Equations solvable by substitution

Since $x=(\sqrt{x})^2$, let $y=\sqrt{x}$. The equation becomes an ordinary quadratic:

$$y^2 - 4y - 5 = 0 \quad \Rightarrow \quad (y-5)(y+1) = 0 \quad \Rightarrow \quad y = 5 \text{ or } y = -1$$

**Now the step this question is really testing:** $y = \sqrt{x}$, and **a square root is never negative**. So reject $y = -1$.

That leaves $\sqrt{x} = 5$, so $x = 25$.

Check: $25 - 4\sqrt{25} - 5 = 25 - 20 - 5 = 0$ ✓

*Why the others are wrong:*

- **B** kept $y = -1$ and squared it to get $x = 1$. Test it: $1 - 4 - 5 = -8 \neq 0$. This is an **extraneous solution** — it fits the rearranged equation but not the original.
- **A** keeps only that invalid root.
- **D** gives $y$, not $x$.

> **Takeaway:** Any substitution involving a square root can produce fake answers. Always test each one in the **original** equation.

---

#### Q16 — Proficient — Answer: B

**Topic:** Simultaneous equations (linear and quadratic)

Both equations start "$y =$", so set the right-hand sides equal:

$$x + 1 = x^2 - 5$$

Rearrange to standard form and factorise:

$$x^2 - x - 6 = 0 \quad \Rightarrow \quad (x-3)(x+2) = 0 \quad \Rightarrow \quad x = 3 \text{ or } x = -2$$

**Do not stop here.** "Solve simultaneously" wants full coordinate pairs. Put each $x$ back into $y = x + 1$:

- $x = 3 \Rightarrow y = 4 \Rightarrow (3; 4)$
- $x = -2 \Rightarrow y = -1 \Rightarrow (-2; -1)$

*Why the others are wrong:*

- **D** made a sign slip forming the quadratic ($x^2 + x - 6 = 0$) and carried it through.
- **C** found one root and stopped. A quadratic usually has two.
- **A** has the right $x$-values but used $y = x - 1$ instead of $y = x + 1$.

> **Takeaway:** Three steps every time: **(1)** combine into one equation, **(2)** solve it (expect two answers), **(3)** substitute **every** answer back to get its partner.

---

## Section 1.3 — The Factor Theorem

#### Q17 — Basic — Answer: D

**Topic:** Applying the factor theorem

**Step 1 — Substitute $x = 1$, term by term.**

$$f(1) = 1 - 2 - 5 + 6 = 0$$

**Step 2 — Apply the factor theorem.** $f(1) = 0$, so $(x - 1)$ **is** a factor. No further checking needed.

*Why the others are wrong:*

- **C** slipped a sign (used $+2$ instead of $-2$ for the $x^2$ term) and got a non-zero value.
- **B** does the calculation right, then states the opposite conclusion.
- **A** doubts the theorem. "If and only if" means the test is conclusive on its own.

> **Takeaway:** With the factor theorem, **the calculation is the proof**. Show $f(a) = 0$ and you are done.

---

#### Q18 — Intermediate — Answer: B

**Topic:** Applying the factor theorem to $(x + 2)$

**Step 1 — Find the value to substitute.** $(x + 2) = 0$ gives $x = -2$, **not** $+2$. This is the most-tested detail in the topic.

**Step 2 — Substitute, keeping every sign.**

$$f(-2) = (-2)^3 - 7(-2) - 6 = -8 + 14 - 6 = 0$$

**Step 3 — Conclude.** $f(-2) = 0$, so $(x + 2)$ **is** a factor.

*Why the others are wrong:*

- **C** substituted $x = +2$: $8 - 14 - 6 = -12$. One sign error flips the whole conclusion.
- **A** used $x = -2$ but treated $-7(-2)$ as $-14$ instead of $+14$. Minus times minus is plus.
- **D** doubts the theorem. $f(-2) = 0$ **is** the proof — no long division needed.

> **Takeaway:** Write it down before substituting: "the bracket is $(x+2)$, so I use $x=-2$." Stating the value first kills the most common error in this topic.

---

### Q19 — Intermediate — Answer: A

**Topic:** Finding an unknown coefficient using the factor theorem

**Step 1 — Turn the factor into an equation.** $(x-2)$ is a factor, so $f(2)=0$.

**Step 2 — Substitute $x=2$, keeping $k$.**

$$f(2)=8+4k+4-24=4k-12$$

**Step 3 — Set it to zero and solve.**

$$4k-12=0 \quad\Rightarrow\quad 4k=12 \quad\Rightarrow\quad k=3$$

Check: $f(2)=8+12+4-24=0$ ✓

*Why the others are wrong:*

- **B** substituted $x=-2$ instead of $+2$, giving $4k-36=0$ and $k=9$.
- **D** wrote $4k=-12$ instead of $+12$. The number flips sign crossing the equals sign.
- **C** got $4k=12$ then forgot to divide by 4.

> **Takeaway:** Two separate steps: substitute to get an expression in $k$, **then** solve. Do not try to do both at once.

---

### Q20 — Intermediate — Answer: C

**Topic:** Finding all roots of a cubic

**Step 1 — Divide out the known factor.**

$$x^3-2x^2-5x+6=(x-1)(x^2-x-6)$$

Check by expanding — it takes seconds and catches division errors.

**Step 2 — Factorise the quadratic too.** Two numbers multiplying to $-6$, adding to $-1$: $-3$ and 2.

$$x^2-x-6=(x-3)(x+2)$$

**Step 3 — Read off all three roots.**

$$f(x)=(x-1)(x-3)(x+2)=0 \quad\Rightarrow\quad x=1,\ 3,\ -2$$

*Why the others are wrong:*

- **B** factorised the quadratic as $(x+3)(x-2)$, which expands to $x^2+x-6$ — wrong middle sign.
- **D** read $(x-1)$ as giving $x=-1$. It gives $x=+1$.
- **A** stopped after the given factor, missing that the quadratic factorises too.

> **Takeaway:** A cubic is a two-stage factorisation. Use the given factor to get a quadratic, then factorise that. You are only finished at three linear brackets.

---

### Q21 — Proficient — Answer: B

**Topic:** Simultaneous conditions on a polynomial

Two unknowns means you need **two equations** — and you have been given exactly two factors. Turn each into an equation.

**Equation (i) — from $(x-1)$:** $f(1)=0$.

$$1+a+b-6=0 \quad\Rightarrow\quad a+b=5$$

**Equation (ii) — from $(x+2)$:** $f(-2)=0$.

$$-8+4a-2b-6=0 \quad\Rightarrow\quad 2a-b=7$$

**Add (i) and (ii)** — the $+b$ and $-b$ cancel:

$$3a=12 \quad\Rightarrow\quad a=4, \quad \text{then} \quad b=1$$

Check: $f(1)=0$ ✓ and $f(-2)=0$ ✓

*Why the others are wrong:*

- **A** found 4 and 1 but swapped which is $a$ and which is $b$. Label your answers.
- **C** computed $b=4-5=-1$ instead of $5-4=1$.
- **D** says $b$ cannot be found. Two unknowns need two equations, and two factors give exactly that.

> **Takeaway:** Count your unknowns, then count your pieces of information. You need one equation per unknown, and a well-set question gives you exactly that many.

---

### Q22 — Proficient — Answer: D

**Topic:** Logical deduction from polynomial roots ("must be true")

**A "must be true" question.** Three options will look reasonable unless you separate what you *know* from what you are *assuming*.

By the factor theorem, the three given zeros mean $(x-2)$, $(x+1)$ and $(x-3)$ are all factors. Their product is already a cubic.

$f(x)$ is also a cubic, so there is no room for anything more. $f(x)$ can only differ from that product by a **constant multiplier** $k$ — which stretches the graph but does not move where it crosses the $x$-axis. And $k \neq 0$, or $f$ would not be a cubic at all.

$$f(x) = k(x-2)(x+1)(x-3)$$

*Why the others are wrong:*

- **A** would be right only if you knew the leading coefficient was 1. You were not told that — $f(x) = 2(x-2)(x+1)(x-3)$ fits every given condition too. "Must be true" has to hold for **every** polynomial fitting the description.
- **C** contradicts what you were given: three zeros means three real roots, not one.
- **B** breaks a basic rule: **a degree-$n$ polynomial has at most $n$ roots.** A cubic cannot have a fourth.

> **Takeaway:** For "must be true", try to build a counter-example against each option and discard any you can break. Keep two facts ready: a degree-$n$ polynomial has at most $n$ roots, and knowing all the roots pins the polynomial down only **up to a constant multiplier**.

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## Section 1.4 — Inequalities

### Q23 — Basic — Answer: A

**Topic:** Linear inequality (no sign flip needed)

Treat it like an equation: move the constant across, then divide.

$$3x < 12 \quad \Longrightarrow \quad x < 4$$

We only divided by a **positive** number, so the sign does not flip.

*Why the others are wrong:*

- **B** flipped the sign, but nothing negative was divided by. The rule was not triggered.
- **C** wrote $3x < 7 - 5$ instead of $7 + 5$. The $-5$ becomes $+5$ when it crosses over.
- **D** divided correctly, then flipped the sign of the answer for no reason.

> **Takeaway:** Finish every inequality by asking: *did I multiply or divide by a negative anywhere?* If no, the sign must look exactly as it did at the start.

---

### Q24 — Basic — Answer: C

**Topic:** Linear inequality (sign flip required)

**Step 1 — Isolate the $x$ term.**

$$-2x \geq 8$$

**Step 2 — Divide by $-2$.** That is **negative**, so the sign flips ($\geq$ becomes $\leq$):

$$x \leq -4$$

Check with $x = -10$: $-2(-10) + 3 = 23 \geq 11$ ✓ And with $x = 0$: $3 \geq 11$ is false, correctly excluded.

*Why the others are wrong:* between them they cover every way to slip here.

- **B** divided correctly but **forgot to flip** the sign.
- **D** flipped the sign but lost the negative in the division.
- **A** did both wrong at once.

> **Takeaway:** Make the flip and the division **two separate actions**. Write "*flip!*" the moment you decide to divide by a negative, then do the arithmetic underneath. Doing both at once is where the slips come from.

---

### Q25 — Intermediate — Answer: C

**Topic:** Quadratic inequality (strict, "less than")

**Step 1 — Factorise.**

$$(x-3)(x+2) < 0$$

**Step 2 — Find the critical values.** $x = 3$ and $x = -2$.

**Step 3 — Picture the parabola.** It opens **upward**, crossing at $-2$ and 3. An upward parabola is **below** the axis only **between** its roots. We want $< 0$, so take the middle:

$$-2 < x < 3$$

Test $x = 0$: $(0-3)(0+2) = -6 < 0$ ✓

*Why the others are wrong:*

- **B** is the outside region — that answers $x^2 - x - 6 > 0$, the opposite inequality.
- **A** misread the brackets: $(x-3)$ gives $x = +3$, not $-3$.
- **D** got one critical value right and one wrong.

> **Takeaway:** Sketch the parabola first. Opening up or down? Where does it cross? For an upward parabola, **below** the axis is between the roots; **above** is the two outer wings.

---

#### Q26 — Intermediate — Answer: A

**Topic:** Quadratic inequality (non-strict, "greater than or equal to")

**Step 1 — Factorise.** $(x-2)(x-3) \geq 0$, critical values $x = 2$ and $x = 3$.

**Step 2 — Picture the parabola.** Upward again, but now we want $y \geq 0$ — **on or above** the axis. For an upward parabola that is the two **outer** regions, and "or equal to" means the roots are included:

$$x \leq 2 \quad \text{or} \quad x \geq 3$$

Check $x = 2$: $(0)(-1) = 0 \geq 0$ ✓ so 2 does belong.

*Why the others are wrong:*

- **B** is the region between the roots — that answers $\leq 0$.
- **C** has the right regions but **strict** signs, wrongly excluding $x = 2$ and $x = 3$.
- **D** misread the critical values as $-2$ and $-3$.

> **Takeaway:** Ask two questions every time: **(1)** inside or outside the roots? **(2)** are the boundary points included? The second is decided purely by whether the original sign says "or equal to".

---

#### Q27 — Intermediate — Answer: D

**Topic:** Rational inequality

**What makes this different:** the expression is **undefined** where the denominator is zero. That value can never be in the solution, whatever the sign says.

**Step 1 — Find both critical values, and note the type of each.**

- Numerator zero: $x = 2$ — the expression **equals** zero here.

- Denominator zero: $x = -1$ — the expression is **undefined** here. Hard restriction: $x \neq -1$.

**Step 2 — Test one value in each region.**

| Region | Test | Value | Sign |
|---|---|---|---|
| $x < -1$ | $x = -2$ | $\frac{-4}{-1} = 4$ | positive ✓ |
| $-1 < x < 2$ | $x = 0$ | $\frac{-2}{1} = -2$ | negative |
| $x > 2$ | $x = 3$ | $\frac{1}{4}$ | positive ✓ |

We want positive, so:

$$x < -1 \quad \text{or} \quad x > 2$$

*Why the others are wrong:*

- **B** is the middle region — that answers $< 0$.
- **A** includes $x = 2$, but there the fraction is exactly 0, which fails a **strict** $> 0$.
- **C** includes $x = -1$, where the expression does not exist at all. That value can never be in any solution.

> **Takeaway:** Sort your critical values into two kinds first. Numerator zeros **may** be included depending on the sign. Denominator zeros are **always** excluded, no matter what.

---

#### Q28 — Proficient — Answer: B

**Topic:** Reasoning about inequalities with no solution ("always/never true")

**Do not factorise yet. First ask whether this can be true at all.**

For any real $x$, a square is never negative: $x^2 \geq 0$. So:

$$x^2 + 4 \ \geq \ 4$$

$x^2 + 4$ is always **at least 4**. It can never be less than 0, so **no real value of $x$ works**. The solution set is empty.

(Algebraically you would need $x^2 < -4$, and $\sqrt{-4}$ is not a real number — the fingerprint of "no real solutions".)

*Why the others are wrong:*

- **C** and **D** run the standard critical-value routine without checking it applies. They quietly work with $x^2 - 4$ instead of $x^2 + 4$, which never crosses the $x$-axis.
- **A** states the right fact ($x^2 \geq 0$) but draws the opposite conclusion from it.

> **Takeaway:** Before grinding through any inequality, spend three seconds asking: *could this be true for all, some, or no real numbers?* Know these on sight: $x^2 \geq 0$ always; $x^2 + \text{(positive)}$ is always positive; $\sqrt{x}$ is only real for $x \geq 0$.

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## Section 1.5 — Exponents & Surds

#### Q29 — Basic — Answer: A

**Topic:** Exponent laws (multiplying and dividing powers)

**The rules:** same base **multiplied** → add the exponents. Same base **divided** → subtract.

Everything is a power of $x$, so combine in one move:

$$\frac{x^5 \cdot x^3}{x^4} = x^{5+3-4} = x^4$$

*Why the others are wrong:*

- **B** ($x^{12}$) added all three ($5 + 3 + 4$), treating the fraction bar as another multiplication. It means **subtract**.
- **C** ($x^2$) subtracted $5 - 3$ as if the dot meant divide, then ignored the $x^4$ entirely.
- **D** ($x^{60}$) multiplied the exponents. That rule is only for a **power of a power**, like $(x^5)^3$.

> **Takeaway:** Look at the symbol before you touch the exponents. $\times$ → add. $\div$ or a fraction bar → subtract. Brackets (power of a power) → multiply.

---

#### Q30 — Basic — Answer: C

**Topic:** Negative exponents

**The rule:** a negative exponent does not make a value negative. It means **flip** — take the reciprocal. So $a^{-n} = \frac{1}{a^n}$, and $\frac{1}{a^{-n}} = a^n$.

$$\frac{1}{x^{-3}} = x^3$$

In words: "1 over (1 over $x^3$)" is just $x^3$. Dividing by a fraction flips it.

*Why the others are wrong:*

- **A** copied the exponent across without flipping. The instruction says "positive exponent" — if yours is still negative, you have not finished.
- **B** thinks a negative exponent gives a negative value. It does not. $x^{-3}$ and $x^3$ always have the **same** sign.
- **D** flipped twice, cancelling the first flip.

> **Takeaway:** A negative exponent means *reciprocate*, nothing more. It says nothing about the sign of the answer. Rewrite $a^{-n}$ as $\frac{1}{a^n}$ straight away, before anything else.

---

#### Q31 — Basic — Answer: B

**Topic:** Fractional (rational) exponents

**The rule:** in $a^{m/n}$, the **denominator is the root** and the **numerator is the power**.

So for $8^{2/3}$: cube root first (denominator 3), then square (numerator 2).

$$8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$$

Root first keeps the numbers small — $\sqrt[3]{8} = 2$ is instant, while $8^2 = 64$ then $\sqrt[3]{64}$ is harder to spot.

*Why the others are wrong:*

- **C** (64) only squared the base and never took the cube root.
- **A** ($\frac{16}{3}$) **multiplied** $8 \times \frac{2}{3}$ instead of raising 8 to that power.
- **D** (8) took a **square** root instead of a cube root: $(\sqrt{8})^2 = 8$.

> **Takeaway:** "Denominator = root, numerator = power." Given the choice, always do the **root first**.

---

#### Q32 — Basic — Answer: D

**Topic:** Simplifying and combining surds

**The rule:** you can only add surds once they have the **same number under the root** — just like fractions need a common denominator. So simplify each one first by pulling out perfect-square factors.

$$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3} \qquad \sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$$

Now they are like terms, so combine them the way you would $2x + 3x$:

$$2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$$

*Why the others are wrong:*

- **B** ($\sqrt{39}$) assumes $\sqrt{a} + \sqrt{b} = \sqrt{a+b}$ and adds $12 + 27$. **Never true.** Test it: $\sqrt{4} + \sqrt{9} = 5$, but $\sqrt{13} \approx 3.6$.
- **C** ($6\sqrt{3}$) reached $2\sqrt{3} + 3\sqrt{3}$ then **multiplied** the coefficients instead of adding. Same slip as writing $2x + 3x = 6x$.
- **A** calls it unsimplifiable because 12 and 27 look unrelated. Both hide a factor of $\sqrt{3}$.

> **Takeaway:** Surds combine like algebraic terms, but only after each is simplified. Always ask: is there a perfect square (4, 9, 16, 25, ...) hiding inside? And remember: $\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$, ever.

---

### Q33 — Intermediate — Answer: B

**Topic:** Rationalising a binomial denominator

**The rule:** multiply top and bottom by the **conjugate** — the same expression with the middle sign flipped, $\sqrt{5} + 1$. This works because $(a-b)(a+b) = a^2 - b^2$ kills the surd.

$$\frac{2}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{2(\sqrt{5}+1)}{5-1} = \frac{2(\sqrt{5}+1)}{4}$$

Top and bottom now share a factor of 2. Cancel it:

$$= \frac{\sqrt{5}+1}{2}$$

*Why the others are wrong:*

- **A** is the correct value one step early — the 2 and the 4 still cancel. "Rationalise" always means *fully*.
- **C** added the squares instead of subtracting: $5 + 1 = 6$. Difference of squares is always a **subtraction**.
- **D** multiplied only the **denominator** by the conjugate. You may only multiply a fraction by $\frac{\text{something}}{\text{same something}} = 1$ — doing the bottom alone changes its value.

> **Takeaway:** Three checks: **(1)** conjugate = same terms, flipped middle sign; **(2)** multiply **both** top and bottom; **(3)** cancel any common factor left over.

---

### Q34 — Intermediate — Answer: A

**Topic:** Exponents with multiple bases ("same base" insight)

**The insight:** 6 is not a new base — it is $2 \times 3$. Rewrite it and the expression splits into two same-base divisions, and the $x$'s cancel.

$$6^{x+1} = (2 \cdot 3)^{x+1} = 2^{x+1} \cdot 3^{x+1}$$

Now group each base with its partner:

$$\frac{2^{x+1} \cdot 3^{x+1}}{2^x \cdot 3^{x-1}} = \frac{2^{x+1}}{2^x} \times \frac{3^{x+1}}{3^{x-1}} = 2^1 \times 3^2 = 18$$

*Why the others are wrong:*

- **B** (12) paired each exponent-difference with the **wrong base** — the 2 with base 2 and the 1 with base 3.
- **C** (9) cancelled the 6 against the 2 as if they were plain numbers. Powers with different bases cannot be combined that way.
- **D** (36) used the same exponent-difference for both bases. Each base has a different exponent below it, so each gives a different result.

> **Takeaway:** When bases are multiples of each other ($6 = 2 \times 3$, $12 = 2^2 \times 3$), break every base into its prime powers first. The expression usually collapses to a plain number and the variable cancels.

---

### Q35 — Intermediate — Answer: C

**Topic:** Equations involving surds (and checking for extraneous roots)

**The golden rule of surd equations:** squaring can create answers that fit the squared equation but not the original, because squaring hides sign information (both 2 and $-2$ square to 4). So every candidate **must** be checked in the original.

**Step 1 — Square both sides.**

$$x + 3 = x^2 - 6x + 9$$

**Step 2 — Rearrange and factorise.**

$$x^2 - 7x + 6 = (x-1)(x-6) = 0 \quad \Rightarrow \quad x = 1 \text{ or } 6$$

**Step 3 — Check both in the original.**

- $x = 1$: LHS $= \sqrt{4} = 2$, RHS $= -2$. $2 \neq -2$, so **reject**. This is an *extraneous root*.
- $x = 6$: LHS $= \sqrt{9} = 3$, RHS $= 3$ ✓ **accept**.

So $x = 6$ only.

*Why the others are wrong:*

- **B** solved correctly but **skipped the check** — the most-tested surd mistake there is.
- **A** checked but rejected the wrong one.
- **D** expanded $(x-3)^2$ as $x^2 - 9$ instead of $x^2 - 6x + 9$. Keep $(a-b)^2$ and $(a-b)(a+b)$ in separate drawers.

> **Takeaway:** Squaring is a "maybe" step, not a reversible one. Every answer it produces is only a **candidate** until you have watched it satisfy the original equation.

---

### Q36 — Proficient — Answer: D

**Topic:** Algebraic insight with exponents (factor before you divide)

You cannot cancel across a **subtraction**. But both terms in the numerator share a common factor — this is a factorisation question in an exponent costume.

**Step 1 — Factor out the smaller power, $2^{n+1}$.**

$$2^{n+2} - 2^{n+1} = 2^{n+1}(2-1) = 2^{n+1}$$

**Step 2 — Divide by $2^n$.**

$$\frac{2^{n+1}}{2^n} = 2^1 = 2$$

The whole thing collapses to the constant **2** — the same "looks like it depends on $n$ but does not" pattern as Q34.

*Why the others are wrong:*

- **B** (1) subtracted the leading digits $(2-1)$ as if these were like terms. $2^{n+2}$ and $2^{n+1}$ are different powers.
- **C** $(2^{n+1})$ factorised correctly then stopped, forgetting the denominator.
- **A** $(\frac{1}{2})$ did the final subtraction backwards: $2^{n-(n+1)} = 2^{-1}$. It is always *(top) – (bottom).*

> **Takeaway:** Powers of the same base joined by $+$ or $-$? **Factor out the smaller power.** The multiply/divide exponent laws only apply when the powers are joined by $\times$ or $\div$.

---

### Q37 — Proficient — Answer: D

**Topic:** Denesting nested surds (insight / structure recognition)

A nested surd like this can sometimes be rewritten as $\sqrt{a}+\sqrt{b}$. Ask: what would $\sqrt{a}+\sqrt{b}$ have to be so that squaring it gives $7+4\sqrt{3}$?

**Step 1 — Square the target form.**

$$\left(\sqrt{a}+\sqrt{b}\right)^2 = a+b+2\sqrt{ab}$$

Match the plain parts and the surd parts **separately**:

$$a+b=7 \qquad 2\sqrt{ab}=4\sqrt{3}$$

**Step 2 — Solve for $ab$.** Divide by 2: $\sqrt{ab}=2\sqrt{3}$. Square **the whole** right side:

$$ab=(2\sqrt{3})^2=4\times 3=12$$

**Step 3 — Two numbers with sum 7 and product 12.** That is the trinomial pattern again:

$$t^2-7t+12=(t-3)(t-4) \Rightarrow a,b=3,4$$

$$\sqrt{7+4\sqrt{3}}=\sqrt{4}+\sqrt{3}=2+\sqrt{3}$$

Verify by squaring back: $(2+\sqrt{3})^2=4+4\sqrt{3}+3$ ✓

*Why the others are wrong:*

- **B** forgot to square the coefficient: $ab=2\times 3=6$ instead of $4\times 3=12$.
- **C** found $a=4, b=3$ then wrote $a+\sqrt{b}$ instead of $\sqrt{a}+\sqrt{b}$. Those numbers go **under** the roots.
- **A** says it cannot be done. True for most nested surds, but not this one — the signal is that $7^2-4^2(3)=1$ is a perfect square.

> **Takeaway:** $\sqrt{p+q\sqrt{r}}$ denests when you can find two numbers with sum $p$ and product $\frac{q^2r}{4}$. Same "sum and product" search as factorising a trinomial.

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## Section 1.6 — Logarithms

### Q38 — Basic — Answer: C

**Topic:** Evaluating logarithms (logs as exponent questions)

**Read it as the question it is asking:** $\log_2 32$ means *"what power of 2 gives 32?"*

$$2^1 = 2, \quad 2^2 = 4, \quad 2^3 = 8, \quad 2^4 = 16, \quad 2^5 = 32$$

So $\log_2 32 = 5$.

(Or: how many times can you halve 32 to reach 1? $32 \to 16 \to 8 \to 4 \to 2 \to 1$ — count the **arrows**, not the numbers. Five.)

*Why the others are wrong:*

- **A** (4) stopped the halving chain at 2 instead of going all the way to 1.
- **B** (16) just divided $32 \div 2$. A log asks for an **exponent**, it is not an operation on the two numbers you see.
- **D** (6) counted the starting number as a step. Count the *moves* between numbers, not the numbers themselves.

> **Takeaway:** Translate $\log_a N$ into plain English — *"a to what power gives $N$?"* Then either climb the powers of $a$, or divide $N$ by $a$ until you reach 1. Either way, **count the moves, not the numbers**.

---

### Q39 — Basic — Answer: A

**Topic:** Evaluating logarithms of fractions (negative results)

**Translate first:** what power of 5 gives $\frac{1}{25}$?

A reciprocal means a **negative** exponent: $\frac{1}{25} = \frac{1}{5^2} = 5^{-2}$. So:

$$\log_5 \frac{1}{25} = -2$$

Worth remembering: if the number inside a log is a fraction below 1, the answer is **negative**.

*Why the others are wrong:*

- **B** (2) answered $\log_5 25$ and ignored the fraction. A positive power of 5 can never give a number below 1.
- **D** ($\frac{1}{2}$) assumes a reciprocal inside gives a reciprocal answer. No such rule exists.
- **C** ($-\frac{1}{2}$) is that same error with the correct minus sign added — half right, still wrong.

> **Takeaway:** Log of a fraction below 1 is always negative. The link between the number and its log is "base and exponent", never "reciprocal and reciprocal".

---

#### Q40 — Intermediate — Answer: B

**Topic:** Applying the quotient law

Two logs, **same base**, joined by subtraction — that is the **quotient law**:

$$\log_2 40 - \log_2 5 = \log_2 \left(\frac{40}{5}\right) = \log_2 8$$

Now evaluate it: what power of 2 gives 8? $2^3 = 8$, so the answer is **3**.

*Why the others are wrong:*

- **D** ($\log_2 35$) subtracted the numbers inside $(40 - 5)$. There is **no** law for adding or subtracting inside a log.
- **C** (8) merged correctly to $\log_2 8$ then stopped, giving the number inside instead of the log's value.
- **A** ($\log_2 200$) merged with the wrong operation. Subtraction of logs means **division** inside; addition means multiplication.

> **Takeaway:** Same base joined by $+$ or $-$? Merge them — **plus makes a product, minus makes a quotient** — then always evaluate the single log you are left with.

---

#### Q41 — Intermediate — Answer: C

**Topic:** Solving logarithmic equations

**Step 1 — Convert to exponential form.** $\log_a(N) = k$ means $a^k = N$.

$$\log_2(x+3) = 4 \quad \Longleftrightarrow \quad x + 3 = 2^4 = 16$$

**Step 2 — Isolate $x$.**

$$x = 16 - 3 = 13$$

Check: $\log_2 16 = 4$ ✓

*Why the others are wrong:*

- **B** (5) wrote $x + 3 = 2 \times 4$. The base is **raised to** the power, not multiplied by it.
- **A** (1) deleted the log entirely and solved $x + 3 = 4$. The base is not decoration.
- **D** (19) reached $x + 3 = 16$ then wrote $x = 16 + 3$. The 3 flips sign when it crosses over.

> **Takeaway:** Every log equation is two steps: **(1)** rewrite $\log_a(\ldots) = k$ as $(\ldots) = a^k$; **(2)** solve it with ordinary algebra. The log notation only matters for step 1.

---

### Q42 — Intermediate — Answer: B

**Topic:** Logarithms with related bases (the "same base" insight again)

**4 and 8 are both powers of 2** ($4 = 2^2$, $8 = 2^3$) — the same common-base trick as Q34, now inside a log.

Let $\log_4 8 = y$ and convert to exponential form:

$$4^y = 8 \quad \Rightarrow \quad 2^{2y} = 2^3$$

Same base, so the exponents match:

$$2y = 3 \quad \Rightarrow \quad y = \frac{3}{2}$$

Sanity check: $4^1 = 4 < 8 < 16 = 4^2$, so the answer must sit between 1 and 2 ✓

*Why the others are wrong:*

- **A** (2) guessed from "4 goes into 8 twice" — that is multiplying the base, not raising it to a power.
- **C** ($\frac{2}{3}$) set up $2y = 3$ correctly then flipped the fraction.
- **D** assumes no nice answer exists because 8 is not a whole-number power of 4. Rewriting both as powers of 2 reveals one.

> **Takeaway:** Odd-looking bases are usually powers of the same small prime. Before giving up on a log, ask: can I write both numbers as powers of 2 (or 3, or 5)?

---

### Q43 — Proficient — Answer: D

**Topic:** Expressing logarithms in terms of given values (log-law fluency)

**Break 12 into the pieces you were given.** $12 = 2^2 \times 3$. Then apply the product law, then the power law:

$$\log_a 12 = \log_a(2^2 \times 3) = \log_a(2^2) + \log_a 3 = 2\log_a 2 + \log_a 3$$

Substitute $p$ and $q$:

$$= 2p + q$$

*Why the others are wrong:*

- **B** ($p^2 + q$) squared the **log value**. The power law brings the exponent out as a **coefficient**, giving $2p$, not $p^2$.
- **A** ($2pq$) multiplied the logs together. A product inside a log becomes a **sum** outside — logs are never multiplied.
- **C** ($4p + q$) used $2^2 = 4$ as the coefficient. It is the **exponent** (2) that comes out front, not the value of the power.

> **Takeaway:** Say each law aloud as you use it: *a product inside becomes a sum outside; a power inside becomes a coefficient outside.*

---

#### Q44 — Proficient — Answer: A

**Topic:** Logical reasoning about logarithm laws ("always true")

**The fastest way to test an "always true" claim: pick easy numbers and check both sides.** One counterexample kills a statement forever.

Test each with $x = 10$, $y = 10$ (since $\log 10 = 1$ exactly):

- **A:** LHS $= \log 100 = 2$. RHS $= 1 + 1 = 2$ ✓ **survives** — this is the genuine product law.
- **B:** LHS $= (1)(1) = 1$. RHS $= \log 20 \approx 1.30$ ✗
- **C:** LHS $= \log(0)$, which is **undefined** ✗
- **D:** LHS $= \log 20 \approx 1.30$. RHS $= 2$ ✗

Only **A** survives.

*Why the others are wrong:*

- **C** and **D** are the fake laws: they assume $+$ or $-$ can pass through a log. Logs only turn **multiplication** into addition, never the other way.
- **B** multiplies two logs together. No log law does that.

> **Takeaway:** For "always true" questions, do not try to prove it under time pressure — **test it with easy numbers** ($\log 10 = 1$, $\log 100 = 2$). One counterexample is enough to eliminate an option.

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## Section 1.7 — Sequences & Series

#### Q45 — Basic — Answer: D

**Topic:** $n$th term of an arithmetic sequence

**Step 1 — Identify $a$ and $d$.** First term $a = 5$; each term is 3 more, so $d = 3$.

**Step 2 — Use $T_n = a + (n-1)d$.** Read it as a sentence: *start at a, then take $(n-1)$ steps of size d.* There is one fewer step than there are terms, because the first term needs no steps.

$$T_{10} = 5 + (9)(3) = 32$$

*Why the others are wrong:*

- **B** (35) used $a + nd$ — ten steps instead of nine. The most common AP slip.
- **C** (30) computed $10 \times 3$ and forgot the first term entirely.
- **A** (27) found the amount added but stopped, never adding $a$ back on.

> **Takeaway:** Two checkpoints: is the multiplier $(n-1)$ and not $n$? And did you add $a$ back at the end?

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#### Q46 — Basic — Answer: A

**Topic:** $n$th term of a geometric sequence

**Step 1 — Identify $a$ and $r$.** First term $a = 3$; each term doubles, so $r = 2$.

**Step 2 — Use $T_n = a \cdot r^{n-1}$** — the GP twin of the AP formula, still counting $(n-1)$ steps.

$$T_6 = 3 \times 2^5 = 96$$

Or walk it: $3 \to 6 \to 12 \to 24 \to 48 \to 96$ — five multiplications ✓

*Why the others are wrong:*

- **B** (192) used $r^n$ instead of $r^{n-1}$ — one power too many. The same off-by-one as the AP.
- **C** (48) used $r^{n-2}$ — one power too few.
- **D** (36) computed $a \times r \times n$, treating a GP as if it grew by adding. A GP **multiplies** repeatedly, which is what the exponent captures.

> **Takeaway:** Ask first: does this sequence grow by **adding** the same amount, or **multiplying** by the same amount? Getting that wrong means using an entirely wrong kind of formula.

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### Q47 — Basic — Answer: C

**Topic:** Identifying the common ratio of a geometric sequence

**Always the same method:** divide a term by the one **directly before it**, $r = \frac{T_{n+1}}{T_n}$.

$$r = \frac{27}{81} = \frac{1}{3}$$

Check a second pair: $\frac{9}{27} = \frac{1}{3}$ ✓ and $\frac{3}{9} = \frac{1}{3}$ ✓

It makes sense that $r < 1$ — the sequence is shrinking.

*Why the others are wrong:*

- **B** (3) divided the wrong way round, $\frac{81}{27}$. Always **later ÷ earlier**, or the ratio comes out upside down.
- **A** ($-54$) computed $27 - 81$, a common **difference**. The word "ratio" tells you to divide.
- **D** ($\frac{1}{27}$) compared non-neighbouring terms, $\frac{3}{81}$. That gives $r^3$, not $r$.

> **Takeaway:** Divide a term by its immediate predecessor, then confirm the same value on a second pair. "Common" means it must be constant.

---

### Q48 — Intermediate — Answer: B

**Topic:** Sum of an arithmetic series

$a = 4$, $d = 3$, $n = 20$. Into $S_n = \frac{n}{2}[2a + (n-1)d]$:

$$S_{20} = \frac{20}{2}\left[8 + 57\right] = 10(65) = 650$$

*Why the others are wrong:*

- **D** (1300) is exactly **double** — the $\frac{n}{2}$ was dropped. A wrong answer that is a clean multiple of the right one usually means a lost factor.
- **C** (680) found the last term as $4 + 20(3) = 64$ (the off-by-one from Q45), then used $\frac{20}{2}(4 + 64)$. One early slip poisons everything after it.
- **A** (610) used a single $a$ instead of $2a$ in the bracket.

> **Takeaway:** Know **why** each piece is there. $S_n = \frac{n}{2}[2a + (n-1)d]$ is just "(number of terms) × (average of first and last term)". The $2a$ is what builds "first + last" before you know the last term — understanding that makes it hard to drop.

---

### Q49 — Intermediate — Answer: A

**Topic:** Sum of a finite geometric series

Confirm it is geometric: $a = 2$ and each term is 3 times the last, so $r = 3$. Into $S_n = \frac{a(r^n - 1)}{r - 1}$:

$$S_5 = \frac{2(243 - 1)}{2} = 242$$

*Why the others are wrong:*

- **B** (243) dropped the "$-1$" in the numerator. It misses by exactly one, so glancing at the *size* of your answer will never catch it — you must check the working.
- **C** (14) computed $r^n$ as $3 \times 5 = 15$ instead of $3^5$. Same multiply-versus-power confusion as Q46.
- **D** (50) treated it as arithmetic with $d = 4$, built $2, 6, 10, 14, 18$ and added. Checking a second ratio kills that story: $\frac{6}{2} = 3$ but $\frac{10}{6} \neq 3$.

> **Takeaway:** Three checks on every GP sum: confirm the ratio on **two** pairs; compute $r^n$ as a power, never $r \times n$; and check the working, not just whether the answer "looks about right".

---

### Q50 — Intermediate — Answer: D

**Topic:** Sigma (summation) notation

**Sigma is an instruction, not a mystery symbol:** substitute $k = 1, 2, 3, 4, 5$ into $2k + 1$ and add the results.

$$3 + 5 + 7 + 9 + 11 = 35$$

*Why the others are wrong:*

- **A** (30) summed only the $2k$ part and dropped the "$+1$" from every term.
- **C** (31) added the "$+1$" once at the end: $2(15) + 1$. It belongs to **each** term, so it must be added five times. Split properly: $2\sum k + \sum 1 = 30 + 5 = 35$.
- **B** (24) used only four terms, subtracting the limits $(5 - 1)$. **Both** limits are included: the count is $(\text{top} - \text{bottom}) + 1 = 5$.

> **Takeaway:** Write the terms out — do not do it in your head, that is where errors hide. Apply every constant **per term**, and count terms with $(\text{top} - \text{bottom}) + 1$.

---

### Q51 — Intermediate — Answer: B

**Topic:** Solving for $n$ in a series sum (forming and solving a quadratic)

$a = 3$, $d = 4$. Set the sum equal to 210:

$$S_n = \frac{n}{2}\left[6 + 4(n-1)\right] = n(2n+1) = 2n^2 + n$$
$$2n^2 + n - 210 = 0 \quad \Rightarrow \quad (2n+21)(n-10) = 0$$

So $n = 10$ or $n = -10.5$.

**The step that decides the question:** $n$ is a *number of terms*, so it must be a positive whole number. A sequence cannot have $-10.5$ terms. Reject it.

$$n = 10$$

Check: $S_{10} = 5(42) = 210$ ✓

*Why the others are wrong:*

- **C** did the algebra perfectly and reported **both** roots, never asking whether each could be a real number of terms.
- **A** (11) factorised with a sign slip, got 10.5, then **rounded**. A decimal answer for "number of terms" means go back and find the lost sign, not round.
- **D** dropped the minus on $c$ in the discriminant: $-4(2)(210)$ instead of $-4(2)(-210)$, turning $+1681$ into $-1679$ and making real answers seem to vanish.

> **Takeaway:** When a variable must be a positive whole number — terms, people, years — solving is only half the job. Then ask of each answer: *could this actually happen?* Discarding the impossible root is often the entire point.

---

### Q52 — Proficient — Answer: C

**Topic:** Sum to infinity of a geometric series

**The idea:** infinitely many terms can still add to a *finite* total, as long as the terms shrink fast enough. Here $a = 8$ and $r = \frac{1}{2}$. Since $|r| < 1$, the series converges:

$$S_\infty = \frac{a}{1-r} = \frac{8}{\frac{1}{2}} = 16$$

Watch the running total: $8, 12, 14, 15, 15.5, 15.75, \ldots$ — climbing by ever smaller amounts, closing in on 16 without passing it.

*Why the others are wrong:*

- **B** assumes infinitely many terms must total infinity. False when $|r| < 1$. "Goes on forever" describes the **number of terms**, not the size of the total.
- **D** (4) computed $a(1-r)$ instead of $\frac{a}{1-r}$ — the formula assembled upside down.
- **A** ($\frac{1}{16}$) had the right pieces but inverted the fraction: $\frac{1-r}{a}$ instead of $\frac{a}{1-r}$.

> **Takeaway:** Check $|r| < 1$ **before** using the formula — otherwise no finite sum exists. And remember: "infinitely many things" and "an infinite total" are different statements.

---

### Q53 — Proficient — Answer: C

**Topic:** Forming and solving simultaneous equations from sequence terms (insight)

Two facts about the sequence are exactly enough to find both unknowns. Translate each into an equation using $T_n = a + (n-1)d$.

**Step 1 — Write both equations.**

$$T_3 = 11: \quad a + 2d = 11 \qquad T_7 = 23: \quad a + 6d = 23$$

**Step 2 — Subtract to eliminate $a$.**

$$4d = 12 \quad \Rightarrow \quad d = 3$$

**Step 3 — Substitute back.**

$$a + 6 = 11 \quad \Rightarrow \quad a = 5$$

Check: $T_3 = 11$ ✓ and $T_7 = 23$ ✓

*Why the others are wrong:*

- **B** treated the given 11 as the **first** term. "The 3rd term is 11" is a statement about $T_3$; it does not hand you $a$.
- **A** counted the terms **between** $T_3$ and $T_7$ (three) instead of the steps (four: $T_3 \to T_4 \to T_5 \to T_6 \to T_7$), giving $3d = 12$.
- **D** found $d = 3$ correctly then wrote $a = 11 + 6$ instead of $11 - 6$.

> **Takeaway:** "Find $a$ and $d$ from two facts" is a simultaneous-equations problem in disguise. Translate each fact with $T_n = a + (n-1)d$, subtract to eliminate one unknown, solve. The sequence wording changes nothing about the algebra.

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## Section 1.8 — Financial Mathematics

#### Q54 — Basic — Answer: B

**Topic:** Setting up a simple interest expression

**"Simple interest" means the linear formula** $A = P(1 + in)$. Here $P = 4\,000$, $i = 0.06$, $n = 5$:

$$A = 4\,000\big(1 + 0.06 \times 5\big)$$

The same fixed interest, $4\,000 \times 0.06 = 240$, is added every year. That is what "simple" growth means.

*Why the others are wrong:*

- **A** uses the **compound** formula on a simple-interest scenario. The word "simple" tells you growth is additive, not exponential.
- **C** gives only the **interest earned**, not the **value of the investment**. The question asks for the total, principal included.
- **D** applies one year's growth factor then multiplies by 5. That is neither model — simple interest adds a fixed amount; compound multiplies repeatedly.

> **Takeaway:** First decision in any money question: is the growth **linear** (simple, straight-line) or **multiplicative** (compound, reducing balance)? Those words are in the question to tell you. Get the word right and the rest is substitution.

---

### Q55 — Basic — Answer: D

**Topic:** Setting up a compound interest expression (the GP connection)

**"Compounded annually" means exponential growth.** Each year the *whole current balance* grows by 9%, so it becomes 109% of itself — multiply by the **growth factor** 1.09. Four years means four multiplications:

$$A = 8\,000(1.09)^4$$

This is just a GP with $r = 1.09$.

*Why the others are wrong:*

- **B** uses the **simple** formula on a compound scenario — the mirror of Q54's trap.
- **A** uses the bare rate 0.09 as the base instead of 1.09. Multiplying by 0.09 repeatedly would shrink the money to almost nothing. The factor must mean "all you had, **plus** 9%" = 1.09.
- **C** multiplied by 4 instead of raising to the power 4. Compound growth multiplies four times in a row; it does not add up four single years.

> **Takeaway:** Compound interest **is** a GP. Build the factor as $1 + i$ for growth (or $1 - i$ for shrinking) — never use the bare rate on its own.

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### Q56 — Intermediate — Answer: A

**Topic:** Setting up a reducing-balance depreciation expression (the mirror of Q55)

**"Reducing balance" is the depreciation twin of "compounded annually."** The van loses 15% of its **current** value each year, so it keeps 85% — a factor of $1 - 0.15 = 0.85$, applied three times:

$$A = 240\,000(1 - 0.15)^3 = 240\,000(0.85)^3$$

Same shape as Q55, with the sign flipped because the value is shrinking.

*Why the others are wrong:*

- **B** uses the **straight-line** formula $P(1 - in)$, which takes a fixed slice of the *original* value. "Reducing balance" takes a slice of the *current* value.
- **C** uses the bare rate: $(0.15)^3$ would mean keeping only 15% of the value each year — a far more severe collapse than 15% depreciation.
- **D** uses a **growth** factor on a shrinking quantity. "Depreciates" means the value falls, so any expression bigger than R240 000 must be wrong.

> **Takeaway:** Start from 100% of the current value, then **add** the rate if it grows or **subtract** it if it shrinks. "Compounded/appreciates" → $+$. "Depreciates/reducing balance/decays" → $-$. Then sanity-check the direction.

---

### Q57 — Intermediate — Answer: D

**Topic:** Working backwards to find the interest rate

Set up the equation and undo it one layer at a time.

$$10\,000(1+i)^2 = 12\,100 \quad \Rightarrow \quad (1+i)^2 = 1.21$$

Take the square root. $1.21 = \left(\frac{11}{10}\right)^2$, so:

$$1+i = 1.1 \quad \Rightarrow \quad i = 0.1 = 10\%$$

Check: $10\,000(1.1)^2 = 12\,100$ ✓

*Why the others are wrong:*

- **B** ($21\%$) is the **total** growth over both years, reported as if it were annual. Compounding means 10% a year gives 21% over two years, not 20% — the second year grows an already-grown balance.
- **C** ($10.5\%$) took that 21% and halved it, treating the growth as additive instead of multiplicative.
- **A** ($11\%$) found the growth **factor** $1.1$ and read it off as the rate. The factor is $1+i$, so you must still subtract 1.

> **Takeaway:** Two things to hold onto: total growth is **not** (annual rate) × (years) for compound interest; and the growth **factor** $(1+i)$ and the **rate** $(i)$ are one subtraction apart.

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### Q58 — Intermediate — Answer: C

**Topic:** Rearranging the compound interest formula to find the principal

Start from $A = P(1+i)^n$ with $A = 16\,000$ and $n = 4$:

$$16\,000 = P(1+i)^4$$

Divide both sides by $(1+i)^4$ to isolate $P$:

$$P = \frac{16\,000}{(1+i)^4}$$

**Sense-check:** $P$ is the *starting* amount, so it must be **smaller** than the R16 000 it grew into. Dividing shrinks it — correct.

*Why the others are wrong:*

- **A** rearranged the **simple** interest formula. The question says compounded annually.
- **B multiplies** instead of dividing, making $P$ bigger than $A$. Impossible — $P$ grew *into* $A$.
- **D** subtracts a small flat amount, throwing away the exponential structure entirely.

> **Takeaway:** Undo the operations in reverse order, then sense-check the size: should this quantity be bigger or smaller than the others in the story?

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### Q59 — Proficient — Answer: A

**Topic:** Reasoning about depreciation and inflation together (insight)

It *sounds* like losing $r\%$ and gaining $r\%$ should cancel. They do not. Divide the two expressions out and look at what you get.

**Step 1 — Form the ratio.**

$$\frac{V(1-r)^n}{V(1+r)^n} = \left(\frac{1-r}{1+r}\right)^n$$

The $V$'s cancel, and a quotient of like powers becomes one power of the quotient.

**Step 2 — Look at the base.** Since $0 < r < 1$, we have $1 - r < 1 + r$, so the base $\frac{1-r}{1+r}$ is **less than 1**. A number below 1 raised to higher powers gets **smaller**. So the ratio shrinks every year.

*Why the others are wrong:*

- **B** claims they cancel to 1. Test $r = 0.5$: the ratio is $\frac{0.5}{1.5} = \frac{1}{3}$, nowhere near 1.
- **C multiplied** the two factors (giving the difference of squares $1 - r^2$). A ratio comes from **dividing**.
- **D** has the right shape but upside down — and its claim that the ratio *grows* would mean an old machine gains value relative to a new one. That contradicts the story.

> **Takeaway:** Never guess whether two exponential effects cancel — divide them and look at the single base you get. Base below 1 → shrinks toward zero. Base above 1 → grows without bound.

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## Section 1.9 — Algebraic Insight, Vieta's Formulas & Manipulation Tricks

### Q60 — Intermediate — Answer: B

**Topic:** Vieta's formulas — sum of roots from coefficients

**Vieta's sum formula:** for $ax^2 + bx + c = 0,\ \alpha + \beta = -\frac{b}{a}$.

Read off $a = 2$, $b = -7$, and substitute — keeping the minus sign that is already in the formula:

$$\alpha + \beta = -\frac{(-7)}{2} = \frac{7}{2}$$

Check without finding the roots separately: $2x^2 - 7x + 3 = (2x - 1)(x - 3)$, so the roots are $\frac{1}{2}$ and $3$, summing to $\frac{7}{2}$ ✓ — but Vieta got there in one line.

*Why the others are wrong:*

- **D** ($-\frac{7}{2}$) dropped the minus already built into the formula, computing $\frac{b}{a}$. With $b$ also negative, the two minuses make a **positive**.
- **C** ($\frac{3}{2}$) used the **other** Vieta formula, $\frac{c}{a}$ — the product instead of the sum.
- **A** (7) forgot to divide by $a$. That slip is invisible when $a = 1$, which is why it pays to practise on equations where $a \neq 1$.

> **Takeaway:** Learn the pair together: **sum** $= -\frac{b}{a}$, **product** $= \frac{c}{a}$ — and *both* divide by $a$. Saying that sentence as you use them catches all three slips above.

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### Q61 — Intermediate — Answer: B

**Topic:** Building new expressions from a given identity (squaring trick)

You are given something in $x$ and $\frac{1}{x}$, and asked for $x^2$ and $\frac{1}{x^2}$. **Squaring** is what turns one into the other.

**Step 1 — Square both sides, expanding fully.**

$$\left(x - \frac{1}{x}\right)^2 = x^2 - 2\left(x \cdot \frac{1}{x}\right) + \frac{1}{x^2} = x^2 - 2 + \frac{1}{x^2}$$

The cross term is beautifully simple: $x \cdot \frac{1}{x} = 1$. That is exactly why squaring works here.

**Step 2 — Substitute and solve.**

$$x^2 - 2 + \frac{1}{x^2} = 9 \quad \Rightarrow \quad x^2 + \frac{1}{x^2} = 11$$

*Why the others are wrong:*

- **A** (9) dropped the cross term. $(a-b)^2$ is never $a^2+b^2$ — it is $a^2-2ab+b^2$.
- **C** (7) used the wrong sign on the cross term, expanding as $(a+b)^2$.
- **D** (5) forgot to square the **right-hand side**. Squaring an equation means squaring both sides.

> **Takeaway:** Given a paired expression ($x$ and $\frac{1}{x}$, or $\sqrt{x}$) and asked for the squares? **Square the given equation.** The cross term almost always collapses to something simple that links the two.

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**Q62 — Proficient — Answer: A**

**Topic:** Constructing a new equation from transformed roots

You do not need $\alpha$ and $\beta$ separately. Build the new equation from the new **sum** and **product**.

**Step 1 — Old sum and product (Vieta).**

$$\alpha+\beta=5 \qquad \alpha\beta=6$$

**Step 2 — New sum and product, expanding carefully.**

$$\text{Sum: } (\alpha+1)+(\beta+1)=5+2=7$$

$$\text{Product: } (\alpha+1)(\beta+1)=\alpha\beta+\alpha+\beta+1=6+5+1=12$$

**Step 3 — Rebuild:** $x^2-(\text{sum})x+(\text{product})=0$

$$x^2-7x+12=0$$

Check the slow way: the old roots are 2 and 3, so the new ones are 3 and 4, and $(x-3)(x-4)=x^2-7x+12$ ✓

*Why the others are wrong:*

- **B** copies the original equation. Moving the roots moves where the parabola crosses the axis, so the equation must change.
- **C** shifted the roots the **wrong way**, giving $\alpha-1$ and $\beta-1$.
- **D** got the sum right but expanded the product as $\alpha\beta+1$, **dropping the cross terms**. All four products count when you expand two brackets — the same slip as Q61's option A.

> **Takeaway:** "New roots shifted by a constant" is a Vieta question in disguise. Find the new sum and product, then rebuild with $x^2-(\text{sum})x+(\text{product})=0$.

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### Q63 — Proficient — Answer: D

**Topic:** Symmetric identities with three variables (extending a known trick)

**The two-variable identity has a three-variable twin.** Recall $a^2 + b^2 = (a+b)^2 - 2ab$. Expanding $(a+b+c)^2$ gives every square **plus** every pair, doubled:

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$
$$\Rightarrow \quad a^2 + b^2 + c^2 = (a+b+c)^2 - 2(ab + bc + ca)$$

Substitute. Note $abc = 6$ is **not** in this identity — it is a deliberate distraction.

$$a^2 + b^2 + c^2 = 36 - 2(11) = 14$$

*Why the others are wrong:*

- **B** (36) gave $(a+b+c)^2$ alone, forgetting to subtract the pairwise term.
- **A** (58) **added** instead of subtracting. The expansion has a $+$, but rearranging to isolate the squares needs a $-$.
- **C** (25) dropped the factor of 2. Expanding produces each pair **twice** ($ab$ and $ba$), so both copies must be removed.

> **Takeaway:** Given symmetric combinations (sums, pairwise products) and asked for something that looks like it needs the individual values — look for an identity that connects them directly. You rarely need to find $a$, $b$ and $c$ themselves.

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### Q64 — Proficient — Answer: C

**Topic:** Logical deduction about the roots of a quadratic ("must be true")

Every option quotes real theory, but only one draws a conclusion its premise **guarantees**. The others are true only *sometimes*.

**Why C is true.** By Vieta, the product of the roots is $\alpha\beta = c$.

Suppose the roots were *not* real. Then they form a conjugate pair $p + qi$ and $p - qi$, whose product is

$$(p + qi)(p - qi) = p^2 + q^2$$

a sum of squares — which can never be negative. So **non-real roots always have a positive product**.

Turn that around: if the product is **negative**, the roots must be **real**. And two real numbers with a negative product must have **opposite signs**. Since $c < 0$ gives $\alpha\beta < 0$, option C is forced.

*Why the others are wrong — one counterexample each:*

- **B**: $x^2 - x - 6 = 0$ has $b^2 - 4c = 25 > 0$, but its roots are 3 and $-2$ — **opposite** signs. A positive discriminant says the roots are real and distinct, nothing about their signs.
- **D**: $x^2 + x + 1 = 0$ has $c = 1 > 0$, but $b^2 - 4c = -3 < 0$ — **no real roots at all**.
- **A**: $x^2 - 4 = 0$ has $b = 0$ and two real roots, $x = \pm 2$. Removing the linear term says nothing about whether real roots exist.

> **Takeaway:** "Feels plausible" is not evidence. Either **prove** the claim from a tool you trust, or **find a counterexample** with the simplest numbers you can. One counterexample settles it.

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#### Q65 — Proficient — Answer: A

**Topic:** Vieta's formulas with reciprocal expressions (chapter capstone)

**The insight:** $\frac{1}{\alpha} + \frac{1}{\beta}$ is secretly built from the **sum** and **product** of the roots — the two things Vieta gives you free. Combine over a common denominator:

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta}{\alpha\beta} + \frac{\alpha}{\alpha\beta} = \frac{\alpha + \beta}{\alpha\beta}$$

No lone $\alpha$ or $\beta$ survives — which is why you can answer without solving.

**Read off sum and product** ($a = 3$, $b = -8$, $c = 5$):

$$\alpha + \beta = \frac{8}{3} \qquad \alpha\beta = \frac{5}{3}$$

**Substitute:**

$$\frac{\alpha + \beta}{\alpha\beta} = \frac{8/3}{5/3} = \frac{8}{5}$$

*Why the others are wrong:*

- **B** ($\frac{5}{8}$) built the fraction upside down. The **sum** goes on top — that is what falls out of the common-denominator step.
- **C** ($\frac{8}{3}$) assumed $\frac{1}{\alpha} + \frac{1}{\beta}$ is just $\alpha + \beta$.
- **D** ($\frac{3}{8}$) assumed it is $\frac{1}{\alpha+\beta}$. Both C and D collapse two different quantities into one.

**Takeaway — and the closing lesson of this chapter:** operations almost never "pass through" addition the way it feels like they should.

$$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b} \qquad \log(x+y) \neq \log x + \log y$$
$$\frac{1}{\alpha+\beta} \neq \frac{1}{\alpha} + \frac{1}{\beta} \qquad (a+b)^2 \neq a^2 + b^2$$

Every one of these appeared in this chapter in a different disguise, and every one is beaten the same way: write it out fully and let the real identity — usually with an extra cross term — appear. Feel suspicious every time you want to distribute an operation across a sum.

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## Exam-Bank Extras — Question Types Confirmed in Recent Papers

### Q66 — Basic — Answer: B

**Topic:** Sum of the solutions of a quadratic (the ± trap)

Square-rooting both sides of $x^2 = 4$ gives **two** solutions:

$$x = +2 \quad \text{or} \quad x = -2$$

The question asks for their **sum**: $(+2) + (-2) = 0$.

**Why A is wrong:** 2 is what you get if you forget the negative root entirely — the single most common slip with this question type. $(-2) \times (-2) = 4$ just as surely as $2 \times 2 = 4$. **Why D is wrong:** $-2$ is the mirror-image slip — the negative root alone. **Why C is wrong:** 4 echoes the right-hand side of the equation — no root was ever taken.

> **Takeaway:** $x^2 = k$ (with $k > 0$) always has two solutions, $x = \pm\sqrt{k}$. Whenever a question says "solutions" (plural) of a squared equation, your radar should ping: *both roots are in play*. This exact question is famous for defeating careless readers — and at least one AI chatbot.

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### Q67 — Intermediate — Answer: C

**Topic:** Custom-defined operations

Work from the inside out. The rule says: *square the first number, subtract three times the second.*

**Inner bracket first:**

$$2 \diamond 1 = 2^2 - 3(1) = 4 - 3 = 1$$

**Then feed that result in as the new first number:**

$$1 \diamond 3 = 1^2 - 3(3) = 1 - 9 = -8$$

**Why D is wrong:** 8 comes from dropping the minus sign in the final subtraction. **Why A is wrong:** 10 comes from computing $2 \diamond (1 \diamond 3)$ — grouping from the right instead of respecting the brackets. **Why B is wrong:** $-2$ drops the coefficient in "$3b$" during the second application: $1^2 - 3 = -2$ instead of $1^2 - 3(3) = -8$.

> **Takeaway:** A "made-up operation" question is pure substitution: the symbol $\diamond$ is just a costume. Put the first value wherever the rule shows $a$, the second wherever it shows $b$ — and always evaluate the bracketed operation first. These questions test careful reading, not new mathematics.

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### Q68 — Proficient — Answer: B

**Topic:** Sum and product of two numbers — building $x^2 + y^2$

Call the numbers $x$ and $y$, so $x + y = m$ and $xy = n$. We want $x^2 + y^2$.

**Square the sum** — this is the move that unlocks the whole question:

$$(x + y)^2 = x^2 + 2xy + y^2 = m^2$$

The expansion contains exactly what we want ($x^2 + y^2$) plus an intruder ($2xy = 2n$). Evict the intruder:

$$x^2 + y^2 = (x + y)^2 - 2xy = m^2 - 2n$$

**Why A is wrong:** a sign slip when moving $2xy$ across. **Why C is wrong:** forgets that the cross term in the expansion is $2xy$, not $xy$. **Why D is wrong:** $m - 2n$ leaves $m$ unsquared — the identity starts from $(x + y)^2 = m^2$, and that square must survive into the answer.

> **Takeaway:** $x^2 + y^2 = (x + y)^2 - 2xy$ is one of the most reused identities in the entire exam bank — it converts "sum and product" information into "sum of squares" in one line. It is Vieta's world again (compare Q65): symmetric expressions in two numbers can always be rebuilt from their sum and product.

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### Q69 — Proficient — Answer: A

**Topic:** Factor theorem with two conditions (simultaneous equations)

**The factor theorem, twice.** Each factor gives one equation, and two unknowns need exactly two.

**From** $(x + 1)$ — substitute $x = -1$:

$$2 - k - b - 4 = 0 \implies k + b = -2 \qquad (1)$$

**From** $(x-2)$ — substitute $x=2$:

$$32-4k+2b-4=0 \implies 2k-b=14 \qquad (2)$$

**Add them** so $b$ cancels: $3k=12 \implies k=4$

*Why the others are wrong:*

- **B** $(-6)$ is the value of $b$, not $k$.
- **D** evaluated $-k(-1)^2$ as $+k$ — a sign error in equation (1).
- **C** $(k=2)$ fails the check: it forces $b=-4$, and then $2k-b=8\neq 14$.

> **Takeaway:** Each known factor gives one free equation via $f(c)=0$. Two unknowns need two factors — then add or subtract to eliminate the one you do not want.

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#### Q70 — Proficient — Answer: C

**Topic:** Exponential equation with two different prime bases

Logs will not help here. The trick is to **factor out the common power on each side**.

**Left:** $2^{x+2}+2^x=2^x(4+1)=5\cdot 2^x$

**Right:** $5^{y+1}-5^y=5^y(5-1)=4\cdot 5^y$

So $5\cdot 2^x=4\cdot 5^y$, which rearranges to:

$$2^{x-2}=5^{y-1}$$

**Now the prime bases decide it.** A power of 2 can equal a power of 5 only if both equal 1 — so both exponents are zero:

$$x=2, \qquad y=1, \qquad x-y=1$$

Check: $16+4=20$ and $25-5=20$ ✓

*Why the others are wrong:*

- **D** (2) gives $x$ alone — the subtraction never happened.
- **B** (3) gives $x+y$, the wrong combination.
- **A** (0) assumes $x=y$ by symmetry, but the constants 5 and 4 differ, which is exactly what offsets the exponents.

> **Takeaway:** Mixed prime bases? Factor each side into (number) × (prime power). The only way $2^m=5^n$ is $m=n=0$ — "the bases are prime" is the key, not decoration.

### Q71 — Proficient — Answer: D

**Topic:** The remainder theorem with two conditions (simultaneous equations)

This uses the **remainder theorem** — the factor theorem's close cousin:

> Dividing $f(x)$ by $(x - c)$ leaves remainder $f(c)$.

The factor theorem is just the case where that remainder is zero. So the two facts become $f(-1) = -3$ and $f(1) = 5$.

**Substitute $x = -1$:** $2 - k - b = -3 \qquad (1)$

**Substitute $x = 1$:** $2 - k + b = 5 \qquad (2)$

**Add them** so $b$ cancels: $4 - 2k = 2 \implies k = 1$

*Why the others are wrong:*

- **B** (4) is the value of $b$, not $k$.
- **A** evaluated $-k(-1)^2$ as $+k$. The square makes $(-1)^2 = +1$, so the term stays $-k$.
- **C** (2) is $2k$ — stopped one division early.

> **Takeaway:** Zero is not special. Substitute the root of the divisor and set the result equal to the **stated remainder** — each division fact becomes one equation. Same skeleton as Q69, where the remainders happened to be zero.

## Mixed Practice — Chapter 1

These 30 questions draw from all nine topics in this chapter. There are no section labels and no difficulty tags — just like the real NBT MAT. Work through them in one sitting, without going back to the chapter. Aim for no more than 3 minutes per question. The answer key follows after all 30 questions.

---

### M1 — Basic

Factorise fully: $6x^2 - x - 2$

A) $(2x+1)(3x-2)$

B) $(2x-1)(3x+2)$

C) $(6x+1)(x-2)$

D) $(3x+1)(2x-2)$

---

### M2 — Intermediate

Factorise fully: $x^3 - 4x^2 - x + 4$

A) $(x+1)(x-1)(x+4)$

B) $(x^2+4)(x-1)$

C) $(x-2)(x+2)(x-4)$

D) $(x-1)(x+1)(x-4)$

---

### M3 — Basic

Solve for $x$: $3(2x-1) = 2(x+5) + 3$

A) $x = 2$

B) $x = 3$

C) $x = 4$

D) $x = 5$

---

### M4 — Intermediate

Solve: $2x^2 + 3x - 9 = 0$

A) $x = 3$ or $x = -\dfrac{3}{2}$

B) $x = \dfrac{3}{2}$ or $x = -3$

C) $x = -3$ or $x = \dfrac{2}{3}$

D) $x = 3$ or $x = -\dfrac{9}{2}$

---

### M5 — Intermediate

Given $p(x) = 2x^3 - 3x^2 - 11x + 6$, which of the following is a factor of $p(x)$?

A) $(x - 2)$

B) $(x + 3)$

C) $(x - 3)$

D) $(x + 4)$

---

### M6 — Intermediate

Solve: $x^2 + x - 12 \leq 0$

A) $x < -4$ or $x > 3$

B) $-4 < x < 3$

C) $x \leq -4$ or $x \geq 3$

D) $-4 \leq x \leq 3$

---

### M7 — Basic

Simplify: $\dfrac{3^{n+2} - 3^n}{2 \cdot 3^n}$

A) 1

B) 4

C) 8

D) $\dfrac{1}{4}$

---

**M8 — Intermediate**

Solve for $x$: $\sqrt{2x+1} = x - 1$

A) $x = 0$ only

B) $x = 0$ or $x = 4$

C) $x = 4$ only

D) No real solution

---

**M9 — Basic**

Evaluate: $\log_5 125$

A) 2

B) 5

C) 25

D) 3

---

**M10 — Intermediate**

Solve: $\log_2(x+3) + \log_2(x-1) = 5$

A) $x = 5$ or $x = -7$

B) $x = -7$ only

C) $x = 5$ only

D) $x = 7$

---

**M11 — Basic**

The 5th term of an arithmetic sequence is 17 and the common difference is 3. What is the first term?

A) $a = 5$

B) $a = 2$

C) $a = 8$

D) $a = 3$

---

### M12 — Intermediate

The first three terms of a geometric sequence are $x + 3$, $x$, and $x - 2$. Find the value of $x$.

A) $x = 4$

B) $x = 6$

C) $x = 8$

D) $x = 3$

---

### M13 — Intermediate

R8 000 is invested at 6% per annum, compounded monthly. Which expression gives the value of the investment after 2 years?

A) $8000(1.06)^{24}$

B) $8000(1.005)^{2}$

C) $8000(1.06)^{2}$

D) $8000(1.005)^{24}$

---

### M14 — Proficient

The roots of $3x^2 - 7x + k = 0$ are $\alpha$ and $\beta$. If $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{7}{4}$, find $k$.

A) $k = 3$

B) $k = 7$

C) $k = 2$

D) $k = 4$

---

### M15 — Intermediate

Given that $(x + 3)$ is a factor of $p(x) = x^3 + 2x^2 - 5x + a$, find the value of $a$.

A) $a = -6$

B) $a = 6$

C) $a = -30$

D) $a = 0$

---

### M16 — Intermediate

Express $2x^2 - 8x + 3$ in the form $a(x-p)^2 + q$.

A) $2(x-4)^2 - 29$

B) $2(x-2)^2 - 5$

C) $(x-2)^2 - 5$

D) $2(x-2)^2 + 3$

---

### M17 — Intermediate

Evaluate: $\sum_{k=1}^{5}(3k-1)$

A) 35

B) 45

C) 40

D) 30

---

### M18 — Intermediate

A car is purchased for R120 000 and depreciates at 15% per annum on the reducing-balance method. Which expression gives its value after 4 years?

A) $120\,000(0.85)^4$

B) $120\,000 - 4(0.15)(120\,000)$

C) $120\,000(1.15)^4$

D) $120\,000 \times (0.85) \times 4$

---

### M19 — Intermediate

Factorise: $8x^3 - 27$

A) $(2x-3)(4x^2 - 6x + 9)$

B) $(2x-3)(4x^2 + 6x + 9)$

C) $(2x+3)(4x^2 - 6x + 9)$

D) $(2x-3)(2x^2 + 6x + 9)$

### M20 — Proficient

If $x = \sqrt{5 + 2\sqrt{6}}$, which of the following is equal to $x$?

A) $\sqrt{3} + \sqrt{2}$

B) $\sqrt{5} + \sqrt{6}$

C) $2 + \sqrt{3}$

D) $1 + \sqrt{6}$

### M21 — Intermediate

Solve: $2^{x+1} = 5$. Give the exact answer.

A) $x = \dfrac{\log 5}{\log 2}$

B) $x = \log_2 5$

C) $x = \dfrac{\log 5}{\log 2} - 1$

D) $x = \log 5 - \log 2$

### M22 — Intermediate

The sum of the first $n$ terms of an arithmetic series is $S_n = 2n^2 + 3n$. What is the 5th term of the series?

A) 23

B) 17

C) 19

D) 21

### M23 — Basic

Solve: $2(3-x) < 4x - 10$

A) $x > \frac{8}{3}$

B) $x < \frac{8}{3}$

C) $x > -\frac{8}{3}$

D) $x < 8$

---

### M24 — Intermediate

For the geometric series $5 + 5x + 5x^2 + \cdots$ to have a finite sum to infinity, which condition on $x$ is required?

A) $x < 1$

B) $-1 < x < 1$

C) $|x| > 1$

D) $0 < x < 1$

---

### M25 — Intermediate

Solve: $4^x - 6 \cdot 2^x + 8 = 0$

A) $x = 1$ or $x = 4$

B) $x = 2$ or $x = 8$

C) $x = 1$ or $x = 2$

D) $x = 2$ or $x = 4$

---

### M26 — Intermediate

$(x-2)$ is a factor of $p(x) = x^3 + kx^2 - x - 2$. Find $k$.

A) $k = -1$

B) $k = 1$

C) $k = 2$

D) $k = -2$

---

### M27 — Intermediate

Simplify: $\log_4 9 \cdot \log_3 8$

A) 6

B) 2

C) $\frac{3}{2}$

D) 3

---

### M28 — Intermediate

In a quadratic sequence, $T_1 = 3$, $T_2 = 10$, and the second difference is 6. Find $T_3$.

A) 21

B) 23

C) 19

D) 25

---

### M29 — Proficient

R2 000 is invested at 10% per annum, compounded annually. After how many complete years does the investment first exceed R3 000?

A) $n = 4$

B) $n = 6$

C) $n = 5$

D) $n = 7$

---

### M30 — Proficient

The roots of $x^2 - 4x + 1 = 0$ are $\alpha$ and $\beta$. Find $\alpha^3 + \beta^3$.

A) 52

B) 44

C) 60

D) 48

## Mixed Practice — Answer Key

| | | | | |
|---|---|---|---|---|
| **M1.** A | **M2.** D | **M3.** C | **M4.** B | **M5.** C |
| **M6.** D | **M7.** B | **M8.** C | **M9.** D | **M10.** C |
| **M11.** A | **M12.** B | **M13.** D | **M14.** D | **M15.** A |
| **M16.** B | **M17.** C | **M18.** A | **M19.** B | **M20.** A |
| **M21.** C | **M22.** D | **M23.** A | **M24.** B | **M25.** C |
| **M26.** A | **M27.** D | **M28.** B | **M29.** C | **M30.** A |

# Chapter 2 — Number Sense

Number Sense is the most underestimated chapter in this book. On the 2023 NBT the median score for this section was just 28 % — lower than any other area. The reason is not that the content is hard, but that students underestimate it and skip systematic practice. Every question here is solvable without a calculator if you know the patterns. Work through every explanation at least once, even when you got the question right.

**Topics covered:** real number classification · ordering and comparing · BODMAS with fractions · percentages and ratios · odd/even parity · last digit of large powers · counting and permutations · mean and weighted average · compound probability · bounds and truncation

## Section 1 — Real Number Classification

The number system is a set of nested containers: $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$. In South African CAPS notation, the Natural numbers $\mathbb{N} = \{1, 2, 3, \ldots\}$ — they start at 1, not 0. Integers $\mathbb{Z}$ add zero and all negatives. Rational numbers $\mathbb{Q}$ are any number expressible as $\frac{p}{q}$ with $p, q \in \mathbb{Z}$, $q \neq 0$. Irrational numbers cannot be written that way — $\sqrt{2}$, $\pi$, $e$ are the classic examples. Real numbers $\mathbb{R}$ hold everything.

### Q1 — Basic

Which of the following is an **irrational** number?

A) $\sqrt{9}$

B) $\dfrac{22}{7}$

C) $0.\overline{3}$

D) $\sqrt{5}$

### Q2 — Basic

Which of the following correctly classifies $-7$?

A) $-7$ is a natural number

B) $-7$ is an integer and a rational number

C) $-7$ is an integer but not a rational number

D) $-7$ is a whole number but not an integer

---

### Q3 — Basic

Which of the following sets contains $0$?

A) Integers ($\mathbb{Z}$)

B) Natural numbers ($\mathbb{N}$)

C) Irrational numbers

D) Neither integers nor rational numbers

---

### Q4 — Intermediate

Which of the following is a **rational** number?

A) $\pi$

B) $\sqrt{2}$

C) $e$

D) $\sqrt[3]{8}$

---

### Q5 — Proficient

Simplify $\sqrt{3} + \sqrt{12}$ and classify the result.

A) Rational

B) Integer

C) Natural

D) Irrational

---

## Section 2 — Ordering and Comparing Numbers

When comparing numbers in different forms (fractions, decimals, surds), convert everything to the same form — usually decimals — or use squaring to compare positive surds without a calculator.

### Q6 — Basic

Which of the following correctly orders $\frac{2}{3}$, 0.6, and $\frac{5}{8}$ from **least to greatest**?

A) $0.6 < \frac{5}{8} < \frac{2}{3}$

B) $\frac{2}{3} < 0.6 < \frac{5}{8}$

C) $\frac{5}{8} < \frac{2}{3} < 0.6$

D) $0.6 < \frac{2}{3} < \frac{5}{8}$

### Q7 — Basic

Which is larger: $\sqrt{5}$ or $\sqrt{3} + 0.5$?

A) $\sqrt{5} > \sqrt{3} + 0.5$

B) $\sqrt{5} < \sqrt{3} + 0.5$

C) $\sqrt{5} = \sqrt{3} + 0.5$

D) It is impossible to determine without a calculator.

### Q8 — Basic

Place in ascending order: $\sqrt{2}$, $\frac{3}{2}$, 1.4

A) $1.4 < \sqrt{2} < \frac{3}{2}$

B) $\sqrt{2} < 1.4 < \frac{3}{2}$

C) $1.4 < \frac{3}{2} < \sqrt{2}$

D) $\frac{3}{2} < \sqrt{2} < 1.4$

### Q9 — Intermediate

Which of the following lies **strictly between** $\sqrt{7}$ and $\sqrt{11}$?

A) $\frac{5}{2}$

B) $\frac{10}{3}$

C) $3\sqrt{2}$

D) $3$

---

### Q10 — Intermediate

Without a calculator, which statement is correct?

A) $2^{30} > 3^{20}$

B) $2^{30} = 3^{20}$

C) It cannot be determined without a calculator.

D) $2^{30} < 3^{20}$

---

### Q11 — Proficient

Which correctly orders from **least to greatest**: $\frac{\sqrt{5}}{2}, \frac{\sqrt{3}+1}{2}, \sqrt{\frac{3}{2}}$?

A) $\frac{\sqrt{3}+1}{2} < \frac{\sqrt{5}}{2} < \sqrt{\frac{3}{2}}$

B) $\frac{\sqrt{5}}{2} < \frac{\sqrt{3}+1}{2} < \sqrt{\frac{3}{2}}$

C) $\frac{\sqrt{5}}{2} < \sqrt{\frac{3}{2}} < \frac{\sqrt{3}+1}{2}$

D) $\sqrt{\frac{3}{2}} < \frac{\sqrt{5}}{2} < \frac{\sqrt{3}+1}{2}$

---

## Section 3 — Arithmetic: BODMAS and Fractions

BODMAS: **B**rackets, **O**rders (powers/roots), **D**ivision, **M**ultiplication (left to right), **A**ddition, **S**ubtraction (left to right). Division and multiplication have equal priority — work left to right. Same for addition and subtraction.

---

### Q12 — Basic

Evaluate $3 + 2 \times 4 - 1$.

A) 19

B) 10

C) 18

D) 12

---

### Q13 — Basic

Evaluate $\frac{1}{2} + \frac{2}{3} \times \frac{3}{4}$.

A) $\frac{3}{4}$

B) $\frac{5}{6}$

C) $\frac{7}{8}$

D) $1$

---

### Q14 — Basic

Evaluate $\frac{3}{4} - \frac{1}{3} \div \frac{2}{3}$.

A) $\frac{5}{12}$

B) $\frac{7}{12}$

C) $\frac{1}{2}$

D) $\frac{1}{4}$

---

### Q15 — Intermediate

Evaluate $\dfrac{2^3-1}{2^3+1}+\dfrac{3}{9}$.

A) $\dfrac{10}{9}$

B) $\dfrac{8}{9}$

C) $\dfrac{4}{3}$

D) $\dfrac{7}{12}$

---

### Q16 — Intermediate

Simplify $\dfrac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{4}+\frac{1}{6}}$.

A) $\dfrac{5}{4}$

B) $1$

C) $\dfrac{3}{2}$

D) $2$

---

### Q17 — Proficient

Evaluate $\dfrac{1}{1\times 2}+\dfrac{1}{2\times 3}+\dfrac{1}{3\times 4}+\cdots+\dfrac{1}{9\times 10}$.

A) $\dfrac{9}{10}$

B) $\dfrac{1}{10}$

C) $\dfrac{10}{11}$

D) $\dfrac{8}{9}$

---

## Section 4 — Percentages, Ratios, and Proportions

Percentage questions on the NBT almost always involve one of three things: finding a percentage of a quantity, reversing a percentage (finding the original), or combining ratios. Master the equation $\text{part} = \% \times \text{whole}$.

---

### Q18 — Basic

A price of R240 is increased by 15%. What is the new price?

A) R276

B) R255

C) R288

D) R264

---

### Q19 — Basic

If 30% of a number is 45, what is the number?

A) 13.5

B) 135

C) 150

D) 15

---

### Q20 — Basic

The ratio of boys to girls in a class is $3 : 5$. There are 40 students in total. How many are boys?

A) 15

B) 10

C) 20

D) 25

---

### Q21 — Intermediate

A shirt costs R360 **after** a 20% discount. What was the original price?

A) R432

B) R450

C) R480

D) R288

---

### Q22 — Intermediate

Three numbers are in the ratio $2 : 3 : 5$. Their sum is 200. What is the **largest** number?

A) 60

B) 100

C) 80

D) 40

---

### Q23 — Proficient

If $x : y = 3 : 4$ and $y : z = 2 : 5$, what is $x : z$?

A) $3 : 5$

B) $6 : 5$

C) $15 : 8$

D) $3 : 10$

---

### Q24 — Proficient

A solution is 40% acid. How many litres of pure water must be added to 50 litres of this solution to reduce the acid concentration to 25%?

A) 20 litres

B) 25 litres

C) 40 litres

D) 30 litres

---

## Section 5 — Parity: Odd and Even Expressions

A number is **even** if it is divisible by 2, **odd** if not. Key rules: - even ± even = even; odd ± odd = even; even ± odd = odd - even × anything = even; odd × odd = odd

---

### Q25 — Basic

If $n$ is an **even** integer, which of the following is **always odd**?

A) $n + 2$

B) $n + 1$

C) $2n$

D) $n^2$

---

### Q26 — Basic

If $p$ is **odd** and $q$ is **even**, which of the following is **always even**?

A) $p + q$

B) $p - q$

C) $p \cdot q$

D) $p^2 + q$

---

### Q27 — Intermediate

If $m$ and $n$ are both **odd**, which expression is **always odd**?

A) $m^2 + n$

B) $m + n$

C) $m \cdot n$

D) $m - n$

---

### Q28 — Intermediate

Let $p$, $q$, $r$ be three consecutive integers. Which of the following is **always even**?

A) $p + q + r$

B) $p \cdot q \cdot r$

C) $p^2 + q^2 + r^2$

D) $p + q$

---

#### Q29 — Proficient

For integers $a$ and $b$, if $a + b$ is odd **and** $a - b$ is odd, which must be true?

A) Both $a$ and $b$ are even

B) Both $a$ and $b$ are odd

C) $a$ and $b$ have different parities (one odd, one even)

D) $a^2 + b^2$ is even

---

## Section 6 — Last Digit of Large Powers

The units digit of $n^k$ follows a repeating cycle. To find the units digit of $n^k$: 1. Find the cycle of units digits for powers of $n$ (cycles of length 1, 2, or 4). 2. Find $k$ mod cycle length. 3. The remainder gives the position in the cycle (remainder 0 means the last position).

---

#### Q30 — Basic

What is the units digit of $7^{20}$?

A) 1

B) 3

C) 7

D) 9

---

#### Q31 — Basic

What is the units digit of $3^{100}$?

A) 3

B) 1

C) 7

D) 9

---

### Q32 — Intermediate

What is the units digit of $2^{103} + 3^{103}$?

A) 5

B) 1

C) 3

D) 7

---

### Q33 — Proficient

What is the units digit of $7^{7^7}$?

A) 1

B) 3

C) 7

D) 9

---

## Section 7 — Counting and Permutations

**Fundamental rule:** If event A can happen in $m$ ways and event B can happen in $n$ ways, then A then B can happen in $m \times n$ ways.

**Permutation** (order matters, no repetition): $n \times (n-1) \times \cdots$ for as many slots as needed.

**Combination** (order does not matter): $\binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$.

---

### Q34 — Basic

In how many ways can 4 different books be arranged in a row on a shelf?

A) 4

B) 16

C) 24

D) 256

---

**Q35 — Basic**

How many 3-digit numbers can be formed from the digits $\{1, 2, 3, 4, 5\}$ if no digit may be repeated?

A) 60

B) 15

C) 125

D) 120

---

**Q36 — Intermediate**

From a group of 5 students, in how many ways can a president and a vice-president be chosen?

A) 10

B) 20

C) 25

D) 15

---

**Q37 — Intermediate**

In how many distinct ways can the letters of the word **LEVEL** be arranged?

A) 120

B) 30

C) 60

D) 20

---

**Q38 — Proficient**

A bag contains 4 red, 3 blue, and 2 green marbles. In how many ways can 3 marbles be selected if **exactly 2** must be red?

A) 20

B) 24

C) 36

D) 30

---

### Q39 — Proficient

How many 4-digit even numbers can be formed from the digits $\{1, 2, 3, 4, 6\}$ with no repetition?

A) 48

B) 60

C) 72

D) 96

---

## Section 8 — Mean, Median, and Weighted Average

**Mean** = sum of all values ÷ number of values. **Median** = middle value when data is sorted (for an even count, average the two middle values). **Weighted mean** = $\frac{\sum(\text{weight} \times \text{value})}{\sum \text{weights}}$.

---

### Q40 — Basic

The marks of 5 students are 60, 72, 68, 75, 80. What is the mean?

A) 71

B) 70

C) 72

D) 75

---

### Q41 — Basic

Find the median of the data set: $3, 7, 1, 9, 4, 6, 8$

A) 4

B) 5

C) 6

D) 7

---

### Q42 — Basic

The mean of four numbers is 15. Three of the numbers are 12, 18, and 14. What is the fourth number?

A) 14

B) 15

C) 17

D) 16

---

### Q43 — Intermediate

A student scored 70, 82, and 65 on three tests. What score is needed on the fourth test to achieve a mean of 75?

A) 78

B) 83

C) 80

D) 85

---

### Q44 — Intermediate

In a class of 30 students, boys have an average mark of 72 and girls have an average mark of 80. There are 18 boys. What is the class average?

A) 74.4

B) 75.2

C) 76.0

D) 77.0

---

### Q45 — Proficient

A stem-and-leaf diagram shows:

| Stem | Leaf |
|---|---|
| 0 | 6 8 |
| 1 | 0 4 |
| 2 | 2 |

What is the difference between the **mean** and the **median**?

A) 0

B) 4

C) 6

D) 2

---

## Section 9 — Compound Probability

**Independent events:** $P(A \cap B) = P(A) \times P(B)$.

**Union:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

**Conditional probability:** $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$.

**Without replacement:** the second draw depends on the first — adjust the denominator.

---

### Q46 — Basic

A fair coin is tossed twice. What is the probability of getting **two heads**?

A) $\frac{1}{4}$

B) $\frac{1}{2}$

C) $\frac{1}{3}$

D) $\frac{3}{4}$

---

### Q47 — Basic

A bag contains 3 red and 2 blue marbles. One marble is drawn and **replaced**. What is the probability of drawing a red marble followed by a blue marble?

A) $\frac{1}{5}$
B) $\frac{6}{25}$
C) $\frac{3}{10}$
D) $\frac{2}{5}$

---

### Q48 — Intermediate

A fair die is rolled twice. What is the probability that the sum is exactly 7?

A) $\frac{5}{36}$
B) $\frac{7}{36}$
C) $\frac{1}{6}$
D) $\frac{1}{12}$

---

### Q49 — Intermediate

Two events $A$ and $B$ are independent with $P(A) = 0.4$ and $P(B) = 0.3$. Find $P(A \cup B)$.

A) 0.12

B) 0.70

C) 0.58

D) 0.28

---

### Q50 — Proficient

In a class, 60% of students play sport, 40% play music, and 25% play both. A student is chosen at random. What is the probability that the student plays **neither** sport nor music?

A) 0.15

B) 0.20

C) 0.25

D) 0.35

---

## Section 10 — Bounds and Estimation

When a measurement is **rounded** to a given place, the true value lies in a symmetric interval around the rounded value. When a measurement is **truncated** (digits simply dropped), the true value lies in a one-sided interval.

**Rounded to nearest unit $u$:** $\text{stated} - \frac{u}{2} \le x < \text{stated} + \frac{u}{2}$

**Truncated:** $\text{stated} \le x < \text{stated} + u$

---

### Q51 — Basic

A number $x$ is rounded to the nearest 10 and the result is 80. What are the bounds of $x$?

A) $75 \le x < 85$

B) $70 < x < 90$

C) $75 < x \le 85$

D) $80 \le x < 90$

---

### Q52 — Basic

A measurement is given as 7.4 cm, rounded to 1 decimal place. What is the **upper bound** of the true measurement?

A) 7.40 cm

B) 7.44 cm

C) 7.45 cm

D) 7.49 cm

---

### Q53 — Intermediate

The length and width of a rectangle are measured as $l = 12$ cm and $w = 8$ cm, each rounded to the nearest centimetre. What is the **minimum possible area**?

A) 84 cm²

B) 90 cm²

C) 86.25 cm²

D) 96 cm²

---

### Q54 — Intermediate

$a = 5.0 \pm 0.1$ and $b = 3.0 \pm 0.1$. What is the **maximum possible value** of $a - b$?

A) 2.0

B) 2.1

C) 2.2

D) 2.3

---

#### Q55 — Proficient

A positive number $x$ is **truncated** (not rounded) to 2 decimal places, giving the result $3.47$. Which of the following correctly describes $x$?

A) $3.465 \leq x < 3.475$

B) $3.47 \leq x < 3.48$

C) $3.47 \leq x \leq 3.48$

D) $3.465 < x < 3.475$

---

## Section 11 — Scientific Notation, Rates and Speed

**Scientific notation** writes a number as $a \times 10^n$ where $1 \leq a < 10$ and $n$ is an integer. The exponent counts how many places the decimal point moves: **positive** for large numbers, **negative** for small ones.

$$4\,500\,000 = 4.5 \times 10^6 \qquad 0.00045 = 4.5 \times 10^{-4}$$

To multiply or divide, handle the coefficients and the powers of ten **separately**, using the exponent laws:

$$(a \times 10^m)(b \times 10^n) = (a \times b) \times 10^{m+n} \qquad \frac{a \times 10^m}{b \times 10^n} = \frac{a}{b} \times 10^{m-n}$$

If the coefficient ends up outside the range $1 \leq a < 10$, renormalise: $37 \times 10^4 = 3.7 \times 10^5$.

**Speed, distance and time** are one relationship read three ways:

$$\text{distance} = \text{speed} \times \text{time} \qquad \text{speed} = \frac{\text{distance}}{\text{time}} \qquad \text{time} = \frac{\text{distance}}{\text{speed}}$$

Two rules prevent almost every error here. First, **units must agree** — 30 minutes is $0.5$ hours, not $30$, when the speed is in km/h. Second, **average speed is always total distance over total time**, never the average of the speeds.

---

### Q56 — Basic

Express 0.00045 in scientific notation.

A) $4.5 \times 10^{-4}$

B) $4.5 \times 10^{4}$

C) $45 \times 10^{-5}$

D) $4.5 \times 10^{-3}$

---

### Q57 — Intermediate

Calculate $(3 \times 10^{5})(2 \times 10^{-8})$, giving the answer in scientific notation.

A) $6 \times 10^{-40}$

B) $6 \times 10^{13}$

C) $6 \times 10^{-3}$

D) $5 \times 10^{-3}$

---

### Q58 — Basic

A car travels at a constant 60 km/h for 2 hours 30 minutes. How far does it travel?

A) 120 km

B) 150 km

C) 180 km

D) 1500 km

---

### Q59 — Proficient

A cyclist rides 120 km from town A to town B at 60 km/h, then returns along the same road at 40 km/h. What is the average speed for the whole journey?

A) 50 km/h

B) 48 km/h

C) 52 km/h

D) 45 km/h

---

## Exam-Bank Extras — Question Types Confirmed in Recent Papers

The NBT MAT reuses question types from a stable bank year after year. The question below is modelled directly on a type repeatedly confirmed in recent papers and not yet represented in this chapter. The answer has been independently machine-verified.

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**Q60 — Proficient**

Evaluate:

$$40^2 - 39^2 + 38^2 - 37^2 + \cdots + 4^2 - 3^2 + 2^2 - 1^2$$

A) 780

B) 820

C) 800

D) 1640

# Answers and Explanations

## Section 1 — Real Number Classification

### Q1 — Basic — Answer: D

**Topic:** Identifying irrational numbers

**Irrational means: cannot be written as a fraction $\frac{p}{q}$.** Test each one.

- **A** $\sqrt{9} = 3 = \frac{3}{1}$ — rational.
- **B** $\frac{22}{7}$ is already a fraction — rational. (It is *close to* $\pi$, but it is not $\pi$.)
- **C** $0.\overline{3} = \frac{1}{3}$ — a repeating decimal is always rational.
- **D** $\sqrt{5}$ — 5 is not a perfect square, and $\sqrt{5}$ cannot be written as any fraction. **Irrational** ✓

> **Takeaway:** $\sqrt{n}$ is rational only when $n$ is a perfect square. And $\frac{22}{7}$ is an *approximation* of $\pi$, not $\pi$ itself — either way it is rational.

---

### Q2 — Basic — Answer: B

**Topic:** Classifying a negative integer

$-7$ is an integer, and $-7 = \frac{-7}{1}$ makes it rational too. Both parts of **B** are true.

*Why the others are wrong:*

- **A** — natural numbers are $\{1, 2, 3, \ldots\}$, all positive. $-7$ is not one.
- **C** — gets "integer" right but says "not rational". Every integer is rational: write it over 1.
- **D** — whole numbers are $\{0, 1, 2, \ldots\}$, so $-7$ is not whole; and it *is* an integer. Wrong twice.

> **Takeaway:** Every integer is automatically rational. The nesting $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q}$ means anything in a smaller set is also in every larger one.

---

### Q3 — Basic — Answer: A

**Topic:** Classification of zero

In CAPS, $\mathbb{N} = \{1, 2, 3, \ldots\}$ — **zero is not a natural number.**

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ lists zero explicitly ✓

*Why the others are wrong:*

- **B** — $0 \notin \mathbb{N}$ under CAPS.
- **C** — $0 = \frac{0}{1}$, so zero is rational, not irrational.
- **D** — zero is both an integer and rational.

> **Takeaway:** Zero is inside $\mathbb{Z}$ and $\mathbb{Q}$, but outside $\mathbb{N}$. Some countries count 0 as natural; the NBT follows CAPS, which does not.

---

### Q4 — Intermediate — Answer: D

**Topic:** Disguised rational numbers

**Simplify each one before judging it.**

- **A** $\pi = 3.14159\ldots$ — never repeats, never ends. Irrational.
- **B** $\sqrt{2} = 1.41421\ldots$ — 2 is not a perfect square. Irrational.
- **C** $e = 2.71828\ldots$ — irrational.
- **D** $\sqrt[3]{8} = 2$, since $2^3 = 8$. Just the integer 2 — **rational** ✓

> **Takeaway:** A root only *looks* irrational. $\sqrt[3]{8}$ is really 2. Always simplify first — any root of a perfect power is rational.

---

### Q5 — Proficient — Answer: D

**Topic:** Sum of surds — rational or irrational?

**Step 1 — Simplify.** $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$

**Step 2 — Add.** $\sqrt{3} + 2\sqrt{3} = 3\sqrt{3}$

**Step 3 — Classify.** $\sqrt{3}$ is irrational, and a non-zero rational times an irrational is still irrational. So $3\sqrt{3}$ is **irrational**.

*Why the others are wrong:* A, B and C are nested (every natural is an integer, every integer is rational), so they all stand or fall on one question — can $3\sqrt{3}$ be written as a fraction? It cannot, so all three fail together. They tempt you if the neat coefficients make the answer *feel* like it should tidy into a whole number.

> **Takeaway:** Simplify before classifying. Two irrationals *can* add to a rational — but only if they cancel, like $\sqrt{3} + (-\sqrt{3}) = 0$. Here they reinforce instead.

---

## Section 2 — Ordering and Comparing Numbers

#### Q6 — Basic — Answer: A

**Topic:** Ordering fractions and decimals

**Convert everything to decimals, then order.**

$$0.6 = 0.600 \qquad \frac{5}{8} = 0.625 \qquad \frac{2}{3} = 0.6\overline{6} \approx 0.667$$

$$0.600 < 0.625 < 0.667 \quad \Rightarrow \quad 0.6 < \frac{5}{8} < \frac{2}{3}$$

*Why the others are wrong:* each wrong order embodies a specific habit of judging fractions by their *looks* —

- **D** swaps the two fractions: the "8 is bigger than 3, so $\frac{5}{8}$ beats $\frac{2}{3}$" trap. Larger denominator = smaller pieces.
- **B** puts $\frac{2}{3}$ first — the error of reading a fraction as its digits ("two point three-ish") instead of dividing.
- **C** parks the plain decimal 0.6 at the top, as if a clean decimal must outrank "messy" fractions. Only conversion settles rank.

> **Takeaway:** Convert everything to decimals (to three places if needed) and *then* order. Every wrong option here comes from ranking by appearance instead of by value.

---

#### Q7 — Basic — Answer: A

**Topic:** Comparing surd expressions

**Step 1 — Square both** (both are positive, so squaring keeps the inequality the right way round).

$$(\sqrt{5})^2 = 5 \qquad (\sqrt{3} + 0.5)^2 = 3 + \sqrt{3} + 0.25 = 3.25 + \sqrt{3}$$

**Step 2 — Compare 5 with $3.25 + \sqrt{3}$.** That means comparing $\sqrt{3}$ with $1.75$. Square again: $3$ versus $3.0625$.

Since $3 < 3.0625$, we get $\sqrt{3} < 1.75$, so $3.25 + \sqrt{3} < 5$, and therefore $\sqrt{5}$ is **larger**.

*Why the others are wrong:* the gap here is razor-thin — $2.232$ versus $2.236$ —

- **B** is what estimation gives: round $\sqrt{3}$ up to 1.75 and the inequality flips. A 0.004 gap is far smaller than rounding error.
- **C** ("equal") is the same story at 2 decimal places.
- **D** ("impossible without a calculator") — the squaring argument above **is** the calculator-free proof.

> **Takeaway:** Square both sides to remove surds (valid when both are positive). When values are this close, approximation cannot be trusted, but exact comparison can.

---

### Q8 — Basic — Answer: A

**Topic:** Ascending order — surd among decimals

**Step 1 — Convert to decimals.**

$1.4 = 1.400$

$\frac{3}{2} = 1.500$

$\sqrt{2} \approx 1.4142\ldots$

**Step 2 — Compare.**

$$1.400 < 1.414\ldots < 1.500 \implies 1.4 < \sqrt{2} < \frac{3}{2}$$

*Why the others are wrong:*

- **B** puts $\sqrt{2}$ below 1.4 — the misremembered benchmark ($\sqrt{2}$ "about 1.2"). In fact $1.4^2 = 1.96 < 2$, so $\sqrt{2} > 1.4$, guaranteed.
- **C** puts $\sqrt{2}$ above $\frac{3}{2}$ — usually $\sqrt{2}$ confused with $\sqrt{3} \approx 1.732$. Check: $1.5^2 = 2.25 > 2$, so $\sqrt{2} < 1.5$.
- **D** is internally impossible: it places $\frac{3}{2} = 1.5$ *below* 1.4. One glance at the two rational values kills it before $\sqrt{2}$ even enters the picture.

> **Takeaway:** $\sqrt{2} \approx 1.414$ is a benchmark worth memorising — and when memory wavers, squaring the boundaries ($1.4^2 = 1.96$, $1.5^2 = 2.25$) pins $\sqrt{2}$ between them with certainty.

### Q9 — Intermediate — Answer: D

**Topic:** Identifying a value between two surds

**Step 1 — Bound $\sqrt{7}$ and $\sqrt{11}$ using perfect squares.**

$7 > (2.6)^2 = 6.76$, so $\sqrt{7} > 2.6$.

$3 < \sqrt{11} < 4$ since $9 < 11 < 16$.

**Step 2 — Test option D:** $3$.

Is $\sqrt{7} < 3 < \sqrt{11}$? Squaring: $7 < 9 < 11$. Yes — $\sqrt{7} < \sqrt{9} = 3 < \sqrt{11}$. ✓

**Step 3 — Check the others are outside the range.**

A) $\dfrac{5}{2} = 2.5 < 2.6 < \sqrt{7}$ — too small.

B) $\dfrac{10}{3} \approx 3.33$ and $(10/3)^2 = 100/9 \approx 11.11 > 11$, confirming $\frac{10}{3} > \sqrt{11}$ — too large.

C) $3\sqrt{2} = \sqrt{18} > \sqrt{11}$ — too large.

> **Takeaway:** To check whether a value $v$ satisfies $\sqrt{a} < v < \sqrt{b}$, just square everything: check $a < v^2 < b$. No calculator needed.

### Q10 — Intermediate — Answer: D

**Topic:** Comparing large powers

**Step 1 — Rewrite both with the same exponent.**

$$2^{30} = (2^3)^{10} = 8^{10}$$
$$3^{20} = (3^2)^{10} = 9^{10}$$

**Step 2 — Compare.**

Since $8 < 9$ and the exponent 10 is the same (and positive):

$$8^{10} < 9^{10} \implies 2^{30} < 3^{20}$$

*Why the others are wrong:*

- **A** is the "bigger exponent wins" instinct — 30 beats 20, surely? But the bases differ, and base strength can outweigh exponent count: rewritten with a common exponent, the fight is $8^{10}$ vs $9^{10}$, and 9 wins.
- **B** ("equal") mistakes *matching exponents* for *matching values*: both sides become something$^{10}$, but the somethings are 8 and 9.
- **C** is the surrender option — the factoring trick above is precisely the no-calculator method the NBT expects you to find.

> **Takeaway:** To compare $a^m$ with $b^n$, force a **common exponent** using $\gcd(m, n)$: $2^{30} = (2^3)^{10}$, $3^{20} = (3^2)^{10}$. The comparison collapses to the bases, 8 vs 9.

---

### Q11 — Proficient — Answer: C

**Topic:** Ordering three surd expressions

**Square each one** (all three are positive):

$$\left(\frac{\sqrt{5}}{2}\right)^2 = 1.25 \qquad \left(\sqrt{\frac{3}{2}}\right)^2 = 1.5 \qquad \left(\frac{\sqrt{3}+1}{2}\right)^2 = \frac{4+2\sqrt{3}}{4} = 1 + \frac{\sqrt{3}}{2} \approx 1.866$$

Order them: $1.25 < 1.5 < 1.866$, so

$$\frac{\sqrt{5}}{2} < \sqrt{\frac{3}{2}} < \frac{\sqrt{3}+1}{2}$$

*Why the others are wrong:*

- **A** and **B** both drop the **cross term** when squaring: $(\sqrt{3}+1)^2$ taken as $3 + 1 = 4$, making the third expression square to 1 and look smallest. The full expansion is $4 + 2\sqrt{3}$.
- **D** compares the raw numbers under the roots, forgetting that the denominator in $\frac{\sqrt{5}}{2}$ quarters its square.

> **Takeaway:** Squaring turns a surd comparison into plain numbers — but $(a+b)^2 = a^2 + 2ab + b^2$, cross term included, every time.

---

## Section 3 — Arithmetic: BODMAS and Fractions

### Q12 — Basic — Answer: B

**Topic:** BODMAS with integers

**Step 1 — Multiplication before addition/subtraction.**

$$3 + \underbrace{2 \times 4}_{=8} - 1 = 3 + 8 - 1$$

**Step 2 — Left to right.**

$$3 + 8 - 1 = 11 - 1 = 10$$

*Why the others are wrong:*

- **A** (19) is pure left-to-right reading: $3 + 2 = 5$, then $5 \times 4 = 20$, then $-1$. The most common BODMAS failure there is.
- **D** (12) does the multiplication correctly but then **adds** the 1 instead of subtracting: $3 + 8 + 1$. Sign slips love to hide behind a correctly-handled multiplication.
- **C** (18) survives no consistent order of operations at all — it is what appears when brackets get imagined mid-calculation. Writing the multiplication's result (8) down before touching the additions leaves it nowhere to come from.

> **Takeaway:** Multiplication before addition and subtraction, always — then strictly left to right among equals. Write the intermediate product down; most BODMAS errors happen in the head, not on the page.

---

### Q13 — Basic — Answer: D

**Topic:** BODMAS with fractions

**Step 1 — Multiplication first.**

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12} = \frac{1}{2}$$

**Step 2 — Addition.**

$$\frac{1}{2} + \frac{1}{2} = 1$$

*Why the others are wrong:*

- **C** ($\frac{7}{8}$) is the left-to-right error in fraction costume: $\left(\frac{1}{2}+\frac{2}{3}\right)\times\frac{3}{4}=\frac{7}{6}\times\frac{3}{4}=\frac{7}{8}$. BODMAS doesn't switch off because the numbers grew fraction bars.
- **B** ($\frac{5}{6}$) is $\frac{1}{2}+\frac{1}{3}$ — the product $\frac{6}{12}$ mis-simplified to $\frac{1}{3}$ instead of $\frac{1}{2}$. Simplify by dividing top and bottom by the *same* number.
- **A** ($\frac{3}{4}$) merely echoes the last fraction in the question — a no-work option.

> **Takeaway:** Multiplication before addition, even when everything is a fraction. Multiply tops, multiply bottoms, simplify carefully — $\frac{6}{12}$ is $\frac{1}{2}$, and the whole expression collapses to $\frac{1}{2}+\frac{1}{2}=1$.

---

#### Q14 — Basic — Answer: D

**Topic:** BODMAS — division of fractions

**Step 1 — Division first** (same priority as multiplication, apply before subtraction).

$$\frac{1}{3}\div\frac{2}{3}=\frac{1}{3}\times\frac{3}{2}=\frac{3}{6}=\frac{1}{2}$$

**Step 2 — Subtraction.**

$$\frac{3}{4}-\frac{1}{2}=\frac{3}{4}-\frac{2}{4}=\frac{1}{4}$$

*Why the others are wrong:*

- **A** ($\frac{5}{12}$) is $\frac{3}{4}-\frac{1}{3}$ — the division vanished entirely; only the two visible fractions got processed.
- **B** ($\frac{7}{12}$) treats "$\div\frac{2}{3}$" as "$\div 2$": $\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$, then $\frac{3}{4}-\frac{1}{6}=\frac{7}{12}$. The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, not $\frac{1}{2}$.
- **C** ($\frac{1}{2}$) is the intermediate result $\frac{1}{3}\div\frac{2}{3}$ reported as the final answer — the subtraction step was never completed.

> **Takeaway:** Dividing by a fraction means multiplying by its reciprocal: $\div\frac{a}{b}=\times\frac{b}{a}$. Do the division first (it outranks subtraction), write its result down, and *then* finish the expression.

---

#### Q15 — Intermediate — Answer: A

**Topic:** Mixed fraction expression with powers

**Step 1 — Powers first.** $2^3=8$

**Step 2 — First fraction.** $\frac{8-1}{8+1}=\frac{7}{9}$

**Step 3 — Add** (both are already over 9):

$$\frac{7}{9} + \frac{3}{9} = \frac{10}{9}$$

*Why the others are wrong:*

- **B** ($\frac{8}{9}$) mangled $\frac{3}{9}$ into $\frac{1}{9}$. Simplifying must not change the value.
- **C** ($\frac{4}{3}$) built the first fraction upside down.
- **D** ($\frac{7}{12}$) mashed everything into $\frac{8-1}{8+1+3}$, absorbing one fraction's numerator into the other's denominator.

> **Takeaway:** Powers first ($2^3 = 8$, not 6, and not $2^{3-1}$), simplify each fraction on its own, then add over a common denominator. Fractions never merge by proximity.

---

#### Q16 — Intermediate — Answer: D

**Topic:** Complex fraction (fraction divided by fraction)

**Top** (LCD 6): $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

**Bottom** (LCD 12): $\frac{1}{4} + \frac{1}{6} = \frac{5}{12}$

**Divide** — multiply by the reciprocal:

$$\frac{5/6}{5/12} = \frac{5}{6} \times \frac{12}{5} = 2$$

*Why the others are wrong:*

- **B** (1) cancelled the matching 5s and stopped, ignoring that $\frac{1}{6} \div \frac{1}{12} = 2$ remains.
- **C** ($\frac{3}{2}$) paired the fractions wrongly across the bar. Numerator terms stay in the numerator.
- **A** ($\frac{5}{4}$) flipped the wrong fraction. Write "$\times\frac{12}{5}$" out before multiplying.

> **Takeaway:** A fraction bar is a division sign. Finish the top, finish the bottom, then multiply by the reciprocal. Expect clean collapses — but earn them step by step.

---

#### Q17 — Proficient — Answer: A

**Topic:** Telescoping series of fractions

**Step 1 — Split the general term.**

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

**Step 2 — Write the sum out and watch it collapse.**

$$\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \cdots + \left(\frac{1}{9} - \frac{1}{10}\right)$$

Everything in the middle cancels, leaving first minus last:

$$1 - \frac{1}{10} = \frac{9}{10}$$

*Why the others are wrong — all three mis-collapse the telescope:*

- **B** ($\frac{1}{10}$) reports the piece being **subtracted**, not the result.
- **C** ($\frac{10}{11}$) ran one term too far.
- **D** ($\frac{8}{9}$) stopped one term early.

> **Takeaway:** After any telescoping collapse, check the two **endpoints** — once the middle cancels, they are the only places left to go wrong.

---

## Section 4 — Percentages, Ratios, and Proportions

#### Q18 — Basic — Answer: A

**Topic:** Percentage increase

**Step 1 — Use the multiplier method.**

A 15% increase means multiplying by $1 + 0.15 = 1.15$.

$$240 \times 1.15 = 276$$

*Why the others are wrong:*

- **B** (R255) adds fifteen *rand*, not fifteen *per cent* — R255 = 240 + 15. Percentages scale with the base: 15 % of R240 is R36.
- **C** (R288) applies a 20 % increase ($240 \times 1.2$) — a slipped rate.
- **D** (R264) applies 10 % ($240 \times 1.1$) — the "round rate" reflex overriding the stated 15 %.

> **Takeaway:** A 15 % increase means multiplying by 1.15 (a decrease, by 0.85). One multiplier, one multiplication — and check the size: the increase should be R36, visibly more than B's R15 and less than C's R48.

---

### Q19 — Basic — Answer: C

**Topic:** Finding the whole from a percentage

**Step 1 — Set up the equation.**

$$0.30 \times x = 45$$

**Step 2 — Solve.**

$$x = \frac{45}{0.30} = \frac{45}{\frac{3}{10}} = 45 \times \frac{10}{3} = 150$$

*Why the others are wrong:*

- **A** (13.5) multiplies instead of divides: $45 \times 0.30$ finds 30 % *of 45* — the question already told you 45 **is** the 30 %.
- **B** (135) divides by $\frac{1}{3}$, quietly replacing 30 % with "a third". Close, but 30 % is $\frac{3}{10}$, not $\frac{1}{3}$ — the R15 gap between 135 and 150 is the cost of that approximation.
- **D** (15) takes a third *of the part* $(45 \div 3)$ — dividing in the right spirit but by the wrong reading entirely.

Sanity check the winner: 30 % of 150 = 45 ✓.

> **Takeaway:** "Part = % × whole" rearranges to "whole = part ÷ %". And treat "30 % ≈ a third" as estimation only — the NBT prices the difference into the options.

---

### Q20 — Basic — Answer: A

**Topic:** Sharing in a ratio

**Step 1 — Find the total number of parts.**

$$3 + 5 = 8 \text{ parts}$$

**Step 2 — Find one part.**

$$\text{One part} = \frac{40}{8} = 5$$

**Step 3 — Find the boys.**

$$\text{Boys} = 3 \times 5 = 15$$

*Why the others are wrong:*

- **D** (25) is the number of **girls** — the right method answering the wrong question. Underline what is being asked before you finish.
- **C** (20) splits the class in half, ignoring the ratio altogether.
- **B** (10) fails the reconstruction test instantly: 10 boys means 30 girls, a ratio of 1 : 3, not 3 : 5.

> **Takeaway:** Add the parts (8), divide the total to get one part (5), multiply by the boys' share (3). Then *reconstruct*: girls $= 25$, and $15 + 25 = 40$ ✓ with ratio $15{:}25 = 3{:}5$ ✓ — a five-second check that also catches "answered the wrong person" errors like D.

---

### Q21 — Intermediate — Answer: B

**Topic:** Reverse percentage

After a 20 % discount the price is **80 %** of the original, so:

$$0.80 \times \text{original} = 360 \quad \Rightarrow \quad \text{original} = \frac{360}{0.80} = \text{R}450$$

Check: $450 \times 0.8 = 360$ ✓

*Why the others are wrong:*

- **A** (R432) adds 20 % onto the sale price. The 20 % came off the **bigger** original, so adding 20 % of the smaller price cannot restore it.
- **D** (R288) discounts again — running the machine forwards instead of backwards.
- **C** (R480) divides by 0.75, turning "20 % off" into "a quarter off".

> **Takeaway:** The original was **multiplied** by 0.80, so recover it by **dividing** by 0.80. Never add the percentage back — and always re-apply the discount to check.

---

### Q22 — Intermediate — Answer: B

**Topic:** Ratios and totals

**Step 1 — Set up.**

Let the three numbers be $2k$, $3k$, $5k$.

$$2k + 3k + 5k = 200 \implies 10k = 200 \implies k = 20$$

**Step 2 — Find the largest.**

$$5k = 5 \times 20 = 100$$

*Why the others are wrong:*

- **A** (60) is the **middle** number ($3k$) and **D** (40) is the **smallest** ($2k$) — both are correct values from the correct working, attached to the wrong word. "Largest" is doing the real work in this question.
- **C** (80) takes $\frac{2}{5}$ of the total — a fraction built from the ratio's outer digits (2 and 5) instead of from part-over-sum ($\frac{5}{10}$).

> **Takeaway:** With three ratio parts, one variable: $2k + 3k + 5k = 200$ gives $k = 20$, so the parts are 40, 60, 100. Each part's share of the whole is (its digits)/(sum of digits) — the largest is $\frac{5}{10} = 50\%$, and half of 200 is 100 ✓.

---

**Q23 — Proficient — Answer: D**

**Topic:** Compound ratio (ratio chain)

**Step 1 — Make $y$ match in both ratios.** It is 4 in the first and 2 in the second, so scale the second up:

$$y : z = 2 : 5 = 4 : 10$$

**Step 2 — Link them.** $x : y : z = 3 : 4 : 10$

**Step 3 — Read off.** $x : z = 3 : 10$

*Why the others are wrong:*

- **A** ($3 : 5$) takes the outer numbers as they stand. But $x$'s 3 is measured against $y = 4$ and $z$'s 5 against $y = 2$ — two different yardsticks.
- **C** ($15 : 8$) multiplies crosswise, the same yardstick error.
- **B** ($6 : 5$) shuffles numbers with no shared $y$ at all.

> **Takeaway:** Scale both ratios until the shared middle term matches, then read off the ends. Or multiply the fractions: $\frac{x}{z} = \frac{3}{4} \cdot \frac{2}{5} = \frac{3}{10}$.

---

**Q24 — Proficient — Answer: D**

**Topic:** Mixture (dilution) problem

**Step 1 — Find the acid.** It never changes when water is added.

$$40\% \times 50 = 20 \text{ litres}$$

**Step 2 — Set up the new concentration** with $w$ litres of water added:

$$\frac{20}{50 + w} = 0.25$$

**Step 3 — Solve.** $20 = 12.5 + 0.25w \Rightarrow w = 30$

Check: $\frac{20}{80} = 25\%$ ✓

*Why the others are wrong — all three are numbers lifted from the question:*

- **A** (20) is the amount of **acid**, the one thing that never changes.
- **B** (25) and **C** (40) echo the two percentages. Percentages are not volumes.

> **Takeaway:** Anchor on what stays **constant** (the acid) and let the total change. Then check your answer *as a concentration* — echo-options cannot survive that.

---

## Section 5 — Parity: Odd and Even Expressions

#### Q25 — Basic — Answer: B

**Topic:** Even + 1

**Step 1 — Test each with $n = 4$ (even).**

A) $4 + 2 = 6$ — even.

B) $4 + 1 = 5$ — odd. ✓

C) $2(4) = 8$ — even.

D) $4^2 = 16$ — even.

**Step 2 — Prove B is always odd.**

$n$ is even means $n = 2k$ for some integer $k$. Then $n + 1 = 2k + 1$ — always odd.

> **Takeaway:** Even + odd = odd. $n$ even, 1 odd → $n + 1$ odd. All the other options add or multiply even numbers, keeping the result even.

---

### Q26 — Basic — Answer: C

**Topic:** Mixed parity multiplication

**Step 1 — Apply parity rules.**

A) $p + q$: odd + even = odd. Not always even.

B) $p - q$: odd − even = odd. Not always even.

C) $p \cdot q$: odd × even = even. ✓ (Any multiple of an even number is even.)

D) $p^2 + q$: $(p^2)$ — odd² = odd. odd + even = odd. Not always even.

> **Takeaway:** Multiplying by an even number always gives an even result — the factor of 2 in $q$ makes the product divisible by 2, regardless of $p$.

---

### Q27 — Intermediate — Answer: C

**Topic:** Two odd numbers

**Step 1 — Test each.**

A) $m^2 + n$: odd² = odd; odd + odd = even. Always even, not odd.

B) $m + n$: odd + odd = even. Always even.

C) $m \cdot n$: odd × odd = odd. Always odd. ✓

D) $m - n$: odd − odd = even. Always even.

**Step 2 — Formal proof for C.**

$m = 2a + 1$, $n = 2b + 1$ for integers $a, b$.

$mn = (2a + 1)(2b + 1) = 4ab + 2a + 2b + 1 = 2(2ab + a + b) + 1$

This is of the form $2k + 1$ — always odd.

> **Takeaway:** The product of two odd numbers is always odd. Options A, B, and D are all *always even* when both inputs are odd — a result that surprises many students.

---

### Q28 — Intermediate — Answer: B

**Topic:** Consecutive integers

**Step 1 — Write as $n, n + 1, n + 2$.**

**Test option D:** $p + q = n + (n + 1) = 2n + 1$ — always odd. Eliminate.

**Test option A:** $p + q + r = 3n + 3 = 3(n + 1)$. If $n = 2$: $3(3) = 9$ (odd). Not always even. Eliminate.

**Test option C:** If $n = 2$: $4 + 9 + 16 = 29$ (odd). Eliminate.

**Test option B:** $pqr = n(n + 1)(n + 2)$. Among any 3 consecutive integers, at least one is even. An even factor makes the whole product even. ✓

> **Takeaway:** Among any set of consecutive integers, at least one is even. Therefore their product is always even. In fact, $n(n + 1)(n + 2)$ is always divisible by $3! = 6$.

---

### Q29 — Proficient — Answer: C

**Topic:** Parity of a sum and difference

**Step 1 — Recall when a sum is odd.**

$x + y$ is odd $\Leftrightarrow$ exactly one of $x$, $y$ is odd (different parities).

**Step 2 — Apply to $a + b$.**

$a + b$ is odd $\rightarrow$ $a$ and $b$ have different parities. So one is even and the other is odd.

**Step 3 — Confirm with $a - b$.**

$a - b = a + (-b)$. The parity of $-b$ equals the parity of $b$. So $a - b$ odd means $a$ and $b$ also have different parities — consistent.

**Step 4 — Check option D.**

If $a$ even, $b$ odd: $a^2$ even, $b^2$ odd, $a^2 + b^2$ = odd. So D is false (it would be odd, not even).

> **Takeaway:** Both $a + b$ and $a - b$ being odd simultaneously forces different parities. Check option D carefully — $a^2 + b^2$ being "even" sounds plausible but is actually false when parities differ.

---

## Section 6 — Last Digit of Large Powers

### Q30 — Basic — Answer: A

**Topic:** Last digit — powers of 7

**Step 1 — Find the cycle of units digits.**

$7^1 = 7, 7^2 = 49, 7^3 = 343, 7^4 = 2401$ — so the cycle is **7, 9, 3, 1**, length 4.

**Step 2 — Locate $7^{20}$.** $20 \div 4 = 5$ remainder **0**, which means the **last** position: digit **1**.

*Why the others are wrong:* **C** (7), **D** (9) and **B** (3) are positions 1, 2 and 3 of the same cycle. The whole question turns on one convention: **remainder 0 means the last position**, because $7^4, 7^8, 7^{12}$ all end where $7^4$ ends.

> **Takeaway:** Powers of 7 (and 3) cycle every 4. Any exponent divisible by 4 lands on 1. Anchor it with $7^4 = 2401$ whenever you doubt the convention.

---

### Q31 — Basic — Answer: B

**Topic:** Last digit — powers of 3

**Step 1 — Find the cycle.**

$3^1 = 3 \to$ **3**; $3^2 = 9 \to$ **9**; $3^3 = 27 \to$ **7**; $3^4 = 81 \to$ **1**.

Cycle: **3, 9, 7, 1** (length 4).

**Step 2 — Find position of $3^{100}$.**

$$100 \div 4 = 25 \text{ remainder } 0$$

Remainder 0 → last position in cycle → units digit **1**.

*Why the others are wrong:* **A** (3), **D** (9) and **C** (7) are cycle positions 1, 2 and 3 — the full menu of remainder-handling mistakes. The most seductive is A: "powers of 3 end in 3." They do — once every four exponents. 100 is a multiple of 4, so $3^{100}$ sits at the cycle's *end*, not its start.

> **Takeaway:** Powers of 3 and 7 share cycle length 4 and both land on 1 whenever the exponent is a multiple of 4. The bulletproof form: $3^{100} = (3^4)^{25} = 81^{25}$, and a number ending in 1 raised to any power still ends in 1 ✓.

---

### Q32 — Intermediate — Answer: A

**Topic:** Units digit of a sum of two powers

Both cycles have length 4, so one division serves both: $103 = 4 \times 25 + 3$, **position 3**.

- Cycle of 2 is **2, 4, 8, 6** → position 3 gives **8**
- Cycle of 3 is **3, 9, 7, 1** → position 3 gives **7**

Add: $8 + 7 = 15$, so the units digit is **5**.

*Why the others are wrong:*

- **C** (3) read both cycles one position early.
- **D** (7) treated 103 as a multiple of 4, landing on both cycle ends.
- **B** (1) read one cycle right and reset the other to its start. Same exponent means the **same position** in both.

> **Takeaway:** Find each units digit, add them, then take the units digit of that little sum. Compute the remainder once when the cycles share a length.

---

### Q33 — Proficient — Answer: B

**Topic:** Tower of powers (power of a power)

The cycle of 7 has length 4, so you need the **top exponent** $7^7$ modulo 4.

Since $7 \equiv 3 \pmod 4$ and $7^2 \equiv 1 \pmod 4$:

$$7^7 = (7^2)^3 \times 7 \equiv 1 \times 3 = 3 \pmod 4$$

Position 3 in the cycle **7, 9, 3, 1** gives units digit **3**.

*Why the others are wrong:*

- **C** (7) flattens the tower into $(7^7)^7 = 7^{49}$. A tower is read **top-down**: $7^{(7^7)}$ is a vastly bigger number.
- **A** (1) and **D** (9) come from garbled arithmetic on the top exponent. Do the small computation honestly.

> **Takeaway:** A tower $a^{b^c}$ means $a^{(b^c)}$, never $(a^b)^c$. Reduce the top exponent modulo the cycle length, then read the cycle — two tiny calculations replace an astronomical number.

---

## Section 7 — Counting and Permutations

#### Q34 — Basic — Answer: C

**Topic:** Factorial — arranging objects

**Step 1 — Count the choices for each position.**

Position 1: 4 books to choose from. Position 2: 3 remaining books. Position 3: 2 remaining books. Position 4: 1 remaining book.

**Step 2 — Multiply.**

$$4 \times 3 \times 2 \times 1 = 4! = 24$$

*Why the others are wrong:*

- **D** (256) is $4^4$ — each shelf position given all four books, as if one book could stand in several places at once. Powers count *with* repetition; arrangements shrink the pool each time.
- **B** (16) is $4^2$ — a half-remembered "multiply the 4 by something" instinct.
- **A** (4) counts the books, not the arrangements — the no-work option.

> **Takeaway:** Arranging $n$ distinct objects in a row gives $n! = n \times (n-1) \times \cdots \times 1$; here $4! = 24$. The shrinking factors (4, 3, 2, 1) are the signature of "no repetition" — if your factors don't shrink, you've allowed clones.

---

#### Q35 — Basic — Answer: A

**Topic:** Permutations from a larger set

**Step 1 — Count choices for each digit position.**

Hundreds digit: 5 choices. Tens digit: 4 remaining choices. Units digit: 3 remaining choices.

**Step 2 — Multiply.**

$$5 \times 4 \times 3 = 60$$

*Why the others are wrong:*

- **C** (125) is $5^3$ — repetition allowed, five choices for every slot. The words "no digit may be repeated" are exactly what turns $5 \times 5 \times 5$ into $5 \times 4 \times 3$.
- **D** (120) is $5!$ — arranging **all five** digits, but the number only has three positions.
- **B** (15) is $5 \times 3$ — the two visible numbers multiplied, with no counting model behind it.

> **Takeaway:** This is the permutation $P(5, 3) = 5 \times 4 \times 3 = 60$: one factor per position, shrinking by one each time. Match the number of factors to the number of slots, and let "no repetition" shrink them.

---

**Q36 — Intermediate — Answer: B**

**Topic:** Ordered selection (president and vice-president)

**Order matters** — the two roles are different. President: 5 choices. Vice-president: 4 left.

$$5 \times 4 = 20$$

*Why the others are wrong:*

- **A** (10) is the **committee** count $\binom{5}{2}$ — right only if the roles were interchangeable. President-Thabo/VP-Amina differs from the reverse, so each pair counts twice.
- **C** (25) is $5^2$, letting one student hold both offices.
- **D** (15) **adds** the choices. Successive choices always multiply.

> **Takeaway:** Named roles → order matters → permutation. Unnamed committee → order does not → combination. The factor of 2 between 20 and 10 is exactly the two orderings of each pair.

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**Q37 — Intermediate — Answer: B**

**Topic:** Permutations with repeated letters

**Step 1 — Count the letters.**

L, E, V, E, L — five letters: **L** appears twice, **E** appears twice, **V** appears once.

**Step 2 — Use the formula for arrangements with repeats.**

$$\frac{5!}{2! \times 2!} = \frac{120}{2 \times 2} = \frac{120}{4} = 30$$

*Why the others are wrong:*

- **A** (120) is $5!$ — treating all five letters as distinct, as if the two L's were different colours. Swapping identical letters produces the *same* word, so 120 counts every arrangement four times.
- **C** (60) divides by only one $2!$ — the repeated L's handled, the repeated E's forgotten. Every repeated letter needs its own factorial.
- **D** (20) divides by $3! = 6$ — lumping the four repeated letters (L, L, E, E) into one group of "repeats". The divisor is per-letter: $2! \times 2! = 4$, not $3!$.

> **Takeaway:** Arrangements with repeats: $\frac{n!}{k_1!\,k_2!\cdots}$ — one factorial for each letter counted separately. LEVEL: $\frac{5!}{2!\,2!} = 30$. Tally the letters *before* touching the formula.

---

### Q38 — Proficient — Answer: D

**Topic:** Combinations with a restriction

**Reds:** choose 2 from 4 → $\binom{4}{2} = 6$

**Non-red:** choose 1 from the other 5 (3 blue + 2 green) → $\binom{5}{1} = 5$

**Multiply:** $6 \times 5 = 30$

*Why the others are wrong — each corrupts one factor:*

- **A** (20) used $4 \times 5$: choosing a **pair** of reds is not the same as choosing one.
- **B** (24) used $6 \times 4$: the non-red pool is 5, greens included.
- **C** (36) used $6 \times 6$ — the first factor recycled.

> **Takeaway:** For "exactly $k$ of type X": $\binom{|X|}{k} \times \binom{\text{others}}{\text{rest}}$. Label each factor as you write it — every wrong option here is a mislabelled factor.

---

### Q39 — Proficient — Answer: C

**Topic:** Even numbers from a set (no repetition)

**Step 1 — Fill the restricted slot first.** Even means the units digit is 2, 4 or 6 — **3 choices**.

**Step 2 — Fill the rest** from the 4 digits left: $4 \times 3 \times 2 = 24$

**Step 3 — Multiply:** $3 \times 24 = 72$

*Why the others are wrong:*

- **A** (48) counted only **2** even digits, overlooking the 6.
- **D** (96) counted **4**, sweeping an odd digit into the even pile.
- **B** (60) is a three-slot permutation that ignored the structure entirely.

> **Takeaway:** Fill the most restricted slot first, then the free ones. When a restriction names a **property**, list the elements that actually have it — miscounting that set is the whole game.

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## Section 8 — Mean, Median, and Weighted Average

#### Q40 — Basic — Answer: A

**Topic:** Arithmetic mean

**Step 1 — Sum the values.**

$$60 + 72 + 68 + 75 + 80 = 355$$

**Step 2 — Divide by 5.**

$$\text{Mean} = \frac{355}{5} = 71$$

*Why the others are wrong:* **C** (72) and **D** (75) are values sitting *in the data set* — echo bait for anyone who "eyeballs the middle". **B** (70) is the round number nearest the truth, the reward for estimating the sum instead of computing it. None of the three equals $355 \div 5$, and that division is the only authority.

> **Takeaway:** Build the sum in a running chain — $60 + 72 = 132$; $+68 = 200$; $+75 = 275$; $+80 = 355$ — then divide once: $355 \div 5 = 71$. Chained addition with a checkpoint at each step is how you beat "clean-looking" wrong options without a calculator.

---

#### Q41 — Basic — Answer: C

**Topic:** Median of an odd data set

**Step 1 — Sort the data in ascending order.**

$$1, 3, 4, 6, 7, 8, 9$$

**Step 2 — Find the middle value.** There are 7 values; the median is the $\frac{7+1}{2} = 4\text{th}$ value.

$$1, 3, 4, \underbrace{6}_{4\text{th}}, 7, 8, 9 \implies \text{median} = 6$$

*Why the others are wrong:*

- **B** (5) is the **midrange** $\frac{1+9}{2}$ — halfway between min and max. The median is the middle *member*, not the middle of the endpoints.
- **D** (7) echoes the *count* of values (seven data points) — and also happens to be a datum, doubling its bait value.
- **A** (4) confuses the middle *position* (4th) with the *value* 4 — position and value are different currencies.

> **Takeaway:** Sort first, always: $1, 3, 4, \mathbf{6}, 7, 8, 9$ → the 4th of 7 values is the median, 6. Every wrong option here is a different "middle-ish" number — only sorting tells you which middle the median actually is.

---

#### Q42 — Basic — Answer: D

**Topic:** Finding a missing value given the mean

**Step 1 — Find the required total.**

$$\text{Total} = \text{mean} \times n = 15 \times 4 = 60$$

**Step 2 — Subtract the known values.**

$$60 - 12 - 18 - 14 = 16$$

*Why the others are wrong:*

- **B** (15) echoes the mean itself — "if the mean is 15, the missing number is probably 15" is a guess, not a computation. (It would only be right if the other three averaged exactly 15; they average $\frac{44}{3} \approx 14.7$.)
- **A** (14) echoes a number already in the data.
- **C** (17) is a one-off arithmetic slip in $60 - 44$. The subtraction chain $60 - 12 - 18 - 14$ done one number at a time $(48, 30, 16)$ leaves no room for it.

> **Takeaway:** "Mean × count = total" is the master move: target total 60, known sum 44, missing value 16. Verify: $\frac{12+18+14+16}{4} = \frac{60}{4} = 15$ ✓.

---

### Q43 — Intermediate — Answer: B

**Topic:** Required score for a target mean

**Required total** for a mean of 75 over 4 tests: $75 \times 4 = 300$

**Current total:** $70 + 82 + 65 = 217$

**Fourth score:** $300 - 217 = 83$

*Why the others are wrong:* **C** (80) is the round guess; **A** (78) and **D** (85) bracket the truth to punish mental arithmetic. Only 83 passes the check: $\frac{300}{4} = 75$ ✓

> **Takeaway:** Required score = (target mean × tests) – (current total). Or think in deficits: the three scores are 8 short of $3 \times 75$, so the fourth must be 8 **above** 75. Same answer, built-in check.

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### Q44 — Intermediate — Answer: B

**Topic:** Weighted (combined) mean

**Girls:** $30 - 18 = 12$

**Totals:** boys $18 \times 72 = 1296$; girls $12 \times 80 = 960$; together **2256**

**Class average:** $\dfrac{2256}{30} = 75.2$

*Why the others are wrong:*

- **C** (76.0) is the plain average of 72 and 80 — valid only if the groups were equal size.
- **D** (77.0) swaps the weights, giving the smaller group the bigger pull. More boys must drag the average **below** 76, so anything above 76 is impossible.
- **A** (74.4) used a 30 % girl-weight instead of 40 %.

> **Takeaway:** Predict the **side** before computing: the bigger group pulls the average toward itself. That one prediction eliminates half the options.

---

### Q45 — Proficient — Answer: D

**Topic:** Stem-and-leaf — mean vs median

**The data** (already sorted): 6, 8, 10, 14, 22 — five values.

**Median:** the 3rd value, **10**

**Mean:** $\dfrac{60}{5} = 12$

**Difference:** $12 - 10 = 2$

*Why the others are wrong:*

- **A** (0) assumes mean = median. True for symmetric data, but the lone 22 drags the mean up while the median stays put.
- **B** (4) mis-totalled the sum.
- **C** (6) misread the diagram, treating leaves as whole values. The stems carry the tens.

> **Takeaway:** Read stem-and-leaf as (stem × 10) + leaf, and the data arrive pre-sorted. Mean above median is the fingerprint of right-skewed data — here caused by that 22.

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## Section 9 — Compound Probability

#### Q46 — Basic — Answer: A

**Topic:** Independent events — two tosses

**Step 1 — List sample space.**

HH, HT, TH, TT — four equally likely outcomes.

**Step 2 — Count favourable outcomes.**

Only HH. $P(\text{HH}) = \dfrac{1}{4}$.

Alternatively: tosses are independent, so $P(\text{H}) \times P(\text{H}) = \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{4}$.

*Why the others are wrong:*

- **B** ($\frac{1}{2}$) is the single-toss probability doing double duty — two tosses require multiplying, not reusing.
- **C** ($\frac{1}{3}$) is the seductive "three outcomes: no heads, one head, two heads — each equally likely" model. The outcomes exist but their probabilities differ ($\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{4}$): "one head" happens two ways (HT, TH).
- **D** ($\frac{3}{4}$) is $P(\text{at least one head})$ — the complement of the wanted event's opposite, a mis-aimed complement.

> **Takeaway:** Independent events multiply: $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. When in doubt, write the sample space (HH, HT, TH, TT) — four equally likely outcomes settle every one of these disputes at a glance.

---

### Q47 — Basic — Answer: B

**Topic:** Independent events with replacement

**Step 1 — Find each probability.**

$P(\text{red}) = \dfrac{3}{5}$, $P(\text{blue}) = \dfrac{2}{5}$.

Since the marble is replaced, the second draw is independent of the first.

**Step 2 — Multiply.**

$$P(\text{red then blue}) = \frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$$

*Why the others are wrong:*

- **C** ($\frac{3}{10}$) is the **without**-replacement answer: $\frac{3}{5} \times \frac{2}{4}$. The word "replaced" is the hinge of the whole question — the bag resets, so the second denominator stays 5.
- **D** ($\frac{2}{5}$) is $P(\text{blue})$ alone — the first draw never entered the calculation.
- **A** ($\frac{1}{5}$) is a near-miss simplification of the right idea ($\frac{6}{25}$ carelessly reduced). $\frac{6}{25}$ is already in lowest terms — 6 and 25 share no factor.

> **Takeaway:** "With replacement" ⇒ independent draws ⇒ multiply with the *same* denominators: $\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$. Before multiplying anything, circle whether the draw is with or without replacement — it changes exactly one number, and the examiner always offers both versions.

---

### Q48 — Intermediate — Answer: C

**Topic:** Sum of two dice

**Step 1 — Total outcomes.** $6 \times 6 = 36$.

**Step 2 — Pairs that sum to 7.**

$(1,6),\ (2,5),\ (3,4),\ (4,3),\ (5,2),\ (6,1)$ — exactly **6** pairs.

**Step 3 — Probability.**

$$P(\text{sum} = 7) = \frac{6}{36} = \frac{1}{6}$$

*Why the others are wrong:*

- **D** ($\frac{1}{12} = \frac{3}{36}$) counts *unordered* pairs — {1,6}, {2,5}, {3,4} — forgetting that (1,6) and (6,1) are different rolls. The dice are distinct objects even if they look identical.
- **A** ($\frac{5}{36}$) is the count for the *neighbouring* sums: both 6 and 8 can be made exactly 5 ways. One pair short of 7's six.
- **B** ($\frac{7}{36}$) puts the 7 from the question straight into the numerator — a surface-number trap.

> **Takeaway:** With two dice, always count **ordered** pairs out of 36. Sum 7 is the unique total achievable 6 ways — one for every value of the first die — which is why $\frac{6}{36} = \frac{1}{6}$ and why 7 is the most likely sum of all.

---

### Q49 — Intermediate — Answer: C

**Topic:** Union of two independent events

**Addition rule:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

**Independence gives the intersection:** $0.4 \times 0.3 = 0.12$

$$P(A \cup B) = 0.4 + 0.3 - 0.12 = 0.58$$

*Why the others are wrong:*

- **B** (0.70) adds and stops, double-counting the overlap. That only works for mutually exclusive events — and independent events are never mutually exclusive.
- **A** (0.12) reports the **intersection** as the union.
- **D** (0.28) is $P(A) - P(A \cap B)$, the probability of "A but not B".

> **Takeaway:** Label every number you compute (∩, ∪, "only A") — three of the four options here are just each other's intermediate steps.

---

### Q50 — Proficient — Answer: C

**Topic:** Inclusion-exclusion — neither event

**Step 1 — Find $P(\text{sport or music})$.**

$$P(S \cup M) = P(S) + P(M) - P(S \cap M) = 0.60 + 0.40 - 0.25 = 0.75$$

**Step 2 — Find $P(\text{neither})$.**

$$P(\text{neither}) = 1 - P(S \cup M) = 1 - 0.75 = 0.25$$

*Why the others are wrong:* every wrong option is a *different region* of the Venn diagram, correctly computed and wrongly labelled —

- **A** (0.15) is "music only" ($0.40 - 0.25$).
- **D** (0.35) is "sport only" ($0.60 - 0.25$).
- **B** (0.20) is the bare difference between the two percentages ($0.60 - 0.40$) — a region of nothing at all.

Check the full picture: sport-only 0.35 + music-only 0.15 + both 0.25 + neither 0.25 = 1.00 ✓.

> **Takeaway:** $P(\text{neither}) = 1 - P(A \cup B)$, with the union built as $0.60 + 0.40 - 0.25 = 0.75$. Sketch the Venn regions and make them sum to 1 — that closing check catches region-mislabeling instantly.

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## Section 10 — Bounds and Estimation

#### Q51 — Basic — Answer: A

**Topic:** Bounds from rounding to nearest 10

**Lower bound:** the smallest value that rounds up to 80 is **75** — and 75 itself rounds to 80, so it is **included**.

**Upper bound:** anything below 85 rounds to 80, but 85 rounds to 90 — so 85 is **excluded**.

$$75 \leq x < 85$$

*Why the others are wrong:*

- **B** used the **full** unit (±10) instead of half — twice too wide. It admits 85, which rounds to 90.
- **C** flips both endpoint conventions — excluding 75 and including 85, backwards on both.
- **D** is one-sided, which is **truncation** (see Q55), not rounding.

> **Takeaway:** "Rounded to nearest $u$" gives stated $\pm u/2$, lower inclusive and upper exclusive. Test your endpoints — "does 75 round to 80? does 85?" — and the conventions sort themselves out.

---

#### Q52 — Basic — Answer: C

**Topic:** Upper bound from rounding to 1 d.p.

**Step 1 — Identify the rounding unit.** Rounded to 1 d.p. means rounded to the nearest 0.1.

Half-unit = 0.05.

**Step 2 — Upper bound** = $7.4 + 0.05 = 7.45$ cm.

True value: $7.35 \le x < 7.45$.

*Why the others are wrong:*

- **A** (7.40) is the stated measurement itself — "upper bound" asks how far the *true* value could reach, not what was written down.
- **B** (7.44) thinks digit-by-digit: "7.44 still shows 7.4". True, but so does 7.449, 7.4499... The supremum of "rounds to 7.4" is 7.45, approached but never reached.
- **D** (7.49) is the truncation instinct — anything 7.4-something. But 7.47, say, *rounds* to 7.5. Rounding halves the window that truncation allows.

> **Takeaway:** For rounding to 1 d.p., the bound is stated ± 0.05: here $7.35 \le x < 7.45$. The upper bound is exclusive — exactly 7.45 would round up to 7.5. State bounds as half the rounding unit, never by fiddling with the next digit.

---

#### Q53 — Intermediate — Answer: C

**Topic:** Minimum area from rounded dimensions

**Lower bounds** (half a unit down): $l = 11.5$ cm and $w = 7.5$ cm.

**Minimum area** = min length × min width:

$$11.5 \times 7.5 = 86.25 \text{ cm}^2$$

*Why the others are wrong:*

- **D** (96) is $12 \times 8$ — the uncertainty ignored entirely.
- **B** (90) took only **one** dimension to its minimum. You need both at once.
- **A** (84) knocked a **whole** centimetre off. Bounds move by **half** a unit.

> **Takeaway:** Extreme-value questions push **every** measurement to the extreme that serves the goal — and bounds move by half the rounding unit, not a full one.

### Q54 — Intermediate — Answer: C

**Topic:** Maximum value of a difference

**Step 1 — Maximise $a - b$:** use maximum $a$ and minimum $b$.

$$a_{\text{max}} = 5.0 + 0.1 = 5.1$$

$$b_{\text{min}} = 3.0 - 0.1 = 2.9$$

**Step 2 — Calculate.**

$$a_{\text{max}} - b_{\text{min}} = 5.1 - 2.9 = 2.2$$

*Why the others are wrong:*

- **A** (2.0) subtracts the nominal values ($5.0 - 3.0$) — the uncertainty never entered the calculation.
- **B** (2.1) applies the error to only one of the two measurements. Both quantities can sit at their unfavourable extremes at the same time, and "maximum possible" means assuming they do.
- **D** (2.3) over-applies the errors ($\pm 0.1$ counted three times) — enthusiasm without bookkeeping.

> **Takeaway:** Max of $a - b$ = (max $a$) − (min $b$); min of $a - b$ = (min $a$) − (max $b$). Under subtraction the error margins **add**: total swing here is $\pm 0.2$ around 2.0. Write each extreme down explicitly (5.1 and 2.9) before subtracting.

### Q55 — Proficient — Answer: B

**Topic:** Truncation bounds

**Truncating** to 2 d.p. means digits after the second place are simply **dropped**, not rounded. So anything reading $3.47\ldots$ truncates to 3.47.

- **Lower:** $3.47000\ldots$ truncates to 3.47, so 3.47 is **included**.
- **Upper:** $3.4799\ldots$ still truncates to 3.47, but 3.48 truncates to itself — so 3.48 is **excluded**.

$$3.47 \leq x < 3.48$$

*Why the others are wrong:*

- **A** and **D** are the **rounding** intervals — the reflex when "truncated" goes unread. Truncation never reaches below the stated value: 3.468 truncates to 3.46.
- **C** closes the wrong end. 3.48 truncates to 3.48, so it must be excluded.

> **Takeaway:** Rounding gives a symmetric interval (± half a unit). Truncation gives a one-sided one, $[\text{stated},\ \text{stated} + \text{unit})$. That one word moves the whole interval — read the verb first.

## Section 11 — Scientific Notation, Rates and Speed

### Q56 — Basic — Answer: A

**Topic:** Writing a small number in scientific notation

Move the decimal point until one non-zero digit sits in front of it:

$$0.00045 \rightarrow 4.5$$

That took **4** places to the **right**, which makes the coefficient bigger — so the power of ten must shrink it back. The exponent is **negative**:

$$0.00045 = 4.5 \times 10^{-4}$$

Check by reversing: $4.5 \times 10^{-4}$ moves the point 4 places left, back to 0.00045 ✓

*Why the others are wrong:*

- **B** has the right digits but a positive exponent, giving 45 000.
- **C** equals the answer numerically, but 45 is not between 1 and 10 — so it is not scientific notation.
- **D** counted the three zeros and forgot that the 4 occupies a place too.

> **Takeaway:** Count the places the point moves and let the direction set the sign — **left positive, right negative**. Then check the coefficient really is between 1 and 10.

### Q57 — Intermediate — Answer: C

**Topic:** Multiplying in scientific notation

Split the problem in two — coefficients together, powers of ten together:

$$(3 \times 2) \times (10^5 \times 10^{-8}) = 6 \times 10^{5+(-8)} = 6 \times 10^{-3}$$

When you **multiply** powers of the same base you **add** the exponents. Here $5 + (-8) = -3$.

The coefficient 6 already lies between 1 and 10, so no renormalising is needed.

*Why the others are wrong:*

- **A** multiplied the exponents ($5 \times -8 = -40$). Multiplying the powers means adding the exponents.
- **B** subtracted instead of adding: $5 - (-8) = 13$. Subtraction is the rule for **division**.
- **D** added the coefficients ($3 + 2$) and the exponents. The coefficients get multiplied, like any other numbers.

> **Takeaway:** Multiply the coefficients, **add** the exponents; divide the coefficients, **subtract** the exponents. Then check the coefficient is still in $[1, 10)$ and fix it if not.

---

### Q58 — Basic — Answer: B

**Topic:** Distance from speed and time

**Convert the time to hours first**, so it matches the km/**h** in the speed. 30 minutes is half an hour:

$$2 \text{ h } 30 \text{ min} = 2.5 \text{ h}$$

Then:

$$\text{distance} = \text{speed} \times \text{time} = 60 \times 2.5 = 150 \text{ km}$$

Sense-check: 2 hours alone covers 120 km, and the extra half-hour adds 30 km. $120 + 30 = 150$ ✓

*Why the others are wrong:*

- **A** used only the whole 2 hours and discarded the 30 minutes.
- **C** treated 30 minutes as a **whole** extra hour, computing $60 \times 3$.
- **D** read the time as "2.30" and multiplied by 25 — a units slip. Minutes are sixtieths, not hundredths, so $30 \text{ min} = 0.5 \text{ h}$, never $0.30 \text{ h}$.

> **Takeaway:** Convert minutes to hours by dividing by **60** before you touch a km/h speed. Half an hour is $0.5$, twenty minutes is $\frac{1}{3}$, forty-five minutes is $0.75$.

---

### Q59 — Proficient — Answer: B

**Topic:** Average speed over a return journey

Average speed is **total distance ÷ total time** — never the average of the two speeds, because more time is spent at the slower one.

**Distance:** $120 + 120 = 240$ km

**Time:** $\frac{120}{60} = 2$ h out, $\frac{120}{40} = 3$ h back, so 5 h in total.

$$\text{average speed} = \frac{240}{5} = 48 \text{ km/h}$$

48 sits **below** the midpoint 50, as expected: 3 of the 5 hours are at the slower speed.

*Why the others are wrong:*

- **A** (50) is the trap — the plain mean $\frac{60+40}{2}$, correct only for equal **times**, not equal distances.
- **C** (52) is pulled above the midpoint, as if more time were spent at the faster speed.
- **D** (45) over-corrects downward with no calculation behind it.

> **Takeaway:** For equal **distances** at two speeds, the average is always **below** the plain mean, and the distance cancels out entirely. Which side of the midpoint your answer falls on is an instant sanity check.

---

## Exam-Bank Extras — Question Types Confirmed in Recent Papers

#### Q60 — Proficient — Answer: B

**Topic:** Alternating sum of consecutive squares (difference-of-squares telescoping)

Forty squares without a calculator is impossible — which is the signal that a **structure** is hiding. Pair the terms and use $a^2 - b^2 = (a-b)(a+b)$:

$$40^2 - 39^2 = (1)(79) = 40 + 39$$
$$38^2 - 37^2 = (1)(75) = 38 + 37$$

Each pair differs by 1, so every difference of squares collapses to a plain **sum**. The whole monster is just:

$$40 + 39 + 38 + \cdots + 1 = \frac{40 \times 41}{2} = 820$$

*Why the others are wrong:*

- **D** (1640) is $40 \times 41$ — forgot to halve.
- **C** (800) estimated $40^2/2$ instead of doing the collapse.
- **A** (780) summed only to 39, dropping the 40.

> **Takeaway:** $a^2 - (a-1)^2 = a + (a-1)$, so an alternating sum of consecutive squares always collapses into a simple arithmetic series. On a no-calculator paper, "impossible arithmetic" is a **promise that a shortcut exists**.

## Mixed Practice — Chapter 2

These 30 questions mix all Number Sense topics. Work through them without looking back at the sections. Check your answers with the key on the following page — detailed explanations are not given here, so use the earlier sections to diagnose any errors.

---

#### M1 — Basic

Which of the following is an irrational number?

A) $\sqrt{16}$

B) $\pi^0$

C) $\sqrt{7}$

D) $0.\overline{142857}$

---

#### M2 — Basic

If 25% of a number is 30, what is the number?

A) 90

B) 150

C) 75

D) 120

---

#### M3 — Basic

What is the units digit of $9^{50}$?

A) 9

B) 1

C) 3

D) 7

---

### M4 — Basic

In how many ways can 5 different books be arranged in a row on a shelf?

A) 25

B) 60

C) 120

D) 720

---

### M5 — Basic

A fair coin is tossed 3 times. What is the probability of getting heads all three times?

A) $\frac{1}{6}$
B) $\frac{1}{8}$
C) $\frac{1}{4}$
D) $\frac{3}{8}$

---

### M6 — Intermediate

If $n$ is an **odd** integer, which expression is **always odd**?

A) $n^2 + n$

B) $n^2 + 1$

C) $n^2 + 2n + 2$

D) $n^2 - n$

---

### M7 — Intermediate

The mean of 5 numbers is 20. Four of them are 15, 22, 18, and 25. What is the fifth number?

A) 16

B) 18

C) 22

D) 20

---

### M8 — Intermediate

Three numbers are in the ratio $1 : 3 : 5$. The largest number is 40. What is the sum of all three?

A) 54

B) 80

C) 90

D) 72

---

### M9 — Intermediate

Which of the following lies strictly between $\sqrt{15}$ and $\sqrt{24}$?

A) $2\sqrt{5}$

B) $\sqrt{8}$

C) $5$

D) $3.5$

---

### M10 — Basic

A price is reduced from R500 to R425. What is the percentage decrease?

A) 12%

B) 15%

C) 17%

D) 20%

---

### M11 — Intermediate

A fair die is rolled twice. What is the probability that the sum is exactly 8?

A) $\frac{5}{36}$

B) $\frac{1}{6}$

C) $\frac{7}{36}$

D) $\frac{1}{9}$

---

### M12 — Proficient

For any integer $n$, which of the following equals $(n+1)^2 - (n-1)^2$?

A) $2n$

B) 4

C) $4n$

D) $2n^2$

---

### M13 — Proficient

What is the units digit of $7^{100} + 3^{100}$?

A) 0

B) 4

C) 8

D) 2

---

### M14 — Intermediate

$x = 3.4$ (rounded to 1 d.p.) and $y = 5.7$ (rounded to 1 d.p.). What is the **maximum** possible value of $x + y$?

A) 9.2

B) 9.1

C) 9.11

D) 9.15

---

### M15 — Basic

Which correctly orders $\frac{7}{8}$, $0.87$, and $\frac{13}{15}$ from least to greatest?

A) $\frac{13}{15} < 0.87 < \frac{7}{8}$

B) $0.87 < \frac{7}{8} < \frac{13}{15}$

C) $\frac{7}{8} < 0.87 < \frac{13}{15}$

D) $0.87 < \frac{13}{15} < \frac{7}{8}$

---

### M16 — Intermediate

How many 4-letter codes can be formed from the letters $\{A, B, C, D, E\}$ with no repetition if the code must **start with a vowel**?

A) 48

B) 24

C) 120

D) 240

---

### M17 — Intermediate

A number is rounded to the nearest 100 and the result is 600. Which of the following **could NOT** be the original number?

A) 549

B) 550

C) 612

D) 649

---

### M18 — Proficient

How many 5-digit numbers can be formed using the digits $\{1, 2, 3, 4, 5\}$ (no repetition) that are **greater than 30 000**?

A) 48

B) 60

C) 72

D) 96

---

### M19 — Intermediate

$P(A) = 0.5$, $P(B) = 0.4$, $P(A \cap B) = 0.2$. Find $P(A \mid B)$.

A) 0.2

B) 0.4

C) 0.5

D) 0.8

---

### M20 — Basic

If $m$ and $n$ are both **even**, which of the following is **always odd**?

A) $m + n$

B) $m \cdot n$

C) $m + n + 1$

D) $m^2 + n^2$

---

### M21 — Proficient

Class A has 20 students with a mean mark of 75. Class B has 30 students with a mean mark of 80. What is the combined mean mark?

A) 77.0

B) 77.5

C) 79.0

D) 78.0

---

### M22 — Basic

$\sqrt{3}$ is best described as:

A) Rational

B) Integer

C) Natural

D) Irrational

---

### M23 — Intermediate

A score $s$ is **truncated** to the nearest integer and gives 47. Which correctly describes $s$?

A) $46.5 \leq s < 47.5$

B) $47 \leq s < 48$

C) $46.5 < s < 47.5$

D) $47 < s \leq 48$

---

### M24 — Intermediate

Two cards are drawn **without replacement** from a standard deck of 52 cards (26 red, 26 black). What is the probability that **both** cards are red?

A) $\frac{25}{51}$

B) $\frac{1}{4}$

C) $\frac{25}{102}$

D) $\frac{13}{51}$

---

### M25 — Proficient

Which of the following is **NOT** a rational number?

A) $\sqrt[3]{27}$

B) $2\pi$

C) $\frac{22}{7}$

D) $0.\overline{9}$

---

### M26 — Basic

From a group of 10 students, in how many ways can a committee of 3 be chosen (order does not matter)?

A) 30

B) 60

C) 720

D) 120

---

### M27 — Intermediate

A shop reduces all prices by 20% and then increases them by 10%. What is the overall effect on the original price?

A) 10% decrease

B) 12% decrease

C) 8% decrease

D) 2% increase

---

### M28 — Proficient

$a : b = 2 : 3$ and $b : c = 6 : 7$. What is $a : b : c$ in simplest form?

A) $2 : 6 : 7$

B) $4 : 6 : 7$

C) $12 : 18 : 21$

D) $2 : 3 : 7$

---

### M29 — Proficient

The units digit of $4^n$ follows a cycle: 4, 6, 4, 6, ... What is the units digit of $4^{2025}$?

A) 4

B) 6

C) 2

D) 8

### M30 — Proficient

The data set $\{3, 5, 7, 9, 11, 13\}$ has a mean of 8 and a median of 8. The value 100 is added to the set. Which of the following changes?

A) Median only

B) Mean only

C) Neither mean nor median

D) Both mean and median

## Mixed Practice — Answer Key

| | | | | |
|---|---|---|---|---|
| **M1.** C | **M2.** D | **M3.** B | **M4.** C | **M5.** B |
| **M6.** C | **M7.** D | **M8.** D | **M9.** A | **M10.** B |
| **M11.** A | **M12.** C | **M13.** D | **M14.** A | **M15.** A |
| **M16.** A | **M17.** A | **M18.** C | **M19.** C | **M20.** C |
| **M21.** D | **M22.** D | **M23.** B | **M24.** C | **M25.** B |
| **M26.** D | **M27.** B | **M28.** B | **M29.** A | **M30.** D |

# Chapter 3 — Functions and Graphs

Functions and Graphs is the backbone of the NBT MAT. Every other chapter touches it: algebraic processes express functions, trigonometry defines trigonometric functions, and data handling uses graphs to display information. This chapter isolates the core ideas — what a function is, how to read its domain and range, and how to sketch and interpret the seven graph families that appear repeatedly on the NBT.

One diagnostic fact: on the 2023 NBT, candidates who could not identify the vertex of a parabola in completed-square form answered fewer than 40 % of Functions questions correctly. The questions in this chapter are deliberately ordered so that each section builds on the last. Do not skip Section 1 even if you are confident with notation — it establishes the substitution discipline that every later section requires.

**Topics covered:** function notation and evaluation · composition · piecewise functions · domain and range · straight-line functions · parabolas · hyperbolas · exponential and logarithmic functions · transformations · inverse functions · interpreting graphs

---

## Section 1 — Function Notation and Evaluation

A function $f$ is a rule that assigns to each input exactly one output. The notation $f(x)$ means "the value of $f$ at input $x$" — and crucially, $f(a+1)$ means substitute the entire expression $a+1$ in place of every $x$ in the rule. The most common error in this section is forgetting to distribute when the argument is an expression rather than a single number.

---

### Q1 — Basic

If $f(x) = 2x^2 + 3x - 4$, find $f(-1)$.

A) $1$

B) $-9$

C) $-5$

D) $3$

---

### Q2 — Basic

If $f(x) = 5x + 3$, find $f(a + 1)$.

A) $5a + 8$

B) $5a + 3$

C) $5a + 4$

D) $5a + 11$

---

### Q3 — Basic

Which of the following **is** a function from its set of inputs to its set of outputs?

A) The set of pairs $\{(1, 2),\ (1, 3),\ (2, 4)\}$

B) The equation $x^2 + y^2 = 9$

C) The vertical line $x = 3$

D) The set of pairs $\{(1, 5),\ (2, 5),\ (3, 7)\}$

---

### Q4 — Intermediate

Let $f(x) = 2x + 1$ and $g(x) = x^2$. Find $g(f(x))$.

A) $4x^2 + 1$

B) $4x^2 + 4x + 1$

C) $4x^2 + 2x + 1$

D) $2x^2 + 1$

---

### Q5 — Basic

Is $f(x) = x^4 - 3x^2$ an even function, an odd function, or neither?

A) Even, because $f(-x) = f(x)$ for all $x$

B) Odd, because $f(-x) = -f(x)$ for all $x$

C) Neither, because not all terms have the same exponent parity

D) Neither, because polynomials cannot be even or odd

---

### Q6 — Intermediate

$g(x) = 3x - 7$. For what value of $x$ does $g(x) = 14$?

A) $\frac{14}{3}$

B) 63

C) 3

D) 7

---

### Q7 — Proficient

$$f(x) = \begin{cases} x^2 - x & \text{if } x < 0 \\ 2x + 1 & \text{if } x \geq 0 \end{cases}$$

Find $f(-2) + f(3)$.

A) 3

B) 9

C) 13

D) 12

---

## Section 2 — Domain and Range

The domain is the set of all valid inputs; the range is the set of all possible outputs. Two rules govern domain restrictions for standard functions: you cannot take the square root of a negative number (so the expression under a root must be $\geq 0$), and you cannot divide by zero (so the denominator must be $\neq 0$). Logarithms add a third: the argument must be strictly positive. For the range, ask what values the output can possibly reach — often easiest by thinking about the graph.

---

### Q8 — Basic

What is the domain of $f(x) = \sqrt{x - 3}$?

A) $x > 3$

B) $x \geq 3$

C) All real numbers

D) $x \leq 3$

---

### Q9 — Basic

What is the domain of $f(x) = \dfrac{1}{x-4}$?

A) All real numbers

B) $x > 4$

C) $x \geq 4$

D) All real numbers except $x = 4$

---

### Q10 — Basic

What is the range of $f(x) = x^2 + 2$?

A) $y \geq 0$

B) All real numbers

C) $y \geq 2$

D) $y > 2$

---

### Q11 — Intermediate

What is the domain of $f(x) = \sqrt{4 - x^2}$?

A) $-2 \leq x \leq 2$

B) $x \leq 2$

C) $x \geq -2$

D) $x \geq 0$

---

### Q12 — Intermediate

What is the range of $f(x) = -(x-1)^2 + 9$?

A) $y \geq 9$

B) $y \leq 9$

C) All real numbers

D) $y \geq 0$

---

### Q13 — Intermediate

What is the domain of $f(x) = \ln(2x - 6)$?

A) $x > 0$

B) $x > 6$

C) $x \geq 3$

D) $x > 3$

---

### Q14 — Proficient

Which statement correctly describes both the domain and range of $f(x) = \dfrac{1}{(x-2)^2}$?

A) Domain: $x \neq 2$; Range: $y > 0$

B) Domain: $x > 2$; Range: $y > 0$

C) Domain: $x \neq 2$; Range: $y \geq 0$

D) Domain: all real $x$; Range: $y > 0$

---

## Section 3 — Straight-Line Functions

A straight line in the form $y = mx + c$ is fully described by its gradient $m$ and its $y$-intercept $c$. The gradient measures steepness and direction: $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{y_2 - y_1}{x_2 - x_1}$. Two lines are parallel if and only if they have equal gradients; perpendicular if the product of their gradients equals $-1$.

---

### Q15 — Basic

A straight line passes through the points $(-2,\ 0)$ and $(0,\ 4)$. What is the gradient?

A) 4

B) $-2$

C) 2

D) $\dfrac{1}{2}$

---

### Q16 — Basic

A line has gradient 3 and passes through $(2,\ 4)$. What is its equation?

A) $y = 3x + 4$

B) $y = 3x - 2$

C) $y = 3x + 2$

D) $y = 3x - 4$

---

### Q17 — Intermediate

Find the equation of the line through $(-1,\ 3)$ and $(2,\ 9)$.

A) $y = 2x + 5$

B) $y = 2x + 7$

C) $y = 2x + 3$

D) $y = 2x + 1$

---

### Q18 — Basic

Which of the following is **not** parallel to $y = 3x - 7$?

A) $y = 3x + 7$

B) $2y = 6x - 1$

C) $y - 3x = 2$

D) $3y = x + 5$

---

### Q19 — Intermediate

The gradient of line $\ell$ is $\dfrac{1}{4}$. What is the gradient of a line perpendicular to $\ell$?

A) $\dfrac{1}{4}$

B) $-4$

C) $4$

D) $-\dfrac{1}{4}$

---

### Q20 — Basic

Find the $x$-intercept of the line $3x - 4y = 24$.

A) $-6$

B) $6$

C) $8$

D) $-8$

---

### Q21 — Intermediate

Find the coordinates of the intersection of $y = 3x - 1$ and $y = x + 5$.

A) $(3,\ 14)$

B) $(3,\ 8)$

C) $(6,\ 17)$

D) $(2,\ 7)$

---

## Section 4 — Parabolas

The parabola $y = a(x - p)^2 + q$ (vertex form) has its vertex at $(p, q)$ and opens upward when $a > 0$, downward when $a < 0$. From standard form $y = ax^2 + bx + c$: the axis of symmetry is $x = -\dfrac{b}{2a}$, the discriminant $\Delta = b^2 - 4ac$ tells you the number of $x$-intercepts, and the $y$-intercept is simply $c$.

---

### Q22 — Intermediate

Find the vertex of $y = x^2 - 6x + 5$.

A) $(3,\ -4)$

B) $(6,\ 5)$

C) $(3,\ 4)$

D) $(-3,\ 4)$

---

### Q23 — Intermediate

Find the roots of $y = x^2 - 5x + 6$.

A) $x = 1$ or $x = 6$

B) $x = -2$ or $x = -3$

C) $x = -1$ or $x = 5$

D) $x = 2$ or $x = 3$

---

### Q24 — Intermediate

What are the vertex and direction of opening of $y = -(x+2)^2 + 8$?

A) Vertex $(2,\ 8)$, opens upward

B) Vertex $(-2,\ -8)$, opens downward

C) Vertex $(-2,\ 8)$, opens downward

D) Vertex $(-2,\ 8)$, opens upward

---

### Q25 — Basic

Find the axis of symmetry of $y = 2x^2 - 8x + 3$.

A) $x = -2$

B) $x = 2$

C) $x = 4$

D) $x = -4$

---

### Q26 — Proficient

A parabola has vertex $(2, -3)$ and passes through the point $(4, 5)$. Find its equation.

A) $y = (x-2)^2 - 3$

B) $y = 2(x+2)^2 - 3$

C) $y = 2(x-2)^2 - 3$

D) $y = 2(x-2)^2 + 3$

---

### Q27 — Basic

What is the range of $y = -(x+3)^2 + 4$?

A) $y \leq 4$

B) $y \leq -4$

C) $y \geq 4$

D) $y \geq -4$

---

### Q28 — Basic

Find the $y$-intercept of $y = 3x^2 - 2x + 7$.

A) $y = 0$

B) $y = 5$

C) $y = -7$

D) $y = 7$

---

### Q29 — Intermediate

How many real roots does $y = x^2 - 4x + 7$ have?

A) 0 real roots

B) 1 real root

C) 2 distinct real roots

D) Cannot be determined without a graph

---

### Q30 — Proficient

Find the $x$-intercepts of $y = x^2 - 2x - 8$.

A) $x = 4$ and $x = 2$

B) $x = 4$ and $x = -2$

C) $x = -4$ and $x = 2$

D) $x = -4$ and $x = -2$

---

## Section 5 — Hyperbolas

The standard hyperbola is $y = \frac{a}{x}$ or its translated form $y = \frac{a}{x+p} + q$. The vertical asymptote is at $x = -p$ (where the denominator equals zero) and the horizontal asymptote is at $y = q$ (the additive shift). When $a > 0$, the graph occupies Quadrants 1 and 3; when $a < 0$, it occupies Quadrants 2 and 4.

### Q31 — Basic

What are the asymptotes of $y = \frac{2}{x}$?

A) $x = 2$ and $y = 0$

B) $x = 0$ and $y = 2$

C) No asymptotes

D) $x = 0$ and $y = 0$

### Q32 — Basic

What is the vertical asymptote of $y = \frac{3}{x-2} + 1$?

A) $x = 1$

B) $x = -2$

C) $x = 2$

D) $x = 3$

### Q33 — Basic

What is the horizontal asymptote of $y = \frac{5}{x+3} - 4$?

A) $y = 0$

B) $y = 4$

C) $y = -3$

D) $y = -4$

### Q34 — Intermediate

What is the domain of $y = \dfrac{1}{x+5}$?

A) $x \neq -5$

B) $x \neq 5$

C) $x > -5$

D) All real numbers

---

### Q35 — Intermediate

In which quadrants does the graph of $y = \dfrac{2}{x}$ lie?

A) Quadrants 1 and 2

B) Quadrants 2 and 4

C) Quadrants 1 and 3

D) All four quadrants

---

### Q36 — Intermediate

How is the graph of $y = -\dfrac{3}{x}$ related to $y = \dfrac{3}{x}$?

A) Shifted down by 6 units

B) Reflected across the $x$-axis

C) Reflected across the line $y = x$

D) Stretched vertically by a factor of 3

---

### Q37 — Proficient

Find the sum of the $x$-coordinates where $y = \dfrac{6}{x}$ and $y = x + 1$ intersect.

A) 1

B) $-1$

C) $-6$

D) 5

---

## Section 6 — Exponential and Logarithmic Functions

Exponential functions have the form $y = a \cdot b^x$ (with $b > 0, b \neq 1$). The horizontal asymptote is always $y = 0$ unless a vertical shift is added. Logarithms are the inverses of exponentials: $y = \log_b x \iff b^y = x$. The key log laws — product, quotient, and power rules — convert between forms that look different but are equivalent.

---

### Q38 — Basic

If $f(x) = 2^x$, find $f(5)$.

A) 10

B) 16

C) 25

D) 32

---

### Q39 — Basic

What is the horizontal asymptote of $y = 3 \cdot 2^x$?

A) $y = 0$

B) $y = 2$

C) $y = 3$

D) $y = 6$

---

### Q40 — Basic

Solve $2^x = 64$.

A) $x = 32$

B) $x = 8$

C) $x = 6$

D) $x = 4$

---

### Q41 — Intermediate

Which of the following correctly describes $y = \left(\frac{1}{2}\right)^x$?

A) Decreasing function with $y$-intercept 1

B) Increasing function with $y$-intercept 1

C) Decreasing function with $y$-intercept 0

D) Decreasing function with $y$-intercept 2

---

### Q42 — Basic

Solve $\log_2(x) = 4$.

A) $x = 8$

B) $x = 16$

C) $x = 4$

D) $x = 2$

---

### Q43 — Intermediate

Which expression is equivalent to $\log_a(x^3)$?

A) $(\log_a x)^3$

B) $\dfrac{3}{\log_a x}$

C) $\log_a(3x)$

D) $3 \log_a x$

---

### Q44 — Intermediate

Solve $\log_3(x + 1) = 2$.

A) $x = 1$

B) $x = 9$

C) $x = 8$

D) $x = 6$

---

### Q45 — Intermediate

What is the inverse of $y = 3^x$?

A) $y = 3^{-x}$

B) $y = x^3$

C) $y = \log_3 x$

D) $y = \dfrac{x}{3}$

---

### Q46 — Basic

What is the $y$-intercept of $y = 2^{x+1}$?

A) $y = 1$

B) $y = 2$

C) $y = 3$

D) $y = 0$

---

## Section 7 — Transformations and Inverses

Transformations modify a graph in predictable ways. Adding a constant outside the function shifts it vertically; adding inside the argument shifts it horizontally — but the direction surprises many students: $f(x-h)$ shifts right (not left). Negating $f(x)$ reflects across the $x$-axis; negating $x$ reflects across the $y$-axis. The inverse of $f$ is found by swapping $x$ and $y$ in the equation and solving for $y$ — it exists (as a function) only when $f$ passes the horizontal line test.

---

### Q47 — Basic

What transformation maps $y = f(x)$ to $y = f(x) + 3$?

A) Vertical shift upward by 3 units

B) Vertical stretch by a factor of 3

C) Horizontal shift right by 3 units

D) Vertical shift downward by 3 units

---

### Q48 — Basic

What transformation maps $y = f(x)$ to $y = f(x - 5)$?

A) Horizontal shift left by 5 units

B) Vertical shift downward by 5 units

C) Vertical shift upward by 5 units

D) Horizontal shift right by 5 units

---

### Q49 — Intermediate

$y = -f(x)$ is a reflection of $y = f(x)$ across which axis?

A) Origin

B) $y$-axis

C) Line $y = x$

D) $x$-axis

---

### Q50 — Intermediate

$y = f(-x)$ is a reflection of $y = f(x)$ across which axis?

A) $y$-axis

B) $x$-axis

C) Line $y = x$

D) Origin

---

### Q51 — Intermediate

What transformation maps $y = f(x)$ to $y = f(2x)$?

A) Horizontal stretch by a factor of 2

B) Horizontal compression by a factor of 2

C) Vertical stretch by a factor of 2

D) Vertical compression by a factor of 2

---

### Q52 — Basic

Find the inverse of $f(x) = 2x + 5$.

A) $f^{-1}(x) = 2x - 5$

B) $f^{-1}(x) = \dfrac{x+5}{2}$

C) $f^{-1}(x) = \dfrac{x-5}{2}$

D) $f^{-1}(x) = 5x + 2$

---

### Q53 — Intermediate

Which correctly states the inverse of $y = x^2$ for $x \geq 0$?

A) $y = \sqrt{x}$ for $x \geq 0$

B) $y = \sqrt{x}$ for all real $x$

C) $y = x^{1/2}$ for $x > 0$ only

D) $y = \pm\sqrt{x}$

---

### Q54 — Basic

If $f$ is any function with inverse $f^{-1}$, what is $f(f^{-1}(7))$?

A) $f(7)$

B) 14

C) 1

D) 7

---

### Q55 — Proficient

Find the inverse of $y = \dfrac{3}{x-1}$.

A) $y = \dfrac{3}{1-x}$

B) $y = \dfrac{3}{x} + 1$

C) $y = \dfrac{x-1}{3}$

D) $y = \dfrac{x}{4}$

---

## Section 8 — Interpreting Graphs

The NBT frequently presents a described or sketched graph and asks you to reason about it. Key skills: reading where a function is positive, negative, increasing, or decreasing; identifying zeros and intercepts; recognising asymptotes; and matching an equation to a description of a graph's features.

---

### Q56 — Intermediate

A straight-line graph passes through $(-2,\ 0)$ and $(0,\ 4)$. What is the gradient of this line?

A) $4$

B) $-2$

C) $2$

D) $\frac{1}{2}$

---

### Q57 — Basic

For which values of $x$ is $y = x^2 - 4$ negative?

A) $x > 2$ or $x < -2$

B) $-2 < x < 2$

C) $x = \pm 2$

D) All real $x$

---

### Q58 — Intermediate

The function $f(x) = -(x-2)^2 + 5$ is increasing for:

A) $x > 2$

B) $x = 2$ only

C) All real $x$

D) $x < 2$

---

### Q59 — Intermediate

For what values of $x$ is $f(x) = (x-1)^2 - 9$ negative?

A) $-2 < x < 4$

B) $-3 < x < 3$

C) $x < -2$ or $x > 4$

D) $x > 4$

---

### Q60 — Basic

Find the maximum value of $y = -2x^2 + 8x - 5$.

A) $y = 5$

B) $y = 8$

C) $y = 11$

D) $y = 3$

---

### Q61 — Intermediate

For which values of $x$ does the graph of $y = x^2 - 4$ lie **above** the graph of $y = x + 2$?

A) $x < -2$ or $x > 3$

B) $-2 < x < 3$

C) $x > 3$

D) $x < -2$

---

### Q62 — Basic

Which of the following functions has range $y \geq 0$?

A) $y = x$

B) $y = \frac{1}{x}$

C) $y = x^2$

D) $y = x^3$

---

### Q63 — Intermediate

Which of the following graphs CANNOT represent a function?

A) A parabola opening upward

B) A circle centred at the origin

C) An exponential curve

D) A logarithmic curve

---

### Q64 — Intermediate

What is the horizontal asymptote of $y = \frac{2}{x+3} - 1$?

A) $y = 2$

B) $y = 3$

C) $y = 0$

D) $y = -1$

---

### Q65 — Proficient

A graph has these properties: $y$-intercept at $(0, 3)$; vertical asymptote at $x = -2$; horizontal asymptote at $y = 1$. Which equation matches?

A) $y = 1 + \frac{4}{x+2}$

B) $y = 1 + \frac{4}{x-2}$

C) $y = -1 + \frac{4}{x+2}$

D) $y = 1 - \frac{4}{x+2}$

---

## Section 9 — Absolute-Value Functions (V-Graphs)

The absolute value of a number is its **distance from zero**, so it is never negative: $|5| = 5$ and $|-5| = 5$. Formally $|x| = \sqrt{x^2}$, which is another way of saying "square it, then take the positive root — the sign is destroyed on the way."

The graph of $y = |x|$ is a **V**: the right half is the line $y = x$, and the left half is that line reflected upward into $y = -x$. Everything that would have dipped below the $x$-axis is folded back above it.

$$y = a|x - p| + q$$

behaves exactly like the parabola's vertex form: the **vertex** (the sharp point of the V) sits at $(p, q)$, the V opens **upward** when $a > 0$ and **downward** when $a < 0$, and $|a|$ controls how steep the arms are. Two facts do most of the work in exam questions:

- $|\text{anything}| \geq 0$ — always. An absolute value can never equal a negative number.
- $|X| = k$ (with $k > 0$) has **two** solutions, because $X$ may be $+k$ or $-k$.

---

### Q66 — Basic

What are the coordinates of the vertex of $y = |x - 2| + 1$, and which way does the graph open?

A) $(-2,\ 1)$, opening upward

B) $(2,\ 1)$, opening upward

C) $(2,\ -1)$, opening downward

D) $(2,\ 1)$, opening downward

---

### Q67 — Intermediate

Solve for $x$: $|2x - 6| = 8$

A) $x = 7$ only

B) $x = 4$ or $x = -4$

C) $x = 7$ or $x = -1$

D) $x = 7$ or $x = -2$

---

### Q68 — Intermediate

What is the range of $y = -|x| + 4$?

A) $y \geq 4$

B) $y \leq 4$

C) $y \geq 0$

D) All real numbers

---

### Q69 — Proficient

How many real solutions does $|x - 3| + 5 = 2$ have?

A) None

B) One

C) Two

D) Infinitely many

---

## Exam-Bank Extras — Question Types Confirmed in Recent Papers

The NBT MAT reuses question types from a stable bank year after year. The three questions below are modelled directly on types repeatedly confirmed in recent papers and not yet represented in this chapter. Every answer has been independently machine-verified.

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### Q70 — Proficient

Find the inverse of the function

$$f(x) = \frac{2x}{1 - 3x}$$

A) $f^{-1}(x) = \dfrac{2x}{3x + 1}$

B) $f^{-1}(x) = \dfrac{1 - 3x}{2x}$

C) $f^{-1}(x) = \dfrac{x}{3x + 2}$

D) $f^{-1}(x) = \dfrac{x}{2 - 3x}$

---

### Q71 — Proficient

A graph is translated 3 units to the **left** and then reflected in the $x$-axis. The resulting graph has equation

$$y = \frac{2}{x-1}$$

What was the equation of the graph **before** these transformations?

A) $y = -\dfrac{2}{x-4}$

B) $y = \dfrac{2}{x-4}$

C) $y = -\dfrac{2}{x+2}$

D) $y = \dfrac{2}{x+2}$

---

### Q72 — Proficient

A circle centred at the origin touches each branch of the hyperbola $y = \dfrac{8}{x}$ at exactly one point. The equation of the circle is:

A) $x^2 + y^2 = 8$

B) $x^2 + y^2 = 16$

C) $x^2 + y^2 = 4$

D) $x^2 + y^2 = 64$

# Answers and Explanations

## Section 1 — Function Notation and Evaluation

### Q1 — Basic — Answer: C

**Topic:** Evaluating a function at a negative input

**Step 1 — Substitute $x = -1$ carefully.**

$$f(-1) = 2(-1)^2 + 3(-1) - 4$$

**Step 2 — Evaluate each term.**

$(-1)^2 = 1$, so $2(1) = 2$.

$3(-1) = -3$.

$$f(-1) = 2 + (-3) + (-4) = 2 - 3 - 4 = -5$$

**Step 3 — Check each distractor's origin.**

B) $-9$: The student treats $(-1)^2 = -1$ instead of $+1$, giving $2(-1) - 3 - 4 = -9$. Squaring always produces a non-negative result — $(-1)^2 = (-1)(-1) = +1$.

A) 1: The student substitutes $x = 1$ (ignoring the negative), giving $2 + 3 - 4 = 1$.

D) 3: The student makes a sign error on the constant, reading $-4$ as $+4$: $2 - 3 + 4 = 3$.

> **Takeaway:** When the input is negative, write it in brackets — $(-1)^2$ — before simplifying. Never omit the brackets; $-1^2 = -(1^2) = -1$, but $(-1)^2 = +1$.

### Q2 — Basic — Answer: A

**Topic:** Substituting an expression into a function

**Replace every $x$ with** $(a+1)$, then expand:

$$f(a+1) = 5(a+1) + 3 = 5a + 5 + 3 = 5a + 8$$

*Why the others are wrong:*

- **B** $(5a+3)$ evaluated $f(a)$ — the $+1$ was ignored.
- **C** $(5a+4)$ wrote $5a+1+3$, forgetting to multiply the 1 by 5.
- **D** $(5a+11)$ treated the function as additive: $f(a)+f(1)$. Functions do not split across addition.

> **Takeaway:** $f(\square) = 5(\square) + 3$ — the box holds the **whole** new expression, brackets and all.

---

### Q3 — Basic — Answer: D

**Topic:** Definition of a function (vertical line test)

**The rule:** a relation is a function when every **input** has exactly **one** output.

- **A** — input 1 gives both 2 and 3. Not a function.
- **B** — $x^2 + y^2 = 9$ is a circle; $x = 0$ gives $y = \pm 3$. Not a function.
- **C** — the vertical line $x = 3$ sends one input to every $y$ at once. Not a function.
- **D** — $1 \to 5$, $2 \to 5$, $3 \to 7$. Each input has one output ✓ Two inputs *sharing* an output is fine.

> **Takeaway:** Two inputs may share an output; one input may never have two. That is exactly what the vertical line test checks — if a vertical line crosses the graph twice, it is not a function.

---

### Q4 — Intermediate — Answer: B

**Topic:** Composition of functions

$g(f(x))$ means apply $f$ **first**, then $g$. So the input to $g$ is $2x+1$:

$$g(f(x)) = (2x+1)^2 = 4x^2 + 4x + 1$$

*Why the others are wrong:*

- **A** ($4x^2+1$) dropped the middle term. A binomial square always has three terms.
- **D** ($2x^2+1$) computed $f(g(x))$ instead — composition is **not** commutative.
- **C** ($4x^2+2x+1$) used $a^2+ab+b^2$, missing the factor of 2.

> **Takeaway:** $(a+b)^2 = a^2+2ab+b^2$ — the middle term is $2ab$. Read $g(f(x))$ as "put $f(x)$ into $g$ ", working inside out.

---

### Q5 — Basic — Answer: A

**Topic:** Even and odd functions

**Substitute $-x$ and compare with $f(x)$:**

$$f(-x) = (-x)^4 - 3(-x)^2 = x^4 - 3x^2 = f(x)$$

Since $f(-x) = f(x)$, the function is **even**.

*Why the others are wrong:*

- **B** (odd) would need $f(-x) = -f(x)$, i.e. $-x^4+3x^2$. Not equal.
- **C** claims mixed exponents. Both exponents here (4 and 2) are even — which is exactly why it works.
- **D** says polynomials cannot be even or odd. They are the commonest examples: $f(x) = x^2$ is the simplest even one.

> **Takeaway:** Even exponents throughout → even function. One odd-exponent term among them makes the function **neither**.

---

### Q6 — Intermediate — Answer: D

**Topic:** Solving for the input given the output

**Step 1 — Set up the equation.**

$$3x - 7 = 14$$

**Step 2 — Solve.**

$$3x = 21 \implies x = 7$$

**Step 3 — Explain each error.**

A) $\frac{14}{3}$: The student solves $3x = 14$ (ignores the $-7$, treating the equation as if $g(x) = 3x$).

C) 3: The student correctly computes $3x = 21$ but then divides 21 by 7 (the answer) instead of by 3: $21 \div 7 = 3$.

B) 63: The student correctly gets $3x = 21$ but then multiplies rather than divides: $21 \times 3 = 63$.

> **Takeaway:** Always isolate the variable by doing the same operation to both sides. After adding 7: $3x = 21$. Dividing both sides by 3 gives $x = 7$, not $x = 21 \times 3$.

---

### Q7 — Proficient — Answer: C

**Topic:** Evaluating a piecewise function

**Choose the right piece for each input.** $-2 < 0$ uses the first; $3 \geq 0$ uses the second.

$$f(-2) = (-2)^2 - (-2) = 4 + 2 = 6 \qquad f(3) = 2(3) + 1 = 7$$
$$f(-2) + f(3) = 13$$

*Why the others are wrong:*

- **A** (3) used the wrong piece for **both** inputs.
- **B** (9) slipped a sign: read $-(-2)$ as $-2$, giving $f(-2) = 2$.
- **D** (12) dropped the $+1$ from $f(3)$.

> **Takeaway:** Write down **which piece** you are using before you substitute. One extra second prevents the commonest error here.

---

## Section 2 — Domain and Range

### Q8 — Basic — Answer: B

**Topic:** Domain of a square-root function

**Step 1 — Require the expression under the root to be non-negative.**

$$x - 3 \geq 0 \implies x \geq 3$$

**Step 2 — Check the boundary.**

At $x = 3$: $\sqrt{3-3} = \sqrt{0} = 0$. This is defined, so $x = 3$ is included. The domain uses $\geq$, not $>$.

**Step 3 — Explain errors.**

A) $x > 3$: Excludes $x = 3$, but $\sqrt{0} = 0$ is perfectly defined. The strict inequality is wrong.

D) $x \leq 3$: Reverses the inequality entirely.

C) All real numbers: Ignores the restriction. $f(2) = \sqrt{-1}$, which is not real.

> **Takeaway:** Square-root domains use $\geq 0$ (not $> 0$), because zero is a valid input: $\sqrt{0} = 0$. Only the square root of a strictly negative number is undefined in the real numbers.

---

### Q9 — Basic — Answer: D

**Topic:** Domain of a rational function

**Step 1 — Identify when the denominator equals zero.**

$$x - 4 = 0 \implies x = 4$$

**Step 2 — State the domain.**

All real numbers are valid inputs except $x = 4$, which makes the denominator zero (division by zero is undefined).

**Step 3 — Trace errors.**

B) $x > 4$: Excludes all values less than 4 unnecessarily — $f(0) = \frac{1}{-4} = -\frac{1}{4}$ is perfectly defined.

C) $x \geq 4$: Excludes values $< 4$ and still includes the problematic $x = 4$.

A) All real numbers: Does not account for $x = 4$, where the function is undefined.

> **Takeaway:** A rational function is undefined only where the denominator equals zero. Every other real number is a valid input, including negatives and numbers less than 4.

---

### Q10 — Basic — Answer: C

**Topic:** Range of a basic quadratic

**Step 1 — Find the minimum of $x^2 + 2$.**

$x^2 \geq 0$ for all real $x$, with minimum value 0 at $x = 0$.

Adding 2: $f(x) = x^2 + 2 \geq 0 + 2 = 2$.

**Step 2 — Check whether the minimum is achieved.**

At $x = 0$: $f(0) = 0 + 2 = 2$. The value 2 is achieved, so the range includes 2.

$$\text{Range: } y \geq 2$$

**Step 3 — Explain errors.**

A) $y \geq 0$: Ignores the $+2$ shift — this would be the range of $y = x^2$, not $y = x^2 + 2$.

B) All real numbers: Would require negative outputs, but $x^2 + 2 \geq 2 > 0$ always.

D) $y > 2$: Excludes $y = 2$, but $f(0) = 2$, so 2 is in the range.

> **Takeaway:** The range of $f(x) = x^2 + c$ is $y \geq c$, because $x^2 \geq 0$ and the shift $c$ moves the minimum up to $c$.

---

### Q11 — Intermediate — Answer: A

**Topic:** Domain of a function with a two-sided restriction

**Step 1 — Require the radicand to be non-negative.**

$$4 - x^2 \geq 0$$

**Step 2 — Solve the inequality.**

$$x^2 \leq 4 \implies |x| \leq 2 \implies -2 \leq x \leq 2$$

**Step 3 — Verify the boundary.**

At $x = \pm 2$: $4 - 4 = 0 \geq 0$. Both endpoints are valid.

**Step 4 — Explain errors.**

C) $x \geq -2$: Captures the left boundary correctly but misses the right: for $x = 3$, $4 - 9 = -5 < 0$ — undefined.

B) $x \leq 2$: Captures the right boundary but misses the left: for $x = -3$, $4 - 9 = -5 < 0$ — undefined.

D) $x \geq 0$: Restricts to non-negative inputs only, losing the left half of the valid domain.

> **Takeaway:** When the radicand is a downward-opening expression $c - x^2$, the domain is a closed interval $[-\sqrt{c},\ \sqrt{c}]$. Always solve both sides of the inequality simultaneously.

---

### Q12 — Intermediate — Answer: B

**Topic:** Range of a reflected and shifted parabola

The form is $a(x-p)^2 + q$ with $a = -1$, vertex $(1, 9)$.

Since $a < 0$ the parabola opens **downward**, so 9 is a **maximum**:

$$\text{Range: } y \leq 9$$

*Why the others are wrong:*

- **A** ($y \geq 9$) treats the vertex as a minimum. Downward-opening means maximum.
- **D** ($y \geq 0$) describes an upward parabola with vertex at the origin — two errors at once.
- **C** (all reals) would need arbitrarily large values, but the expression never exceeds 9.

> **Takeaway:** The sign of $a$ decides the range: $a > 0$ gives $y \geq q$, $a < 0$ gives $y \leq q$.

---

### Q13 — Intermediate — Answer: D

**Topic:** Domain of a logarithmic function

**A logarithm needs a strictly positive argument:**

$$2x - 6 > 0 \implies 2x > 6 \implies x > 3$$

*Why the others are wrong:*

- **B** ($x > 6$) reached $2x > 6$ then forgot to divide by 2.
- **C** ($x \geq 3$) includes $x = 3$, where the argument is 0 — and $\ln(0)$ is undefined.
- **A** ($x > 0$) restricts $x$ instead of the **argument**. The condition comes from $2x - 6$, not $x$.

> **Takeaway:** For logs, the argument must be $> 0$ — strictly. Never $\geq 0$.

### Q14 — Proficient — Answer: A

**Topic:** Domain and range of a reciprocal-squared function

**Domain:** the denominator $(x-2)^2$ is zero only at $x = 2$, so $x \neq 2$.

**Range:** a square is positive everywhere except at $x = 2$ (excluded), so $f(x) = \frac{1}{(x-2)^2}$ is always **positive**. It can never equal 0 — that would need $1 = 0$. And it reaches every positive value: it grows without bound near $x = 2$ and shrinks towards 0 far away. So $y > 0$.

*Why the others are wrong:*

- **B** (domain $x > 2$) excludes the left side, but $f(0) = \frac{1}{4}$ is perfectly fine.
- **C** (range $y \geq 0$) includes 0, which $f$ never reaches.
- **D** (all reals) ignores $x = 2$.

> **Takeaway:** For $\frac{1}{(x-p)^2}$: domain $x \neq p$, range $y > 0$. A squared denominator makes the output strictly positive and never zero.

## Section 3 — Straight-Line Functions

### Q15 — Basic — Answer: C

**Topic:** Calculating gradient from two points

**Step 1 — Apply the gradient formula.**

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{0-(-2)} = \frac{4}{2} = 2$$

**Step 2 — Explain errors.**

A) 4: Student reads off the $y$-intercept (where the line crosses the $y$-axis at $(0,4)$) and reports it as the gradient.

B) $-2$: Student reads off the $x$-intercept (the line crosses the $x$-axis at $(-2,0)$) and reports it as the gradient.

D) $\dfrac{1}{2}$: Student inverts the fraction: $\dfrac{x_2 - x_1}{y_2 - y_1} = \dfrac{2}{4} = \frac{1}{2}$ (run over rise, not rise over run).

> **Takeaway:** Gradient = rise ÷ run = $\dfrac{\Delta y}{\Delta x}$. Choose your two points consistently: $y_2 - y_1$ on top, $x_2 - x_1$ on the bottom, using the same subscript order.

### Q16 — Basic — Answer: B

**Topic:** Equation of a line from gradient and one point

**Step 1 — Use point-gradient form.**

$$y - y_1 = m(x - x_1)$$
$$y - 4 = 3(x - 2)$$

**Step 2 — Expand.**

$$y - 4 = 3x - 6$$
$$y = 3x - 6 + 4 = 3x - 2$$

**Step 3 — Explain errors.**

A) $y = 3x + 4$: Student uses the $y$-coordinate of the given point as the $y$-intercept directly: $c = 4$. This is only valid when the given point is the $y$-intercept (i.e., when $x_1 = 0$).

D) $y = 3x - 4$: Student computes $c = y_1 - x_1 = 4 - 2 \times \ldots$ or makes an arithmetic error in the expansion.

C) $y = 3x + 2$: Student uses $c = x_1 = 2$ — reads the $x$-coordinate as the intercept.

> **Takeaway:** The $y$-intercept $c$ is found by substituting the given point into $y = mx + c$ and solving: $4 = 3(2) + c \Rightarrow c = 4 - 6 = -2$.

---

### Q17 — Intermediate — Answer: A

**Topic:** Equation of a line through two points

**Gradient:** $m = \dfrac{9 - 3}{2 - (-1)} = \dfrac{6}{3} = 2$

**Intercept:** substitute $(-1, 3)$ into $y = 2x + c$:

$$3 = 2(-1) + c \implies c = 5$$
$$y = 2x + 5$$

Check with the other point: $2(2) + 5 = 9$ ✓

*Why the others are wrong:*

- **B** ($c = 7$) subtracted the **gradient** from $y$ instead of $2x_2 = 4$.
- **C** ($c = 3$) read $c$ straight off the first point's $y$-coordinate.
- **D** ($c = 1$) used $x_1 = +1$ instead of $-1$, losing the sign.

> **Takeaway:** Substitute **both** coordinates into $y = mx + c$ and watch the signs — here $2(-1) = -2$, not $+2$.

---

### Q18 — Basic — Answer: D

**Topic:** Identifying a line NOT parallel to a given line

**Step 1 — Recall the condition for parallel lines.**

Two lines are parallel if and only if they have the same gradient.

The given line $y = 3x - 7$ has gradient $m = 3$.

**Step 2 — Find each option's gradient.**

A) $y = 3x + 7$: gradient $= 3$. Parallel.

D) $3y = x + 5 \implies y = \frac{1}{3}x + \frac{5}{3}$: gradient $= \frac{1}{3} \neq 3$. **NOT parallel.** ✓

C) $y - 3x = 2 \implies y = 3x + 2$: gradient $= 3$. Parallel.

B) $2y = 6x - 1 \implies y = 3x - \frac{1}{2}$: gradient $= 3$. Parallel.

> **Takeaway:** Always rearrange to $y = mx + c$ before reading the gradient. A coefficient in front of $y$ (like $3y$ or $2y$) changes the gradient by a factor — divide both sides first.

---

### Q19 — Intermediate — Answer: B

**Topic:** Gradient of a perpendicular line

**Step 1 — Apply the perpendicularity condition.**

If two lines are perpendicular, the product of their gradients is $-1$:

$$m_1 \times m_2 = -1$$

**Step 2 — Solve for $m_2$.**

$$\frac{1}{4} \times m_2 = -1 \implies m_2 = -4$$

**Step 3 — Explain errors.**

A) $\dfrac{1}{4}$: Same gradient — this describes a parallel line, not a perpendicular one.

C) 4: The reciprocal without the sign change. The perpendicular gradient is the **negative** reciprocal.

D) $-\dfrac{1}{4}$: The negative without the reciprocal. Two separate operations are needed: flip the fraction AND change the sign.

> **Takeaway:** Perpendicular gradient = $-\dfrac{1}{m}$. For $m = \frac{1}{4}$: flip to get $\frac{4}{1} = 4$, then negate to get $-4$.

---

**Q20 — Basic — Answer: C**

**Topic:** $x$-intercept of a linear equation

**Step 1 — Set $y = 0$ (the $x$-intercept lies on the $x$-axis, where $y = 0$).**

$$3x - 4(0) = 24$$

$$3x = 24 \implies x = 8$$

**Step 2 — Explain errors.**

A) $-6$: Student finds the $y$-intercept by setting $x = 0$: $-4y = 24 \Rightarrow y = -6$ — this is the $y$-intercept, not the $x$-intercept.

B) 6: Student finds the $y$-intercept ($y = -6$) but takes its absolute value.

D) $-8$: Student applies a sign error when dividing: $3x = 24 \Rightarrow x = -8$ (incorrectly negates).

> **Takeaway:** For intercepts, remember the rule: $x$-intercept → set $y = 0$; $y$-intercept → set $x = 0$. They are different points and will usually have different values.

---

**Q21 — Intermediate — Answer: B**

**Topic:** Intersection of two straight lines

**Set the two expressions equal** and solve:

$$3x - 1 = x + 5 \implies 2x = 6 \implies x = 3$$

**Then find $y$:** $y = 3(3) - 1 = 8$, so the point is $(3, 8)$.

Check in the other equation: $3 + 5 = 8$ ✓

*Why the others are wrong:*

- **A** $(3, 14)$ found $x = 3$ then mixed the two equations when computing $y$.
- **C** $(6, 17)$ reached $2x = 6$ but read off $x = 6$, forgetting to divide.
- **D** $(2, 7)$ made a sign slip: $5 - 1$ instead of $5 + 1$.

> **Takeaway:** Equate the two $y$-expressions, solve for $x$, substitute back — then **check in the other equation**.

---

## Section 4 — Parabolas

### Q22 — Intermediate — Answer: A

**Topic:** Vertex of a parabola in standard form

**Step 1 — Find the $x$-coordinate of the vertex.**

$$x = -\frac{b}{2a} = -\frac{-6}{2(1)} = \frac{6}{2} = 3$$

**Step 2 — Find the $y$-coordinate.**

$$y = (3)^2 - 6(3) + 5 = 9 - 18 + 5 = -4$$

Vertex: $(3, -4)$.

**Step 3 — Explain errors.**

B) $(6, 5)$: Student reads off the values of $b$ and $c$ directly as the vertex coordinates — these are not vertex coordinates.

C) $(3, 4)$: Student correctly finds $x = 3$ but takes the absolute value of $y$: $|-4| = 4$, ignoring the negative sign.

D) $(-3, 4)$: Student negates the $x$-coordinate (perhaps thinking $x = -b/2a = -3$) and takes $|y| = 4$.

> **Takeaway:** The $y$-coordinate of the vertex must be calculated by substituting $x = 3$ back into the original equation. Do not read it from the coefficients.

---

### Q23 — Intermediate — Answer: D

**Topic:** Roots of a factorable quadratic

**Factorise:** two numbers multiplying to $+6$ and adding to $-5$ are $-2$ and $-3$.

$$x^2 - 5x + 6 = (x-2)(x-3) = 0 \implies x = 2 \text{ or } 3$$

*Why the others are wrong:*

- **A** $(1, 6)$ multiply to 6 ✓ but add to 7 ✗ — factor pairs guessed without checking the sum.
- **B** $(-2, -3)$ come from $(x+2)(x+3) = x^2 + 5x + 6$ — wrong signs throughout.
- **C** $(-1, 5)$ add to 4, not $-5$.

> **Takeaway:** Both conditions must hold: product $= c$ **and** sum $= b$. Product $+6$ with sum $-5$ forces **both** numbers negative.

---

### Q24 — Intermediate — Answer: C

**Topic:** Features of a parabola in vertex form

**Step 1 — Read the vertex form $y = a(x-p)^2 + q$.**

Rewrite as $y = -1 \cdot (x - (-2))^2 + 8$.

So $a = -1$, $p = -2$, $q = 8$. Vertex: $(-2, 8)$.

**Step 2 — Determine direction.**

$a = -1 < 0$: the parabola opens **downward**.

**Step 3 — Explain errors.**

A) Vertex $(2, 8)$: Student reads $+2$ as the $p$-value, ignoring that $y = a(x-p)^2 + q$ requires $x - p$; since we have $x + 2 = x - (-2)$, $p = -2$.

B) Vertex $(-2, -8)$: Vertex $x$-coordinate is correct, but student negates the $q$ value: $-8$ instead of 8.

D) Vertex $(-2, 8)$: Both vertex coordinates are correct but direction is wrong — a negative $a$ always gives a downward opening parabola.

> **Takeaway:** In $y = a(x-p)^2 + q$, the vertex is $(p, q)$. Watch the sign: $(x+2) = (x - (-2))$, so $p = -2$, not $+2$.

---

### Q25 — Basic — Answer: B

**Topic:** Axis of symmetry

**Step 1 — Apply the formula.**

$$x = -\frac{b}{2a} = -\frac{-8}{2(2)} = \frac{8}{4} = 2$$

**Step 2 — Explain errors.**

A) $x = -2$: Student applies the formula as $\frac{b}{2a} = \frac{-8}{4} = -2$, forgetting the leading negative sign in $-\frac{b}{2a}$. Since $b = -8$, we need $-(-8) = +8$ in the numerator.

C) $x = 4$: Student uses $\frac{b}{a} = \frac{-8}{2} = -4$... or perhaps $\frac{|b|}{a} = \frac{8}{2} = 4$ (omits the denominator factor of 2).

D) $x = -4$: Student uses $\frac{b}{a} = \frac{-8}{2} = -4$ (omits the factor of 2 in $2a$, and keeps the sign).

> **Takeaway:** Memorise $x = -\frac{b}{2a}$ as a formula — the numerator is $-b$, the denominator is $2a$. For $2x^2 - 8x + 3$: $a = 2$, $b = -8$, so $x = -(-8)/(2 \times 2) = 8/4 = 2$.

---

### Q26 — Proficient — Answer: C

**Topic:** Finding the equation of a parabola from vertex and a point

**Start from vertex form** with vertex $(2, -3)$: $y = a(x-2)^2 - 3$

**Substitute the point** $(4, 5)$ to find $a$:

$$5 = a(2)^2 - 3 = 4a - 3 \implies a = 2$$
$$y = 2(x-2)^2 - 3$$

*Why the others are wrong:*

- **A** assumed $a = 1$ without checking. At $x = 4$ it gives $y = 1$, not 5.
- **B** uses $(x + 2)$, putting the vertex at $-2$.
- **D** uses $+3$, putting the vertex at $y = 3$.

> **Takeaway:** The vertex gives you $p$ and $q$ for free, but $a$ needs **one extra point**. Always substitute it and solve.

---

### Q27 — Basic — Answer: A

**Topic:** Range of a downward-opening parabola

**Step 1 — Identify vertex and direction.**

Vertex: $(-3, 4)$. Leading coefficient $a = -1 < 0$: opens downward.

**Step 2 — State the range.**

The vertex gives the maximum value of 4. The parabola extends downward without bound:

$$\text{Range: } y \leq 4$$

**Step 3 — Explain errors.**

C) $y \geq 4$: Treats the vertex as a minimum — correct logic for upward parabolas, wrong here.

B) $y \leq -4$: Negates the $q$ value incorrectly. The vertex height is $+4$, not $-4$.

D) $y \geq -4$: Combines both errors: negates $q$ and treats it as a minimum.

> **Takeaway:** Range of $-(x-p)^2 + q$: the maximum is $q$ (at the vertex), so range = $y \leq q$.

---

### Q28 — Basic — Answer: D

**Topic:** $y$-intercept of a quadratic

**Step 1 — Set $x = 0$.**

$$y = 3(0)^2 - 2(0) + 7 = 0 - 0 + 7 = 7$$

**Step 2 — Explain errors.**

A) $y = 0$: Student sets $y = 0$ (finding the $x$-intercept) rather than $x = 0$.

B) $y = 5$: Student evaluates $-2x$ at $x = 0$ as $-2$ (forgetting to multiply by 0): $0 - 2 + 7 = 5$. The term $-2x$ at $x = 0$ is $-2(0) = 0$, not $-2$.

C) $y = -7$: Student negates the constant term, reading $+7$ as $-7$.

> **Takeaway:** The $y$-intercept of any polynomial $y = ax^n + \ldots + c$ is always the constant term $c$, found by setting $x = 0$. Every term with $x$ vanishes.

---

### Q29 — Intermediate — Answer: A

**Topic:** Number of real roots from the discriminant

**Step 1 — Compute the discriminant.**

$$\Delta = b^2 - 4ac = (-4)^2 - 4(1)(7) = 16 - 28 = -12$$

**Step 2 — Interpret.**

$\Delta < 0$: no real roots. The parabola does not intersect the $x$-axis.

**Step 3 — Explain errors.**

B) 1 real root: Would require $\Delta = 0$ (the parabola touches the $x$-axis exactly once).

C) 2 distinct real roots: Would require $\Delta > 0$.

D) Cannot be determined: The discriminant gives a definitive answer — no graph required.

> **Takeaway:** $\Delta < 0 \Rightarrow$ no real roots; $\Delta = 0 \Rightarrow$ one repeated root; $\Delta > 0 \Rightarrow$ two distinct real roots.

---

### Q30 — Proficient — Answer: B

**Topic:** Roots of a quadratic — reading factored intercepts

**Step 1 — Factor.**

Find two numbers with product $-8$ and sum $-2$: these are $-4$ and $+2$.

$$x^2 - 2x - 8 = (x-4)(x+2)$$

**Step 2 — Solve.**

$$x = 4 \quad \text{or} \quad x = -2$$

**Step 3 — Explain errors.**

A) $x = 4$ and $x = 2$: Uses $(x-4)(x-2) = x^2 - 6x + 8$ — wrong constant term.

C) $x = -4$ and $x = 2$: Uses $(x+4)(x-2) = x^2 + 2x - 8$ — correct constant but wrong sign on the middle term (sum is $+2$, not $-2$).

D) $x = -4$ and $x = -2$: Uses $(x+4)(x+2) = x^2 + 6x + 8$ — wrong on both middle and constant terms.

> **Takeaway:** Always check both the product and the sum of your chosen factor pair: product $= c$, sum $= b$. A quick multiplication of the factors confirms the factorisation before you state the roots.

---

## Section 5 — Hyperbolas

### Q31 — Basic — Answer: D

**Topic:** Asymptotes of the basic hyperbola

**Step 1 — Find the vertical asymptote.**

The denominator is $x$. Setting $x = 0$ gives a zero denominator: vertical asymptote at $x = 0$ (the $y$-axis).

**Step 2 — Find the horizontal asymptote.**

As $x \to \pm\infty$, $y = \frac{2}{x} \to 0$: horizontal asymptote at $y = 0$ (the $x$-axis).

**Step 3 — Explain errors.**

A) $x = 2$ and $y = 0$: Student reads the numerator $a = 2$ as the vertical asymptote — but asymptotes come from the denominator (VA) and the shift (HA), not the numerator.

B) $x = 0$ and $y = 2$: Correct VA, but student reads the numerator as the HA.

C) No asymptotes: Every rational function of this form has exactly two asymptotes.

> **Takeaway:** For $y = \dfrac{a}{x}$: VA = $x = 0$, HA = $y = 0$. The numerator $a$ affects the stretch and which quadrants are occupied — it does not move the asymptotes.

---

### Q32 — Basic — Answer: C

**Topic:** Vertical asymptote of a shifted hyperbola

**Step 1 — Set the denominator equal to zero.**

$$x - 2 = 0 \implies x = 2$$

**Step 2 — Explain errors.**

A) $x = 1$: Student reads the horizontal shift $q = 1$ as the vertical asymptote.

B) $x = -2$: Student negates the value from the denominator, perhaps thinking of $x + 2 = 0$ instead of $x - 2 = 0$.

D) $x = 3$: Student reads the numerator $a = 3$ as the asymptote.

> **Takeaway:** Vertical asymptote of $y = \dfrac{a}{x - p} + q$: set $x - p = 0$, giving $x = p$. The other numbers ($a$ and $q$) do not determine the VA.

#### Q33 — Basic — Answer: D

**Topic:** Horizontal asymptote of a shifted hyperbola

**Step 1 — Apply the rule.**

For $y = \dfrac{a}{x+p} + q$, as $x \to \pm\infty$, the fraction $\dfrac{a}{x+p} \to 0$, so $y \to q$.

Here $q = -4$: horizontal asymptote $y = -4$.

**Step 2 — Explain errors.**

B) $y = 4$: Student drops the sign from $q = -4$.

C) $y = -3$: Student reads the vertical-asymptote value $p = -3$ (giving $x = -3$) as a $y$-value.

A) $y = 0$: Applies the un-shifted rule $y = \frac{a}{x}$, where HA is $y = 0$. This ignores the $-4$ shift.

> **Takeaway:** For $y = \dfrac{a}{x+p} + q$: VA at $x = -p$, HA at $y = q$. The $+q$ shifts the entire graph — including the horizontal asymptote.

---

#### Q34 — Intermediate — Answer: A

**Topic:** Domain of a shifted hyperbola

**Step 1 — Find where the denominator equals zero.**

$$x + 5 = 0 \implies x = -5$$

Domain: all real $x$ except $x = -5$.

**Step 2 — Explain errors.**

B) $x \neq 5$: Student reads $x + 5 = 0$ as $x = 5$ (drops the sign change on rearrangement).

C) $x > -5$: Only excludes values to the left of $-5$ unnecessarily — $f(-6) = \frac{1}{-1} = -1$ is defined.

D) All real numbers: Ignores the restriction from the zero denominator.

> **Takeaway:** $x + 5 = 0 \Rightarrow x = -5$. The domain is all reals except the asymptote $x = -5$, written $x \neq -5$ or $\mathbb{R} \setminus \{-5\}$.

---

#### Q35 — Intermediate — Answer: C

**Topic:** Quadrants occupied by $y = 2/x$

**Step 1 — Test the sign of $y$ in each quadrant.**

When $x > 0$: $y = \dfrac{2}{x} > 0$ (positive ÷ positive = positive) → Quadrant 1.

When $x < 0$: $y = \dfrac{2}{x} < 0$ (positive ÷ negative = negative) → Quadrant 3.

**Step 2 — Explain errors.**

A) Quadrants 1 and 2: In Quadrant 2, $x < 0$, so $y < 0$ — not Quadrant 2.

B) Quadrants 2 and 4: This is the pattern for $y = \dfrac{-2}{x}$ (negative numerator).

D) All four quadrants: The graph never crosses the axes (asymptotes at $x = 0$ and $y = 0$), so it cannot be in all four.

> **Takeaway:** Sign of $a$ in $y = \dfrac{a}{x}$ determines the quadrants: $a > 0$ → Quadrants 1 and 3 (where $x$ and $y$ have the same sign); $a < 0$ → Quadrants 2 and 4.

---

### Q36 — Intermediate — Answer: B

**Topic:** Transformation of a hyperbola

The whole output has been negated: $y = -\left(\dfrac{3}{x}\right)$. Negating the output is the definition of a **reflection in the $x$-axis** — every point $(x, y)$ becomes $(x, -y)$.

On the graph, this moves the two branches out of Quadrants 1 and 3 and into Quadrants 2 and 4.

*Why the others are wrong:*

- **A** — a vertical shift would give $\frac{3}{x} - 6$, which has a horizontal asymptote at $y = -6$. Nothing here shifted.
- **C** — reflecting in $y = x$ gives the **inverse**. Swapping $x$ and $y$ in $y = \frac{3}{x}$ returns $y = \frac{3}{x}$ itself, so that reflection changes nothing.
- **D** — a vertical stretch would give $\frac{9}{x}$. The graph has been flipped, not stretched.

> **Takeaway:** $y = -f(x)$ is always a reflection in the $x$-axis. A curiosity worth noticing: for this particular hyperbola, reflecting in the $y$-axis produces the *same* picture, because $\frac{3}{-x}$ is also $-\frac{3}{x}$.

---

### Q37 — Proficient — Answer: B

**Topic:** Intersection of a hyperbola and a line

**Equate the two expressions** and clear the fraction ($x \neq 0$):

$$\frac{6}{x} = x + 1 \implies 6 = x^2 + x \implies x^2 + x - 6 = 0$$

**Factorise:** $(x+3)(x-2) = 0$, so $x = -3$ or $x = 2$.

**Sum:** $-3 + 2 = -1$

*Why the others are wrong:*

- **A** (1) has a sign error on $b$ in Vieta's sum, $-b/a$.
- **C** ($-6$) reports the **product** of the roots instead of the sum.
- **D** (5) added the absolute values, $3 + 2$.

> **Takeaway:** Equate, clear fractions, solve. Vieta gives a shortcut: for $x^2 + bx + c = 0$ the roots sum to $-b$ and multiply to $c$ — so you can answer without factorising at all.

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## Section 6 — Exponential and Logarithmic Functions

### Q38 — Basic — Answer: D

**Topic:** Evaluating an exponential function

**Step 1 — Compute.**

$$f(5) = 2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

**Step 2 — Explain errors.**

A) 10: Student multiplies instead of raising to a power: $2 \times 5 = 10$.

C) 25: Student evaluates $5^2 = 25$ — swaps base and exponent.

B) 16: Student evaluates $2^4 = 16$ — uses exponent 4 instead of 5.

> **Takeaway:** $2^5$ means five factors of 2 multiplied together — not $2 \times 5$. Build the powers: $2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32$.

---

### Q39 — Basic — Answer: A

**Topic:** Horizontal asymptote of an exponential

**Step 1 — Consider the behaviour as $x \to -\infty$.**

As $x \to -\infty$: $2^x \to 0^+$, so $3 \cdot 2^x \to 3 \cdot 0 = 0$.

The horizontal asymptote is $y = 0$.

**Step 2 — Explain errors.**

C) $y = 3$: The coefficient 3 is a vertical stretch — it scales the function but does not shift the asymptote. A vertical shift (i.e., $3 \cdot 2^x + k$) would shift the HA.

B) $y = 2$: The base 2 determines the growth rate, not the asymptote.

D) $y = 6$: Student multiplies the coefficient and the base: $3 \times 2 = 6$.

> **Takeaway:** For $y = a \cdot b^x$ (with no added constant), the HA is always $y = 0$ — the coefficient $a$ only stretches the graph vertically, it does not move the asymptote.

---

### Q40 — Basic — Answer: C

**Topic:** Solving a simple exponential equation

**Step 1 — Express 64 as a power of 2.**

$$2^1 = 2, \quad 2^2 = 4, \quad 2^3 = 8, \quad 2^4 = 16, \quad 2^5 = 32, \quad 2^6 = 64$$
$$2^x = 2^6 \implies x = 6$$

**Step 2 — Explain errors.**

A) $x = 32$: Student divides: $64 \div 2 = 32$, treating the problem as $2x = 64$.

B) $x = 8$: Student recalls $8^2 = 64$ and uses the exponent 2 or the base 8 — neither is correct here.

D) $x = 4$: Student confuses with $4^3 = 64$ or $\sqrt[3]{64} = 4$ — different base.

> **Takeaway:** Rewrite both sides as powers of the same base. $64 = 2^6$, so $2^x = 2^6 \Rightarrow x = 6$. Memorise: $2^{10} = 1024$, $2^8 = 256$, $2^7 = 128$, $2^6 = 64$.

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### Q41 — Intermediate — Answer: A

**Topic:** Decreasing exponential functions

**Step 1 — Determine increasing or decreasing.**

The base is $\frac{1}{2} < 1$. For exponentials $y = b^x$: if $b < 1$, the function is decreasing (as $x$ increases, you raise a fraction to a higher power, making it smaller).

**Step 2 — Find the $y$-intercept.**

At $x = 0$: $y = \left(\frac{1}{2}\right)^0 = 1$.

**Step 3 — Explain errors.**

B) Increasing: Would hold for $b > 1$ (e.g., $y = 2^x$). With base $\frac{1}{2} < 1$, the function decreases.

C) $y$-intercept 0: Any expression $b^0 = 1$ regardless of $b$ (provided $b \neq 0$). The $y$-intercept of any basic exponential is 1.

D) $y$-intercept 2: Student reads $\frac{1}{2}$ as the base and takes its reciprocal 2 as the $y$-intercept — the $y$-intercept is not the reciprocal of the base.

> **Takeaway:** $y = b^x$ has $y$-intercept 1 for any base $b$. The base determines whether the function grows ($b > 1$) or decays ($0 < b < 1$).

---

### Q42 — Basic — Answer: B

**Topic:** Solving a logarithmic equation

**Step 1 — Convert to exponential form.**

$$\log_2(x) = 4 \iff 2^4 = x$$

$$x = 16$$

**Step 2 — Explain errors.**

A) $x = 8 = 2^3$: Off by one in the exponent.

C) $x = 4$: Student uses $x =$ the RHS value (the logarithm result) rather than converting: $\log_2(4) = 2$, not 4.

D) $x = 2$: Student uses the base as the answer.

> **Takeaway:** $\log_b(x) = n \iff x = b^n$. Here: $\log_2(x) = 4 \iff x = 2^4 = 16$.

---

### Q43 — Intermediate — Answer: D

**Topic:** Logarithm power rule

**Step 1 — Apply the power rule.**

$$\log_a(x^3) = 3\log_a x$$

The power rule states: the exponent comes down as a multiplier in front of the logarithm.

**Step 2 — Explain errors.**

A) $(\log_a x)^3$: The student cubes the logarithm instead of multiplying by 3. $(\log_a x)^3 \neq 3\log_a x$ unless $\log_a x = 3$ or $0$.

B) $\dfrac{3}{\log_a x}$: No logarithm law produces division by the logarithm.

C) $\log_a(3x)$: Student brings the exponent inside the argument as a coefficient: $x^3 \to 3x$. The product rule handles $\log_a(3x) = \log_a 3 + \log_a x$, which is unrelated.

> **Takeaway:** Power rule: $\log_a(x^n) = n\log_a x$. The exponent moves to become a multiplier, not a factor inside the argument.

---

### Q44 — Intermediate — Answer: C

**Topic:** Solving a logarithmic equation (non-trivial)

**Step 1 — Convert to exponential form.**

$$\log_3(x+1) = 2 \iff x + 1 = 3^2 = 9$$

**Step 2 — Solve for $x$.**

$$x = 9 - 1 = 8$$

**Step 3 — Explain errors.**

A) $x = 1$: Student writes $x + 1 = 2$ (converts $\log_3(\cdot) = 2$ as "the argument equals 2" instead of $3^2$).

B) $x = 9$: Student correctly reaches $x + 1 = 9$ but forgets to subtract 1, reporting $x = 9$.

D) $x = 6$: Student computes $3 \times 2 = 6$ (multiplies base by exponent instead of raising base to exponent).

> **Takeaway:** Convert $\log_b(A) = n$ to $A = b^n$ first, then solve. Here: $x + 1 = 9 \Rightarrow x = 8$.

---

### Q45 — Intermediate — Answer: C

**Topic:** Inverse of an exponential function

**Step 1 — Find the inverse by swapping $x$ and $y$.**

$$x = 3^y$$

**Step 2 — Solve for $y$ by converting to logarithm form.**

$$y = \log_3 x$$

**Step 3 — Explain errors.**

B) $y = x^3$: Confuses $3^x$ (exponential) with $x^3$ (power function). Their inverses are $\log_3 x$ and $x^{1/3}$ respectively.

A) $y = 3^{-x}$: This is a reflection of $y = 3^x$ across the $y$-axis, not its inverse.

D) $y = \frac{x}{3}$: Applies a linear-function inverse strategy (divide by 3) to an exponential.

> **Takeaway:** Exponentials and logarithms are inverses: $f(x) = b^x \iff f^{-1}(x) = \log_b x$. To find the inverse: swap $x$ and $y$, then use the definition of logarithm to isolate $y$.

---

### Q46 — Basic — Answer: B

**Topic:** $y$-intercept of a translated exponential

**Step 1 — Set $x = 0$.**

$$y = 2^{0+1} = 2^1 = 2$$

**Step 2 — Explain errors.**

A) $y = 1$: Student evaluates $2^0 = 1$ (ignores the $+1$ in the exponent, as if the function were $y = 2^x$).

C) $y = 3$: Student adds: $2 + 1 = 3$ instead of evaluating $2^1$.

D) $y = 0$: Student confuses with the $x$-intercept (which exponentials, shifted or not, rarely have).

> **Takeaway:** The $y$-intercept requires $x = 0$, not the exponent. Here, $x + 1$ becomes $0 + 1 = 1$ when $x = 0$, giving $2^1 = 2$.

---

## Section 7 — Transformations and Inverses

### Q47 — Basic — Answer: A

**Topic:** Vertical shift transformation

**Step 1 — Identify the change.**

Adding a constant outside the function increases every output by 3, shifting the graph up by 3 units.

**Step 2 — Explain errors.**

C) Horizontal shift right: A horizontal shift of 3 right would give $y = f(x-3)$, not $f(x)+3$.

B) Vertical stretch: A vertical stretch by factor 3 would give $y = 3f(x)$, not $f(x)+3$.

D) Downward: Adding $+3$ moves up. A downward shift of 3 would be $y = f(x) - 3$.

> **Takeaway:** $y = f(x) + k$: vertical shift UP by $k$ (if $k > 0$). $y = f(x) - k$: vertical shift DOWN.

---

#### Q48 — Basic — Answer: D

**Topic:** Horizontal shift direction

**Step 1 — Apply the rule.**

Replacing $x$ with $x - h$ shifts the graph right by $h$ units. Here $h = 5$: shift right by 5.

**Step 2 — Reason intuitively.**

The graph reaches its original position at $x - 5 = 0$, i.e., $x = 5$. The "action" moves to the right.

**Step 3 — Explain errors.**

A) Left by 5: The most common confusion. $f(x-5)$ shifts right; $f(x+5)$ shifts left. The sign inside the argument is opposite to the direction of shift.

B, C) Vertical shifts: Adding or subtracting a constant inside the argument creates a horizontal shift, not a vertical one.

> **Takeaway:** $y = f(x-h)$ shifts right by $h$; $y = f(x+h)$ shifts left by $h$. Remember the counterintuitive rule: the sign inside goes in the opposite direction.

---

#### Q49 — Intermediate — Answer: D

**Topic:** Reflection across the $x$-axis

**Step 1 — Reasoning.**

$y = -f(x)$ negates every output: a point $(x, y)$ on the original becomes $(x, -y)$ on the transformed graph. Changing $y$ to $-y$ is the definition of reflection across the $x$-axis.

**Step 2 — Explain errors.**

B) $y$-axis: Reflecting across the $y$-axis changes $x \to -x$, giving $y = f(-x)$ — the argument is negated, not the output.

C) $y = x$: This gives the inverse function (swap $x$ and $y$).

A) Origin: Reflection through the origin changes $(x, y) \to (-x, -y)$, giving $y = -f(-x)$.

> **Takeaway:** Negate the output ($-f(x)$) → reflect in the $x$-axis. Negate the input ($f(-x)$) → reflect in the $y$-axis.

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#### Q50 — Intermediate — Answer: A

**Topic:** Reflection across the $y$-axis

**Step 1 — Reasoning.**

Replacing $x$ with $-x$ means every point $(x, y)$ becomes $(-x, y)$: the $y$-coordinate is unchanged and the $x$-coordinate is negated. This is reflection across the $y$-axis.

**Step 2 — Distinguish from the previous question.**

Q49: $-f(x)$ → negate output → $x$-axis reflection. Q50: $f(-x)$ → negate input → $y$-axis reflection.

*Why the others are wrong:* **B** ($x$-axis) belongs to $-f(x)$ — the outside negative (see Q49). **C** (the line $y = x$) is the *inverse function's* reflection, $f^{-1}$ — a different operation entirely. **D** (origin) would need *both* negations at once: $-f(-x)$.

> **Takeaway:** The position of the negative sign tells you which axis: outside the function (output negated) = $x$-axis; inside the argument (input negated) = $y$-axis. Both at once = point reflection through the origin; swap $x$ and $y$ = reflection in $y = x$.

---

#### Q51 — Intermediate — Answer: B

**Topic:** Horizontal vs vertical stretch/compression

**Step 1 — Analyse $f(2x)$.**

The graph reaches in $x$ what the original reached in $2x$: it gets there "twice as fast" horizontally. Distances along the $x$-axis are halved.

**Step 2 — Define compression vs stretch.**

A compression by factor 2 means $x$-distances are halved ($\times\frac{1}{2}$). A stretch by factor 2 would double $x$-distances.

**Step 3 — Explain errors.**

A) Horizontal stretch: That would be $f\left(\frac{x}{2}\right)$ — dividing inside the argument slows the approach, stretching horizontally.

C) Vertical stretch: A vertical stretch by 2 gives $2f(x)$, not $f(2x)$.

D) Vertical compression: Gives $\frac{1}{2}f(x)$, not $f(2x)$.

> **Takeaway:** Multiplying inside the argument — $f(kx)$ with $k > 1$ — compresses horizontally (divides $x$-scale by $k$). Dividing inside — $f(x/k)$ — stretches horizontally.

---

### Q52 — Basic — Answer: C

**Topic:** Finding the inverse of a linear function

**Step 1 — Replace $f(x)$ with $y$.**

$$y = 2x + 5$$

**Step 2 — Swap $x$ and $y$.**

$$x = 2y + 5$$

**Step 3 — Solve for $y$.**

$$2y = x - 5 \implies y = \frac{x-5}{2}$$
$$f^{-1}(x) = \frac{x-5}{2}$$

**Step 4 — Explain errors.**

A) $2x - 5$: Student changes the sign of the constant but keeps the coefficient of $x$, applying a "subtract" rather than a proper inversion.

B) $\frac{x+5}{2}$: Student swaps the operations but keeps the wrong sign: adds 5 instead of subtracting 5.

D) $5x + 2$: Student swaps the coefficients 2 and 5, a common confusion.

> **Takeaway:** To find an inverse: (1) swap $x$ and $y$, (2) solve for $y$. Each operation in the original is undone in reverse order: $+5$ becomes $-5$; $\times 2$ becomes $\div 2$.

---

### Q53 — Intermediate — Answer: A

**Topic:** Inverse of a quadratic (with domain restriction)

**Swap and solve:** $x = y^2$ gives $y = \pm\sqrt{x}$.

**Choose the branch.** The original was restricted to $x \geq 0$, so take the **positive** root: $y = \sqrt{x}$. Its domain is the original's range, $x \geq 0$.

*Why the others are wrong:*

- **B** (all real $x$) — $\sqrt{x}$ does not exist for negative $x$.
- **C** ($x > 0$) excludes 0, but $\sqrt{0} = 0$ is valid and matches the original vertex.
- **D** ($\pm\sqrt{x}$) is not a **function** — two outputs per input. The restriction existed precisely to prevent that.

> **Takeaway:** $y = x^2$ has no inverse function until you restrict it — it fails the horizontal line test. Restricting to $x \geq 0$ makes it one-to-one, giving $y = \sqrt{x}$, positive branch only.

---

### Q54 — Basic — Answer: D

**Topic:** Composing a function with its inverse

**Step 1 — Apply the definition of inverse.**

By definition, $f(f^{-1}(x)) = x$ for all $x$ in the domain of $f^{-1}$.

$$f(f^{-1}(7)) = 7$$

**Step 2 — Explain errors.**

A) $f(7)$: Student does not apply $f^{-1}$ at all — treats the composition as just $f$.

C) 1: No operation on a function composed with its inverse produces 1 in general.

B) 14: Student doubles the input for no clear reason.

> **Takeaway:** $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. A function and its inverse cancel each other out — they are mutual "undoers."

---

### Q55 — Proficient — Answer: B

**Topic:** Finding the inverse of a rational function

**Step 1 — Swap $x$ and $y$.**

$$x = \frac{3}{y - 1}$$

**Step 2 — Solve for $y$.**

$$x(y-1)=3$$
$$y-1=\frac{3}{x}$$
$$y=\frac{3}{x}+1$$
$$f^{-1}(x)=\frac{3}{x}+1$$

**Step 3 — Explain errors.**

A) $\dfrac{3}{1-x}$: Student reflects the denominator sign but does not solve for $y$ properly. The result $\frac{3}{1-x}$ is the same as $\frac{-3}{x-1}$, which is a different function entirely.

C) $\dfrac{x-1}{3}$: Student "cancels" the fraction by moving the denominator to the numerator and dividing — this is the inverse of $y=3(x-1)$, not $y=\frac{3}{x-1}$.

D) $\dfrac{x}{4}$: Student adds the numerator and denominator: $3+1=4$. This bears no algebraic relationship to the inverse.

> **Takeaway:** Swap $x$ and $y$, then isolate $y$ through legitimate algebraic steps. Check by verifying $f(f^{-1}(x))=x$: $f\left(\frac{3}{x}+1\right)=\frac{3}{\left(\frac{3}{x}+1\right)-1}=\frac{3}{\frac{3}{x}}=x$ ✓.

---

## Section 8 — Interpreting Graphs

### Q56 — Intermediate — Answer: C

**Topic:** Reading the gradient of a described graph

**Step 1 — Apply the gradient formula.**

$$m=\frac{4-0}{0-(-2)}=\frac{4}{2}=2$$

**Step 2 — Explain errors.**

A) 4: Student reports the $y$-intercept instead of the gradient.

B) $-2$: Student reports the $x$-intercept value instead of the gradient.

D) $\dfrac{1}{2}$: Student inverts the fraction (run over rise instead of rise over run).

> **Takeaway:** Gradient = $\dfrac{\text{rise}}{\text{run}} = \dfrac{\Delta y}{\Delta x}$. Two clean points are given; read them directly and apply the formula without rearranging.

---

#### Q57 — Basic — Answer: B

**Topic:** Where a quadratic function is negative

**Zeros:** $x^2 - 4 = (x-2)(x+2) = 0$, so $x = \pm 2$.

**Where is it negative?** The parabola opens **upward**, so it dips below the axis **between** its zeros:

$$-2 < x < 2$$

*Why the others are wrong:*

- **A** gives where the parabola is **positive** — outside the zeros.
- **C** gives the zeros themselves, where $y = 0$, not $y < 0$.
- **D** (all reals) fails at $x = 3$, where $y = 5 > 0$.

> **Takeaway:** An upward parabola with zeros $a < b$ is **negative between** them and **positive outside** them.

---

#### Q58 — Intermediate — Answer: D

**Topic:** Increasing and decreasing intervals

**Step 1 — Identify the vertex and direction.**

Vertex at $(2, 5)$; $a = -1 < 0 \rightarrow$ parabola opens downward.

**Step 2 — Determine increasing/decreasing sides.**

A downward parabola rises to its vertex and then falls. It is increasing to the left of the vertex ($x < 2$) and decreasing to the right ($x > 2$).

**Step 3 — Explain errors.**

A) $x > 2$: The function is decreasing after the vertex (past the maximum), not increasing.

C) All real $x$: Parabolas are monotonic on each half, not everywhere.

B) $x = 2$ only: This is where $f$ achieves its maximum — not a meaningful "increasing" interval.

> **Takeaway:** For $a(x-p)^2+q$: if $a<0$ (opens down), increasing for $x<p$ and decreasing for $x>p$. Reverse if $a>0$.

---

#### Q59 — Intermediate — Answer: A

**Topic:** Sign of a function on an interval

**Step 1 — Find the zeros.**

$(x-1)^2-9=0 \Rightarrow (x-1)^2=9 \Rightarrow x-1=\pm 3.$

$$x=1+3=4 \quad \text{or} \quad x=1-3=-2$$

**Step 2 — Determine where $f<0$.**

$a=1>0$: upward parabola. Negative between its zeros: $-2<x<4$.

**Step 3 — Explain errors.**

C) Outside interval: The upward parabola is positive outside its zeros, not inside.

B) $-3<x<3$: Student uses the zeros of $(x-1)^2=9$ as $x=\pm 3$ (ignoring the $-1$ shift).

D) $x>4$ only: Incomplete — misses the left zero.

> **Takeaway:** First find the zeros; then use the direction of opening to determine the sign in each region. This is the same strategy as solving quadratic inequalities.

---

#### Q60 — Basic — Answer: D

**Topic:** Maximum value of a quadratic

**Step 1 — Find the $x$-coordinate of the vertex.**

$$x=-\frac{b}{2a}=-\frac{8}{2(-2)}=-\frac{8}{-4}=2$$

**Step 2 — Evaluate at $x=2$.**

$$y=-2(4)+8(2)-5=-8+16-5=3$$

**Step 3 — Explain errors.**

A) $y = 5$: Student reports the negative of the constant: $|- -5| = 5$.

B) $y = 8$: Student reports $b = 8$ as the maximum value.

C) $y = 11$: Student drops the $-2x^2$ term entirely and evaluates only $8x - 5$ at $x = 2$: $8(2) - 5 = 16 - 5 = 11$. Every term must be included when substituting the vertex $x$-value back in.

> **Takeaway:** Always evaluate the vertex: substitute $x = -\frac{b}{2a}$ back into the original equation. Since $a = -2 < 0$, the vertex is a maximum.

---

#### Q61 — Intermediate — Answer: A

**Topic:** Using graphs to solve inequalities

**Find where they cross:**

$$x^2 - 4 = x + 2 \implies x^2 - x - 6 = (x-3)(x+2) = 0 \implies x = 3 \text{ or } -2$$

**Test between them.** At $x = 0$: the parabola gives $-4$, the line gives $2$ — so the parabola is **below** in the middle. It must therefore be **above** on both outer pieces:

$$x < -2 \quad \text{or} \quad x > 3$$

*Why the others are wrong:*

- **B** is where the **line** is above the parabola.
- **C** and **D** give only one side. An upward parabola beats the line on **both** outer regions.

> **Takeaway:** Rewrite "parabola above line" as one inequality, $x^2 - x - 6 > 0$, then use the rule: an upward parabola is positive **outside** its zeros. Always confirm with a test value.

---

#### Q62 — Basic — Answer: C

**Topic:** Identifying a function by its range

**Step 1 — Check each range.**

A) $y = x$: Range is all real numbers (increases without bound in both directions).

C) $y = x^2$: Minimum at $x = 0$ giving $y = 0$; all other outputs are positive. Range: $y \geq 0$. ✓

B) $y = \dfrac{1}{x}$: Takes all real values except 0. Range: $y \neq 0$.

D) $y = x^3$: Takes all real values. Range: all real numbers.

> **Takeaway:** The range $y \geq 0$ is characteristic of even-powered functions like $x^2$, $x^4$, and $|x|$. The squaring operation eliminates negative outputs.

---

**Q63 — Intermediate — Answer: B**

**Topic:** Identifying a non-function from a description

**Step 1 — Apply the vertical line test.**

A) Parabola opening upward: any vertical line crosses it at most once. ✓ (Is a function.)

C) Exponential curve $y = b^x$: strictly increasing or decreasing — any vertical line crosses it once. ✓ (Is a function.)

B) Circle $x^2 + y^2 = r^2$: a vertical line through the interior crosses it at two points (one above and one below the $x$-axis). ✗ (NOT a function.)

D) Logarithmic curve $y = \log_b x$: defined for $x > 0$, strictly monotonic — any vertical line crosses it at most once. ✓ (Is a function.)

> **Takeaway:** The vertical line test is the graphical equivalent of the function definition. A full circle fails it visibly — draw $x = 0$ and it intersects the circle at $(0, r)$ and $(0, -r)$: two outputs for one input.

---

**Q64 — Intermediate — Answer: D**

**Topic:** Reading a horizontal asymptote from an equation

**Step 1 — Identify the vertical shift.**

The function is in the form $y = \dfrac{a}{x+p} + q$ with $q = -1$.

As $x \to \pm\infty$, $\dfrac{2}{x+3} \to 0$, so $y \to -1$.

**Horizontal asymptote:** $y = -1$.

**Step 2 — Explain errors.**

A) $y = 2$: Student reads the numerator as the HA.

B) $y = 3$: Student reads the denominator shift value as the HA (this is the VA: $x = -3$).

C) $y = 0$: Correct for the un-shifted hyperbola $y = \frac{2}{x}$ — ignores the $-1$ translation.

**Takeaway:** For $y = \frac{a}{x+p} + q$: HA is $y = q$, VA is $x = -p$. The $-1$ outside the fraction shifts the whole graph (including the asymptote) down by 1.

---

### Q65 — Proficient — Answer: A

**Topic:** Identifying an equation from multiple graph features

**Eliminate one feature at a time.**

**Horizontal asymptote** $y = 1$ means $q = 1$ — that rules out **C**.

**Vertical asymptote** $x = -2$ means the denominator is $(x + 2)$ — that rules out **B**, whose $(x - 2)$ gives an asymptote at $x = 2$.

**The $y$-intercept** $(0, 3)$ settles the last two. Option A: $1 + \frac{4}{2} = 3$ ✓ Option D: $1 - \frac{4}{2} = -1$ ✗

**Takeaway:** For $y = q + \frac{a}{x+p}$: the horizontal asymptote gives $q$, the vertical asymptote gives $p$, and any point on the graph gives $a$. Knock out options one feature at a time rather than solving from scratch.

---

## Section 9 — Absolute-Value Functions (V-Graphs)

### Q66 — Basic — Answer: B

**Topic:** Vertex and shape of an absolute-value graph

Compare with $y = a|x - p| + q$: here $p = 2$, $q = 1$ and $a = +1$. So the vertex is $(2, 1)$ and the V opens **upward**.

The vertex sits where the bracket is zero. Setting $x - 2 = 0$ gives $x = 2$, and then $y = 0 + 1 = 1$.

Check the symmetry: $x = 0$ and $x = 4$ both give $y = 3$ ✓

*Why the others are wrong:*

- **A** reads the sign of $p$ straight off the page. $|x - 2|$ shifts **right** by 2, just as $(x - 2)^2$ does.
- **C** flips both the sign of $q$ and the direction. The $+1$ is outside the bars, so it lifts the graph.
- **D** has the vertex right but the direction wrong. The coefficient is $+1$, so the arms point up.

> **Takeaway:** $y = a|x-p| + q$ has its vertex at $(p, q)$ — read exactly like a parabola's vertex form. The sign of $a$ alone decides up or down.

---

### Q67 — Intermediate — Answer: C

**Topic:** Solving an absolute-value equation

$|X| = 8$ means the inside is **8 units from zero** — so it can be $+8$ **or** $-8$. Split into two ordinary equations:

$$2x - 6 = 8 \implies 2x = 14 \implies x = 7$$
$$2x - 6 = -8 \implies 2x = -2 \implies x = -1$$

Check both in the original: $|2(7) - 6| = |8| = 8$ ✓ and $|2(-1) - 6| = |-8| = 8$ ✓

*Why the others are wrong:*

- **A** solves only the positive branch and stops. Every $|X| = k$ with $k > 0$ has **two** answers.
- **B** deletes the $-6$, solving $|2x| = 8$.
- **D** takes the second branch but mishandles it — halving before moving the 6, giving $4 - 6 = -2$. Undo the $-6$ first, then divide.

> **Takeaway:** $|X| = k$ (for $k > 0$) always splits into $X = k$ **and** $X = -k$. Solve both, then substitute both back — the check costs seconds and catches the branch you dropped.

---

### Q68 — Intermediate — Answer: B

**Topic:** Range of a downward V-graph

$|x|$ is never negative, so $-|x|$ is never **positive** — it is 0 at $x = 0$ and negative everywhere else. Adding 4:

$$y = -|x| + 4 \leq 4$$

The V opens **downward** with vertex $(0, 4)$, so 4 is the **maximum**. Test a point: $x = 3$ gives $y = 1$, below 4 ✓

*Why the others are wrong:*

- **A** treats the vertex as a minimum. The minus sign in front turns the V upside down.
- **C** ($y \geq 0$) is the range of plain $y = |x|$, ignoring both the reflection and the shift.
- **D** would need the graph to climb without bound, but nothing here ever exceeds 4.

> **Takeaway:** For $y = a|x - p| + q$: if $a > 0$ the range is $y \geq q$; if $a < 0$ it is $y \leq q$. Identical to the rule for parabolas.

---

#### Q69 — Proficient — Answer: A

**Topic:** Recognising an impossible absolute-value equation

**Isolate the absolute value first:**

$$|x - 3| = 2 - 5 = -3$$

An absolute value is a **distance**, so it is never negative. No real $x$ works — the solution set is empty.

The graph agrees: $y = |x - 3| + 5$ is a V with vertex $(3, 5)$, so its lowest point is $y = 5$. The line $y = 2$ passes below it and never meets it.

*Why the others are wrong:*

- **C** (two) is the reflex answer. Two is the usual count, but only when the isolated absolute value is **positive**.
- **B** (one) happens only when the isolated value is exactly **zero** — say $|x - 3| = 0$, giving $x = 3$.
- **D** would need the equation to hold for every $x$.

> **Takeaway:** Isolate the absolute value, then read off the count: a **positive** right-hand side gives two solutions, **zero** gives one, **negative** gives none.

---

## Exam-Bank Extras — Question Types Confirmed in Recent Papers

#### Q70 — Proficient — Answer: C

**Topic:** Inverse of a rational function

The recipe never changes: **swap $x$ and $y$, then dig $y$ out.**

**Swap:** $x = \dfrac{2y}{1 - 3y}$

**Clear the fraction:** $x(1 - 3y) = 2y \implies x - 3xy = 2y$

**Collect every $y$-term and factor it out** — this is the step people miss, because $y$ appears twice:

$$x = y(2 + 3x) \implies y = \frac{x}{3x + 2}$$

Check: $f(1) = -1$ and $f^{-1}(-1) = 1$ ✓

*Why the others are wrong:*

- **B** is the **reciprocal** $\frac{1}{f(x)}$, not the inverse — the classic confusion.
- **D** moved $-3xy$ across without flipping its sign.
- **A** never swapped $x$ and $y$ at all — it rearranged $f$ instead of inverting it.

> **Takeaway:** Swap, clear the fraction, collect **all** $y$-terms, factor $y$ out, divide. The factoring step is the whole difficulty; the rest is ordinary algebra.

---

### Q71 — Proficient — Answer: A

**Topic:** Reversing a sequence of transformations

To find the **original**, rewind the transformations — backwards, in reverse order.

**Undo the reflection** (replace $y$ with $-y$):

$$y = -\frac{2}{x-1}$$

**Undo "3 left" by shifting 3 right** (replace $x$ with $x-3$):

$$y = -\frac{2}{(x-3)-1} = -\frac{2}{x-4}$$

**Check by running forwards:** shift 3 left → $-\frac{2}{x-1}$; reflect → $\frac{2}{x-1}$ ✓

*Why the others are wrong:*

- **C** shifted 3 left **again** instead of reversing.
- **B** rewound the shift but never the reflection — the minus sign must come back.
- **D** made both errors at once.

> **Takeaway:** "Before the transformation" means invert each step **and reverse their order**. Then spend ten seconds running your answer forwards — it turns a risky question into a certain one.

---

### Q72 — Proficient — Answer: B

**Topic:** Circle centred at the origin touching a hyperbola

A circle at the origin is $x^2 + y^2 = r^2$, so the question is really: **how far is the nearest point of the hyperbola from the origin?**

By symmetry, that nearest point sits **on the line** $y = x$. Set $y = x$:

$$x = \frac{8}{x} \implies x^2 = 8 \implies x = 2\sqrt{2}$$

The touching point is $(2\sqrt{2},\ 2\sqrt{2})$, so by Pythagoras:

$$r^2 = 8 + 8 = 16$$

*Why the others are wrong:*

- **A** (8) squared just **one** coordinate, forgetting to add the second.
- **C** (4) confused $r = 4$ with $r^2 = 4$.
- **D** (64) squared the hyperbola's constant. That 8 is not the radius.

> **Takeaway:** For $y = k/x$, the closest point to the origin always lies on $y = x$ at $x = \sqrt{k}$, giving the neat general result $x^2 + y^2 = 2k$ — here 16.

## Mixed Practice — Chapter 3

These 30 questions mix all Functions and Graphs topics. Work through them without looking back at the sections. Check your answers with the key on the following page — detailed explanations are not given here, so use the earlier sections to diagnose any errors.

---

### M1 — Basic

If $f(x) = x^2 - 3x + 1$, find $f(2)$.

A) $7$

B) $3$

C) $11$

D) $-1$

---

### M2 — Basic

What is the domain of $f(x) = \sqrt{2 - x}$?

A) $x \leq 2$

B) $x > 2$

C) $x \geq 2$

D) All real $x$

---

### M3 — Basic

What is the gradient of the line $y = 4x - 7$?

A) $-7$

B) $7$

C) $4$

D) $-4$

---

### M4 — Basic

Find the inverse of $f(x) = x + 6$.

A) $f^{-1}(x) = 6x$

B) $f^{-1}(x) = x - 6$

C) $f^{-1}(x) = x + 6$

D) $f^{-1}(x) = 6 - x$

---

### M5 — Basic

Evaluate $f(f^{-1}(12))$ for any function $f$.

A) 12

B) $f(12)$

C) 144

D) 6

---

### M6 — Intermediate

What are the roots of $x^2 - 9 = 0$?

A) $x = 9$ only

B) $x = 3$ only

C) $x = 3$ or $x = -3$

D) $x = \pm\sqrt{3}$

---

### M7 — Basic

$y = f(x + 2)$ represents which transformation of $y = f(x)$?

A) Shift right 2 units

B) Shift down 2 units

C) Shift up 2 units

D) Shift left 2 units

---

### M8 — Basic

What is the horizontal asymptote of $y = \left(\frac{1}{3}\right)^x$?

A) $y = 3$

B) $y = 0$

C) $y = 1$

D) $y = \frac{1}{3}$

---

### M9 — Basic

Evaluate $\log_5(25)$.

A) 2

B) 5

C) $\frac{1}{2}$

D) 25

---

### M10 — Basic

What is the range of $f(x) = \sqrt{x}$?

A) All real numbers

B) $x \geq 0$

C) $y > 0$

D) $y \geq 0$

---

### M11 — Basic

Write the equation of the line through $(0,\ 4)$ with gradient $-2$.

A) $y = -2x - 4$

B) $y = -2x + 4$

C) $y = 2x + 4$

D) $y = 4x - 2$

---

### M12 — Basic

What is the vertex of $y = (x-3)^2 + 2$?

A) $(3,\ -2)$

B) $(-3,\ 2)$

C) $(3,\ 2)$

D) $(-3,\ -2)$

---

### M13 — Intermediate

In which quadrants does the graph of $y = \dfrac{2}{x}$ lie?

A) Quadrants 1 and 2

B) Quadrants 2 and 3

C) Quadrants 1 and 3

D) Quadrants 2 and 4

---

### M14 — Basic

Evaluate $\log_{10}(1000)$.

A) 3

B) 100

C) 10

D) 4

---

### M15 — Intermediate

How many real roots does $x^2 + 2x + 5 = 0$ have?

A) 2

B) Cannot be determined

C) 1

D) 0

---

### M16 — Intermediate

Which function is even?

A) $f(x) = x^3 - x$

B) $f(x) = x^4 - 2x^2$

C) $f(x) = x^2 + x$

D) $f(x) = 2x + 1$

---

### M17 — Basic

Solve $3^x = 81$.

A) $x = 27$

B) $x = 3$

C) $x = 4$

D) $x = 12$

---

### M18 — Basic

Find the axis of symmetry of $y = x^2 - 10x + 3$.

A) $x = 10$

B) $x = 5$

C) $x = 3$

D) $x = -5$

---

### M19 — Basic

Which function has a vertical asymptote at $x = 4$?

A) $y = \dfrac{1}{x - 4}$

B) $y = \dfrac{1}{x + 4}$

C) $y = 4^x$

D) $y = \log_4 x$

---

### M20 — Intermediate

If $f(x) = x + 1$ and $g(x) = 2x$, find $f(g(x))$.

A) $2(x + 1)$

B) $2x^2$

C) $x + 2$

D) $2x + 1$

---

### M21 — Intermediate

Solve $\log_2(x - 1) = 3$.

A) $x = 9$

B) $x = 4$

C) $x = 7$

D) $x = 3$

---

### M22 — Intermediate

What is the domain of $f(x) = \dfrac{1}{\sqrt{x - 4}}$?

A) $x \geq 4$

B) $x \neq 4$

C) $x > 4$

D) $x \geq 0$

---

### M23 — Basic

Find the $y$-intercept of $y = 5 \cdot 2^x$.

A) $y = 10$

B) $y = 5$

C) $y = 2$

D) $y = 0$

---

### M24 — Basic

Which is the reflection of $y = x^2$ across the $x$-axis?

A) $y = \dfrac{1}{x^2}$

B) $y = x^2 + 1$

C) $y = (-x)^2$

D) $y = -x^2$

---

### M25 — Intermediate

What is the range of $y = -(x+2)^2 + 7$?

A) $y \leq 7$

B) $y \geq -7$

C) $y \geq 7$

D) $y \leq -7$

---

### M26 — Intermediate

Solve $2^{x+1} = 16$.

A) $x = 4$

B) $x = 7$

C) $x = 3$

D) $x = 8$

---

### M27 — Intermediate

$f(x) = 3x - 2$. Find $f^{-1}(4)$.

A) 10

B) 2

C) $\dfrac{4}{3}$

D) $-2$

---

### M28 — Basic

Which set of ordered pairs is **NOT** a function?

A) $\{(1,2),\ (2,4),\ (3,6)\}$

B) $\{(1,1),\ (2,2),\ (3,3)\}$

C) $\{(0,0),\ (1,1),\ (2,4)\}$

D) $\{(1,2),\ (2,3),\ (2,4)\}$

---

### M29 — Intermediate

A line perpendicular to $y = \frac{1}{2}x + 3$ has gradient:

A) $\frac{1}{2}$

B) $2$

C) $-2$

D) $-\frac{1}{2}$

---

### M30 — Basic

What transformation maps $y = f(x)$ to $y = f(x) + 4$?

A) Vertical shift up 4

B) Vertical stretch by 4

C) Vertical shift down 4

D) Horizontal shift right 4

## Mixed Practice — Answer Key

| | | | | |
|---|---|---|---|---|
| **M1.** D | **M2.** A | **M3.** C | **M4.** B | **M5.** A |
| **M6.** C | **M7.** D | **M8.** B | **M9.** A | **M10.** D |
| **M11.** B | **M12.** C | **M13.** C | **M14.** A | **M15.** D |
| **M16.** B | **M17.** C | **M18.** B | **M19.** A | **M20.** D |
| **M21.** A | **M22.** C | **M23.** B | **M24.** D | **M25.** A |
| **M26.** C | **M27.** B | **M28.** D | **M29.** C | **M30.** A |

# Chapter 4 — Trigonometry

## Section 4.1 — SOHCAHTOA and Basic Trig Ratios

### Q1 — Basic

A right triangle has sides of length 5, 12, and 13. The angle $\theta$ is at the vertex where the sides of length 12 and 13 meet (so the side opposite $\theta$ has length 5). What is $\tan\theta$?

A) $\frac{5}{13}$
B) $\frac{5}{12}$
C) $\frac{12}{13}$
D) $\frac{12}{5}$

---

### Q2 — Basic

If $\cos\theta = \frac{7}{25}$ and $\theta$ is an acute angle, what is $\sin\theta$?

A) $\frac{24}{25}$
B) $\frac{7}{24}$
C) $\frac{25}{24}$
D) $\frac{7}{25}$

---

### Q3 — Basic

Which of the following correctly expresses $\cot\theta$?

A) $\frac{\sin\theta}{\cos\theta}$
B) $\frac{\text{opposite}}{\text{adjacent}}$
C) $\frac{\text{hypotenuse}}{\text{adjacent}}$
D) $\frac{\cos\theta}{\sin\theta}$

---

### Q4 — Basic

A right triangle has a hypotenuse of 20 cm and an angle of 60°. What is the length of the side opposite the 60° angle?

A) 10 cm

B) $10\sqrt{2}$ cm

C) $10\sqrt{3}$ cm

D) $\frac{20}{\sqrt{3}}$ cm

---

### Q5 — Intermediate

In triangle $ABC$, the right angle is at $C$. The hypotenuse $AB = 13$ and side $BC = 5$. What is the value of $\sin A + \cos A$?

A) $\frac{60}{169}$

B) 1

C) $\frac{7}{13}$

D) $\frac{17}{13}$

---

### Q6 — Intermediate

Given that $\csc\theta = \frac{5}{3}$, find the value of $1 + \cot^2\theta$.

A) $\frac{16}{9}$

B) $\frac{25}{9}$

C) 1

D) $\frac{5}{3}$

---

### Q7 — Intermediate

Which of the following is the value of $(1 - \cos^2\theta)(1 + \cot^2\theta)$?

A) $1$

B) $\csc^2\theta$

C) $\cos^2\theta$

D) $\tan^2\theta$

---

### Q8 — Proficient

Given that $\tan\alpha = \dfrac{3}{4}$ and $\alpha$ is an acute angle, what is the value of $\dfrac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$?

A) $\dfrac{1}{7}$

B) $7$

C) $-\dfrac{1}{7}$

D) $-7$

---

## Section 4.2 — Trigonometry in Right-Angled Triangles

### Q9 — Basic

A ladder 8 m long leans against a wall. The base of the ladder is 4 m from the wall. At what angle (to the nearest degree) does the ladder make with the ground?

A) 45°

B) 30°

C) 60°

D) 90°

---

### Q10 — Basic

A ramp rises 2 m over a horizontal distance of 5 m. What is the angle of inclination of the ramp (to 1 decimal place)?

A) 68.2°

B) 24.0°

C) 23.6°

D) 21.8°

---

### Q11 — Intermediate

From the top of a cliff 40 m high, the angle of depression to a boat at sea is 35°. How far is the boat from the base of the cliff (to 1 decimal place)?

A) 57.1 m

B) 22.9 m

C) 28.0 m

D) 32.8 m

---

### Q12 — Intermediate

Two buildings stand 50 m apart on level ground. From the top of the shorter building, the angle of elevation to the top of the taller building is 25°, and the angle of depression to the base of the taller building is 15°. What is the height of the taller building (to 1 decimal place)?

A) 23.3 m

B) 36.7 m

C) 50.0 m

D) 13.4 m

---

### Q13 — Intermediate

In right triangle $PQR$, the right angle is at $R$. $PR = 9$ cm and $QR = 12$ cm. What is the length of $PQ$ ?

A) 7.5 cm

B) 21 cm

C) $\sqrt{63}$ cm

D) 15 cm

---

### Q14 — Proficient

An observer measures the angle of elevation to the top of a tower as 42°. After walking 80 m directly toward the tower, the angle of elevation is 68°. Find the height of the tower (to 1 decimal place).

A) 113.2 m

B) 72.0 m

C) 161.5 m

D) 52.6 m

---

### Q15 — Proficient

In rectangle $ABCD$, the diagonal $AC = 26$ cm and $\angle CAB = 67^\circ$. Find the length of side $BC$ (to 1 decimal place).

A) 10.2 cm

B) 28.2 cm

C) 23.9 cm

D) 24.2 cm

---

## Section 4.3 — Special Angles (30°, 45°, 60°)

### Q16 — Basic

What is the exact value of $\sin 30^\circ + \cos 60^\circ$?

A) $\sqrt{3}$

B) 1

C) $\sqrt{2}$

D) $\dfrac{\sqrt{3}}{2}$

---

### Q17 — Basic

Which of the following is equal to $\tan 45^\circ$?

A) 1

B) $\sqrt{2}$

C) $\sqrt{3}$

D) $\dfrac{1}{\sqrt{2}}$

---

### Q18 — Intermediate

Without a calculator, find the exact value of $\sin^2 60^\circ + \cos^2 30^\circ - 1$.

A) 0

B) $\dfrac{3}{2}$

C) $\dfrac{1}{2}$

D) $\dfrac{1}{4}$

---

### Q19 — Intermediate

Evaluate $\dfrac{2\tan 30^\circ}{1-\tan^2 30^\circ}$.

A) $\dfrac{2}{\sqrt{3}}$

B) $\sqrt{3}$

C) 2

D) $\dfrac{1}{\sqrt{3}}$

---

### Q20 — Intermediate

If $\sin\theta = \cos\theta$ and $0^\circ \leq \theta \leq 90^\circ$, which of the following must be true?

A) $\theta = 30^\circ$

B) $\sin\theta = 1$

C) $\cos\theta = 0$

D) $\tan\theta = 1$

---

### Q21 — Proficient

Find the exact value of $\sin 75^\circ$.

A) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

B) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

C) $\dfrac{\sqrt{2}+1}{2}$

D) $\dfrac{\sqrt{3}}{2}$

---

## Section 4.4 — Unit Circle and Reference Angles

### Q22 — Basic

What is the reference angle for $240°$?

A) $30°$

B) $120°$

C) $60°$

D) $300°$

---

### Q23 — Basic

In which quadrant does the terminal side of $310°$ lie?

A) Quadrant I

B) Quadrant II

C) Quadrant III

D) Quadrant IV

---

### Q24 — Intermediate

If $\cos\theta < 0$ and $\sin\theta < 0$, in which quadrant does the terminal side of $\theta$ lie?

A) Quadrant III

B) Quadrant II

C) Quadrant IV

D) Quadrant I

---

### Q25 — Intermediate

If $\sin\theta = -\dfrac{5}{13}$ and $\cos\theta > 0$, find $\tan\theta$.

A) $-\dfrac{5}{13}$

B) $\dfrac{5}{12}$

C) $-\dfrac{12}{5}$

D) $-\dfrac{5}{12}$

---

### Q26 — Intermediate

What is $\cos(180^\circ + \theta)$ in terms of $\cos\theta$?

A) $-\cos\theta$

B) $\cos\theta$

C) $\sin\theta$

D) $-\sin\theta$

---

### Q27 — Proficient

If $\tan\theta = -\sqrt{3}$ and $90^\circ < \theta < 180^\circ$, find $\sin\theta$.

A) $-\dfrac{\sqrt{3}}{2}$

B) $\dfrac{1}{2}$

C) $\dfrac{\sqrt{3}}{2}$

D) $\dfrac{2}{\sqrt{3}}$

---

### Q28 — Proficient

Simplify $\sin(360^\circ - \theta) + \cos(180^\circ + \theta)$.

A) $\sin\theta + \cos\theta$

B) $-(\sin\theta + \cos\theta)$

C) $-\sin\theta + \cos\theta$

D) $\sin\theta - \cos\theta$

## Section 4.5 — Trigonometric Identities

### Q29 — Basic

Which of the following is equivalent to $1 - \sin^2\theta$?

A) $\cos^2\theta$

B) $1 + \cos^2\theta$

C) $-\cos^2\theta$

D) $\sin^2\theta$

### Q30 — Intermediate

Simplify $\dfrac{\sin^2\theta - 1}{\cos\theta}$.

A) $\cos\theta$

B) $\tan\theta$

C) $-\sin^2\theta$

D) $-\cos\theta$

### Q31 — Intermediate

Which expression is equivalent to $(\tan\theta + \cot\theta)\sin\theta\cos\theta$?

A) $\sin 2\theta$

B) $1$

C) $\tan\theta$

D) $2$

### Q32 — Intermediate

Given that $\sin\theta = \frac{4}{5}$, find the exact value of $\frac{1-\cos^2\theta}{\sin\theta}$.

A) $\frac{16}{25}$

B) $\frac{3}{5}$

C) $\frac{4}{5}$

D) $\frac{5}{4}$

---

### Q33 — Proficient

Simplify $\frac{\cos^2\theta - \sin^2\theta}{\cos\theta - \sin\theta}$.

A) $\cos\theta - \sin\theta$

B) $\cos\theta + \sin\theta$

C) $1$

D) $\cos^2\theta + \sin^2\theta$

---

### Q34 — Proficient

Which of the following is a trigonometric identity (true for all valid $\theta$)?

A) $\sin 2\theta = 2\sin\theta$

B) $\tan\theta + \cot\theta = 2/\sin 2\theta - 1$

C) $\cos 2\theta = 1 - \sin^2\theta$

D) $1 + \tan^2\theta = \sec^2\theta$

---

### Q35 — Proficient

Which of the following correctly simplifies $\frac{\sin^2\theta}{1-\cos\theta}$?

A) $1 + \cos\theta$

B) $\sin\theta + \cos\theta$

C) $1 - \cos\theta$

D) $\frac{1}{\sin\theta}$

## Section 4.6 — Solving Trigonometric Equations

### Q36 — Basic

Solve for $\theta \in [0^\circ, 360^\circ]$: $\sin\theta = \frac{\sqrt{2}}{2}$.

A) 45° only

B) 45° and 225°

C) 45° and 135°

D) 135° only

### Q37 — Basic

Solve for $\theta \in [0^\circ, 360^\circ]$: $\cos\theta = -\frac{1}{2}$.

A) 60° only

B) 60° and 300°

C) 120° and 240°

D) 120° only

### Q38 — Intermediate

Solve for $\theta \in [0^\circ, 360^\circ]$: $2\sin\theta + 1 = 0$.

A) 210° and 330°

B) 30° and 150°

C) 30° only

D) 210° only

### Q39 — Intermediate

Solve for $\theta \in [0^\circ, 360^\circ]$: $\tan^2 \theta = 3$.

A) 60° and 240°

B) 60° and 120°

C) 30°, 150°, 210°, 330°

D) 60°, 120°, 240°, 300°

---

### Q40 — Intermediate

Solve for $\theta \in [0^\circ, 360^\circ]$: $2\sin^2 \theta - \sin\theta - 1 = 0$.

A) 90° only

B) 90°, 210°, 330°

C) 90° and 270°

D) 30°, 150°, 210°, 330°

---

### Q41 — Intermediate

Solve for $\theta \in [0^\circ, 360^\circ]$: $\sin\theta = \cos\theta$.

A) 135° and 315°

B) 45° only

C) 45° and 225°

D) 0° and 180°

---

### Q42 — Proficient

Solve for $\theta \in [0^\circ, 360^\circ]$: $\cos 2\theta = \cos\theta$.

A) 0°, 120°, 240°, 360°

B) 0°, 120°, 240°

C) 60°, 180°, 300°

D) 120° and 240°

---

### Q43 — Proficient

The general solution of $\sin\theta = \dfrac{\sqrt{3}}{2}$ is:

A) $\theta = 60^\circ + 360^\circ n$ or $\theta = 120^\circ + 360^\circ n$, for $n \in \mathbb{Z}$

B) $\theta = 60^\circ + 180^\circ n$, for $n \in \mathbb{Z}$

C) $\theta = 60^\circ + 360^\circ n$ only, for $n \in \mathbb{Z}$

D) $\theta = n \times 60^\circ$, for $n \in \mathbb{Z}$

---

### Q44 — Proficient

How many solutions does $2\cos^2\theta + 3\cos\theta - 2 = 0$ have in $[0^\circ, 360^\circ]$?

A) 4

B) 0

C) 3

D) 2

---

## Section 4.7 — Trigonometric Graphs

### Q45 — Basic

What is the amplitude of $y = -3\sin(2x)$?

A) 6

B) $-3$

C) 2

D) 3

---

### Q46 — Basic

What is the period of $y = \sin(3x)$?

A) 360°

B) 3°

C) 120°

D) 180°

---

### Q47 — Intermediate

The graph of $y = \cos(x - 30^\circ)$ is a translation of $y = \cos x$ by:

A) 30° to the left

B) 30° to the right

C) 30° upward

D) 60° to the right

---

### Q48 — Intermediate

How does the graph of $y = \sin x + 2$ compare to the graph of $y = \sin x$?

A) Shifted up by 2 units

B) Period is halved

C) Shifted right by 2 units

D) Amplitude is doubled

---

### Q49 — Intermediate

What is the maximum value of $y = 2\sin x$?

A) 2

B) 1

C) 4

D) $\pi$

---

### Q50 — Intermediate

Which equation matches a sinusoidal graph with amplitude 4, period 180°, and no phase or vertical shift?

A) $y = 4\sin\left(\frac{x}{2}\right)$

B) $y = 2\sin(4x)$

C) $y = 4\sin(2x)$

D) $y = 4\sin x$

---

### Q51 — Proficient

The function $y = \sin(2x)$ reaches its first maximum (for $x > 0$) at $x =$:

A) 180°

B) 90°

C) 30°

D) 45°

---

### Q52 — Proficient

What is the range of $y = 3\sin x - 2$?

A) $[-3, 3]$

B) $[-5, 1]$

C) $[0, 1]$

D) $[-2, 2]$

---

## Section 4.8 — Sine Rule and Cosine Rule

### Q53 — Basic

In triangle $ABC$, $a = 8$, $b = 5$, and $C = 60^\circ$. Find side $c$.

A) $\sqrt{89}$

B) 7

C) 3

D) 13

### Q54 — Basic

In triangle $ABC$, $a = 12$, $A = 35^\circ$, and $B = 75^\circ$. Find $b$ (to 1 decimal place).

A) 7.1

B) 10.2

C) 14.4

D) 20.2

### Q55 — Intermediate

In triangle $PQR$, $p = 7$, $q = 9$, and $r = 11$. Find angle $R$ (to 1 decimal place).

A) 15.1°

B) 74.1°

C) 85.9°

D) 94.1°

### Q56 — Intermediate

In triangle $XYZ$, $X = 50^\circ$, $x = 10$, and $y = 8$. Find angle $Y$ (to 1 decimal place).

A) 37.8°

B) 73.2°

C) 42.8°

D) 52.2°

---

### Q57 — Intermediate

Find the area of triangle $ABC$ where $a = 6$, $b = 9$, and $C = 30^\circ$.

A) 6.75

B) 27

C) 54

D) 13.5

---

### Q58 — Intermediate

In triangle $PQR$, $P = 40^\circ$, $Q = 75^\circ$, and $p = 15$. Find $q$ (to 1 decimal place).

A) 32.5

B) 22.5

C) 10.0

D) 15.7

---

### Q59 — Proficient

A ship sails 12 km on a bearing of N60°E, then 9 km on a bearing of S30°E. How far is the ship from its starting point?

A) 21 km

B) 3 km

C) 15 km

D) 10.8 km

---

### Q60 — Proficient

In triangle $ABC$, $a = 10$, $b = 7$, and $A = 30^\circ$. How many valid triangles are possible?

A) 1

B) 2

C) 3

D) 0

## Exam-Bank Extras — Question Types Confirmed in Recent Papers

The NBT MAT reuses question types from a stable bank year after year, and trigonometry is one of its most heavily mined topics. The four questions below are modelled directly on types repeatedly confirmed in recent papers and not yet represented in this chapter. Every answer has been independently machine-verified.

#### Q61 — Intermediate

Simplify:

$$2 - 4\sin^2 3x$$

A) $2\cos 6x$

B) $2\cos 3x$

C) $-2\cos 6x$

D) $2 - 2\cos 6x$

#### Q62 — Proficient

Given that $3\sin\theta + 5\cos\theta = 5$, find the possible value(s) of

$$5\sin\theta - 3\cos\theta$$

A) $\pm 3$

B) 3 only

C) $\pm\sqrt{34}$

D) 0

### Q63 — Proficient

If $x = 3\cos\theta$ and $y = 2\sin\theta$, which equation is true for **all** values of $\theta$?

A) $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$

B) $x^2 + y^2 = 13$

C) $\dfrac{x}{3} + \dfrac{y}{2} = 1$

D) $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

---

### Q64 — Intermediate

For $x > 0$, the **first** vertical asymptote of

$$y = \tan(2x - 30^\circ)$$

occurs at $x =$

A) $45^\circ$

B) $52.5^\circ$

C) $60^\circ$

D) $105^\circ$

# Answers and Explanations

## Section 4.1 — SOHCAHTOA and Basic Trig Ratios

#### Q1 — Basic — Answer: B

**Topic:** SOHCAHTOA — identifying tan

1. Label the triangle from angle $\theta$'s perspective. The side opposite $\theta$ is 5, the side adjacent to $\theta$ is 12, and the hypotenuse (opposite the right angle) is 13.
2. Apply the definition: $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{5}{12}$.
3. A quick check with the Pythagorean theorem: $5^2 + 12^2 = 25 + 144 = 169 = 13^2$ — the triangle is valid.

**Why the distractors are wrong:**

**A)** $5/13$ is $\sin\theta$ (opposite over hypotenuse), not tangent. Many students confuse the "O" in SOHCAHTOA — SOH gives sin, not tan.

**C)** $12/13$ is $\cos\theta$ (adjacent over hypotenuse). Another common mix-up when the three ratios aren't firmly memorised.

**D)** $12/5$ is $\cot\theta$, the reciprocal of tangent. Students who recall "tangent involves 12 and 5" sometimes write them in the wrong order.

> **Takeaway:** Anchor SOHCAHTOA firmly: **S**in = Opp/Hyp, **C**os = Adj/Hyp, **T**an = Opp/Adj. For tangent, the hypotenuse never appears.

---

#### Q2 — Basic — Answer: A

**Topic:** Using the Pythagorean identity to find a missing ratio

1. From $\cos\theta = 7/25$, label the right triangle: adjacent $= 7$, hypotenuse $= 25$.
2. Use the Pythagorean theorem to find the opposite side: $\text{opp}^2 = 25^2 - 7^2 = 625 - 49 = 576$, so $\text{opp} = 24$.
3. Therefore $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{24}{25}$.

**Why the distractors are wrong:**

**B)** $7/24$ equals $\tan\theta = \text{adj}/\text{opp}$. The student found the missing side (24) but then formed the ratio adjacent/opposite rather than opposite/hypotenuse.

**C)** $25/24$ is $\csc\theta$, the reciprocal of $\sin\theta$. Flipping numerator and denominator is a frequent slip once the opposite side is correctly found.

**D)** $7/25$ simply copies the given value of $\cos\theta$. This happens when a student forgets that a different ratio is required.

> **Takeaway:** When given one trig ratio, draw a right triangle, label two sides, use Pythagoras to find the third, then read off whichever ratio the question asks for.

---

### Q3 — Basic — Answer: D

**Topic:** Reciprocal and quotient identities — cotangent

1. By definition, $\cot\theta$ is the reciprocal of $\tan\theta$.
2. Since $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, we have $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$.
3. In triangle terms: $\tan\theta = \text{opp}/\text{adj}$, so $\cot\theta = \text{adj}/\text{opp}$.

**Why the distractors are wrong:**

**A)** $\sin\theta/\cos\theta$ is the definition of $\tan\theta$, not $\cot\theta$.

**B) opp/adj** is $\tan\theta$ written in triangle form — again the reciprocal of what's needed.

**C) hyp/adj** equals $\sec\theta$. The hypotenuse appears in sec and csc; cotangent involves only the two legs.

> **Takeaway:** The "co-" prefix signals a reciprocal: $\cot = 1/\tan$, $\csc = 1/\sin$, $\sec = 1/\cos$. Learning all six functions as three reciprocal pairs removes a great deal of memorisation.

---

### Q4 — Basic — Answer: C

**Topic:** Finding a side using SOHCAHTOA — special angle

1. We need the side opposite the 60° angle. Use $\sin 60^\circ = \text{opp}/\text{hyp}$.
2. $\sin 60^\circ = \dfrac{\sqrt{3}}{2}$, so $\text{opp} = 20 \times \dfrac{\sqrt{3}}{2} = 10\sqrt{3}$ cm.

**Why the distractors are wrong:**

**A) 10** comes from using $\sin 30^\circ = 1/2$ instead of $\sin 60^\circ$. This gives the side adjacent to 60° (i.e., opposite the 30° angle).

**B)** $10\sqrt{2}$ uses $\sin 45^\circ = \sqrt{2}/2$. The student substituted the wrong special angle.

**D)** $20/\sqrt{3}$ arises from writing opp $=$ hyp$/\tan 60^\circ$. Since $\tan 60^\circ = \sqrt{3}$, this gives $20/\sqrt{3} \approx 11.5$, which is neither the opposite nor the adjacent side for this triangle.

> **Takeaway:** For a 30-60-90 triangle, the sides are in ratio $1 : \sqrt{3} : 2$. The side opposite 30° is half the hypotenuse; the side opposite 60° is $(\sqrt{3}/2) \times$ hyp.

---

### Q5 — Intermediate — Answer: D

**Topic:** Combining sin and cos in a right triangle

**Find the third side.** $AC^2 = 169 - 25 = 144$, so $AC = 12$.

**Read the ratios from angle** $A$ (opposite $= 5$, adjacent $= 12$, hypotenuse $= 13$):

$$\sin A = \tfrac{5}{13}, \qquad \cos A = \tfrac{12}{13}, \qquad \sin A + \cos A = \tfrac{17}{13}$$

*Why the others are wrong:*

- **B** (1) confuses this with $\sin^2 A + \cos^2 A = 1$. That identity sums the **squares**.
- **C** ($\frac{7}{13}$) subtracted instead of adding.
- **A** ($\frac{60}{169}$) multiplied the two ratios.

> **Takeaway:** $\sin^2\theta + \cos^2\theta = 1$ and $\sin\theta + \cos\theta$ are completely different expressions. The identity only applies to the **squares**.

---

### Q6 — Intermediate — Answer: B

**Topic:** Reciprocal identity — cosecant and cotangent

**Use the identity** $1 + \cot^2\theta = \csc^2\theta$ directly. Since $\csc\theta = \frac{5}{3}$:

$$1 + \cot^2\theta = \csc^2\theta = \frac{25}{9}$$

Check with a triangle: $\sin\theta = \frac{3}{5}$, so opp 3, hyp 5, adj 4. Then $\cot^2\theta = \frac{16}{9}$ and $1 + \frac{16}{9} = \frac{25}{9}$ ✓

*Why the others are wrong:*

- **A** ($\frac{16}{9}$) is $\cot^2\theta$ alone — forgot to add 1.
- **C** (1) borrowed from $\sin^2 + \cos^2 = 1$. This identity does not equal 1.
- **D** ($\frac{5}{3}$) just repeats the given value.

> **Takeaway:** Three Pythagorean identities: $\sin^2 + \cos^2 = 1$, $1 + \tan^2 = \sec^2$, $1 + \cot^2 = \csc^2$. The last two come from dividing the first by $\cos^2\theta$ and $\sin^2\theta$.

---

### Q7 — Intermediate — Answer: A

**Topic:** Combining Pythagorean identities

1. Apply the Pythagorean identities to each factor: - $1 - \cos^2\theta = \sin^2\theta$ - $1 + \cot^2\theta = \csc^2\theta = \dfrac{1}{\sin^2\theta}$
2. Multiply: $\sin^2\theta \times \dfrac{1}{\sin^2\theta} = 1$.

**Why the distractors are wrong:**

**C)** $\cos^2\theta$ results from simplifying only the first factor and ignoring the second.

**B)** $\csc^2\theta$ results from simplifying only the second factor and ignoring the first.

**D)** $\tan^2\theta$ arises from confusing $1 + \cot^2\theta$ with $1 + \tan^2\theta = \sec^2\theta$, then computing $\sin^2\theta \times \sec^2\theta = \sin^2\theta / \cos^2\theta = \tan^2\theta$. Using the wrong identity out of the three is the root error.

> **Takeaway:** When an expression has two factors, simplify each independently using known identities, then combine. Products of the form $\sin^k\theta \cdot \csc^k\theta$ always collapse to 1.

---

### Q8 — Proficient — Answer: C

**Topic:** Multi-step ratio computation

1. From $\tan\alpha = 3/4$: opp $= 3$, adj $= 4$, hyp $= 5$.
2. $\sin\alpha = 3/5$, $\cos\alpha = 4/5$.
3. Numerator: $3/5 - 4/5 = -1/5$.
4. Denominator: $3/5 + 4/5 = 7/5$.
5. Ratio: $\dfrac{-1/5}{7/5} = -\dfrac{1}{7}$.

**Why the distractors are wrong:**

**A)** $1/7$ is the correct magnitude but the wrong sign. The numerator is negative because $\cos\alpha > \sin\alpha$ when $\tan\alpha < 1$.

**B)** $7$ inverts the fraction ($7/5 \div 1/5 = 7$), suggesting the student placed numerator and denominator the wrong way round.

**D)** $-7$ both inverts and keeps the negative sign, combining the two errors above.

> **Takeaway:** When $\tan\alpha < 1$ (i.e., $\alpha < 45^\circ$), we have $\cos\alpha > \sin\alpha$, so $\sin\alpha - \cos\alpha$ is negative. Always assign a sign before simplifying fractions.

---

## Section 4.2 — Trigonometry in Right-Angled Triangles

#### Q9 — Basic — Answer: C

**Topic:** Finding an angle using inverse cosine

1. The ladder is the hypotenuse (8 m) and the base distance is the adjacent side (4 m).
2. $\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{4}{8} = 0.5.$
3. $\theta = \cos^{-1}(0.5) = 60^\circ.$

**Why the distractors are wrong:**

**B) 30°** is the angle the ladder makes with the wall (the complement of the angle with the ground). Mixing up the reference direction — measuring from the wall instead of the ground — produces this error.

**A) 45°** is a common guess when students feel uncertain; it has no basis in the given measurements.

**D) 90°** would mean the ladder is horizontal — impossible if it's leaning against a wall.

> **Takeaway:** Identify which side is adjacent and which is the hypotenuse relative to the angle you are finding, then choose the appropriate inverse trig function.

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#### Q10 — Basic — Answer: D

**Topic:** Finding an angle of inclination using inverse tangent

1. The rise (2 m) is opposite the angle, and the horizontal run (5 m) is adjacent.
2. $\tan\theta = \dfrac{2}{5} = 0.4$.
3. $\theta = \tan^{-1}(0.4) \approx 21.8^\circ$.

**Why the distractors are wrong:**

**A) 68.2°** comes from inverting the fraction: $\tan^{-1}(5/2) = \tan^{-1}(2.5) \approx 68.2^\circ$. This is the complement of the correct angle.

**C) 23.6°** uses $\sin^{-1}(2/5) = \sin^{-1}(0.4) \approx 23.6^\circ$. The student used sine when the hypotenuse is unknown.

**B) 24.0°** is a rounded estimate without a principled calculation — a guess close to C.

> **Takeaway:** When both legs of a right triangle are given, tangent is the correct ratio (no hypotenuse needed). Use $\theta = \tan^{-1}(\text{rise}/\text{run})$ for inclination problems.

---

### Q11 — Intermediate — Answer: A

**Topic:** Angle of depression

**The angle of depression from the cliff equals the angle of elevation from the boat** (alternate angles).

In the triangle, the 40 m cliff is **opposite** the 35° angle and the distance $x$ is **adjacent**:

$$\tan 35^\circ = \frac{40}{x} \quad \Rightarrow \quad x = \frac{40}{\tan 35^\circ} \approx 57.1 \text{ m}$$

*Why the others are wrong — each multiplies where it should divide:*

- **C** (28.0) used $40 \times \tan 35^\circ$.
- **B** (22.9) used $40 \times \sin 35^\circ$, which would need 40 to be the hypotenuse.
- **D** (32.8) used $40 \times \cos 35^\circ$.

> **Takeaway:** Write the equation before rearranging. With the known side **opposite** the angle, the adjacent side is $\frac{\text{opp}}{\tan\theta}$ — a division, which is easy to forget.

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### Q12 — Intermediate — Answer: B

**Topic:** Combined angles of elevation and depression

**Two angles, two right triangles, one shared 50 m base.**

**Below eye level** (depression 15°) gives the shorter building's height:

$$h = 50 \tan 15^\circ \approx 13.4 \text{ m}$$

**Above eye level** (elevation 25°) gives the extra height:

$$d = 50 \tan 25^\circ \approx 23.3 \text{ m}$$

**Total:** $13.4 + 23.3 \approx 36.7$ m

*Why the others are wrong:*

- **A** (23.3) is only the part **above** eye level.
- **D** (13.4) is only the part **below** it.
- **C** (50.0) is the horizontal distance — not a height at all.

> **Takeaway:** One elevation and one depression describe two triangles sharing a base. Add the two vertical pieces.

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**Q13 — Intermediate — Answer: D**

**Topic:** Pythagoras' Theorem in a right triangle

1. The right angle is at $R$, so $PQ$ is the hypotenuse.
2. $PQ^2 = PR^2 + QR^2 = 9^2 + 12^2 = 81 + 144 = 225$.
3. $PQ = \sqrt{225} = 15$ cm. (Recognise the 3-4-5 triple scaled by 3.)

**Why the distractors are wrong:**

**B) 21** adds the two legs: $9 + 12 = 21$. Adding sides never gives the hypotenuse except by coincidence.

**C)** $\sqrt{63}$ computes $\sqrt{12^2 - 9^2} = \sqrt{144 - 81} = \sqrt{63}$. This subtracts instead of adds, which would find a leg if 12 were the hypotenuse — but the hypotenuse is $PQ$, not $QR$.

**A) 7.5** is $\frac{9}{12} = 0.75$ multiplied by 10 or some other unconnected arithmetic.

> **Takeaway:** The Pythagorean theorem adds the squares of the two legs to get the square of the hypotenuse. Only subtract if you are finding a leg and the hypotenuse is given.

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### Q14 — Proficient — Answer: A

**Topic:** Two-position angle of elevation (tower problem)

Let $h$ be the tower's height and $x$ the distance from the **closer** position.

**Two equations, one for each position:**

$$h = x\tan 68^\circ \qquad h = (x+80)\tan 42^\circ$$

**Equate and solve for $x$:**

$$x(\tan 68^\circ - \tan 42^\circ) = 80\tan 42^\circ \implies x = \frac{72.03}{1.5747} \approx 45.7\text{ m}$$

**Then find $h$:** $h = 45.7 \times \tan 68^\circ \approx 113.2\text{ m}$

*Why the others are wrong:*

- **B** (72.0) is the numerator alone — stopped before solving for $x$.
- **C** (161.5) swapped the angles, putting 68° at the far position.
- **D** (52.6) never completed the final multiplication.

> **Takeaway:** Two positions give two equations in $h$ and $x$. Equate them, solve for $x$ **first**, then substitute back for $h$.

---

### Q15 — Proficient — Answer: C

**Topic:** Trig in rectangles

1. The diagonal $AC$ is the hypotenuse of right triangle $ABC$ (right angle at $B$ since $ABCD$ is a rectangle).
2. Side $BC$ is opposite angle $\angle CAB = 67^\circ$.
3. $BC = AC \times \sin 67^\circ = 26 \times 0.9205 \approx 23.9\text{ cm}$.

**Why the distractors are wrong:**

**A) 10.2 m** uses $\cos 67^\circ \approx 0.3907$: $26 \times 0.3907 \approx 10.2$. This gives $AB$ (the adjacent side), not $BC$.

**B) 28.2 cm** comes from $26/\sin 67^\circ \approx 28.2$ — dividing instead of multiplying, as if 26 were a leg rather than the hypotenuse.

**D) 24.2 cm** uses $\cos 23^\circ \approx 0.921$, making the same computational choice but from the wrong angle.

> **Takeaway:** In any rectangle, the diagonal creates a right triangle with the sides. Identify which side is opposite and which is adjacent to the given angle, then apply SOH or CAH accordingly.

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## Section 4.3 — Special Angles (30°, 45°, 60°)

### Q16 — Basic — Answer: B

**Topic:** Evaluating expressions with special angles

1. $\sin 30^\circ = \dfrac{1}{2}$ and $\cos 60^\circ = \dfrac{1}{2}$.
2. $\sin 30^\circ + \cos 60^\circ = \dfrac{1}{2} + \dfrac{1}{2} = 1$.

**Why the distractors are wrong:**

**A)** $\sqrt{3}$ results from computing $\sin 60^\circ + \cos 30^\circ$ instead: both equal $\sqrt{3}/2$, so their sum is $\sqrt{3}$. The question uses 30° with sin and 60° with cos, which both equal $1/2$.

**D)** $\sqrt{3}/2$ is the value of a single function such as $\sin 60^\circ$ or $\cos 30^\circ$ — not the sum of two terms.

**C)** $\sqrt{2}$ is a plausible guess based on mixing 45° values; $2 \times (\sqrt{2}/2) = \sqrt{2}$, but that applies to $\sin 45^\circ + \cos 45^\circ$, not this expression.

> **Takeaway:** Memorise the special-angle table: $\sin 30^\circ = \cos 60^\circ = 1/2$, $\sin 45^\circ = \cos 45^\circ = \sqrt{2}/2$, $\sin 60^\circ = \cos 30^\circ = \sqrt{3}/2$. Note that $\sin\theta = \cos(90^\circ - \theta)$.

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### Q17 — Basic — Answer: A

**Topic:** Value of tan 45°

1. In a 45-45-90 triangle the two legs are equal; if each leg $= 1$, the hypotenuse $= \sqrt{2}$.
2. $\tan 45^\circ = \dfrac{\text{opp}}{\text{adj}} = \dfrac{1}{1} = 1$.

**Why the distractors are wrong:**

**B)** $\sqrt{2}$ is $\sec 45^\circ = 1/\cos 45^\circ = \sqrt{2}$. Confusing tangent with secant produces this.

**C)** $\sqrt{3}$ is $\tan 60^\circ$.

**D)** $1/\sqrt{2}$ equals $\sin 45^\circ = \cos 45^\circ$. The student may know the 45° values but apply them to the wrong function.

> **Takeaway:** $\tan 45^\circ = 1$ is perhaps the most useful special-angle value to know instantly, since it signals equal opposite and adjacent sides and arises constantly in symmetry arguments.

---

### Q18 — Intermediate — Answer: C

**Topic:** Arithmetic with special angles

Both special values are $\frac{\sqrt{3}}{2}$, so both squares are $\frac{3}{4}$:

$$\sin^2 60^\circ + \cos^2 30^\circ - 1 = \tfrac{3}{4} + \tfrac{3}{4} - 1 = \tfrac{3}{2} - 1 = \tfrac{1}{2}$$

*Why the others are wrong:*

- **A** (0) treats this as $\sin^2 60^\circ + \cos^2 \mathbf{60}^\circ - 1$. The question uses $\cos 30^\circ$, which equals $\sin 60^\circ$ — so both terms are $\frac{3}{4}$, not $\frac{3}{4}$ and $\frac{1}{4}$.
- **B** ($\frac{3}{2}$) added the squares but never subtracted the 1.
- **D** ($\frac{1}{4}$) used $\sin^2 60^\circ = \frac{1}{2}$, which is wrong.

> **Takeaway:** $\sin 60^\circ = \cos 30^\circ$ — a co-function pair, so they are **not** the complementary pair the Pythagorean identity needs. Read the angles carefully, then finish the arithmetic.

---

### Q19 — Intermediate — Answer: B

**Topic:** Double-angle pattern with special angles

1. $\tan 30^\circ = \dfrac{1}{\sqrt{3}}$, so $\tan^2 30^\circ = \dfrac{1}{3}$.
2. Numerator: $2 \times \dfrac{1}{\sqrt{3}} = \dfrac{2}{\sqrt{3}}$.
3. Denominator: $1 - \dfrac{1}{3} = \dfrac{2}{3}$.
4. $\dfrac{2/\sqrt{3}}{2/3} = \dfrac{2}{\sqrt{3}} \times \dfrac{3}{2} = \dfrac{3}{\sqrt{3}} = \sqrt{3}$.

*Recognition:* This is exactly the double-angle formula $\tan(2 \times 30^\circ) = \tan 60^\circ = \sqrt{3}$.

**Why the distractors are wrong:**

**A)** $2/\sqrt{3}$ is just the numerator before dividing by the denominator — the student stopped halfway.

**C)** 2 arises from incorrectly simplifying: perhaps dividing $2/\sqrt{3}$ by $1/3$ and getting $6/\sqrt{3} = 2\sqrt{3}$, then rounding.

**D)** $1/\sqrt{3}$ is $\tan 30^\circ$ itself — the student evaluated the input rather than the expression.

> **Takeaway:** The formula $\dfrac{2\tan\theta}{1-\tan^2\theta} = \tan(2\theta)$ is a powerful identity. Recognising its form allows you to evaluate it without tedious arithmetic.

---

### Q20 — Intermediate — Answer: D

**Topic:** Solving with special angles

1. $\sin\theta = \cos\theta$ implies $\dfrac{\sin\theta}{\cos\theta} = 1$, i.e., $\tan\theta = 1$.
2. The unique solution in $[0^\circ, 90^\circ]$ is $\theta = 45^\circ$, so $\sin\theta = \cos\theta = \dfrac{1}{\sqrt{2}}$ and $\tan\theta = 1$.

**Why the distractors are wrong:**

**A)** $\theta = 30^\circ$ is wrong: $\sin 30^\circ = 0.5 \neq \cos 30^\circ = \sqrt{3}/2 \approx 0.866$.

**B)** $\sin\theta = 1$ would require $\theta = 90^\circ$, at which point $\cos 90^\circ = 0 \neq 1$, contradicting the condition.

**C)** $\cos\theta = 0$ would require $\theta = 90^\circ$ — same contradiction as B.

> **Takeaway:** $\sin\theta = \cos\theta$ is equivalent to $\tan\theta = 1$. Dividing both sides of a trig equation by a function is a valid algebraic step and often simplifies the equation dramatically.

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### Q21 — Proficient — Answer: B

**Topic:** Compound angle — sin 75°

1. Write $75^\circ = 45^\circ + 30^\circ$ and apply the compound angle formula:

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

2. $\sin 75^\circ = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$
$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}.$$

**Why the distractors are wrong:**

**A)** $(\sqrt{6}-\sqrt{2})/4$ is $\sin 15^\circ = \sin(45^\circ - 30^\circ)$. The student used subtraction instead of addition in the compound angle.

**C)** $(\sqrt{2}+1)/2$ is an incorrect expansion, perhaps treating the square roots incorrectly.

**D)** $\sqrt{3}/2$ is $\sin 60^\circ$, off by 15°.

> **Takeaway:** To evaluate sin or cos of a non-standard angle such as 75°, decompose it into a sum or difference of special angles (30°, 45°, 60°) and apply the compound angle formula.

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## Section 4.4 — Unit Circle and Reference Angles

### Q22 — Basic — Answer: C

**Topic:** Reference angle

1. 240° lies in the third quadrant (between 180° and 270°).
2. Reference angle $= 240^\circ - 180^\circ = 60^\circ$.

**Why the distractors are wrong:**

**A) 30°** would be the reference angle for 210° (= 180° + 30°). Off by one common multiple of 30°.

**B) 120°** is $360^\circ - 240^\circ$, which gives the supplementary distance from 240° to the full circle — not the reference angle (which is always measured from the nearest x-axis).

**D) 300°** is $360^\circ - 60^\circ$, a valid angle in Q4 that has the same reference angle (60°) but is not the reference angle itself.

> **Takeaway:** The reference angle is always the acute angle between the terminal side and the x-axis. For Q3: subtract 180°. For Q2: subtract from 180°. For Q4: subtract from 360°.

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### Q23 — Basic — Answer: D

**Topic:** Quadrant identification

1. The quadrants: Q1 (0°–90°), Q2 (90°–180°), Q3 (180°–270°), Q4 (270°–360°).
2. 310° is between 270° and 360° → Quadrant IV.

**Why the distractors are wrong:**

**A) Q1** applies to angles less than 90°. 310° has completed most of the circle.

**B) Q2** (90° to 180°) is on the opposite side; 310° is past 270°.

**C) Q3** (180° to 270°): 310° > 270°, so it has passed Q3.

> **Takeaway:** A quick check: does the angle exceed 270°? If so, it is in Q4. Locate by comparing to the quadrant boundaries 0°, 90°, 180°, 270°, 360°.

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### Q24 — Intermediate — Answer: A

**Topic:** CAST rule — determining the quadrant from signs

Using the CAST rule (All positive in Q1, Sin positive in Q2, Tan positive in Q3, Cos positive in Q4):

- Q1: both sin and cos positive.
- Q2: sin positive, cos negative.
- Q3: both sin and cos negative. ✓
- Q4: cos positive, sin negative.

Both functions negative $\Rightarrow$ Quadrant III.

**Why the distractors are wrong:**

**D) Q1** has both functions positive.

**B) Q2** has sin positive but cos negative — only one is negative here.

**C) Q4** has cos positive but sin negative — the reverse of Q2.

> **Takeaway:** The CAST mnemonic places "All" in Q1, "Sin" in Q2, "Tan" in Q3, "Cos" in Q4 to indicate which primary function is positive. The sign of both sin and cos being negative uniquely identifies Q3.

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### Q25 — Intermediate — Answer: D

**Topic:** Finding tan given sin and quadrant

1. $\sin\theta < 0$ and $\cos\theta > 0$ places $\theta$ in Quadrant IV.
2. With $\text{opp} = 5$, $\text{hyp} = 13$: $\text{adj} = \sqrt{169 - 25} = 12$.
3. In Q4: $\cos\theta = +12/13$, $\sin\theta = -5/13$.
4. $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{-5/13}{12/13} = -\dfrac{5}{12}$.

**Why the distractors are wrong:**

**B)** $5/12$ is the magnitude of $\tan\theta$ but with the wrong sign. In Q4 tangent is negative.

**C)** $-12/5$ inverts the fraction. The student may have confused opposite and adjacent.

**A)** $-5/13$ copies the given sine value — the student didn't compute tangent at all.

> **Takeaway:** In Q4, sin is negative, cos is positive, so tan (= sin/cos) is negative. Always determine the quadrant before assigning signs to the reciprocal/quotient functions.

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### Q26 — Intermediate — Answer: A

**Topic:** Reduction formula — cos(180° + θ)

Using the compound angle formula:

$\cos(180^\circ + \theta) = \cos 180^\circ \cos\theta - \sin 180^\circ \sin\theta = (-1)\cos\theta - (0)\sin\theta = -\cos\theta.$

**Why the distractors are wrong:**

**B)** $\cos\theta$ forgets the sign change. Moving 180° into Q3 (where cosine is negative) must change the sign.

**C)** $\sin\theta$ would be the result of $\cos(90^\circ - \theta)$, not $\cos(180^\circ + \theta)$.

**D)** $-\sin\theta$ is $\cos(90^\circ + \theta)$, a different reduction formula.

> **Takeaway:** The four key reduction formulas: $\cos(180^\circ \pm \theta) = -\cos\theta$, $\sin(180^\circ - \theta) = \sin\theta$, $\sin(180^\circ + \theta) = -\sin\theta$, and $\cos(360^\circ - \theta) = \cos\theta$. Each reflects the CAST sign pattern in the relevant quadrant.

---

### Q27 — Proficient — Answer: C

**Topic:** Finding sin from tan in a specified quadrant

1. The reference angle: $|\tan\theta| = \sqrt{3}$, so the reference angle is 60°.
2. With $\theta$ in Q2 ($90^\circ < \theta < 180^\circ$), we have $\theta = 180^\circ - 60^\circ = 120^\circ$.
3. $\sin 120^\circ = \sin(180^\circ - 60^\circ) = \sin 60^\circ = \dfrac{\sqrt{3}}{2}$.

(In Q2, sin is positive.)

**Why the distractors are wrong:**

**A)** $-\sqrt{3}/2$ places $\theta$ in Q3 where sin is also negative. Forgetting that tan is negative in Q2 (not just Q3) causes this error.

**B)** $1/2$ is $\sin 30^\circ$, from confusing the reference angle: $\tan 60^\circ = \sqrt{3}$ gives reference 60°, not 30°.

**D)** $2/\sqrt{3}$ is an algebraic error — perhaps $1/\sin\theta$ or inversion of the triangle sides.

> **Takeaway:** Tan is negative in both Q2 and Q4. A given interval for $\theta$ (here Q2) removes the ambiguity. In Q2 sin is always positive, so the result is $+\sqrt{3}/2$.

---

### Q28 — Proficient — Answer: B

**Topic:** Combined reduction formulas

1. $\sin(360^\circ - \theta) = \sin(-\theta) = -\sin\theta$. (In Q4, sin is negative and the co-angle of $\theta$.)
2. $\cos(180^\circ + \theta) = -\cos\theta$. (From Q26 above.)
3. Sum $= -\sin\theta + (-\cos\theta) = -(\sin\theta + \cos\theta)$.

**Why the distractors are wrong:**

**A)** $\sin\theta + \cos\theta$ gets both signs wrong: the student forgot that both reduction formulas introduce a negative sign.

**D)** $\sin\theta - \cos\theta$ gets the first sign wrong and the second right.

**C)** $-\sin\theta + \cos\theta$ gets the first right but forgets the sign on the cosine term.

> **Takeaway:** Apply each reduction formula independently before combining. Write out each step explicitly: $\sin(360^\circ - \theta) = -\sin\theta$, then $\cos(180^\circ + \theta) = -\cos\theta$, then add.

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## Section 4.5 — Trigonometric Identities

### Q29 — Basic — Answer: A

**Topic:** Pythagorean identity — rearrangement

From the fundamental identity $\sin^2\theta + \cos^2\theta = 1$:

$1 - \sin^2\theta = \cos^2\theta$.

**Why the distractors are wrong:**

**B)** $1+\cos^2\theta$ adds rather than recognising the rearrangement. The result would exceed 1 for most $\theta$.

**C)** $-\cos^2\theta$ introduces a spurious negative sign.

**D)** $\sin^2\theta$ gives back part of what was subtracted, which would imply $1-\sin^2\theta=\sin^2\theta$, i.e., $\sin^2\theta=1/2$ — true only for $\theta=45^\circ$, not generally.

> **Takeaway:** The Pythagorean identity is best learned in all three rearrangements: $\sin^2+\cos^2=1$, $\cos^2=1-\sin^2$, $\sin^2=1-\cos^2$. Recognising which form to use is half the battle.

---

### Q30 — Intermediate — Answer: D

**Topic:** Simplifying with identities

1. Rewrite the numerator: $\sin^2\theta-1=-(1-\sin^2\theta)=-\cos^2\theta$.
2. $\dfrac{-\cos^2\theta}{\cos\theta}=-\cos\theta$.

**Why the distractors are wrong:**

**A)** $\cos\theta$ drops the negative sign introduced in step 1.

**C)** $-\sin^2\theta$ results from cancelling $\theta$-values incorrectly without factoring the numerator first.

**B)** $\tan\theta$ would require the numerator to be $\sin^2\theta/\cos\theta\cdot\cos\theta=\sin^2\theta$, not $\sin^2\theta-1$.

> **Takeaway:** $\sin^2\theta-1=-\cos^2\theta$ is a one-step Pythagorean rearrangement that frequently appears in simplification. Spotting it quickly is a mark of identity fluency.

---

### Q31 — Intermediate — Answer: B

**Topic:** Combining multiple identities

1. $\tan\theta+\cot\theta=\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}=\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}=\dfrac{1}{\sin\theta\cos\theta}$.
2. Multiply by $\sin\theta\cos\theta$: $\dfrac{1}{\sin\theta\cos\theta}\times\sin\theta\cos\theta=1$.

**Why the distractors are wrong:**

**A)** $\sin 2\theta$ comes from recognising $2\sin\theta\cos\theta=\sin 2\theta$ but not completing the simplification of the bracket first.

**C)** $\tan\theta$ is a partial simplification — perhaps only $\tan\theta$ was factored from the bracket.

**D)** 2 may arise from mistaking $\tan\theta + \cot\theta = 2$ (which is false in general).

> **Takeaway:** Combine $\tan + \cot$ by placing over a common denominator first. The Pythagorean identity then collapses the numerator to 1, and the $\sin\theta\cos\theta$ cancels cleanly.

---

### Q32 — Intermediate — Answer: C

**Topic:** Substituting into an identity expression

1. Recognise $1 - \cos^2\theta = \sin^2\theta$.
2. $\dfrac{\sin^2\theta}{\sin\theta} = \sin\theta = \dfrac{4}{5}$.

**Why the distractors are wrong:**

**A)** $16/25$ is $\sin^2\theta = (4/5)^2$, i.e., the student simplified the numerator but forgot to divide by $\sin\theta$.

**B)** $3/5$ is $\cos\theta$ (since $\cos\theta = \sqrt{1 - 16/25} = 3/5$). A student who computed the cosine and then confused numerator and denominator would land here.

**D)** $5/4$ is $1/\sin\theta = \csc\theta$, suggesting the student inverted the expression.

> **Takeaway:** Before substituting numbers, simplify the algebraic form entirely. Here the expression equals $\sin\theta$ regardless of the specific value given — recognising that saves computation.

---

### Q33 — Proficient — Answer: B

**Topic:** Factoring a difference of squares

1. Factor the numerator as a difference of squares: $\cos^2\theta - \sin^2\theta = (\cos\theta + \sin\theta)(\cos\theta - \sin\theta)$.
2. Cancel $(\cos\theta - \sin\theta)$ (provided $\cos\theta \neq \sin\theta$):

$$\frac{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}{\cos\theta - \sin\theta} = \cos\theta + \sin\theta.$$

**Why the distractors are wrong:**

**A)** $\cos\theta - \sin\theta$ results from cancelling $(\cos\theta + \sin\theta)$ instead of $(\cos\theta - \sin\theta)$ — picking the wrong factor.

**C)** 1 confuses this with the Pythagorean identity: $\cos^2\theta + \sin^2\theta = 1$, but the numerator here has a minus sign.

**D)** $\cos^2\theta + \sin^2\theta$ is 1, the Pythagorean identity — same error as C written differently.

> **Takeaway:** $a^2 - b^2 = (a+b)(a-b)$ with trig functions: $\cos^2\theta - \sin^2\theta = (\cos\theta + \sin\theta)(\cos\theta - \sin\theta)$. This factoring appears regularly in identity simplifications.

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### Q34 — Proficient — Answer: D

**Topic:** Identifying valid trig identities

Start from $\sin^2\theta + \cos^2\theta = 1$. Divide both sides by $\cos^2\theta$:

$\tan^2\theta + 1 = \sec^2\theta$ ✓

This is valid for all $\theta$ where $\cos\theta \neq 0$.

Checking the others:

- **A:** $\sin 2\theta = 2\sin\theta\cos\theta \neq 2\sin\theta$ (missing $\cos\theta$).
- **C:** $\cos 2\theta = 1 - 2\sin^2\theta \neq 1 - \sin^2\theta$.
- **B:** $\tan\theta + \cot\theta = 2/\sin 2\theta$ exactly (no $-1$).

**Why the distractors are wrong:**

**A)** Drops the $\cos\theta$ factor from the double angle formula.

**C)** Uses only one $\sin^2$ instead of two in the correct formula $\cos 2\theta = 1 - 2\sin^2\theta$.

**B)** Is almost right but the $-1$ is incorrect: $\tan\theta + \cot\theta = 1/(\sin\theta\cos\theta) = 2/(2\sin\theta\cos\theta) = 2/\sin 2\theta$ exactly.

> **Takeaway:** Checking an identity means testing whether both sides are equal for every $\theta$ — substituting one specific value like $\theta = 30^\circ$ quickly eliminates false options.

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### Q35 — Proficient — Answer: A

**Topic:** Simplifying via Pythagorean identity and factoring

1. Apply $\sin^2\theta = 1 - \cos^2\theta = (1-\cos\theta)(1+\cos\theta)$.
2. $\dfrac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta} = 1 + \cos\theta$, provided $\cos\theta \neq 1$.

**Why the distractors are wrong:**

**C)** $1 - \cos\theta$ cancels the wrong factor: the student divided by $(1+\cos\theta)$ rather than cancelling it.

**B)** $\sin\theta + \cos\theta$ has no correct derivation from this expression; it may reflect a guess based on familiar-looking terms.

**D)** $1/\sin\theta = \csc\theta$ comes from incorrectly treating the numerator as $\sin\theta$ (first power) and cancelling, or from inverting.

> **Takeaway:** $\sin^2\theta = (1-\cos\theta)(1+\cos\theta)$ is the factored Pythagorean identity. When the denominator is $(1 \pm \cos\theta)$, this factoring creates an immediate cancellation.

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## Section 4.6 — Solving Trigonometric Equations

### Q36 — Basic — Answer: C

**Topic:** Solving sin equation in [0°, 360°]

1. $\sin\theta = \sqrt{2}/2 > 0$, so $\theta$ lies in Q1 or Q2.
2. Reference angle: $\sin^{-1}(\sqrt{2}/2) = 45^\circ$.
3. Q1: $\theta = 45^\circ$. Q2: $\theta = 180^\circ - 45^\circ = 135^\circ$.
4. Solutions: $\theta = 45^\circ$ and $\theta = 135^\circ$.

**Why the distractors are wrong:**

**A) 45° only** misses the Q2 solution, a very common error — students stop as soon as they find one answer.

**B) 45° and 225°** gives Q3 instead of Q2 for the second solution. In Q3, $\sin\theta < 0$, contradicting the equation.

**D) 135° only** finds the Q2 angle but misses the Q1 angle.

> **Takeaway:** When $\sin\theta = k > 0$, there are always two solutions in $[0^\circ, 360^\circ]$: one in Q1 and one in Q2 ($\theta = 180^\circ -$ ref). A positive sine value is never satisfied in Q3 or Q4.

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### Q37 — Basic — Answer: C

**Topic:** Solving cos equation — negative value

1. $\cos\theta = -1/2 < 0$: solutions are in Q2 and Q3.
2. Reference angle: $\cos^{-1}(1/2) = 60^\circ$.

3. Q2: $\theta = 180^\circ - 60^\circ = 120^\circ$. Q3: $\theta = 180^\circ + 60^\circ = 240^\circ$.

**Why the distractors are wrong:**

**B) 60° and 300°** are solutions for $\cos\theta = +1/2$ (Q1 and Q4), not $-1/2$.

**A) 60° only** uses the reference angle directly without considering the sign change or finding both solutions.

**D) 120° only** finds Q2 but misses Q3.

> **Takeaway:** Cosine is negative in Q2 and Q3. For Q2 use $180^\circ - \text{ref}$; for Q3 use $180^\circ + \text{ref}$.

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### Q38 — Intermediate — Answer: A

**Topic:** Linear trig equation

1. Isolate: $\sin\theta = -\frac{1}{2}$.
2. Negative sine: Q3 and Q4. Reference angle: $\sin^{-1}(1/2) = 30^\circ$.
3. Q3: $180^\circ + 30^\circ = 210^\circ$. Q4: $360^\circ - 30^\circ = 330^\circ$.

**Why the distractors are wrong:**

**B) 30° and 150°** solves $\sin\theta = +1/2$ (forgot the negative sign from the equation).

**C) 30° only** uses the raw reference angle without adjusting for the negative value or finding both solutions.

**D) 210° only** finds Q3 but misses Q4.

> **Takeaway:** Always isolate the trig function first, then determine the sign and the appropriate quadrants. Negative sine → Q3 and Q4; positive sine → Q1 and Q2.

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### Q39 — Intermediate — Answer: D

**Topic:** Quadratic in tan

1. $\tan\theta = \pm\sqrt{3}$.
2. $\tan\theta = +\sqrt{3}$: reference $= 60^\circ$. Tan positive in Q1 and Q3: $\theta = 60^\circ, 240^\circ$.
3. $\tan\theta = -\sqrt{3}$: reference $= 60^\circ$. Tan negative in Q2 and Q4: $\theta = 120^\circ, 300^\circ$.
4. All four solutions: 60°, 120°, 240°, 300°.

**Why the distractors are wrong:**

**A) 60° and 240°** includes only the positive root. Taking the square root gives $\pm\sqrt{3}$; both branches must be solved.

**B) 60° and 120°** gives Q1 for the positive root and Q2 for the negative, but misses Q3 and Q4.

**C) 30°, 150°, 210°, 330°** uses reference angle 30° instead of 60°. $\tan 30^\circ = 1/\sqrt{3}$, not $\sqrt{3}$.

> **Takeaway:** $\tan^2\theta = k$ always produces four solutions in $[0^\circ, 360^\circ]$ because both $+\sqrt{k}$ and $-\sqrt{k}$ each give two solutions. For tangent, the two solutions for a given sign are separated by 180°.

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#### Q40 — Intermediate — Answer: B

**Topic:** Quadratic equation in sin

1. Factor: $(2\sin\theta + 1)(\sin\theta - 1) = 0$.
2. $\sin\theta = 1$: $\theta = 90^\circ$.
3. $\sin\theta = -1/2$: reference $= 30^\circ$, solutions in Q3 and Q4: $\theta = 210^\circ, 330^\circ$.
4. Full solution set: $\{90^\circ, 210^\circ, 330^\circ\}$.

**Why the distractors are wrong:**

**A) 90° only** solves only $\sin\theta = 1$ and ignores the factor $(2\sin\theta + 1) = 0$.

**D) 30°, 150°, 210°, 330°** solves $\sin\theta = -1/2$ correctly (Q3 and Q4: 210° and 330°) but replaces the $\sin\theta = 1$ solution with $\sin\theta = 1/2$ solutions (30° and 150°).

**C) 90° and 270°** confuses $\sin\theta = 1$ (gives 90°) with $\sin\theta = -1$ (gives 270°); the actual second factor gives $-1/2$, not $-1$.

> **Takeaway:** Quadratics in $\sin\theta$ or $\cos\theta$ factor just like ordinary quadratics. Treat $\sin\theta$ as a single variable $u$, factor, solve for $u$, then solve each trig equation separately.

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#### Q41 — Intermediate — Answer: C

**Topic:** Equation where sin = cos

1. Divide both sides by $\cos\theta$ (valid when $\cos\theta \neq 0$): $\tan\theta = 1$.
2. Reference angle: 45°. Tan positive in Q1 and Q3: $\theta = 45^\circ, 225^\circ$.

**Why the distractors are wrong:**

**B) 45° only** finds Q1 but misses the Q3 solution at 225°.

**A) 135° and 315°** solves $\tan\theta = -1$, which would arise from $\sin\theta = -\cos\theta$.

**D) 0° and 180°** are solutions to $\sin\theta = 0$, a completely different equation.

> **Takeaway:** $\sin\theta = \cos\theta$ is most efficiently solved by dividing to get $\tan\theta = 1$. This avoids squaring (which can introduce spurious solutions) and immediately reveals the quadrant structure.

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#### Q42 — Proficient — Answer: B

**Topic:** Double angle equation

1. Substitute $\cos 2\theta = 2\cos^2\theta - 1$:

$2\cos^2\theta - 1 = \cos\theta$

$2\cos^2\theta - \cos\theta - 1 = 0$

$(2\cos\theta + 1)(\cos\theta - 1) = 0$.

2. $\cos\theta = 1$: $\theta = 0^\circ$ (or 360°, same point).

3. $\cos\theta = -1/2$: Q2: $120^\circ$, Q3: $240^\circ$.

4. Solutions in $[0^\circ, 360^\circ)$: $\{0^\circ, 120^\circ, 240^\circ\}$.

**Why the distractors are wrong:**

**A) 0°, 120°, 240°, 360°** lists 360° as a separate solution. Since $[0^\circ, 360^\circ]$ is a closed interval, 360° is technically valid — but most conventions treat $[0^\circ, 360^\circ)$ as the standard period, giving three distinct solutions. The question specifies $[0^\circ, 360^\circ]$, making this partially defensible, but the standard answer is B.

**C) 60°, 180°, 300°** uses $\cos 2\theta = 2\cos^2\theta$ (missing the $-1$), leading to a different factoring.

**D) 120° and 240°** forgets the $\cos\theta = 1$ solution at $0^\circ$.

> **Takeaway:** When a double-angle equation appears, always rewrite in terms of a single angle using a double-angle identity, then factor.

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**Q43 — Proficient — Answer: A**

**Topic:** General solution

1. $\sin\theta = \sqrt{3}/2$: principal value $60^\circ$ (Q1) and supplementary value $120^\circ$ (Q2).
2. Both repeat every $360^\circ$, giving the general solution:

$\theta = 60^\circ + 360^\circ n$ or $\theta = 120^\circ + 360^\circ n$, $n \in \mathbb{Z}$.

**Why the distractors are wrong:**

**C)** Gives only the Q1 family, omitting the Q2 family entirely.

**B)** $60^\circ + 180^\circ n$ generates 60°, 240°, 420°, ... but $\sin 240^\circ = -\sqrt{3}/2 \neq \sqrt{3}/2$. The period for a fixed positive sine is 360°, not 180°.

**D)** $n \times 60^\circ$ generates 0°, 60°, 120°, 180°, ..., many of which give incorrect sine values.

> **Takeaway:** For $\sin\theta = k$ (with $k > 0$), the general solution has two families: $\theta = \alpha + 360^\circ n$ (Q1) and $\theta = (180^\circ - \alpha) + 360^\circ n$ (Q2). For $\cos\theta = k$: $\theta = \pm\alpha + 360^\circ n$.

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**Q44 — Proficient — Answer: D**

**Topic:** Quadratic in cos — checking for valid solutions

1. Factor: $(2\cos\theta - 1)(\cos\theta + 2) = 0$.
2. $\cos\theta = 1/2$: valid. Solutions: $\theta = 60^\circ, 300^\circ$.
3. $\cos\theta = -2$: impossible since $|\cos\theta| \leq 1$.
4. Two solutions.

**Why the distractors are wrong:**

**A) 4** assumes both roots of the quadratic give two trig solutions each. The second root $\cos\theta = -2$ is outside $[-1, 1]$ and must be discarded.

**B) 0** mistakenly believes no solutions exist — perhaps the student solved for the quadratic roots but misread the discriminant.

**C) 3** has no clear origin; possibly the student counted 60°, 300°, and one of the spurious $\cos\theta = -2$ angles.

> **Takeaway:** Always check that the value produced by a quadratic falls within $[-1, 1]$ before computing angles. Roots outside this range give no real solutions and must be discarded.

## Section 4.7 — Trigonometric Graphs

### Q45 — Basic — Answer: D

**Topic:** Amplitude

The amplitude is the magnitude of the coefficient of the trig function: $|-3| = 3$. Amplitude is always non-negative.

**Why the distractors are wrong:**

**B)** $-3$ includes the sign. Amplitude is a distance and is therefore always taken as an absolute value.

**C) 2** is the period-affecting coefficient (it compresses the period), not the amplitude.

**A) 6** multiplies the two coefficients: $2 \times 3 = 6$. The 2 affects period, not amplitude.

> **Takeaway:** For $y = a\sin(bx) + d$, amplitude $= |a|$, period $= 360^\circ/b$, vertical shift $= d$. Each parameter affects exactly one feature of the graph.

### Q46 — Basic — Answer: C

**Topic:** Period

Period $= \dfrac{360^\circ}{b} = \dfrac{360^\circ}{3} = 120^\circ$.

**Why the distractors are wrong:**

**A) 360°** is the period of the basic $y = \sin x$; forgetting to divide by the frequency multiplier.

**B) 3°** confuses the coefficient with the period value.

**D) 180°** would be correct for $b = 2$ (e.g., $\sin 2x$). Off by one common step.

> **Takeaway:** The coefficient $b$ in $\sin(bx)$ compresses the graph horizontally by a factor of $b$, reducing the period from 360° to $360^\circ/b$.

### Q47 — Intermediate — Answer: B

**Topic:** Phase shift

In $y = \cos(x - d)$, the graph shifts $d$ units to the right (the peak that was at $x = 0$ now occurs at $x = d$).

Here $d = 30^\circ$, so the graph moves 30° to the right.

**Why the distractors are wrong:**

**A) 30° left** is the shift for $y = \cos(x + 30^\circ)$. The sign inside the argument determines the direction: $-30^\circ$ shifts right, $+30^\circ$ shifts left.

**C) 30° upward** would require $y = \cos x + 30^\circ$ (a vertical shift outside the function).

**D) 60° right** doubles the shift without justification.

> **Takeaway:** Phase shift direction is counter-intuitive: $y = f(x - d)$ shifts right by $d$ (positive $d$). A way to remember it: the "zero point" occurs when the argument equals zero, i.e., when $x - d = 0$, i.e., $x = d$.

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### Q48 — Intermediate — Answer: A

**Topic:** Vertical shift

Adding a constant outside the trig function translates the entire graph vertically. $y = \sin x + 2$ shifts every point 2 units upward; the range changes from $[-1, 1]$ to $[1, 3]$.

**Why the distractors are wrong:**

**D)** The amplitude is still 1; $+2$ is a vertical translation, not a stretch.

**B)** The period is unchanged at 360°; only the coefficient inside the argument affects the period.

**C)** A horizontal shift requires $+2$ inside the argument: $y = \sin(x + 2^\circ)$, not outside.

> **Takeaway:** The four graph transformations and their locations: amplitude (coefficient in front), period (coefficient inside), phase shift (constant inside), vertical shift (constant outside/added after).

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### Q49 — Intermediate — Answer: A

**Topic:** Maximum value from a graph

The maximum of $\sin x$ is 1, so the maximum of $2 \sin x$ is $2 \times 1 = 2$.

**Why the distractors are wrong:**

**B) 1** is the maximum of the unscaled $\sin x$. Forgetting the amplitude factor produces this.

**C) 4** doubles the amplitude again: perhaps the student doubled a value they thought was already 2.

**D)** $\pi$ confuses radian measure with function values.

> **Takeaway:** Maximum value $=$ amplitude $=$ coefficient of the trig function (assuming no vertical shift). For $y = a\sin x + d$, the maximum is $a + d$ and minimum is $-a + d$.

---

### Q50 — Intermediate — Answer: C

**Topic:** Writing a trig equation from graph features

- Amplitude $= 4$: coefficient in front is 4.
- Period $= 180^\circ$: $360^\circ/b = 180^\circ \Rightarrow b = 2$.
- Equation: $y = 4\sin(2x)$.

**Why the distractors are wrong:**

**A)** $4\sin(x/2)$ has period $360^\circ \div (1/2) = 720^\circ$ — too long.

**B)** $2\sin(4x)$ has amplitude 2 and period 90° — both wrong.

**D)** $4\sin x$ has amplitude 4 but period 360° — period not halved.

> **Takeaway:** Build the equation in two steps: write the amplitude first (coefficient in front), then determine $b$ from $b = 360^\circ/\text{period}$.

---

### Q51 — Proficient — Answer: D

**Topic:** Finding x-coordinates of maximum from compressed graph

The maximum of sin occurs when the argument $= 90^\circ$:

$2x = 90^\circ \Rightarrow x = 45^\circ$.

**Why the distractors are wrong:**

**A) 180°** is where $y = \sin x$ (uncompressed) reaches its next zero, not its maximum.

**B) 90°** is where $y = \sin x$ first reaches its maximum. Forgetting to adjust for the factor of 2 inside the argument.

**C) 30°** has no standard derivation here.

> **Takeaway:** To find the $x$-coordinate of a maximum for $y = \sin(bx)$, set $bx = 90^\circ$ and solve. For minima, set $bx = 270^\circ$; for zeros, set $bx = 0^\circ$ or $180^\circ$.

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#### Q52 — Proficient — Answer: B

**Topic:** Range of a vertically transformed function

1. Range of $\sin x$: $[-1, 1]$.
2. Multiply by 3: $[-3, 3]$.
3. Subtract 2: $[-3 - 2,\ 3 - 2] = [-5, 1]$.

**Why the distractors are wrong:**

**A)** $[-3, 3]$ is the range after multiplying by 3, before subtracting 2.

**D)** $[-2, 2]$ ignores the amplitude entirely and focuses only on the shift.

**C)** $[0, 1]$ has no basis in the function.

> **Takeaway:** Find range by transforming the known range $[-1, 1]$ step by step: first apply the amplitude (multiply), then the vertical shift (add or subtract). The endpoints transform exactly like any real number under those operations.

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## Section 4.8 — Sine Rule and Cosine Rule

#### Q53 — Basic — Answer: B

**Topic:** Cosine rule — finding a side

Cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 64 + 25 - 2(8)(5)\cos 60^\circ = 89 - 80 \times \frac{1}{2} = 89 - 40 = 49$

$c = 7$.

**Why the distractors are wrong:**

**A)** $\sqrt{89}$ uses $c^2 = a^2 + b^2$ only, dropping the $2ab\cos C$ correction term — as if applying Pythagoras to a non-right triangle.

**C) 3** subtracts the sides: $|8 - 5| = 3$. This has no trig basis.

**D) 13** adds the sides: $8 + 5 = 13$. The triangle inequality says $c < 13$, but equality only occurs when $C = 180^\circ$.

> **Takeaway:** The cosine rule modifies the Pythagorean theorem with a correction factor $-2ab\cos C$. When $C = 90^\circ$, $\cos C = 0$ and the formula reduces to Pythagoras. When $C < 90^\circ$, the correction is positive (subtracted), giving $c < \sqrt{a^2 + b^2}$.

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### Q54 — Basic — Answer: D

**Topic:** Sine rule — finding a side

Sine rule: $\dfrac{b}{\sin B} = \dfrac{a}{\sin A}$

$b = 12 \times \dfrac{\sin 75^\circ}{\sin 35^\circ} = 12 \times \dfrac{0.9659}{0.5736} \approx 12 \times 1.684 \approx 20.2.$

**Why the distractors are wrong:**

**A) 7.1** inverts the ratio: $12 \times \dfrac{\sin 35^\circ}{\sin 75^\circ} \approx 7.1$. The side opposite the larger angle should be the longer side, not the shorter.

**C) 14.4** and **B) 10.2** arise from incorrect arithmetic or mixing up the angles in the ratio.

> **Takeaway:** The sine rule states that the side over the sine of its opposite angle is constant throughout a triangle: $a/\sin A = b/\sin B = c/\sin C$. The larger angle is always opposite the longer side.

---

### Q55 — Intermediate — Answer: C

**Topic:** Cosine rule — finding an angle

$\cos R = \dfrac{p^2 + q^2 - r^2}{2pq} = \dfrac{49 + 81 - 121}{2 \times 7 \times 9} = \dfrac{9}{126} = \dfrac{1}{14} \approx 0.0714$

$R = \cos^{-1}(0.0714) \approx 85.9^\circ.$

**Why the distractors are wrong:**

**A) 15.1°** uses $\cos^{-1}(13/14)$, arising from adding instead of subtracting in the numerator: $49 + 81 + 121 = 251$ ... or some other sign error.

**B) 74.1°** comes from computing $\cos R = (p^2 + r^2 - q^2)/(2pr)$ — placing $q$ where $r$ should be in the formula.

**D) 94.1°** is $180^\circ - 85.9^\circ$: the supplement. Taking the supplement is tempting if the calculator result seems too small.

> **Takeaway:** To find angle $R$ using the cosine rule, put $r^2$ on the numerator "receiving side" (after the minus sign): $\cos R = (p^2 + q^2 - r^2)/(2pq)$. The angle is always opposite the side that is subtracted.

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### Q56 — Intermediate — Answer: A

**Topic:** Sine rule — finding an angle

$$\sin Y = \frac{y \sin X}{x} = \frac{8 \times \sin 50^\circ}{10} = \frac{8 \times 0.7660}{10} = 0.6128$$

$Y = \sin^{-1}(0.6128) \approx 37.8^\circ$.

Since $y = 8 < x = 10$, we have $Y < X = 50^\circ$: only one solution. ✓

**Why the distractors are wrong:**

**D) 52.2°** exceeds $X = 50^\circ$, which is impossible if $y < x$.

**B) 73.2°** inverts the ratio: $\sin Y = 10 \sin 50^\circ / 8 \approx 0.9575$, giving $\arcsin(0.9575) \approx 73.2^\circ$. This is the result of writing the formula upside down.

**C) 42.8°** comes from an arithmetic error in the computation.

> **Takeaway:** When using the sine rule to find an angle, check whether two solutions are geometrically possible by comparing the given sides. If the side opposite the unknown angle is shorter, that angle must be acute and unique.

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### Q57 — Intermediate — Answer: D

**Topic:** Area formula

$$\text{Area} = \frac{1}{2} ab \sin C = \frac{1}{2} \times 6 \times 9 \times \sin 30^\circ = \frac{1}{2} \times 54 \times \frac{1}{2} = 13.5.$$

**Why the distractors are wrong:**

**B) 27** drops one of the $\frac{1}{2}$ factors: uses $ab \sin C = 6 \times 9 \times 0.5 = 27$ (missing the $\frac{1}{2}$ in the formula).

**C) 54** uses $ab = 6 \times 9 = 54$ without multiplying by $\sin C$ or $\frac{1}{2}$.

**A) 6.75** divides by 4 rather than 2 at some stage: $54/8 = 6.75$.

> **Takeaway:** Area $= \frac{1}{2}ab\sin C$. The two sides must be those that include the angle $C$ between them (i.e., $C$ is the included angle). If $C = 90^\circ$, this reduces to the familiar $\frac{1}{2} \times \text{base} \times \text{height}$.

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### Q58 — Intermediate — Answer: B

**Topic:** Finding a side using the sine rule with three angles known

1. $R = 180^\circ - 40^\circ - 75^\circ = 65^\circ$.
2. $\dfrac{q}{\sin Q} = \dfrac{p}{\sin P} \Rightarrow q = 15 \times \dfrac{\sin 75^\circ}{\sin 40^\circ} \approx 15 \times \dfrac{0.9659}{0.6428} \approx 15 \times 1.503 \approx 22.5.$

**Why the distractors are wrong:**

**A) 32.5** uses an incorrect ratio or angle, substantially overestimating $q$.

**C) 10.0** inverts the sine ratio: $15 \times \sin 40^\circ / \sin 75^\circ \approx 10.0$. Again, the larger angle (Q = 75°) must be opposite the longer side.

**D) 15.7** uses $\sin 65^\circ / \sin 40^\circ$ instead of $\sin 75^\circ / \sin 40^\circ$, substituting the wrong angle.

> **Takeaway:** In the sine rule $q/\sin Q = p/\sin P$, always pair each side with its opposite angle. First determine all three angles (using the fact that they sum to 180°) before applying the formula.

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### Q59 — Proficient — Answer: C

**Topic:** Navigation — bearings problem

**Resolve each leg into East and North components.**

**Leg 1** (N60°E, 12 km): East $= 12\sin 60^\circ = 6\sqrt{3}$; North $= 12\cos 60^\circ = 6$

**Leg 2** (S30°E, 9 km): East $= 9\sin 30^\circ = 4.5$; North $= -9\cos 30^\circ = -\frac{9\sqrt{3}}{2}$ (southward)

**Add, then use Pythagoras:**

$$\left(6\sqrt{3} + 4.5\right)^2 + \left(6 - \tfrac{9\sqrt{3}}{2}\right)^2 = 144 + 81 = 225$$
$$\text{Distance} = 15 \text{ km}$$

*Why the others are wrong:*

- **A** (21) adds $12 + 9$, ignoring direction.
- **B** (3) subtracts, as if the ship reversed.
- **D** (10.8) uses the cosine rule with **60°** as the included angle: $\sqrt{144 + 81 - 2(12)(9)\cos 60^\circ} = \sqrt{117}$. But the turn from bearing 060° to bearing 150° makes the included angle **90°**, which is why the components method gives a clean 15.

> **Takeaway:** For multi-leg bearing problems, resolve into East/North components, add them, then apply Pythagoras. It sidesteps the hardest part — working out the included angle.

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**Q60 — Proficient — Answer: A**

**Topic:** Ambiguous case of the sine rule

**Sine rule:** $\sin B = \dfrac{b \sin A}{a} = \dfrac{7(0.5)}{10} = 0.35$

That gives $B \approx 20.5^\circ$ **or** $B = 180^\circ - 20.5^\circ = 159.5^\circ$.

**Test the obtuse option:** $30^\circ + 159.5^\circ = 189.5^\circ > 180^\circ$ — impossible. So exactly **one** triangle.

*Why the others are wrong:*

- **D** (0) would need $a < b \sin A = 3.5$. Here $a = 10$, so a triangle certainly exists.
- **B** (2) forgets to test the obtuse angle against the $180^\circ$ limit.
- **C** (3) — SSA can never give three.

> **Takeaway:** With SSA data (angle $A$, its opposite side $a$, and side $b$), the count is decided by comparing $a$ with $b$:

- $a \geq b \rightarrow$ **one** triangle (this question: $10 \geq 7$)
- $b \sin A < a < b \rightarrow$ **two** triangles
- $a = b \sin A \rightarrow$ one right triangle; $a < b \sin A \rightarrow$ none

Either memorise that, or simply test the obtuse angle against $A + B < 180^\circ$ — which works every time.

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## Exam-Bank Extras — Question Types Confirmed in Recent Papers

### Q61 — Intermediate — Answer: A

**Topic:** Double angle in disguise (compression inside the identity)

**Factor until the bracket matches** $\cos 2\theta = 1 - 2\sin^2\theta$. Take out the 2:

$$2 - 4\sin^2 3x = 2\left(1 - 2\sin^2 3x\right)$$

The bracket is $\cos 2\theta$ with $\theta = 3x$, so it becomes $\cos 6x$:

$$= 2\cos 6x$$

*Why the others are wrong:*

- **B** kept the angle as $3x$. The identity **doubles** whatever is inside.
- **C** flipped the sign, misremembering the identity as $2\sin^2\theta - 1$.
- **D** rewrote $4\sin^2 3x$ but forgot it was being subtracted from 2.

> **Takeaway:** Factor until you see $1 - 2\sin^2(\ldots)$, then write cos of **double** the inside angle. Putting $3x$ inside is the exam's favourite way to make that doubling easy to fumble.

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### Q62 — Proficient — Answer: A

**Topic:** Paired linear combinations of sin and cos (square and add)

**Spot the design:** the coefficients 3 and 5 are **swapped** and the sign flipped. That is the fingerprint of **square and add**.

Let $k = 5\sin\theta - 3\cos\theta$ and square both expressions:

$$(3\sin\theta + 5\cos\theta)^2 = 9\sin^2\theta + 30\sin\theta\cos\theta + 25\cos^2\theta = 25$$
$$(5\sin\theta - 3\cos\theta)^2 = 25\sin^2\theta - 30\sin\theta\cos\theta + 9\cos^2\theta = k^2$$

**Add them** — the cross terms cancel, which is exactly why the coefficients were swapped:

$$34(\sin^2\theta + \cos^2\theta) = 25 + k^2 \implies 34 = 25 + k^2 \implies k = \pm 3$$

Both signs genuinely occur, for different values of $\theta$.

*Why the others are wrong:*

- **B** (3 only) discarded the negative root — the same ± trap as $x^2 = 4$.
- **C** ($\pm\sqrt{34}$) is the **maximum** either expression can reach, not this value.
- **D** (0) would need $k^2 = -25$.

> **Takeaway:** Swapped coefficients with a flipped sign → square and add. The cross terms always cancel, leaving $a^2 + b^2$. You never need to find $\theta$.

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### Q63 — Proficient — Answer: D

**Topic:** Eliminating the parameter (angle) from a pair of equations

The answers contain no $\theta$, so the job is to **eliminate the angle** — and the only identity that does that is $\sin^2\theta + \cos^2\theta = 1$. So get both into squared form.

**Isolate and square:**

$$\cos\theta = \frac{x}{3} \Rightarrow \cos^2\theta = \frac{x^2}{9} \qquad \sin\theta = \frac{y}{2} \Rightarrow \sin^2\theta = \frac{y^2}{4}$$

**Add:**

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

*Why the others are wrong:*

- **A** swapped the denominators. The 9 belongs under $x^2$, because $x$ carried the 3.
- **B** gives $9\cos^2\theta + 4\sin^2\theta$, which **changes** with $\theta$.
- **C** added the un-squared ratios, true only at special angles.

> **Takeaway:** For $x = a\cos\theta, y = b\sin\theta$: divide, square, add → $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. "True for all $\theta$" is code for *eliminate the parameter*.

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### Q64 — Intermediate — Answer: C

**Topic:** Locating a vertical asymptote of a tan graph

tan is undefined when its **whole argument** hits $90^\circ$ (or $90^\circ + 180^\circ k$). So set the bracket equal to $90^\circ$:

$$2x - 30^\circ = 90^\circ \implies 2x = 120^\circ \implies x = 60^\circ$$

Confirm it is the **first** positive one: the argument $-90^\circ$ gives $x = -30^\circ$ (negative), and $270^\circ$ gives $x = 150^\circ$ (later).

*Why the others are wrong:*

- **A** ($45^\circ$) ignored the $-30^\circ$ shift, treating it as plain $\tan 2x$.
- **D** ($105^\circ$) set the argument to $180^\circ$ — but $\tan$ is **zero** there, not undefined.
- **B** ($52.5^\circ$) halved the $30^\circ$ somewhere instead of solving $2x = 120^\circ$ cleanly.

> **Takeaway:** For $y = \tan(bx + c)$, solve $bx + c = 90^\circ + 180^\circ k$. Two things cost marks: forgetting the shift, and confusing where $\tan$ is **undefined** ($90^\circ, 270^\circ$) with where it is **zero** ($0^\circ, 180^\circ$).

## Mixed Practice — Chapter 4

### M1 — Basic

**Topic:** Basic SOHCAHTOA

A right triangle has an opposite side of 6 and hypotenuse of 10. What is $\sin\theta$?

A) $\frac{3}{4}$
B) $\frac{4}{5}$
C) $\frac{3}{5}$
D) $\frac{5}{3}$

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### M2 — Basic

**Topic:** Special angle value

$\cos 60^\circ$ equals:

A) $\frac{1}{2}$
B) $\frac{\sqrt{3}}{2}$
C) 1
D) $\frac{\sqrt{2}}{2}$

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### M3 — Basic

**Topic:** CAST rule

In which quadrant is $\tan\theta$ positive and $\sin\theta$ negative?

A) Q1

B) Q2

C) Q4

D) Q3

---

### M4 — Basic

**Topic:** Reference angle

The reference angle for 200° is:

A) 200°

B) 20°

C) 70°

D) 160°

---

### M5 — Basic

**Topic:** Amplitude

What is the amplitude of $y = -4\cos\theta$?

A) $-4$

B) 4

C) 8

D) 2

---

### M6 — Intermediate

**Topic:** Sine rule

In triangle $ABC$, $a = 8$, $A = 45^\circ$, $B = 60^\circ$. Find $b$.

A) $4\sqrt{2}$

B) $8\sqrt{2}$

C) $8\sqrt{3}$

D) $4\sqrt{6}$

---

### M7 — Basic

**Topic:** Solving a sin equation

Solve $\sin\theta = \dfrac{\sqrt{3}}{2}$ for $\theta \in [0^\circ, 360^\circ]$.

A) 60° and 120°

B) 30° and 150°

C) 60° only

D) 30° only

---

### M8 — Basic

**Topic:** Period

What is the period of $y = 2\cos\left(\dfrac{\theta}{2}\right)$?

A) 180°

B) 360°

C) 720°

D) 90°

---

### M9 — Intermediate

**Topic:** Trig values in specified quadrant

Given $\tan\theta = \dfrac{3}{4}$ with $\theta$ in Q3, find $\sin\theta$.

A) $\dfrac{4}{5}$

B) $\dfrac{3}{5}$

C) $-\dfrac{4}{5}$

D) $-\dfrac{3}{5}$

---

### M10 — Intermediate

**Topic:** Identity simplification

Simplify $\sin^2\theta + \cos^2\theta + \tan^2\theta$.

A) 1

B) $\csc^2\theta$

C) $\sec^2\theta$

D) 2

---

### M11 — Intermediate

**Topic:** Solving a cos equation

Solve $2\cos\theta = \sqrt{3}$ for $\theta \in [0^\circ, 360^\circ]$.

A) 30° and 330°

B) 60° and 300°

C) 30° and 150°

D) 60° and 120°

---

### M12 — Intermediate

**Topic:** Area of a triangle

Find the area of triangle $ABC$ where $a = 5$, $b = 8$, $C = 120^\circ$.

A) 20

B) $10\sqrt{3}$

C) 10

D) 40

---

### M13 — Intermediate

**Topic:** Identifying a valid identity

Which identity is correct?

A) $\cos^2\theta - \sin^2\theta = \cos 2\theta$

B) $\sin 2\theta = \sin^2\theta + \cos^2\theta$

C) $\cos 2\theta = 1 - \sin^2\theta$

D) $\sin 2\theta = \sin\theta + \cos\theta$

---

### M14 — Intermediate

**Topic:** Angle of elevation — shadow problem

A tower casts a shadow of length 15 m when the angle of elevation of the sun is 40°. Find the height of the tower (to 1 decimal place).

A) 9.6 m

B) 23.3 m

C) 17.9 m

D) 12.6 m

---

### M15 — Intermediate

**Topic:** Range of transformed function

The range of $y = 2\sin x + 3$ is:

A) $[-1, 1]$

B) $[1, 3]$

C) $[1, 5]$

D) $[-2, 2]$

---

### M16 — Intermediate

**Topic:** Cosine rule — finding an angle

In a triangle with $a = 7$, $b = 7$, $c = 7\sqrt{2}$, find angle $C$.

A) 60°

B) 90°

C) 120°

D) 45°

---

### M17 — Basic

**Topic:** Pythagorean identity — sec and tan

Evaluate $\sec^2 30^\circ - \tan^2 30^\circ$.

A) $\frac{1}{3}$

B) 1

C) $\frac{4}{3}$

D) $\sqrt{3}$

---

### M18 — Basic

**Topic:** Zeros of sin graph

What are the $x$-intercepts of $y = \sin x$ in $[0^\circ, 360^\circ]$?

A) 0°, 180°, 360°

B) 0° only

C) 90°

D) 0° and 90°

---

### M19 — Proficient

**Topic:** Identity simplification

Simplify $\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$.

A) 1

B) $2\sin\theta\cos\theta$

C) $\frac{2}{\sin 2\theta}$

D) $\sin\theta\cos\theta$

---

### M20 — Intermediate

**Topic:** Cosine rule — finding $c^2$

Given $a = 5$, $b = 8$, $C = 60^\circ$, find the value of $c^2$.

A) 89

B) 40

C) 7

D) 49

---

### M21 — Intermediate

**Topic:** Domain of tan

The function $y = \tan\theta$ is undefined when $\theta$ equals (in $[0^\circ, 360^\circ]$):

A) 45° and 225°

B) 0° and 180°

C) 90° and 270°

D) 60° and 120°

---

**M22 — Intermediate**

**Topic:** Cosine rule — finding cos A

In triangle $ABC$, $a = 6$, $b = 4$, $c = 5$. Find $\cos A$.

A) $-\frac{1}{8}$
B) $\frac{1}{8}$
C) $\frac{1}{4}$
D) $-\frac{1}{4}$

---

**M23 — Basic**

**Topic:** Range of cosine

Which of the following is in the range of $y = \cos\theta$?

A) 1.5

B) $-2$

C) 2

D) $-0.5$

---

**M24 — Intermediate**

**Topic:** Solving cos² equation

Solve $\cos^2\theta = \frac{1}{4}$ for $\theta \in [0^\circ, 360^\circ]$.

A) 60°, 120°, 240°, 300°

B) 60° only

C) 60° and 120°

D) 60° and 300° only

---

### M25 — Basic

**Topic:** Co-function identity

$\cos(90^\circ - \theta)$ equals:

A) $\tan\theta$

B) $\cos\theta$

C) $-\sin\theta$

D) $\sin\theta$

---

### M26 — Intermediate

**Topic:** When sin = cos

For which values of $\theta$ in $[0^\circ, 360^\circ]$ is $\sin\theta = \cos\theta$?

A) 45° and 225°

B) 45° only

C) 135° and 315°

D) 90° only

---

### M27 — Intermediate

**Topic:** Minimum of transformed function

What is the minimum value of $y = 2\sin(3x) + 1$?

A) $-2$

B) $0$

C) $-1$

D) $1$

---

### M28 — Basic

**Topic:** Finding a side in a right triangle

In right triangle $DEF$ where $\angle F = 90^\circ$, $\angle D = 35^\circ$, and hypotenuse $DE = 12$ cm. Find $EF$ (to 1 decimal place).

A) 9.8 cm

B) 6.9 cm

C) 10.2 cm

D) 14.6 cm

---

### M29 — Proficient

**Topic:** General solution of cos = 0

The general solution of $\cos\theta = 0$ is:

A) $\theta = 90^\circ + 180^\circ n$, for $n \in \mathbb{Z}$

B) $\theta = 90^\circ + 360^\circ n$, for $n \in \mathbb{Z}$

C) $\theta = 180^\circ n$, for $n \in \mathbb{Z}$

D) $\theta = 90^\circ + 90^\circ n$, for $n \in \mathbb{Z}$

---

### M30 — Intermediate

**Topic:** Intersection of sin and cos graphs

At which values of $x$ in $[0^\circ, 360^\circ]$ do the graphs of $y = \sin x$ and $y = \cos x$ intersect?

A) 45° only

B) 90° and 270°

C) 45° and 225°

D) 0° and 180°

## Mixed Practice — Answer Key

| | | | | |
|---|---|---|---|---|
| **M1.** C | **M2.** A | **M3.** D | **M4.** B | **M5.** B |
| **M6.** D | **M7.** A | **M8.** C | **M9.** D | **M10.** C |
| **M11.** A | **M12.** B | **M13.** A | **M14.** D | **M15.** C |
| **M16.** B | **M17.** B | **M18.** A | **M19.** C | **M20.** D |
| **M21.** C | **M22.** B | **M23.** D | **M24.** A | **M25.** D |
| **M26.** A | **M27.** C | **M28.** B | **M29.** A | **M30.** C |

# Chapter 5 — Geometry

## Lines, Angles, Triangles, Similarity, and Quadrilaterals

## Section 5.1 — Lines, Angles and Parallel Lines

### Q1 — Basic

Two angles are supplementary. One angle measures $(3x + 10)^\circ$ and the other measures $(2x - 10)^\circ$. Find the value of $x$.

A) 34

B) 36

C) 40

D) 38

---

### Q2 — Basic

Two adjacent angles on a straight line measure $3x^\circ$ and $2x^\circ$. Find the value of $x$.

A) 36

B) 30

C) 40

D) 45

---

### Q3 — Basic

Two straight lines intersect. One pair of vertically opposite angles measures $(5x - 15)^\circ$ and $(3x + 15)^\circ$. Find the size of the angle.

A) 15°

B) 45°

C) 60°

D) 75°

---

### Q4 — Intermediate

Two parallel lines are cut by a transversal. A pair of corresponding angles measures $(7x - 11)^\circ$ and $(5x + 29)^\circ$. Find the size of the angle.

A) 20°

B) 100°

C) 120°

D) 129°

---

### Q5 — Intermediate

Two parallel lines are cut by a transversal. Co-interior angles measure $(3x + 20)^\circ$ and $(2x + 40)^\circ$. Find the value of $x$.

A) 28

B) 26

C) 22

D) 24

---

### Q6 — Intermediate

Two parallel lines are cut by a transversal. Alternate angles measure $(4x - 10)^\circ$ and $(2x + 30)^\circ$. Find the size of the angle.

A) 20°

B) 80°

C) 70°

D) 110°

---

### Q7 — Intermediate

Two parallel lines are cut by a transversal. Co-interior angles measure $(3x + 30)^\circ$ and $2x^\circ$. Find the value of $x$.

A) 30

B) 25

C) 36

D) 24

---

### Q8 — Proficient

An exterior angle of a triangle measures $115^\circ$. One of the two non-adjacent interior angles is $48^\circ$. Find the other non-adjacent interior angle.

A) 48°

B) 67°

C) 57°

D) 52°

---

## Section 5.2 — Triangles — Types, Properties, Congruence

### Q9 — Basic

In triangle $ABC$, angle $A = 2x^\circ$, angle $B = 3x^\circ$, and angle $C = x^\circ$. Find the value of $x$.

A) 30

B) 36

C) 25

D) 45

---

**Q10 — Basic**

An isosceles triangle has an apex angle of $50^\circ$. Find the size of each base angle.

A) 55°

B) 75°

C) 70°

D) 65°

---

**Q11 — Intermediate**

A right triangle has legs of length 7 cm and 24 cm. Find the length of the hypotenuse.

A) 20 cm

B) 25 cm

C) 24 cm

D) 26 cm

---

**Q12 — Intermediate**

Two right-angled triangles share the same hypotenuse length and the same length for one leg. Which congruence criterion proves them congruent?

A) SSS

B) SAS

C) RHS

D) AAS

---

**Q13 — Intermediate**

The exterior angle of a triangle is $120^\circ$. One of the non-adjacent interior angles is $45^\circ$. Find the other non-adjacent interior angle.

A) 80°

B) 65°

C) 75°

D) 60°

### Q14 — Intermediate

An isosceles triangle has two equal sides of 10 cm and a base of 12 cm. Find the perpendicular height from the apex to the base.

A) 6 cm

B) 8 cm

C) 10 cm

D) 12 cm

### Q15 — Proficient

Two triangles have sides of 5 cm, 7 cm, and 9 cm respectively. By what criterion are they congruent?

A) SAS

B) RHS

C) AAS

D) SSS

### Q16 — Proficient

A triangle has a base of 15 cm and a perpendicular height of 8 cm. Find its area.

A) 60 $cm^2$

B) 56 $cm^2$

C) 56.5 $cm^2$

D) 45 $cm^2$

## Section 5.3 — Similarity and Proportion

### Q17 — Basic

Two similar triangles have sides in the ratio $2 : 3$. The smaller triangle has a side of $8$ cm. Find the corresponding side of the larger triangle.

A) 9 cm

B) 6 cm

C) 16 cm

D) 12 cm

### Q18 — Basic

Two similar figures have a linear scale factor of $3 : 1$. What is their area ratio?

A) 9:1

B) 3:1

C) 27:1

D) 6:1

### Q19 — Intermediate

Two similar triangles have sides $6$ cm, $8$ cm, $10$ cm and $x$ cm, $12$ cm, $15$ cm respectively. Find $x$.

A) 6

B) 8

C) 9

D) 12

### Q20 — Intermediate

A 6 m pole casts a 4 m shadow. At the same time, a building casts a 20 m shadow. Find the height of the building.

A) 10 m

B) 30 m

C) 24 m

D) 20 m

---

### Q21 — Proficient

In right triangle $ABC$ with the right angle at $C$, the altitude $CD$ is drawn to the hypotenuse $AB$. If $AD = 4$ cm and $DB = 9$ cm, find the length of $CD$.

A) 7 cm

B) 5 cm

C) 6 cm

D) 8 cm

---

### Q22 — Proficient

Two similar cylinders have their heights in the ratio $2 : 5$. What is their volume ratio?

A) 4:25

B) 4:10

C) 2:5

D) 8:125

---

## Section 5.4 — Quadrilaterals

### Q23 — Basic

The angles in a quadrilateral are $80^\circ$, $95^\circ$, $110^\circ$, and $x^\circ$. Find $x$.

A) 65°

B) 75°

C) 70°

D) 80°

---

### Q24 — Basic

A rectangle has length 12 cm and width 5 cm. Find the length of its diagonal.

A) 13 cm

B) 10 cm

C) 12 cm

D) 7 cm

---

### Q25 — Intermediate

A rectangle has length 8 cm and width 6 cm. Find the length of its diagonal.

A) 7 cm

B) 12 cm

C) 10 cm

D) 14 cm

---

### Q26 — Intermediate

A rhombus has diagonals of length 8 cm and 6 cm. Find the length of one side.

A) 4 cm

B) 5 cm

C) 6 cm

D) 7 cm

### Q27 — Intermediate

A parallelogram has a base of 12 cm and a perpendicular height of 7 cm. Find its area.

A) 42 $cm^2$

B) 72 $cm^2$

C) 96 $cm^2$

D) 84 $cm^2$

### Q28 — Intermediate

A trapezium has parallel sides of 10 cm and 14 cm, and a perpendicular height of 8 cm. Find its area.

A) 96 $cm^2$

B) 80 $cm^2$

C) 88 $cm^2$

D) 60 $cm^2$

### Q29 — Proficient

A kite has diagonals of length 10 cm and 8 cm. Find its area.

A) 36 $cm^2$

B) 40 $cm^2$

C) 48 $cm^2$

D) 56 $cm^2$

## Section 5.5 — Circles — Arcs, Sectors, Chords

### Q30 — Basic

Find the circumference of a circle with radius 7 cm. Leave your answer in terms of $\pi$.

A) $7\pi$ cm

B) $49\pi$ cm

C) $28\pi$ cm

D) $14\pi$ cm

---

### Q31 — Basic

A circle has radius 9 cm. A sector subtends an angle of $80^\circ$ at the centre. Find the area of the sector in terms of $\pi$.

A) $9\pi$ cm²

B) $12\pi$ cm²

C) $18\pi$ cm²

D) $24\pi$ cm²

---

### Q32 — Intermediate

A circle has radius 6 cm. An arc subtends an angle of $120^\circ$ at the centre. Find the arc length in terms of $\pi$.

A) $4\pi$ cm

B) $3\pi$ cm

C) $3.5\pi$ cm

D) $2\pi$ cm

---

### Q33 — Intermediate

A chord $AB$ has length 12 cm. The perpendicular distance from the centre of the circle to the chord is 8 cm. Find the radius.

A) 14 cm

B) 8 cm

C) 12 cm

D) 10 cm

---

### Q34 — Intermediate

A central angle subtends an arc of $140^\circ$. Find the inscribed angle that subtends the same arc.

A) 70°

B) 35°

C) 140°

D) 280°

---

### Q35 — Intermediate

A circle has radius 10 cm. A chord of length 16 cm is drawn. Find the perpendicular distance from the centre to the chord.

A) 4 cm

B) 6 cm

C) 5 cm

D) 8 cm

---

### Q36 — Intermediate

$AB$ is a diameter of a circle and $C$ is a point on the circle. In triangle $ABC$, angle $CAB = 35^\circ$. Find angle $ABC$.

A) 35°

B) 45°

C) 55°

D) 50°

---

### Q37 — Proficient

From an external point $P$, a tangent is drawn to a circle with centre $O$ and radius 5 cm. The distance $OP = 13$ cm. Find the length of the tangent.

A) 12 cm

B) 8 cm

C) 10 cm

D) 13 cm

---

### Q38 — Proficient

In a cyclic quadrilateral, two opposite angles measure $3x^\circ$ and $(2x + 10)^\circ$. Find $x$.

A) 30

B) 36

C) 34

D) 40

---

## Section 5.6 — Perimeter and Area

### Q39 — Basic

A rectangle has length 9 cm and width 5 cm. Find its perimeter.

A) 45 cm

B) 25 cm

C) 14 cm

D) 28 cm

---

### Q40 — Basic

A triangle has a base of 10 cm and a perpendicular height of 12 cm. Find its area.

A) 30 $cm^2$

B) 60 $cm^2$

C) 50 $cm^2$

D) 45 $cm^2$

---

### Q41 — Intermediate

A composite shape consists of a rectangle 8 cm $\times$ 5 cm with a triangle placed on top. The triangle has a base of 8 cm (shared with the rectangle) and a height of 3 cm. Find the total area.

A) 60 $cm^2$

B) 48 $cm^2$

C) 56 $cm^2$

D) 52 $cm^2$

---

### Q42 — Intermediate

Find the area of a circle with diameter 10 cm. Leave your answer in terms of $\pi$.

A) $10\pi$ cm²

B) $25\pi$ cm²

C) $50\pi$ cm²

D) $100\pi$ cm²

---

### Q43 — Intermediate

A semicircle has a radius of 6 cm. Find its total perimeter (arc plus diameter).

A) $6 + 6\pi$ cm

B) $12 + 3\pi$ cm

C) $12 + 6\pi$ cm

D) $6 + 12\pi$ cm

---

### Q44 — Intermediate

A circle has radius 12 cm. A sector subtends an angle of $90^\circ$ at the centre. Find the area of the sector in terms of $\pi$.

A) $36\pi$ cm²

B) $24\pi$ cm²

C) $30\pi$ cm²

D) $6\pi$ cm²

---

### Q45 — Proficient

Two concentric circles have radii 8 cm and 5 cm. Find the area of the region between them in terms of $\pi$.

A) $89\pi$ cm²

B) $25\pi$ cm²

C) $64\pi$ cm²

D) $39\pi$ cm²

### Q46 — Proficient

A right triangle has legs of 9 cm and 40 cm. Find the perimeter.

A) 82 cm

B) 88 cm

C) 90 cm

D) 96 cm

## Section 5.7 — Surface Area and Volume

### Q47 — Basic

A rectangular box has length 4 cm, width 3 cm, and height 5 cm. Find its volume.

A) 20 $cm^3$

B) 60 $cm^3$

C) 47 $cm^3$

D) 94 $cm^3$

### Q48 — Basic

A cube has a side length of 4 cm. Find its total surface area.

A) 96 $cm^2$

B) 24 $cm^2$

C) 64 $cm^3$

D) 16 $cm^2$

**Q49 — Intermediate**

A cylinder has a base radius of 3 cm and a height of 8 cm. Find its volume in terms of $\pi$.

A) $36\pi$ cm$^3$

B) $48\pi$ cm$^3$

C) $72\pi$ cm$^3$

D) $24\pi$ cm$^3$

---

**Q50 — Intermediate**

A cylinder has radius 5 cm and height 10 cm. Find its total surface area in terms of $\pi$.

A) $100\pi$ cm$^2$

B) $150\pi$ cm$^2$

C) $200\pi$ cm$^2$

D) $75\pi$ cm$^2$

---

**Q51 — Intermediate**

A cone has a base radius of 6 cm and a height of 8 cm. Find its volume in terms of $\pi$.

A) $48\pi$ cm$^3$

B) $72\pi$ cm$^3$

C) $144\pi$ cm$^3$

D) $96\pi$ cm$^3$

---

**Q52 — Intermediate**

Find the volume of a sphere with radius 3 cm. Leave your answer in terms of $\pi$.

A) $36\pi$ cm$^3$

B) $18\pi$ cm$^3$

C) $27\pi$ cm$^3$

D) $9\pi$ cm$^3$

---

### Q53 — Proficient

A triangular prism has a right-angled triangular cross-section with legs 3 cm and 4 cm. The length of the prism is 10 cm. Find the total surface area.

A) 120 $cm^2$

B) 132 $cm^2$

C) 140 $cm^2$

D) 150 $cm^2$

---

### Q54 — Proficient

A square pyramid has a base of side 9 cm and a height of 4 cm. Find its volume.

A) 72 $cm^3$

B) 216 $cm^3$

C) 144 $cm^3$

D) 108 $cm^3$

---

## Section 5.8 — Coordinate Geometry

### Q55 — Basic

Find the distance between the points $(1, 2)$ and $(4, 6)$.

A) 3

B) 4

C) 5

D) 7

---

### Q56 — Basic

Find the midpoint of the segment joining $(-2,\ 5)$ and $(6,\ -1)$.

A) $(2,\ 2)$

B) $(2,\ 3)$

C) $(4,\ 2)$

D) $(4,\ 4)$

---

### Q57 — Intermediate

Find the gradient of the line passing through $(1,\ 3)$ and $(4,\ 9)$.

A) $-2$

B) 3

C) $\frac{1}{2}$

D) 2

---

### Q58 — Intermediate

Find the equation of the line that passes through $(2,\ 1)$ with gradient 3.

A) $y = 3x + 5$

B) $y = 3x - 1$

C) $y = 3x - 5$

D) $y = 3x + 1$

---

### Q59 — Proficient

A ship sails 9 km due north, then 12 km due east. How far is the ship from its starting point?

A) 15 km

B) 13 km

C) 10 km

D) 21 km

---

### Q60 — Proficient

Triangle $ABC$ has vertices $A(0,\ 0)$, $B(6,\ 0)$, and $C(2,\ 4)$. Find the area of triangle $ABC$.

A) 8 sq units

B) 12 sq units

C) 11 sq units

D) 10 sq units

---

## Exam-Bank Extras — Question Types Confirmed in Recent Papers

The NBT MAT reuses question types from a stable bank year after year. The three questions below are modelled directly on types repeatedly confirmed in recent papers and not yet represented in this chapter. Every answer has been independently machine-verified.

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### Q61 — Intermediate

A water sprinkler stands at the centre of a square field. It sprays water in a full circle whose radius is exactly half the side of the field — so the spray just reaches the middle of each side. What **fraction** of the field is NOT watered?

A) $\frac{\pi}{4}$

B) $\frac{4-\pi}{4}$

C) $\frac{4-\pi}{\pi}$

D) $1-\frac{\pi}{2}$

---

### Q62 — Intermediate

A square lawn is surrounded by a path exactly $1$ m wide on all four sides. The area of the path alone is $40\ \text{m}^2$. Find the area of the lawn.

A) $64\ \text{m}^2$

B) $100\ \text{m}^2$

C) $81\ \text{m}^2$

D) $121\ \text{m}^2$

---

### Q63 — Proficient

A regular pentagon with perimeter 50 cm is inscribed in a circle (all five vertices lie on the circle). The radius of the circle is:

A) $\dfrac{5}{\sin 36^\circ}$

B) $\dfrac{10}{\sin 72^\circ}$

C) $\dfrac{5}{\cos 36^\circ}$

D) $10 \sin 54^\circ$

# Answers and Explanations

## Section 5.1 — Lines, Angles and Parallel Lines

### Q1 — Basic — Answer: B

**Topic:** Supplementary angles — forming and solving an equation

1. Supplementary angles sum to $180^\circ$: $(3x + 10) + (2x - 10) = 180$.
2. Simplify: $5x + 0 = 180$, so $5x = 180$.
3. Divide: $x = 36$. Check: $3(36) + 10 = 118^\circ$ and $2(36) - 10 = 62^\circ$; $118 + 62 = 180^\circ$. ✓

**Why the distractors are wrong:**

**A) 34:** From forgetting the $-10$ constant in the second angle: $3x + 10 + 2x = 180 \Rightarrow 5x + 10 = 180 \Rightarrow 5x = 170 \Rightarrow x = 34$.

**C) 40:** From incorrectly treating supplementary angles as summing to $200^\circ$: $5x = 200 \Rightarrow x = 40$.

**D) 38:** From $5x + 10 = 200$ (combining both errors — missing a constant and wrong total): $5x = 190 \Rightarrow x = 38$.

> **Takeaway:** Supplementary angles add to $180^\circ$; complementary add to $90^\circ$. The anchor: **S**upplementary = **S**traight line = $180^\circ$.

---

### Q2 — Basic — Answer: A

**Topic:** Angles on a straight line

1. Angles on a straight line sum to $180^\circ$: $3x + 2x = 180$.
2. $5x = 180 \Rightarrow x = 36$.
3. Check: $3(36) = 108^\circ$ and $2(36) = 72^\circ$; $108 + 72 = 180^\circ$. ✓

**Why the distractors are wrong:**

**B) 30:** From assuming angles on a line sum to $150^\circ$: $5x = 150 \Rightarrow x = 30$.

**C) 40:** From using $200^\circ$ as the straight-line total: $5x = 200 \Rightarrow x = 40$.

**D) 45:** From setting one angle equal to $90^\circ$ and ignoring the other: $2x = 90 \Rightarrow x = 45$.

> **Takeaway:** Angles on a straight line always sum to $180^\circ$ — this is a fundamental axiom, not something to be derived. It underpins nearly every angle calculation in plane geometry.

---

### Q3 — Basic — Answer: C

**Topic:** Vertically opposite angles

1. Vertically opposite angles are equal: $5x - 15 = 3x + 15$.
2. Solve: $2x = 30 \Rightarrow x = 15$.
3. Angle size: $5(15) - 15 = 75 - 15 = 60^\circ$.

**Why the distractors are wrong:**

**A) 15°:** Reports $x = 15$ as the answer rather than calculating the angle. Finding $x$ is the intermediate step, not the final answer.

**B) 45°:** From computing $3x = 3(15) = 45$ — substituting into only part of the second expression and ignoring the $+15$ constant.

**D) 75°:** From computing $5x = 5(15) = 75$ — substituting into only part of the first expression and ignoring the $-15$ constant.

> **Takeaway:** Always substitute $x$ back into the full original expression to find the angle. Stopping at $x$ is a very common exam mistake.

---

### Q4 — Intermediate — Answer: D

**Topic:** Corresponding angles with parallel lines

1. Corresponding angles (F-angles) between parallel lines are equal: $7x - 11 = 5x + 29$.
2. Solve: $2x = 40 \Rightarrow x = 20$.
3. Angle size: $7(20) - 11 = 140 - 11 = 129^\circ$.

**Why the distractors are wrong:**

**A) 20°:** Reports $x = 20$ instead of computing the angle — the question asks for the angle, not $x$.

**B) 100°:** From substituting only into $5x = 5(20) = 100$ — ignoring the $+29$ constant.

**C) 120°:** From a sign error: $7(20) - 20 = 120$ — subtracting 20 instead of 11.

> **Takeaway:** Corresponding angles are equal only when the lines are parallel (the F-angle property). If the lines are not stated to be parallel, this relationship does not hold.

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**Q5 — Intermediate — Answer: D**

**Topic:** Co-interior (same-side interior) angles

1. Co-interior angles (C-angles) between parallel lines are supplementary: $(3x+20)+(2x+40)=180$.
2. $5x+60=180 \Rightarrow 5x=120 \Rightarrow x=24$.
3. Check: $3(24)+20=92^\circ$ and $2(24)+40=88^\circ$; $92+88=180^\circ$. ✓

**Why the distractors are wrong:**

**B) 26:** From $5x=130 \Rightarrow x=26$ — subtracting only 50 from 180 (using the sum of one constant instead of both: $20+40=60$, not 50).

**C) 22:** From $5x=110 \Rightarrow x=22$ — subtracting 70 from 180 (adding an extra constant that does not exist).

**A) 28:** From forgetting the constant in one angle: $(3x)+(2x+40)=180 \Rightarrow 5x+40=180 \Rightarrow$ $5x=140 \Rightarrow x=28$.

> **Takeaway:** Co-interior angles are **supplementary** ($180^\circ$); alternate and corresponding angles are **equal**. The mnemonic: Co-interior angles are on the **same** side of the transversal and "complete" $180^\circ$.

---

**Q6 — Intermediate — Answer: C**

**Topic:** Alternate (Z-angle) angles

1. Alternate angles (Z-angles) between parallel lines are equal: $4x-10=2x+30$.
2. $2x=40 \Rightarrow x=20$.
3. Angle: $4(20)-10=80-10=70^\circ$.

**Why the distractors are wrong:**

**A) 20°:** Reports $x=20$ rather than the angle.

**B) 80°:** From $4x=4(20)=80$ — forgetting the $-10$ constant in the final substitution.

**D) 110°:** From treating alternate angles as supplementary (co-interior): $4x - 10 + 2x + 30 = 180$ gives an incorrect result, and $180 - 70 = 110^\circ$ is the supplement of the correct answer.

> **Takeaway:** The three parallel-line angle relationships: **Corresponding** (F) = equal; **Alternate** (Z) = equal; **Co-interior** (C) = supplementary. The letter shape drawn between the parallel lines names each relationship.

---

#### Q7 — Intermediate — Answer: A

**Topic:** Co-interior angles — solving for an unknown

1. Co-interior angles sum to $180^\circ$: $(3x + 30) + 2x = 180$.
2. $5x + 30 = 180 \Rightarrow 5x = 150 \Rightarrow x = 30$.
3. Check: $3(30) + 30 = 120^\circ$ and $2(30) = 60^\circ$; $120 + 60 = 180^\circ$. ✓

**Why the distractors are wrong:**

**C) 36:** From forgetting the $+30$ constant entirely: $5x = 180 \Rightarrow x = 36$. Dropping constants is the single most common algebraic error in geometry angle problems.

**B) 25:** From $5x + 30 = 155 \Rightarrow 5x = 125 \Rightarrow x = 25$ — using $155^\circ$ instead of $180^\circ$ as the co-interior total.

**D) 24:** From $5x + 30 = 150 \Rightarrow 5x = 120 \Rightarrow x = 24$ — confusing the co-interior sum ($180^\circ$) with the alternate-angle condition (equal), then using $150^\circ$ as a compromise value.

> **Takeaway:** When setting up an angle equation, write every term explicitly before simplifying. The $+30$ constant does not vanish — it must be subtracted from both sides.

---

#### Q8 — Proficient — Answer: B

**Topic:** Exterior angle theorem

1. The Exterior Angle Theorem: an exterior angle equals the sum of the two non-adjacent (remote) interior angles.
2. Let the unknown angle be $\theta$: $115 = 48 + \theta \Rightarrow \theta = 67^\circ$.
3. Full-triangle verification: the interior angle at the exterior-angle vertex $= 180 - 115 = 65^\circ$. Sum: $48 + 67 + 65 = 180^\circ$. ✓

**Why the distractors are wrong:**

**A) 48°:** Simply copies the given angle — no theorem was applied.

**D) 52°:** From $180 - 48 - 80 = 52$ — incorrectly computing the third angle using a phantom $80^\circ$ value.

**C) 57°:** From computing the vertex interior angle ($180 - 115 = 65^\circ$) then applying an arithmetic slip: $65 - 8 = 57$.

> **Takeaway:** The Exterior Angle Theorem is a direct consequence of two facts: angles in a triangle sum to $180^\circ$, and supplementary angles sum to $180^\circ$. It saves you from needing to find the third interior angle.

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## Section 5.2 — Triangles — Types, Properties, Congruence

### Q9 — Basic — Answer: A

**Topic:** Angle sum of a triangle

1. Angles in any triangle sum to $180^\circ$: $2x + 3x + x = 180$.
2. $6x = 180 \Rightarrow x = 30$.
3. The three angles are $60^\circ$, $90^\circ$, and $30^\circ$ — a valid right triangle.

**Why the distractors are wrong:**

**B) 36:** From miscounting: adding only $2x + 3x = 5x$ and omitting the third angle $x$: $5x = 180 \Rightarrow x = 36$.

**C) 25:** From $6x + 30 = 180 \Rightarrow x = 25$ — adding a phantom constant not present in the original expression.

**D) 45:** From $4x = 180$ (miscounting: perhaps $x + 3x = 4x$ and ignoring $2x$): $x = 45$.

> **Takeaway:** The angle sum of a triangle is always $180^\circ$, regardless of shape or size. Count every angle expression before simplifying — it is easy to miss a term.

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### Q10 — Basic — Answer: D

**Topic:** Isosceles triangle — base angles

1. In an isosceles triangle the two base angles are equal. Let each be $\beta$.
2. Angle sum: $50 + 2\beta = 180 \Rightarrow 2\beta = 130 \Rightarrow \beta = 65^\circ$.

**Why the distractors are wrong:**

**A) 55°:** From $2\beta = 180 - 70 = 110 \Rightarrow \beta = 55$ — using $70^\circ$ as the apex angle instead of $50^\circ$ (a misread).

**C) 70°:** From $2\beta = 140 \Rightarrow \beta = 70$ — subtracting only $40^\circ$ (half the apex angle) instead of the full $50^\circ$ : perhaps the student halved the apex before subtracting.

**B) 75°:** From $2\beta = 150 \Rightarrow \beta = 75$ — subtracting only $30^\circ$ from $180^\circ$ (perhaps using the complement of $60^\circ$ for the apex).

> **Takeaway:** For any isosceles triangle: base angle $= (180^\circ - \text{apex})/2$. The apex and the two base angles partition $180^\circ$; symmetry gives each base angle an equal share of the remainder.

---

### Q11 — Intermediate — Answer: B

**Topic:** Pythagorean theorem — finding the hypotenuse

1. Apply the Pythagorean theorem: $c^2 = 7^2 + 24^2 = 49 + 576 = 625$.
2. $c = \sqrt{625} = 25$ cm.
3. 7-24-25 is a standard Pythagorean triple, alongside 3-4-5 and 5-12-13.

**Why the distractors are wrong:**

**A) 20 cm:** Likely from arithmetic errors in computing $\sqrt{625}$, or from adding the legs instead of using Pythagoras: $7 + 24 - 11 = 20$.

**C) 24 cm:** Simply reads back one of the given legs as the hypotenuse — not applying the theorem at all.

**D) 26 cm:** From misreading $\sqrt{625} = 25$ as $\sqrt{676} = 26$ — confusing 625 with the perfect square $676 = 26^2$.

> **Takeaway:** Memorise the common Pythagorean triples: 3-4-5, 5-12-13, 7-24-25, 8-15-17, 9-40-41 . Recognising them instantly saves time.

---

#### Q12 — Intermediate — Answer: C

**Topic:** Triangle congruence — identifying the correct criterion

You are given a **R**ight angle, the **H**ypotenuse and one **S**ide — that is the **RHS** criterion, which applies only to right-angled triangles.

(Pythagoras would give the third side, but RHS does not require you to find it.)

*Why the others are wrong:*

- **A** SSS needs all three sides stated; only two are.
- **B** SAS needs the angle **between** the two given sides. The right angle sits between the two legs, not between the hypotenuse and a leg.
- **D** AAS needs a second angle, which is not given.

> **Takeaway:** RHS works because a right angle plus the hypotenuse already fix the triangle's shape — one more side pins it down completely. Check the triangle is right-angled before reaching for it.

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#### Q13 — Intermediate — Answer: C

**Topic:** Exterior angle theorem — remote interior angle

1. Exterior angle = sum of two remote interior angles: $120 = 45 + \theta$.
2. $\theta = 75^\circ$.
3. Check: the interior angle at the exterior vertex is $180 - 120 = 60^\circ$. Triangle sum: $45 + 75 + 60 = 180^\circ$. ✓

**Why the distractors are wrong:**

**B) 65°:** Arithmetic error: $120 - 45 - 10 = 65$ (subtracting an extra 10).

**A) 80°:** From $120 - 45 + 5 = 80$ (adding instead of subtracting, with a residual slip).

**D) 60°:** Finds the interior angle at the exterior-angle vertex ($180 - 120 = 60^\circ$) rather than the remote interior angle that was asked for.

> **Takeaway:** The Exterior Angle Theorem has two "remote" angles on the opposite side of the triangle. Identify which remote angle is given and which is unknown before applying the theorem.

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#### Q14 — Intermediate — Answer: B

**Topic:** Height of an isosceles triangle

1. The perpendicular from the apex bisects the base: half-base $= 6$ cm.
2. This forms a right triangle with hypotenuse 10 cm and base leg 6 cm.
3. Height: $h = \sqrt{10^2 - 6^2} = \sqrt{100 - 36} = \sqrt{64} = 8$ cm.

**Why the distractors are wrong:**

**A) 6 cm:** Is the half-base — a correct intermediate value, but not the height. The student stopped one step early.

**C) 10 cm:** Is one of the equal sides (the hypotenuse of the right triangle formed), not the height.

**D) 12 cm:** Is the base length — the starting value, not the answer.

> **Takeaway:** In any isosceles triangle, drop a perpendicular from the apex to the midpoint of the base. This creates two congruent right triangles. Apply Pythagoras to find the height — the half-base is a leg, and the equal side is the hypotenuse.

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#### Q15 — Proficient — Answer: D

**Topic:** Triangle congruence — SSS criterion

1. All three corresponding sides are equal: $5 = 5$, $7 = 7$, $9 = 9$.
2. This matches the **SSS (Side-Side-Side)** criterion: three equal corresponding sides guarantee congruence.

**Why the distractors are wrong:**

**A) SAS:** Requires two sides and the **included** angle. No angle is given.

**B) RHS:** Only applies to right-angled triangles. These triangles are not stated to be right-angled (a 9 cm side opposite a 5 cm and 7 cm pair gives a non-right triangle by the converse Pythagorean: $5^2 + 7^2 = 74 \neq 81 = 9^2$).

**C) AAS:** Requires two angles and a non-included side. No angles are given.

> **Takeaway:** SSS is the most direct congruence criterion — three equal sides uniquely determine a triangle. If you are given all three sides and no angles, SSS is the appropriate criterion.

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### Q16 — Proficient — Answer: A

**Topic:** Area of a triangle using base and height

1. Area $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 15 \times 8$.
2. $= \frac{1}{2} \times 120 = 60$ cm².

**Why the distractors are wrong:**

**D) 45 cm²:** From $\frac{1}{2} \times 15 \times 6 = 45$ — using 6 instead of 8 for the height (misreading), or from $15 \times 3 = 45$ (dividing only the base by 5).

**B) 56 cm²:** From $\frac{1}{2} \times 8 \times 14 = 56$ — using 14 instead of 15 (off-by-one on the base).

**C) 56.5 cm²:** From $\frac{1}{2} \times 15 \times 7.53 \approx 56.5$ — substituting an incorrect height value.

> **Takeaway:** Area $= \frac{1}{2}bh$ requires the height to be the **perpendicular** distance from the base line to the opposite vertex. A slant side is never the height unless the triangle is right-angled with the right angle at the base.

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## Section 5.3 — Similarity and Proportion

### Q17 — Basic — Answer: D

**Topic:** Similar triangles — finding a corresponding side

1. The scale factor from small to large is $\frac{3}{2}$.
2. Corresponding side $= 8 \times \frac{3}{2} = 12$ cm.

**Why the distractors are wrong:**

**B) 6 cm:** From applying the inverse scale factor: $8 \times \frac{2}{3} \approx 5.3$, rounded up, or from subtracting the ratio difference: $8 - 2 = 6$ (treating the ratio as a subtraction instruction).

**C) 16 cm:** From multiplying by 2 instead of by $\frac{3}{2}$: $8 \times 2 = 16$ (confusing "ratio $2 : 3$" with "multiply by the first number").

**A) 9 cm:** From adding 1 for each ratio unit: $8 + (3 - 2) = 9$ — a ratio misconception where the ratio difference is added directly.

> **Takeaway:** To convert between similar figures, multiply by the scale factor (large/small) or divide by (small/large). Always identify which direction — small to large or large to small — before calculating.

---

**Q18 — Basic — Answer: A**

**Topic:** Area ratio of similar figures

1. Areas scale as the square of the linear scale factor: area ratio $= 3^2 : 1^2 = 9 : 1$.
2. If one figure has area $A$, the similar figure scaled by $k$ has area $k^2 A$.

**Why the distractors are wrong:**

**B) 3:1:** Simply copies the linear ratio without squaring — the most common error on similarity-ratio questions.

**C) 27:1:** Is the volume ratio ($3^3$), not the area ratio. Students who know the "cube for volume" rule sometimes apply it to area as well.

**D) 6:1:** From doubling the linear ratio: $3 \times 2 = 6$. There is no geometric justification for this.

> **Takeaway:** The golden rule of similarity: lengths scale by $k$, areas scale by $k^2$, volumes scale by $k^3$. Keep these separate — confusing linear and area ratios is a classic exam trap.

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**Q19 — Intermediate — Answer: C**

**Topic:** Finding a missing side in similar triangles

1. Identify the scale factor: $12/8 = 3/2$ and $15/10 = 3/2$. The scale factor is $\frac{3}{2}$.
2. $x = 6 \times \frac{3}{2} = 9$ cm.

**Why the distractors are wrong:**

**A) 6:** Simply copies the first side of the smaller triangle without scaling.

**B) 8:** Copies the second side of the smaller triangle — another un-scaled value.

**D) 12:** Copies one of the larger triangle's sides rather than computing the unknown.

> **Takeaway:** First establish the scale factor from any pair of known corresponding sides, then apply it to find the unknown. Always verify with a second pair of known sides to confirm the triangles are indeed similar.

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**Q20 — Intermediate — Answer: B**

**Topic:** Shadow proportion

1. The pole and building are similar to their shadows at the same time of day: $\frac{6}{4} = \frac{h}{20}$.
2. Cross-multiply: $4h = 120 \Rightarrow h = 30$ m.

**Why the distractors are wrong:**

**A) 10 m:** From setting $6 - 4 = 2$ (difference method) and adding to $20 - ... = 10$: a non-mathematical approach.

**D) 20 m:** Simply reads back the shadow length as the height — forgetting to apply the scale factor.

**C) 24 m:** From $6 \times 4 = 24$ — multiplying the pole height by the shadow length rather than using the ratio.

> **Takeaway:** Shadow problems are direct proportion: height/shadow = constant for all objects at the same time. Set up the ratio equation before solving — don't guess by adding or multiplying raw values.

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**Q21 — Proficient — Answer: C**

**Topic:** Geometric mean altitude in a right triangle

1. By the geometric mean relation in a right triangle: $CD^2 = AD \times DB$.
2. $CD^2 = 4 \times 9 = 36 \Rightarrow CD = 6$ cm.
3. This result follows from the fact that triangles $ACD$, $CDB$, and $ACB$ are all similar.

**Why the distractors are wrong:**

**B) 5 cm:** From $CD = \frac{AD+DB}{2} = \frac{4+9}{2} = 6.5$, rounded to 5 (arithmetic misapplication of the arithmetic mean).

**A) 7 cm:** From mis-combining the two segments $AD = 4$ and $DB = 9$ by a sum-and-adjust slip; the geometric-mean relation gives $\sqrt{4 \times 9} = 6$, never a sum of the segments.

**D) 8 cm:** From $CD = \frac{4 \times 9}{4.5} = 8$ — inverting the relationship or using an incorrect formula.

> **Takeaway:** When an altitude is drawn from the right angle to the hypotenuse, the altitude is the geometric mean of the two segments it creates: $h^2 = p \cdot q$. This is a consequence of three nested similar triangles.

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#### Q22 — Proficient — Answer: D

**Topic:** Volume ratio of similar solids

1. Volumes of similar solids scale as the cube of the linear scale factor.
2. Volume ratio $= 2^3 : 5^3 = 8 : 125$.

**Why the distractors are wrong:**

**A) 4:25:** Is the area (surface) ratio — $2^2 : 5^2$. Students who learn "$k^2$ for area" sometimes apply it to volume as well.

**C) 2:5:** Is the original linear ratio — volumes were not scaled at all.

**B) 4:10:** From doubling each term of the linear ratio: $2 \times 2 = 4$ and $5 \times 2 = 10$, which has no geometric meaning.

> **Takeaway:** Similar solids: lengths scale by $k$, areas (faces, cross-sections) scale by $k^2$, volumes scale by $k^3$. Each power corresponds to one more dimension being scaled.

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## Section 5.4 — Quadrilaterals

#### Q23 — Basic — Answer: B

**Topic:** Angle sum of a quadrilateral

1. The angles in any quadrilateral sum to $360^\circ$: $80 + 95 + 110 + x = 360$.
2. $285 + x = 360 \Rightarrow x = 75^\circ$.

**Why the distractors are wrong:**

**A) 65°:** From $80 + 95 + 110 + x = 350 \Rightarrow x = 65$ — using $350^\circ$ instead of $360^\circ$ as the quadrilateral angle sum.

**C) 70°:** From $80 + 95 + 110 + x = 355 \Rightarrow x = 70$ — using $355^\circ$ or from an arithmetic error: $80 + 95 = 175$, $175 + 110 = 285$; $360 - 285 = 75$ but student computes $355 - 285 = 70$.

**D) 80°:** Copies the first given angle — the student may have ignored the equation entirely.

> **Takeaway:** Any quadrilateral (regardless of shape) has angle sum $360^\circ$. This is because any quadrilateral can be split into two triangles, each with $180^\circ$: $2 \times 180^\circ = 360^\circ$.

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### Q24 — Basic — Answer: A

**Topic:** Diagonal of a rectangle — Pythagorean triple

1. A rectangle's diagonal is the hypotenuse of a right triangle with legs equal to the length and width.
2. $d = \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13$ cm.
3. 5-12-13 is a standard Pythagorean triple.

**Why the distractors are wrong:**

**D) 7 cm:** From $12 - 5 = 7$ — subtracting the two sides instead of using Pythagoras.

**B) 10 cm:** From $\sqrt{12^2 - 5^2} = \sqrt{119} \approx 10.9$, or confused with the 6-8-10 triple.

**C) 12 cm:** Is the longer side of the rectangle, not the diagonal.

> **Takeaway:** The diagonal of a rectangle is found using Pythagoras: $d = \sqrt{l^2 + w^2}$. For rectangles whose sides form Pythagorean triples (3-4-5, 5-12-13, 8-15-17), the diagonal is exact without needing a calculator.

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### Q25 — Intermediate — Answer: C

**Topic:** Diagonal of a rectangle

1. $d = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10$ cm.
2. 6-8-10 is the 3-4-5 triple scaled by 2.

**Why the distractors are wrong:**

**B) 12 cm:** From $8 + 6 - 2 = 12$ — adding and subtracting rather than using Pythagoras.

**A) 7 cm:** From $8 - 6 + 5 = 7$ or from $\sqrt{8^2 - 6^2} = \sqrt{28} \approx 5.3$, rounded up.

**D) 14 cm:** From $8 + 6 = 14$ — adding the two sides, which gives the perimeter of two sides, not the diagonal.

> **Takeaway:** Knowing scaled Pythagorean triples saves time: 3-4-5 $\times\ 2 =$ 6-8-10. Memorise the most common ones so you can read off the answer immediately.

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#### Q26 — Intermediate — Answer: B

**Topic:** Side length of a rhombus from its diagonals

1. The diagonals of a rhombus bisect each other at right angles: half-diagonals are 4 cm and 3 cm.
2. Each side is the hypotenuse of a right triangle with legs 4 cm and 3 cm.
3. Side $= \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$ cm.

**Why the distractors are wrong:**

**A) 4 cm:** Is half of the longer diagonal — the correct intermediate value, but not the side length.

**C) 6 cm:** Is the shorter diagonal — a given value, not the side.

**D) 7 cm:** From $4 + 3 = 7$ — adding the half-diagonals instead of applying Pythagoras.

> **Takeaway:** The key property: diagonals of a rhombus bisect each other at $90^\circ$. This creates four congruent right triangles whose hypotenuse is the rhombus side.

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#### Q27 — Intermediate — Answer: D

**Topic:** Area of a parallelogram

1. Area of a parallelogram $=$ base $\times$ height $= 12 \times 7 = 84$ cm².
2. Note: the height must be the perpendicular distance between the parallel sides — not the slant side.

**Why the distractors are wrong:**

**A) 42 cm²:** From $\frac{1}{2} \times 12 \times 7 = 42$ — using the triangle area formula instead of the parallelogram formula. A parallelogram is twice the area of the triangle with the same base and height.

**B) 72 cm²:** From $12 \times 6 = 72$ — substituting 6 instead of 7 (misreading the height).

**C) 96 cm²:** From $12 \times 8 = 96$ — substituting 8 instead of 7, perhaps from adding 1 to the height.

> **Takeaway:** Area of a parallelogram $= bh$ (no $\frac{1}{2}$). The formula differs from a triangle ($\frac{1}{2}bh$) because two congruent triangles together form the parallelogram.

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### Q28 — Intermediate — Answer: A

**Topic:** Area of a trapezium

1. Area of a trapezium $= \frac{1}{2}(a+b) \times h = \frac{1}{2}(10+14) \times 8$.
2. $= \frac{1}{2} \times 24 \times 8 = \frac{1}{2} \times 192 = 96$ cm².

**Why the distractors are wrong:**

**D) 60 cm²:** From using only one parallel side: $\frac{1}{2} \times 10 \times 12 = 60$ or $10 \times 6 = 60$.

**B) 80 cm²:** From $10 \times 8 = 80$ — multiplying only one of the parallel sides by the height, ignoring the other.

**C) 88 cm²:** From $\frac{1}{2}(10+12) \times 8 = 88$ — using 12 instead of 14 for the second parallel side (misread).

> **Takeaway:** The trapezium area formula uses the average of the two parallel sides: $A = \frac{1}{2}(a+b)h$. Think of it as the area of a rectangle with width equal to the average parallel side.

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### Q29 — Proficient — Answer: B

**Topic:** Area of a kite using its diagonals

1. The area of a kite (and any quadrilateral with perpendicular diagonals) is: $A = \frac{1}{2}d_1d_2$.
2. $A = \frac{1}{2} \times 10 \times 8 = 40$ cm².

**Why the distractors are wrong:**

**A) 36 cm²:** From $\frac{1}{2}(10+8)^2/\ldots$, or from using only one diagonal: $\frac{1}{2} \times 8 \times 9 = 36$ (using 9 as the half of the full diagonal).

**C) 48 cm²:** From $\frac{1}{2}(10+8) \times \ldots = \frac{1}{2}(18)(5.3) \approx 48$ — confusing the kite formula with the trapezium formula.

**D) 56 cm²:** From $\frac{1}{2} \times (10+8) \times \frac{8}{..}$, or from $8 \times 7 = 56$ (using 7 instead of $\frac{10}{2}$).

> **Takeaway:** A kite's diagonals are perpendicular and one bisects the other. The area formula $\frac{1}{2}d_1 d_2$ applies to any quadrilateral whose diagonals are perpendicular — including rhombuses and squares (which are special kites).

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## Section 5.5 — Circles — Arcs, Sectors, Chords

#### Q30 — Basic — Answer: D

**Topic:** Circumference of a circle

1. Circumference $= 2\pi r = 2 \times \pi \times 7 = 14\pi$ cm.

**Why the distractors are wrong:**

**A) $7\pi$ cm:** From $\pi r$ — forgetting the factor of 2 in the circumference formula.

**C) $28\pi$ cm:** From $4\pi r$ — multiplying by 4 instead of 2, perhaps confusing circumference with the perimeter of a square with side $r$.

**B) $49\pi$ cm:** From $\pi r^2 = 49\pi$ — using the area formula instead of the circumference formula.

> **Takeaway:** Circumference $= 2\pi r$; Area $= \pi r^2$. One involves $r$, the other $r^2$ — keep them visually distinct when writing.

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#### Q31 — Basic — Answer: C

**Topic:** Area of a sector

1. Sector area $= \dfrac{\theta}{360} \times \pi r^2 = \dfrac{80}{360} \times \pi \times 81$.
2. $= \dfrac{2}{9} \times 81\pi = \dfrac{162\pi}{9} = 18\pi$ cm².

**Why the distractors are wrong:**

**A) $9\pi$ cm²:** From halving the sector angle to $40^\circ$ (a mix-up with the inscribed-angle "half" rule): $\dfrac{40}{360} \times \pi \times 81 = \dfrac{1}{9} \times 81\pi = 9\pi$.

**B) $12\pi$ cm²:** From $\dfrac{80}{360} \times \pi \times 54 = 12\pi$ — using $6r = 54$ rather than $r^2 = 81$.

**D) $24\pi$ cm²:** From rounding $\frac{80}{360} \approx \frac{1}{4}$ and computing $\frac{1}{4} \times 96\pi = 24\pi$ — using 96 in place of 81.

> **Takeaway:** Sector area $= \frac{\theta}{360} \times \pi r^2$. The fraction represents "what portion of the full circle" the sector covers.

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### Q32 — Intermediate — Answer: A

**Topic:** Length of an arc

1. Arc length $= \frac{\theta}{360} \times 2\pi r = \frac{120}{360} \times 2\pi \times 6$.
2. $= \frac{1}{3} \times 12\pi = 4\pi$ cm.

**Why the distractors are wrong:**

**D) $2\pi$ cm:** From using half the radius: $\frac{1}{3} \times 2\pi \times 3 = 2\pi$.

**B) $3\pi$ cm:** From using a $90^\circ$ angle instead of $120^\circ$: $\frac{90}{360} \times 2\pi \times 6 = \frac{1}{4} \times 12\pi = 3\pi$.

**C) $3.5\pi$ cm:** Arithmetic slip between the setup $\frac{1}{3} \times 12\pi = 4\pi$ and a miscomputed value.

> **Takeaway:** Arc length uses $2\pi r$ (circumference); sector area uses $\pi r^2$. Both multiply by $\frac{\theta}{360}$. The key distinction: arc length is a one-dimensional measurement; sector area is two-dimensional.

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### Q33 — Intermediate — Answer: D

**Topic:** Finding the radius given a chord and its distance from the centre

1. The perpendicular from the centre bisects the chord: half-chord $= 6$ cm.
2. The radius, half-chord, and perpendicular distance form a right triangle: $r^2 = 6^2 + 8^2 = 36 + 64 = 100$.
3. $r = 10$ cm. (6-8-10 is the 3-4-5 triple scaled by 2.)

**Why the distractors are wrong:**

**B) 8 cm:** Is the given perpendicular distance from the centre — not the radius.

**C) 12 cm:** Is the given chord length — not the radius.

**A) 14 cm:** From $6 + 8 = 14$ — adding the half-chord and the perpendicular distance instead of using the Pythagorean theorem.

> **Takeaway:** The perpendicular from the centre to a chord always bisects the chord. This creates a right triangle: radius (hypotenuse), half-chord (leg), perpendicular distance (leg).

---

### Q34 — Intermediate — Answer: A

**Topic:** Inscribed angle theorem

1. The Inscribed Angle Theorem: an inscribed angle is half the central angle subtending the same arc.
2. Inscribed angle $= \dfrac{140}{2} = 70^\circ$.

**Why the distractors are wrong:**

**B) 35°:** From halving twice: $70 \div 2 = 35$ — applying the theorem twice when it should be applied once.

**C) 140°:** Simply copies the central angle — the theorem was not applied.

**D) 280°:** From computing the reflex arc ($360 - 80 = 280$) and confusing this with the inscribed angle.

> **Takeaway:** Inscribed angle $= \frac{1}{2} \times$ central angle. A key corollary: any angle inscribed in a semicircle equals $90^\circ$ (since the central angle is $180^\circ$).

---

### Q35 — Intermediate — Answer: B

**Topic:** Perpendicular distance from the centre to a chord

1. The perpendicular bisects the chord: half-chord $= 8$ cm.
2. $d^2 = r^2 - (\text{half-chord})^2 = 100 - 64 = 36 \Rightarrow d = 6$ cm.

**Why the distractors are wrong:**

**A) 4 cm:** From $\sqrt{100 - 96} = 2$... or from $16 - 10 - 2 = 4$ (subtracting values with no geometric basis).

**C) 5 cm:** From $\sqrt{100 - 75} = 5$ — using 75 instead of 64 for the half-chord squared.

**D) 8 cm:** Is the half-chord — a correct intermediate value, but the perpendicular distance, not the final answer.

> **Takeaway:** For chord-distance problems: identify the right triangle (radius, half-chord, perpendicular distance), then use $d^2 = r^2 - (c/2)^2$. Always halve the chord length first.

---

#### Q36 — Intermediate — Answer: C

**Topic:** Angle in a semicircle (Thales' theorem)

1. By Thales' theorem, the angle in a semicircle is $90^\circ$: $\angle ACB = 90^\circ$.
2. Angle sum of triangle $ABC$: $90 + 35 + \angle ABC = 180$.
3. $\angle ABC = 55^\circ$.

**Why the distractors are wrong:**

**A) 35°:** Copies the given angle, perhaps assuming the triangle is isosceles when it is not necessarily so.

**B) 45°:** From $90 \div 2 = 45$ — halving the semicircle angle for no valid reason.

**D) 50°:** From $180 - 90 - 40 = 50$ — misreading $35^\circ$ as $40^\circ$.

> **Takeaway:** Thales' theorem — any angle inscribed in a semicircle is $90^\circ$ — is the most frequently tested circle theorem. Identify the diameter first; then the angle opposite it is $90^\circ$.

---

#### Q37 — Proficient — Answer: A

**Topic:** Length of a tangent from an external point

1. The tangent meets the radius at $90^\circ$ at the tangent point $T$: triangle $OTP$ is right-angled at $T$.
2. $PT^2 = OP^2 - OT^2 = 13^2 - 5^2 = 169 - 25 = 144$.
3. $PT = 12$ cm. (This is the 5-12-13 Pythagorean triple.)

**Why the distractors are wrong:**

**B) 8 cm:** From $13 - 5 = 8$ — subtracting the radius from the external distance without using Pythagoras.

**C) 10 cm:** Confused with the 6-8-10 triple, or from misreading $\sqrt{144}$ as $\sqrt{100} = 10$.

**D) 13 cm:** Is the distance $OP$ from the external point to the centre — a given value.

> **Takeaway:** A tangent is perpendicular to the radius at the point of contact. This creates a right triangle: hypotenuse $= OP$, one leg $= r$, other leg $=$ tangent length. Apply Pythagoras.

---

#### Q38 — Proficient — Answer: C

**Topic:** Cyclic quadrilateral — opposite angles

1. Opposite angles in a cyclic quadrilateral are supplementary: $3x + (2x + 10) = 180$.
2. $5x + 10 = 180 \Rightarrow 5x = 170 \Rightarrow x = 34$.
3. The angles are $102^\circ$ and $78^\circ$; $102 + 78 = 180^\circ$. ✓

**Why the distractors are wrong:**

**A) 30:** From $5x + 10 = 160 \Rightarrow x = 30$ — using $160^\circ$ instead of $180^\circ$ as the supplementary sum.

**B) 36:** From $5x = 180 \Rightarrow x = 36$ — omitting the $+10$ constant (the most common algebraic slip).

**D) 40:** From $5x = 200 \Rightarrow x = 40$ — treating opposite angles as summing to $200^\circ$, possibly confusing with the full quadrilateral angle sum divided in two.

> **Takeaway:** In a cyclic quadrilateral, opposite angles are supplementary ($180^\circ$), not equal. This distinguishes cyclic quadrilaterals from parallelograms (where opposite angles are equal).

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## Section 5.6 — Perimeter and Area

#### Q39 — Basic — Answer: D

**Topic:** Perimeter of a rectangle

1. Perimeter $= 2(l + w) = 2(9 + 5) = 2 \times 14 = 28$ cm.

**Why the distractors are wrong:**

**A) 45 cm:** From $9 \times 5 = 45$ — computing the area instead of the perimeter.

**B) 25 cm:** From $9 + 5 + 9 + 2 = 25$ (miscounting sides) or $9 + 5 + 9 + 2 = 25$, adding three sides correctly but using 2 instead of 5 for the final width.

**C) 14 cm:** From $9 + 5 = 14$ — adding only one length and one width, forgetting to multiply by 2.

> **Takeaway:** Perimeter $= 2(l + w)$ counts all four sides. Area $= lw$ counts the enclosed region. Confusing these two is the most common basic formula error in geometry.

---

#### Q40 — Basic — Answer: B

**Topic:** Area of a triangle

1. Area $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 10 \times 12 = 60$ cm².

**Why the distractors are wrong:**

**A) 30 cm²:** From $\frac{1}{2} \times 10 \times 6 = 30$ — halving the height as well as applying the $\frac{1}{2}$ factor (applying $\frac{1}{2}$ twice).

**D) 45 cm²:** From $\frac{1}{2} \times 10 \times 9 = 45$ — using 9 instead of 12 for the height.

**C) 50 cm²:** From $\frac{1}{2} \times 10 \times 10 = 50$ — using 10 for both base and height (misreading).

> **Takeaway:** Area $= \frac{1}{2}bh$ always requires the perpendicular height — the vertical distance from the base to the opposite vertex, not a slant edge.

---

#### Q41 — Intermediate — Answer: D

**Topic:** Area of a composite shape

1. Rectangle: $8 \times 5 = 40$ cm².
2. Triangle: $\frac{1}{2} \times 8 \times 3 = 12$ cm².
3. Total: $40 + 12 = 52$ cm².

**Why the distractors are wrong:**

**B) 48 cm²:** From $40 + \frac{1}{2} \times 8 \times 2 = 40 + 8 = 48$ — using 2 instead of 3 for the triangle height.

**C) 56 cm²:** From $40 + 8 \times 2 = 56$ — computing triangle area as $8 \times 2$ without the $\frac{1}{2}$.

**A) 60 cm²:** From treating the combined height as a single rectangle: $8 \times (5 + 3) - \text{corner} \approx 60$ — not correctly splitting the composite shape.

> **Takeaway:** For composite shapes: decompose into named shapes, compute each area separately, then add or subtract. Label every sub-calculation to avoid mixing components.

---

### Q42 — Intermediate — Answer: B

**Topic:** Area of a circle

1. Diameter $= 10$ cm $\Rightarrow$ radius $r = 5$ cm.
2. Area $= \pi r^2 = 25\pi$ cm².

**Why the distractors are wrong:**

**A) $10\pi$ cm²:** From $\pi \times d = 10\pi$ — using the circumference formula $\pi d$ rather than $\pi r^2$.

**C) $50\pi$ cm²:** From $\pi r^2$ using $r = 10$ (the diameter): $\pi \times 100/2 = 50\pi$ — halving after squaring the diameter.

**D) $100\pi$ cm²:** From $\pi r^2$ using $r = 10$ (the diameter as the radius without halving): $\pi \times 100 = 100\pi$.

> **Takeaway:** Always halve the diameter to find the radius before substituting into any area formula. The diameter appears directly only in $C = \pi d$.

---

### Q43 — Intermediate — Answer: C

**Topic:** Perimeter of a semicircle

1. Straight edge (diameter) $= 2r = 12$ cm.
2. Curved arc $=$ half circumference $= \frac{1}{2} \times 2\pi r = \pi r = 6\pi$ cm.
3. Total perimeter $= 12 + 6\pi$ cm.

**Why the distractors are wrong:**

**A) $6 + 6\pi$ cm:** Uses the radius (6) instead of the diameter (12) for the straight edge.

**B) $12 + 3\pi$ cm:** Uses $\frac{\pi r}{2} = 3\pi$ — halving $\pi r$ again (the half-circumference is $\pi r$, not $\frac{\pi r}{2}$).

**D) $6 + 12\pi$ cm:** Swaps the role of radius and diameter: straight edge $= r = 6$ and arc $= 2\pi r = 12\pi$.

> **Takeaway:** A semicircle's perimeter has two parts: the flat diameter ($2r$) and the curved arc ($\pi r$). Both are needed — don't omit the diameter by treating the semicircle as only a curved boundary.

---

### Q44 — Intermediate — Answer: A

**Topic:** Area of a sector

1. Sector area $= \frac{90}{360} \times \pi r^2 = \frac{1}{4} \times \pi \times 144 = 36\pi$ cm².

**Why the distractors are wrong:**

**D) $6\pi$ cm²:** From computing the arc length instead of the area: $\frac{1}{4} \times 2\pi \times 12 = 6\pi$ — using $2\pi r$ (circumference formula) instead of $\pi r^2$.

**B) $24\pi$ cm²:** From $\frac{1}{4} \times \pi \times 96 = 24\pi$ — using 96 (perhaps $8 \times 12$) instead of $r^2 = 144$.

**C) $30\pi$ cm²:** From $\frac{1}{4} \times \pi \times 120 = 30\pi$ — using 120 instead of 144 for $r^2$ (perhaps from $10 \times 12$).

> **Takeaway:** Sector area $= \frac{\theta}{360} \times \pi r^2$; arc length $= \frac{\theta}{360} \times 2\pi r$. Both have the same fraction, but area uses $\pi r^2$ (two-dimensional) and arc length uses $2\pi r$ (one-dimensional).

---

### Q45 — Proficient — Answer: D

**Topic:** Area of an annulus (ring between two circles)

1. Area of large circle $= \pi(8^2) = 64\pi$ cm².
2. Area of small circle $= \pi(5^2) = 25\pi$ cm².
3. Annulus area $= 64\pi - 25\pi = 39\pi$ cm².

**Why the distractors are wrong:**

**B) $25\pi$ cm²:** Is the area of the small circle alone — the subtraction step was skipped.

**C) $64\pi$ cm²:** Is the area of the large circle alone — the inner region was not removed.

**A) $89\pi$ cm²:** From $64\pi + 25\pi = 89\pi$ — adding the two areas instead of subtracting.

> **Takeaway:** An annulus is a "donut" region. Always compute: annulus area $= \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$. This can be factored: $\pi(R-r)(R+r)$.

---

### Q46 — Proficient — Answer: C

**Topic:** Perimeter of a right triangle — recognising a Pythagorean triple

1. Find the hypotenuse: $c = \sqrt{9^2 + 40^2} = \sqrt{81 + 1600} = \sqrt{1681} = 41$ cm.
2. Perimeter $= 9 + 40 + 41 = 90$ cm.
3. The 9-40-41 triple is worth knowing: $81 + 1600 = 1681 = 41^2$.

**Why the distractors are wrong:**

**A) 82 cm:** From computing $9 + 40 + \sqrt{40^2 - 9^2} = 9 + 40 + 33 = 82$ — incorrectly using the difference of squares for the third side (treating it as a leg rather than the hypotenuse).

**B) 88 cm:** From $9 + 40 + 39 = 88$ — computing $\sqrt{81 + 1521} = \sqrt{1602} \approx 40$ and rounding incorrectly, or from $41 - 2 = 39$.

**D) 96 cm:** From $9 + 40 + 47 = 96$ — arithmetic error in the Pythagorean computation.

> **Takeaway:** Less familiar Pythagorean triples like 9-40-41 appear in exams precisely to test whether you use the theorem correctly. Always compute $c = \sqrt{a^2 + b^2}$ — don't guess.

---

## Section 5.7 — Surface Area and Volume

#### Q47 — Basic — Answer: B

**Topic:** Volume of a rectangular prism

1. Volume of rectangular prism $= l \times w \times h = 4 \times 3 \times 5 = 60$ cm³.

**Why the distractors are wrong:**

**A) 20 cm³:** From $4 \times 5 = 20$ — multiplying only two dimensions instead of all three.

**C) 47 cm³:** From $lw + wh + lh = 12 + 15 + 20 = 47$ — the surface-area sum missing its factor of 2 (and, in any case, an area rather than a volume).

**D) 94 cm³:** From the full surface area formula: $2(lw + wh + lh) = 2(12 + 15 + 20) = 2 \times 47 = 94$ cm² — computing surface area instead of volume.

> **Takeaway:** Volume $= l \times w \times h$; Surface area $= 2(lw + wh + lh)$. Volume counts the interior space (three dimensions multiplied); surface area counts the exterior faces (pairs of rectangles).

---

#### Q48 — Basic — Answer: A

**Topic:** Surface area of a cube

1. A cube has 6 identical square faces.
2. Surface area $= 6 \times s^2 = 6 \times 16 = 96$ cm².

**Why the distractors are wrong:**

**D) 16 cm²:** Is the area of one face ($4^2 = 16$) — forgot to multiply by the 6 faces.

**B) 24 cm²:** From $6 \times 4 = 24$ — using $s$ instead of $s^2$: multiplying the number of faces by the side length, not the face area.

**C) 64 cm³:** Is the volume of the cube ($4^3 = 64$) — computed volume rather than surface area.

> **Takeaway:** Surface area of a cube $= 6s^2$ (six congruent square faces). Volume $= s^3$. A cube has all edges equal, which simplifies both formulas greatly.

---

### Q49 — Intermediate — Answer: C

**Topic:** Volume of a cylinder

1. Volume of a cylinder $= \pi r^2 h = \pi \times 9 \times 8 = 72\pi$ cm³.

**Why the distractors are wrong:**

**B)** $48\pi$ **cm³:** From $\pi \times 6 \times 8 = 48\pi$ — using $2r = 6$ (diameter) instead of $r^2 = 9$.

**A)** $36\pi$ **cm³:** From $\pi \times 9 \times 4 = 36\pi$ — halving the height: $8/2 = 4$.

**D)** $24\pi$ **cm³:** From $\pi \times 3 \times 8 = 24\pi$ — using $r = 3$ instead of $r^2 = 9$.

> **Takeaway:** Volume of a cylinder $= \pi r^2 h$. Think of it as the base circle area ($\pi r^2$) multiplied by the height ($h$). The radius must be squared — forgetting this is the most common error.

---

### Q50 — Intermediate — Answer: B

**Topic:** Surface area of a cylinder

1. Two circular bases: $2 \times \pi r^2 = 2\pi \times 25 = 50\pi$ cm².
2. Curved lateral surface: $2\pi rh = 2\pi \times 5 \times 10 = 100\pi$ cm².
3. Total: $50\pi + 100\pi = 150\pi$ cm².

**Why the distractors are wrong:**

**A)** $100\pi$ **cm²:** Is only the lateral surface area — the two circular bases were not included.

**C)** $200\pi$ **cm²:** From $2 \times \pi rh \times 2 = 200\pi$ or $2\pi r(h + r) \times \frac{4}{3}$... more naturally from $2 \times 100\pi = 200\pi$ — doubling the lateral surface without correctly adding the circles.

**D)** $75\pi$ **cm²:** From $3 \times \pi r^2 = 75\pi$ — using 3 faces instead of the correct formula.

> **Takeaway:** Total surface area of a cylinder $= 2\pi r^2 + 2\pi rh = 2\pi r(r+h)$. The two circular ends plus the rectangle that wraps around (lateral surface). A common omission is forgetting the two circular bases.

---

### Q51 — Intermediate — Answer: D

**Topic:** Volume of a cone

1. Volume of a cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 36 \times 8 = \frac{288\pi}{3} = 96\pi$ cm³.

**Why the distractors are wrong:**

**A) $48\pi$ cm³:** From $\frac{1}{3} \times \pi \times 36 \times 4 = 48\pi$ — halving the height before applying the formula.

**B) $72\pi$ cm³:** From $\frac{1}{3} \times \pi \times 36 \times 6 = 72\pi$ — using the radius (6) instead of the height (8).

**C) $144\pi$ cm³:** From $\pi r^2 h = \pi \times 36 \times 4 = 144\pi$ — forgetting the $\frac{1}{3}$ factor entirely, or using the wrong height.

> **Takeaway:** Volume of a cone $= \frac{1}{3}\pi r^2 h$. It is exactly one-third of the cylinder with the same base and height. The $\frac{1}{3}$ is often forgotten — it is always present for cones and pyramids.

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### Q52 — Intermediate — Answer: A

**Topic:** Volume of a sphere

1. Volume of a sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 27 = \frac{108\pi}{3} = 36\pi$ cm³.

**Why the distractors are wrong:**

**D) $9\pi$ cm³:** From $\pi r^2 \times 1 = 9\pi$ — using the circle area formula instead of the sphere volume.

**B) $18\pi$ cm³:** From $\frac{4}{3}\pi r^2 = \frac{4}{3} \times 9\pi = 12\pi$... or from $2 \times \pi r^2 = 18\pi$ — using $r^2$ instead of $r^3$ in the sphere formula.

**C) $27\pi$ cm³:** From $\pi r^3 = 27\pi$ — forgetting the $\frac{4}{3}$ factor.

> **Takeaway:** Volume of a sphere $= \frac{4}{3}\pi r^3$. The $\frac{4}{3}$ is a unique fraction — it must be memorised. Surface area of a sphere $= 4\pi r^2$ for reference.

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#### Q53 — Proficient — Answer: B

**Topic:** Surface area of a triangular prism

**Find the hypotenuse first** — it becomes the third rectangular face: $\sqrt{3^2 + 4^2} = 5$ cm.

**Two triangular ends:** $2 \times \frac{1}{2} \times 3 \times 4 = 12$ cm²

**Three rectangles:** $30 + 40 + 50 = 120$ cm²

**Total:** 132 cm²

*Why the others are wrong:*

- **A** (120) is the three rectangles only — **both triangular ends dropped**.
- **C** (140) used the **hypotenuse** as the triangle's height: $2 \times \frac{1}{2} \times 4 \times 5 = 20$, plus 120.
- **D** (150) used the prism's **length** as the triangle's height: $2 \times \frac{1}{2} \times 3 \times 10 = 30$, plus 120.

> **Takeaway:** A triangular prism has **five** faces — two triangular ends plus three rectangles. The triangle's height is one of its own short sides (here 4), never the hypotenuse and never the prism's length.

---

#### Q54 — Proficient — Answer: D

**Topic:** Volume of a square pyramid

1. Volume of a pyramid $= \frac{1}{3} \times \text{base area} \times h = \frac{1}{3} \times 81 \times 4$.
2. $= \frac{324}{3} = 108$ cm³.

**Why the distractors are wrong:**

**A) 72 cm³:** From $\frac{1}{3} \times 9 \times 4 \times 6 = 72$ — using 9 (base side) instead of 81 (base area), then compensating with an extra factor.

**C) 144 cm³:** From mis-scaling the base-area × height product ($81 \times 4 = 324$) — for instance multiplying by $\frac{4}{9}$ instead of the correct $\frac{1}{3}$. Only $\frac{1}{3} \times 324 = 108$ is right.

**B) 216 cm³:** From computing the volume of a cube of side 6: $6^3 = 216$ — a completely unrelated calculation.

> **Takeaway:** Volume of a pyramid $= \frac{1}{3} \times B \times h$, where $B$ is the base area. For a square pyramid, $B = s^2$. The $\frac{1}{3}$ factor applies to all pyramids and cones — the pyramid fits exactly three times into a prism of the same base and height.

## Section 5.8 — Coordinate Geometry

### Q55 — Basic — Answer: C

**Topic:** Distance between two points

1. Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4-1)^2 + (6-2)^2}$.
2. $= \sqrt{9+16} = \sqrt{25} = 5$.

**Why the distractors are wrong:**

**A) 3:** Is only the horizontal distance: $|4-1| = 3$.

**B) 4:** Is only the vertical distance: $|6-2| = 4$.

**D) 7:** From $3 + 4 = 7$ — adding the horizontal and vertical distances instead of using Pythagoras.

> **Takeaway:** The distance formula is Pythagoras applied to coordinates: $d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$. The straight-line distance is always the hypotenuse of the right triangle formed by the horizontal and vertical separations.

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### Q56 — Basic — Answer: A

**Topic:** Midpoint of a line segment

1. Midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-2+6}{2}, \frac{5+(-1)}{2}\right)$.
2. $= \left(\frac{4}{2}, \frac{4}{2}\right) = (2,\ 2)$.

**Why the distractors are wrong:**

**D) $(4,4)$:** From $(x_1+x_2,\ y_1+y_2) = (4,4)$ without dividing by 2 — adding coordinates but forgetting to halve.

**B) $(2,3)$:** From correctly computing $x = 2$ but using $y = (5+1)/2 = 3$ (taking the absolute value of $-1$ instead of $-1$ itself).

**C) $(4,2)$:** From $(x_1+x_2, y) = (4,2)$ — not halving the $x$-coordinate.

> **Takeaway:** The midpoint is the average of the $x$-coordinates and the average of the $y$-coordinates. Both coordinates must be divided by 2 after adding.

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### Q57 — Intermediate — Answer: D

**Topic:** Gradient of a line

1. Gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{9-3}{4-1} = \dfrac{6}{3} = 2.$

**Why the distractors are wrong:**

**B) 3:** Reports the horizontal run ($x_2 - x_1 = 4 - 1 = 3$) instead of dividing the rise by the run.

**C) $\frac{1}{2}$:** From inverting the gradient formula: $\dfrac{x_2 - x_1}{y_2 - y_1} = \dfrac{3}{6} = \frac{1}{2}$ — swapping numerator and denominator.

**A) $-2$:** From computing $\dfrac{3-9}{4-1} = \dfrac{-6}{3} = -2$ — subtracting in inconsistent order (using $y_1 - y_2$ but $x_2 - x_1$).

> **Takeaway:** Gradient $= \dfrac{\text{rise}}{\text{run}} = \dfrac{y_2 - y_1}{x_2 - x_1}$. Use a consistent order: either $(y_2 - y_1, x_2 - x_1)$ or $(y_1 - y_2, x_1 - x_2)$ — never mix the two.

---

### Q58 — Intermediate — Answer: C

**Topic:** Equation of a line through a point with a given gradient

**Point-gradient form:** $y - y_1 = m(x - x_1)$

$$y - 1 = 3(x - 2) = 3x - 6 \implies y = 3x - 5$$

*Why the others are wrong:*

- **A** ($+5$) computed $c$ as $mx_1 - y_1$ instead of $y_1 - mx_1 = 1 - 6 = -5$.
- **B** ($-1$) read $3(x-2)$ as $3x - 2$ — the 3 was never distributed to the 2.
- **D** ($+1$) used $y_1$ as the intercept without solving for $c$ at all.

> **Takeaway:** Distribute the gradient to **every** term in the bracket, not just the $x$.

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### Q59 — Proficient — Answer: A

**Topic:** Distance in a coordinate/direction problem

1. North and east are perpendicular directions. The displacement forms a right triangle.

2. Distance $= \sqrt{9^2 + 12^2} = \sqrt{81 + 144} = \sqrt{225} = 15$ km.
3. 9-12-15 is the 3-4-5 triple scaled by 3.

**Why the distractors are wrong:**

**C) 10 km:** From $\sqrt{100} = 10$ — perhaps from $\sqrt{9^2 + 12^2} \approx \sqrt{100}$ after approximating.

**B) 13 km:** Confused with the 5-12-13 triple — misidentifying which triple applies here.

**D) 21 km:** From $9 + 12 = 21$ — adding the distances instead of using Pythagoras.

> **Takeaway:** Compass directions (north/east, north/west, etc.) are always perpendicular, so any north–east (or similar) displacement forms a right triangle. Apply Pythagoras to find the straight-line distance.

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**Q60 — Proficient — Answer: B**

**Topic:** Area of a triangle using coordinates

1. $AB$ lies along the $x$-axis with length 6 (base).
2. The height from $C$ to $AB$ is the $y$-coordinate of $C = 4$.
3. Area $= \frac{1}{2} \times 6 \times 4 = 12$ sq units.
4. Shoelace verification: $\frac{1}{2}|x_A(y_B - y_C) + x_B(y_C - y_A) + x_C(y_A - y_B)| = \frac{1}{2}|0 + 24 + 0| = 12$. ✓

**Why the distractors are wrong:**

**A) 8 sq units:** From $\frac{1}{2} \times 4 \times 4 = 8$ — using the $y$-coordinate of $C$ as both the base and height.

**D) 10 sq units:** From $\frac{1}{2} \times (6 + 4) \times 2 = 10$ — using the sum of base and height incorrectly.

**C) 11 sq units:** Arithmetic error, or from a misapplication of the shoelace formula.

> **Takeaway:** When one side of a triangle is horizontal (or vertical), finding the area is straightforward: base $\times$ height $/2$. For a general triangle, the shoelace formula works for any three coordinate points.

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## Exam-Bank Extras — Question Types Confirmed in Recent Papers

#### Q61 — Intermediate — Answer: B

**Topic:** Circle inscribed in a square (fraction of area not covered)

No numbers are given, so **invent one**. Let the spray radius be $r$; the field's side is then $2r$.

$$\text{Field} = (2r)^2 = 4r^2 \qquad \text{Watered} = \pi r^2$$

The dry part is the four corners:

$$\frac{4r^2 - \pi r^2}{4r^2} = \frac{4-\pi}{4}$$

The $r^2$ cancels — which is why no number was needed. Check the size: $\frac{4-\pi}{4} \approx 21\%$, believable for four corner slivers.

*Why the others are wrong:*

- **A** ($\frac{\pi}{4}$) is the **watered** fraction, not the dry one.
- **C** divides by the circle instead of the **total** area.
- **D** is **negative** (since $\pi > 2$) — an area fraction never can be. A two-second size-check kills it.

> **Takeaway:** Circle inscribed in a square: watered $\approx 78.5\%$, corners $\approx 21.5\%$. And when a ratio question gives no measurements, that is a promise the variable will cancel — pick a letter and push through.

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#### Q62 — Intermediate — Answer: C

**Topic:** Border/path area (algebraic set-up)

Let the lawn's side be $x$. The path adds 1 m on **each** side, so the outer square has side $x+2$ — not $x+1$.

**Path area = outer − lawn:**

$$(x+2)^2 - x^2 = 40$$

Expand, and the $x^2$ terms cancel:

$$4x + 4 = 40 \implies x = 9$$

So the lawn is $9 \times 9 = 81$ m². Check: $121 - 81 = 40$ ✓

*Why the others are wrong:*

- **D** (121) is the **outer** square, not the lawn.
- **A** (64) used $x + 1$, adding the path's width only once.
- **B** (100) expanded $(x + 2)^2 - x^2$ as just $4x$, dropping the $+4$ — the path's four corner squares.

> **Takeaway:** A border of width $w$ makes the outer side $x + 2w$ — it is on **both** sides. The $x^2$ always cancels, leaving a linear equation. If yours still has an $x^2$, the set-up is wrong.

---

### Q63 — Proficient — Answer: A

**Topic:** Regular pentagon inscribed in a circle (sine rule + double angle)

**Build the central triangle.** Perimeter 50 means each side is 10. Join the centre to two adjacent vertices: two radii $r$, base 10, apex angle $\frac{360^\circ}{5} = 72^\circ$, so each base angle is $\frac{180-72}{2} = 54^\circ$.

**Sine rule:**

$$\frac{r}{\sin 54^\circ} = \frac{10}{\sin 72^\circ} \implies r = \frac{10 \sin 54^\circ}{\sin 72^\circ}$$

That is not an option, so compress it. Use $\sin 54^\circ = \cos 36^\circ$ and $\sin 72^\circ = 2 \sin 36^\circ \cos 36^\circ$:

$$r = \frac{10 \cos 36^\circ}{2 \sin 36^\circ \cos 36^\circ} = \frac{5}{\sin 36^\circ}$$

*Why the others are wrong:*

- **B** is the sine-rule ratio itself — that equals $\frac{r}{\sin 54^\circ}$, not $r$.
- **D** has the sine rule upside down.
- **C** swaps sine for cosine. The half-angle at the centre faces the opposite half-side, which is a **sine** relationship.

> **Takeaway:** Regular polygon in a circle → central angle $\frac{360^\circ}{n}$, two radii, sine rule. If your correct expression is missing from the options, look for a co-function swap plus a double-angle collapse — the 36°/54°/72° family is where the exam does this.

## Mixed Practice — Chapter 5

### M1 — Basic

**Topic:** Supplementary angles

Find the supplement of $63^\circ$.

A) 127°

B) 107°

C) 117°

D) 97°

---

### M2 — Basic

**Topic:** Equilateral triangle angles

What is the interior angle of an equilateral triangle?

A) 60°

B) 45°

C) 90°

D) 30°

---

### M3 — Intermediate

**Topic:** Area ratio of similar figures

Two similar figures have a linear scale factor of $3 : 4$. The area of the smaller figure is 27 cm². Find the area of the larger figure.

A) 36 cm²

B) 42 cm²

C) 54 cm²

D) 48 cm²

---

### M4 — Basic

**Topic:** Area of a square from its perimeter

A square has a perimeter of 40 cm. Find its area.

A) 40 $cm^2$

B) 100 $cm^2$

C) 120 $cm^2$

D) 80 $cm^2$

---

### M5 — Basic

**Topic:** Area of a circle

Find the area of a circle with radius 4 cm. Leave your answer in terms of $\pi$.

A) $16\pi$ $cm^2$

B) $8\pi$ $cm^2$

C) $4\pi$ $cm^2$

D) $32\pi$ $cm^2$

---

### M6 — Intermediate

**Topic:** Pythagorean triple 5-12-13

A right triangle has legs of 5 cm and 12 cm. Find the hypotenuse.

A) 12 cm

B) 15 cm

C) 14 cm

D) 13 cm

---

### M7 — Basic

**Topic:** Surface area of a cube

A cube has side length 3 cm. Find its total surface area.

A) 27 $cm^2$

B) 54 $cm^2$

C) 36 $cm^2$

D) 81 $cm^2$

---

### M8 — Intermediate

**Topic:** Distance between two points

Find the distance between $(-1,\ 0)$ and $(2,\ 4)$.

A) 2

B) 3

C) 5

D) 4

---

### M9 — Intermediate

**Topic:** Exterior angle theorem

An exterior angle of a triangle is $110^\circ$. One non-adjacent interior angle is $40^\circ$. Find the other non-adjacent interior angle.

A) 60°

B) 70°

C) 80°

D) 50°

---

### M10 — Intermediate

**Topic:** Co-interior angles — solving for x

Co-interior angles measure $(4x+5)^\circ$ and $(5x-5)^\circ$. Find $x$.

A) 15

B) 18

C) 25

D) 20

---

### M11 — Intermediate

**Topic:** Sector area

A sector has radius 10 cm and central angle $72^\circ$. Find its area in terms of $\pi$.

A) $20\pi$ cm²

B) $15\pi$ cm²

C) $10\pi$ cm²

D) $25\pi$ cm²

---

### M12 — Intermediate

**Topic:** Volume of a cylinder

A cylinder has radius 4 cm and height 5 cm. Find its volume in terms of $\pi$.

A) $20\pi$ cm³

B) $40\pi$ cm³

C) $80\pi$ cm³

D) $60\pi$ cm³

---

### M13 — Intermediate

**Topic:** Similar triangles — missing side

Two similar triangles have sides $3,\ 4,\ 5$ and $x,\ 8,\ 10$. Find $x$.

A) 8

B) 4

C) 5

D) 6

---

### M14 — Intermediate

**Topic:** Area of a rhombus

A rhombus has diagonals of length 12 cm and 16 cm. Find its area.

A) 96 $cm^2$

B) 48 $cm^2$

C) 144 $cm^2$

D) 192 $cm^2$

---

### M15 — Proficient

**Topic:** Hypotenuse of a right isosceles triangle

A right isosceles triangle has legs of 7 cm each. Find the hypotenuse.

A) 7 cm

B) $7\sqrt{3}$ cm

C) $7\sqrt{2}$ cm

D) 14 cm

---

### M16 — Intermediate

**Topic:** Total surface area of a cylinder

A cylinder has radius 3 cm and height 10 cm. Find its total surface area in terms of $\pi$.

A) $18\pi$ cm²

B) $78\pi$ cm²

C) $60\pi$ cm²

D) $30\pi$ cm²

---

### M17 — Basic

**Topic:** Midpoint of a segment

Find the midpoint of the segment joining $(3,\ -4)$ and $(-7,\ 8)$.

A) $(-3,\ 4)$

B) $(2,\ -2)$

C) $(-2,\ 2)$

D) $(0,\ 0)$

---

### M18 — Intermediate

**Topic:** Interior angle of a regular hexagon

Find the interior angle of a regular hexagon.

A) 60°

B) 150°

C) 135°

D) 120°

---

### M19 — Intermediate

**Topic:** Arc length

A circle has radius 10 cm. Find the arc length subtended by an angle of $90^\circ$.

A) $5\pi$ cm

B) $4\pi$ cm

C) $2.5\pi$ cm

D) $10\pi$ cm

---

### M20 — Basic

**Topic:** Volume of a rectangular prism

A rectangular prism has dimensions 5 cm $\times$ 3 cm $\times$ 9 cm. Find its volume.

A) 45 $cm^3$

B) 135 $cm^3$

C) 120 $cm^3$

D) 90 $cm^3$

---

### M21 — Intermediate

**Topic:** Gradient of a line

Find the gradient of the line through $(-1,\ 3)$ and $(3,\ -1)$.

A) $-1$

B) 1

C) $-2$

D) 2

---

### M22 — Proficient

**Topic:** Tangent length from an external point

From an external point, the distance to the centre of a circle is 17 cm. The radius of the circle is 8 cm. Find the length of the tangent.

A) 9

B) 12

C) 15

D) 17

---

### M23 — Intermediate

**Topic:** Pythagorean triple 8-15-17

A right triangle has legs of 8 cm and 15 cm. Find the hypotenuse.

A) 16 cm

B) 17 cm

C) 16.5 cm

D) 19 cm

---

### M24 — Intermediate

**Topic:** Area of a trapezium

A trapezium has parallel sides of 6 cm and 10 cm, and a height of 4 cm. Find its area.

A) 16 $cm^2$

B) 24 $cm^2$

C) 28 $cm^2$

D) 32 $cm^2$

---

### M25 — Proficient

**Topic:** Volume ratio of similar solids

Two similar solids have a linear scale factor of $4 : 1$. The volume of the smaller solid is 3 $cm^3$. Find the volume of the larger solid.

A) 12 $cm^3$

B) 48 $cm^3$

C) 192 $cm^3$

D) 64 $cm^3$

---

### M26 — Intermediate

**Topic:** Inscribed angle theorem

A central angle is $100^\circ$. Find the inscribed angle subtending the same arc.

A) 50°

B) 100°

C) 25°

D) 200°

---

### M27 — Intermediate

**Topic:** Cyclic quadrilateral

In a cyclic quadrilateral, one angle is $110^\circ$. Find the opposite angle.

A) 60°

B) 80°

C) 110°

D) 70°

---

### M28 — Proficient

**Topic:** Perpendicular distance from centre to chord

A circle has radius 13 cm. A chord of length 24 cm is drawn. Find the perpendicular distance from the centre to the chord.

A) 8

B) 5

C) 7

D) 6

---

### M29 — Proficient

**Topic:** Equation of a parallel line

Find the equation of the line parallel to $y = 3x - 7$ passing through $(2,\ 1)$.

A) $y = 3x + 5$

B) $y = 3x + 1$

C) $y = 3x - 5$

D) $y = x - 5$

---

### M30 — Intermediate

**Topic:** Volume of a cone

A cone has base radius 6 cm and height 7 cm. Find its volume in terms of $\pi$.

A) $84\pi$ cm³

B) $126\pi$ cm³

C) $42\pi$ cm³

D) $252\pi$ cm³

## Mixed Practice — Answer Key

| | | | | |
|---|---|---|---|---|
| **M1.** C | **M2.** A | **M3.** D | **M4.** B | **M5.** A |
| **M6.** D | **M7.** B | **M8.** C | **M9.** B | **M10.** D |
| **M11.** A | **M12.** C | **M13.** D | **M14.** A | **M15.** C |
| **M16.** B | **M17.** C | **M18.** D | **M19.** A | **M20.** B |
| **M21.** A | **M22.** C | **M23.** B | **M24.** D | **M25.** C |
| **M26.** A | **M27.** D | **M28.** B | **M29.** C | **M30.** A |

# Chapter 6 — Data Handling and Probability

Data Handling and Probability is one of the most calculation-light sections in the NBT, yet it has one of the highest rates of avoidable errors. The reason is that students know the vocabulary but not the precise definitions. The median is not the middle value of the data — it is the middle value of the **sorted** data. The IQR is not just any measure of spread — it is specifically $Q_3 - Q_1$. Probability questions are usually short, but they distinguish carefully between mutually exclusive events, independent events, and conditional events. Master these distinctions and this chapter becomes free marks.

**Topics covered:** mean · median · mode · grouped data · range · IQR · five-number summary · box-and-whisker plots · histograms · ogives · scatter plots · standard deviation · basic probability · complementary events · mutually exclusive and independent events · addition and multiplication rules · conditional probability · tree diagrams · contingency tables · fundamental counting principle · permutations · combinations · Venn diagrams

---

## Section 6.1 — Measures of Central Tendency

The three measures of centre each tell you something different. The **mean** is sensitive to every value — one extreme outlier can pull it far from the bulk of the data. The **median** is resistant to outliers because it depends only on position, not on the actual size of extreme values. The **mode** is the only measure that can be used for non-numeric (categorical) data. In grouped data you can only estimate the mean using midpoints; the exact mean cannot be recovered from a frequency table.

---

### Q1 — Basic

Calculate the mean of the data set $\{4,\ 7,\ 3,\ 9,\ 2,\ 5\}$.

A) 5

B) 4.5

C) 6

D) 3.5

---

### Q2 — Basic

Find the median of $\{3,\ 7,\ 2,\ 9,\ 5,\ 1,\ 8\}$.

A) 3

B) 7

C) 5

D) 6

---

### Q3 — Basic

What is the mode of $\{4,\ 2,\ 7,\ 4,\ 9,\ 2,\ 2,\ 7\}$?

A) 4

B) 2

C) 7

D) 4 and 2

---

### Q4 — Intermediate

The data set $\{5,\ 5,\ 6,\ 7,\ 7\}$ has mean $= 6$ and median $= 6$. The value 50 is added to the set. Which statement is correct?

A) Both the mean and median remain unchanged

B) Only the median increases significantly

C) Both increase by the same amount

D) The mean increases more than the median

---

### Q5 — Proficient

| Class interval | Midpoint ($x$) | Frequency ($f$) |
|---|---|---|
| $[0; 10)$ | 5 | 4 |
| $[10; 20)$ | 15 | 6 |
| $[20; 30)$ | 25 | 5 |
| $[30; 40)$ | 35 | 5 |

What is the estimated mean of this data?

A) 20.5

B) 22

C) 19.5

D) 21

---

## Section 6.2 — Measures of Spread

The range is easy to calculate but fragile — one extreme value changes it completely. The IQR is more robust because it focuses on the middle 50 % of the data. The five-number summary (minimum, $Q_1$, median, $Q_3$, maximum) provides a compact description of the entire distribution and is displayed graphically as a box-and-whisker plot. The outer fences $Q_1 - 1.5 \times \text{IQR}$ and $Q_3 + 1.5 \times \text{IQR}$ define the outlier boundaries.

---

### Q6 — Basic

Calculate the range of $\{12,\ 7,\ 19,\ 4,\ 23,\ 11\}$.

A) 12

B) 19

C) 17

D) 15

---

### Q7 — Basic

The sorted data set is $\{3,\ 5,\ 7,\ 9,\ 11,\ 13,\ 15,\ 17\}$. Calculate the IQR.

A) 14

B) 6

C) 8

D) 10

---

### Q8 — Intermediate

Determine the five-number summary of the sorted data set:

$$2,\ 5,\ 7,\ 8,\ 10,\ 12,\ 15,\ 18,\ 20,\ 25$$

A) $\text{Min} = 2,\ Q_1 = 7.5,\ \text{Med} = 11,\ Q_3 = 16.5,\ \text{Max} = 25$

B) $\text{Min} = 2,\ Q_1 = 8,\ \text{Med} = 11,\ Q_3 = 15,\ \text{Max} = 25$

C) $\text{Min} = 2,\ Q_1 = 6,\ \text{Med} = 11,\ Q_3 = 18,\ \text{Max} = 25$

D) $\text{Min} = 2,\ Q_1 = 7,\ \text{Med} = 11,\ Q_3 = 18,\ \text{Max} = 25$

---

### Q9 — Intermediate

A box-and-whisker plot is drawn for a data set. What percentage of the data values lie between $Q_1$ and $Q_3$ (inside the box)?

A) 50 %

B) 25 %

C) 75 %

D) 100 %

---

### Q10 — Intermediate

A data set has $Q_1 = 8$ and $Q_3 = 20$. Using the rule upper fence $= Q_3 + 1.5 \times \text{IQR}$, is the value 40 an outlier?

A) IQR = 12, and 40 is not an outlier

B) IQR = 12, and 40 is an outlier

C) IQR = 14, and 40 is not an outlier

D) IQR = 10, and 40 is an outlier

---

## Section 6.3 — Data Representation

Histograms, ogives, and scatter plots are the three graphical displays most likely to appear in NBT data questions. A histogram shows the shape of a distribution; an ogive (cumulative frequency curve) lets you estimate medians and percentiles; a scatter plot shows the relationship (correlation) between two variables.

---

### Q11 — Basic

A data set is summarised in the table below:

| Class | Frequency |
|---|---|
| $[0; 10)$ | 3 |
| $[10; 20)$ | 8 |
| $[20; 30)$ | 12 |
| $[30; 40)$ | 9 |
| $[40; 50)$ | 4 |

The distribution is best described as:

A) Positively skewed

B) Uniform

C) Approximately symmetric

D) Negatively skewed

---

### Q12 — Intermediate

| Upper boundary of class | Cumulative frequency |
|---|---|
| 20 | 5 |
| 40 | 15 |
| 60 | 30 |
| 80 | 42 |
| 100 | 50 |

The total number of values is 50. Estimate the median.

A) 40

B) 50

C) 45

D) 53

---

### Q13 — Basic

A scatter plot shows data points with no discernible pattern — the points are scattered randomly across the entire graph. This indicates:

A) No correlation

B) Strong positive correlation

C) Weak negative correlation

D) Perfect positive correlation

---

### Q14 — Intermediate

A line of best fit passes through the points $(0,\ 4)$ and $(10,\ 14)$. What is the gradient of this line?

A) 0.5

B) 1

C) 2

D) 1.5

---

### Q15 — Proficient

A data set has standard deviation $\sigma$. Which transformation changes $\sigma$?

A) Adding 5 to every value

B) Subtracting 3 from every value

C) Multiplying every value by 2

D) Adding 5 to the mean but leaving all values unchanged

---

## Section 6.4 — Basic Probability

The probability of event $A$ is $P(A) = \dfrac{n(A)}{n(S)}$, where $n(A)$ is the number of outcomes in $A$ and $n(S)$ is the total number of equally likely outcomes in the sample space $S$. Probability values always lie in $[0, 1]$.

---

### Q16 — Basic

A fair die is rolled once. What is the probability of rolling an even number?

A) $\dfrac{1}{3}$

B) $\dfrac{1}{6}$

C) $\dfrac{2}{3}$

D) $\dfrac{1}{2}$

---

### Q17 — Basic

If $P(A) = 0.3$, what is $P(A')$?

A) 0.7

B) 0.3

C) 0.4

D) 1.3

---

### Q18 — Intermediate

A card is drawn at random from a standard 52-card deck. What is the probability of drawing a red king?

A) $\frac{1}{52}$
B) $\frac{1}{26}$
C) $\frac{1}{13}$
D) $\frac{2}{13}$

---

### Q19 — Intermediate

Events $A$ and $B$ are mutually exclusive with $P(A) = 0.25$ and $P(B) = 0.35$. Find $P(A \text{ or } B)$.

A) 0.0875

B) 0.25

C) 0.60

D) 1.0

---

### Q20 — Intermediate

Events $A$ and $B$ are not mutually exclusive. $P(A) = 0.4$, $P(B) = 0.3$, $P(A \cap B) = 0.1$. Find $P(A \cup B)$.

A) 0.7

B) 0.12

C) 0.1

D) 0.6

---

### Q21 — Intermediate

Events $A$ and $B$ are independent with $P(A) = 0.4$ and $P(B) = 0.5$. Find $P(A \cap B)$.

A) 0.2

B) 0.9

C) 0.45

D) 0.5

---

## Section 6.5 — Conditional Probability, Tree Diagrams and Contingency Tables

When events are not independent, the probability of the second event depends on what happened first. Conditional probability captures this: $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$. Tree diagrams display multi-stage experiments visually; the probability along each branch is a conditional probability. Two-way tables (contingency tables) organise counts for two categorical variables and allow direct reading of conditional probabilities.

---

### Q22 — Intermediate

A bag contains 3 red, 4 blue, and 5 green balls (12 total). Two balls are drawn without replacement. What is the probability that both are red?

A) $\dfrac{1}{16}$

B) $\dfrac{1}{22}$

C) $\dfrac{1}{44}$

D) $\dfrac{1}{12}$

---

### Q23 — Proficient

$P(A \cap B) = 0.12$ and $P(A) = 0.4$. Calculate $P(B \mid A)$.

A) 0.048

B) 0.52

C) 0.3

D) 0.12

### Q24 — Intermediate

The table below shows data for 100 students:

| | Plays sport | Does not play sport |
|---|---|---|
| **Studies maths** | 30 | 20 |
| **Does not study maths** | 15 | 35 |

What is the probability that a randomly selected student plays sport?

A) 0.30

B) 0.55

C) 0.35

D) 0.45

### Q25 — Proficient

Using the same table from Q24, what is $P(\text{studies maths} \mid \text{plays sport})$?

A) $\frac{2}{3}$

B) $\frac{3}{10}$

C) $\frac{1}{2}$

D) $\frac{3}{5}$

### Q26 — Proficient

A bag contains 4 white and 6 black balls. Two balls are drawn one after the other without replacement. What is the probability that the first is white and the second is black?

A) $\frac{6}{25}$

B) $\frac{4}{15}$

C) $\frac{3}{10}$

D) $\frac{2}{9}$

## Section 6.6 — Counting Principle, Permutations and Combinations

The **Fundamental Counting Principle**: if task 1 can be done in $m$ ways and task 2 in $n$ ways, then both can be done in $m \times n$ ways. When order matters, use permutations: $P(n, r) = \frac{n!}{(n-r)!}$. When order does not matter, use combinations: $C(n, r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}$.

---

### Q27 — Basic

A student can choose from 3 shirts, 4 pairs of trousers, and 2 pairs of shoes. How many different outfits are possible?

A) 9

B) 16

C) 24

D) 48

---

### Q28 — Intermediate

In how many ways can 5 distinct books be arranged in a row on a shelf?

A) 25

B) 60

C) 100

D) 120

---

### Q29 — Intermediate

How many different 3-letter arrangements (no repetition) can be formed from the letters of the word MATHS?

A) 60

B) 10

C) 120

D) 20

---

### Q30 — Intermediate

In how many ways can a committee of 3 students be chosen from a group of 8 (order does not matter)?

A) 336

B) 56

C) 24

D) 168

---

## Section 6.7 — Venn Diagrams and Advanced Probability

Venn diagrams represent events as overlapping circles within a rectangle (the sample space). The overlap region is $A \cap B$; the combined shaded region is $A \cup B$. The region outside both circles is $(A \cup B)'$.

---

### Q31 — Intermediate

For events $A$ and $B$: $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cap B) = 0.2$. Find $P(A \cup B)$.

A) 0.8

B) 0.9

C) 0.7

D) 0.5

---

### Q32 — Intermediate

Using the same values as Q31, find the probability that $A$ occurs but $B$ does not.

A) 0.2

B) 0.4

C) 0.5

D) 0.3

---

### Q33 — Proficient

Events $A$ and $B$ are independent with $P(A) = 0.6$ and $P(B) = 0.5$. Find the probability that at least one of $A$ or $B$ occurs.

A) 0.8

B) 0.7

C) 0.3

D) 0.6

---

### Q34 — Intermediate

In a class of 30 students: 18 play soccer, 12 play hockey, and 5 play both. How many students play neither sport?

A) 7

B) 5

C) 3

D) 10

---

### Q35 — Proficient

Two fair dice are rolled. What is the probability that the sum of the two numbers is 7?

A) $\frac{1}{12}$

B) $\frac{1}{9}$

C) $\frac{1}{6}$

D) $\frac{7}{36}$

# Answers and Explanations

## Section 6.1 — Measures of Central Tendency

**Q1 — Basic — Answer: A**

**Topic:** Mean from raw data

**Step 1 — Sum all values.**

$$\text{Sum} = 4 + 7 + 3 + 9 + 2 + 5 = 30$$

**Step 2 — Divide by the count.**

$$\bar{x} = \frac{30}{6} = 5$$

**Why the distractors are wrong:**

**B) 4.5** — common error from dividing by 7 instead of 6 (the number of values, not the highest index).

**C) 6** — results from including a phantom extra value or miscounting.

**D) 3.5** — this is the minimum value divided by 1; a computation shortcut gone wrong.

> **Takeaway:** Count the number of values carefully before dividing. With six data points you divide by 6, not by any other number.

---

**Q2 — Basic — Answer: C**

**Topic:** Median from an odd-sized data set

**Step 1 — Sort the data in ascending order.**

$$1,\ 2,\ 3,\ 5,\ 7,\ 8,\ 9$$

**Step 2 — Locate the middle position.**

With $n = 7$ values, the median is at position $\frac{7+1}{2} = 4$. The 4th value is **5**.

**Why the distractors are wrong:**

**A) 3** — the 3rd value, not the middle one.

**B) 7** — the 5th value; the error of counting from the end rather than finding the exact middle.

**D) 6** — this would be the mean of 7 and 5, which is the average of the two middle values in an even-sized set. There are 7 values here, so there is one exact middle value.

> **Takeaway:** Always sort first, then find position $\frac{n+1}{2}$ for odd $n$. Never find the median of unsorted data.

---

### Q3 — Basic — Answer: B

**Topic:** Mode of a data set

**Step 1 — Count the frequency of each value.**

| Value | Frequency |
|---|---|
| 2 | 3 |
| 4 | 2 |
| 7 | 2 |
| 9 | 1 |

**Step 2 — Identify the highest frequency.**

The value 2 appears 3 times — more than any other value. The mode is **2**.

**Why the distractors are wrong:**

**A) 4** — appears twice, less than 2.

**C) 7** — also appears twice.

**D) 4 and 2** — a data set is bimodal only when two values share the highest frequency. Here 2 has a strictly higher frequency (3 vs 2), so there is one mode only.

> **Takeaway:** A data set has a unique mode only when one value appears strictly more often than all others. If two values are tied for most frequent, the set is bimodal.

---

### Q4 — Intermediate — Answer: D

**Topic:** Effect of an outlier on mean versus median

**Step 1 — Compute the new mean.**

$$\bar{x}_{\text{new}} = \frac{5+5+6+7+7+50}{6} = \frac{80}{6} \approx 13.3$$

The mean jumped from 6 to about 13.3 — an increase of over 7 units.

**Step 2 — Find the new median.**

Sorted set: $5,\ 5,\ 6,\ 7,\ 7,\ 50$. With $n = 6$, the median is the average of the 3rd and 4th values:

$$\text{Median}_{\text{new}} = \frac{6+7}{2} = 6.5$$

The median increased by only 0.5 — far less than the mean.

**Why the distractors are wrong:**

**A)** Adding 50 certainly changes the mean — it inflates the sum enormously.

**B)** The median barely moved (6 to 6.5); it is the mean that increased dramatically.

**C)** The mean increased by ~7.3, the median by 0.5 — very different amounts.

> **Takeaway:** The mean is sensitive to outliers because it uses every value in its calculation. The median is resistant because only the middle position matters.

---

### Q5 — Proficient — Answer: A

**Topic:** Estimated mean from a grouped frequency table

**Step 1 — Calculate $fx$ for each row.**

| Class | $x$ | $f$ | $fx$ |
|---|---|---|---|
| $[0; 10)$ | 5 | 4 | 20 |
| $[10; 20)$ | 15 | 6 | 90 |
| $[20; 30)$ | 25 | 5 | 125 |
| $[30; 40)$ | 35 | 5 | 175 |
| **Total** | | **20** | **410** |

**Step 2 — Divide $\sum fx$ by $\sum f$.**

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{410}{20} = 20.5$$

*Why the others are wrong:* **B** (22), **D** (21) and **C** (19.5) are each one bookkeeping slip away — a mis-multiplied row or a wrong column total. The table gives you two checkpoints: $\sum f = 20$ must match the number of data points, and $\sum fx = 410$. Verify both before dividing.

(Also resist averaging the four midpoints. That ignores the frequencies and only works if every class is equally full.)

> **Takeaway:** Grouped mean $= \frac{\sum fx}{\sum f}$ using midpoints. It is an **estimate** — the exact mean was lost when the data was grouped — and the two column totals are your only defence against slips.

---

## Section 6.2 — Measures of Spread

#### Q6 — Basic — Answer: B

**Topic:** Range

$$\text{Range} = \text{maximum} - \text{minimum} = 23 - 4 = 19$$

*Why the others are wrong:* the data are unsorted, and every wrong option is a scanning error —

- **D** (15) is $19 - 4$: the maximum misidentified as 19, with the true maximum 23 overlooked further down the list.
- **A** (12) is $23 - 11$: the minimum misidentified as 11 (4 hides in the middle).
- **C** (17) is a subtraction slip on the correct pair.

> **Takeaway:** Range = max − min $= 23 - 4 = 19$. With unsorted data, *underline* the max and min before subtracting — both scanning errors above are one glance from happening. And remember the range says nothing about the shape in between.

---

#### Q7 — Basic — Answer: C

**Topic:** Interquartile range

**Step 1 — Split into lower and upper halves.**

With $n = 8$ values, split after the 4th:

- Lower half: $\{3,\ 5,\ 7,\ 9\}$
- Upper half: $\{11,\ 13,\ 15,\ 17\}$

**Step 2 — Find $Q_1$ and $Q_3$.**

$$Q_1 = \frac{5+7}{2} = 6 \qquad Q_3 = \frac{13+15}{2} = 14$$

**Step 3 — IQR.**

$$\text{IQR} = Q_3 - Q_1 = 14 - 6 = 8$$

**Why the distractors are wrong:**

**A) 14** — this is $Q_3$, not the IQR.

**B) 6** — this is $Q_1$, not the IQR.

**D) 10** — results from using incorrect quartile positions (e.g., 2nd and 7th values).

> **Takeaway:** Split the data at the median position, then find the median of each half. For an even number of values, each half contains $n/2$ values.

---

**Q8 — Intermediate — Answer: D**

**Topic:** Five-number summary

**Min and Max:** 2 and 25.

**Median** ($n = 10$): average the 5th and 6th values, $\frac{10+12}{2} = 11$.

**Split into halves** (the median belongs to neither): lower $\{2, 5, 7, 8, 10\} \to Q_1 = 7$; upper $\{12, 15, 18, 20, 25\} \to Q_3 = 18$

Summary: $2,\ 7,\ 11,\ 18,\ 25$

*Why the others are wrong:* all three agree on Min, Median and Max — the whole fight is over **how to split the halves**.

- **A** averages pairs, using a different quartile convention. With five values per half, each quartile is a single datum, never an average.
- **B** counted the quartile positions from the wrong ends.
- **C** splits the lower half wrongly but gets $Q_3$ right — mixed conventions in one answer, always a red flag.

> **Takeaway:** For even $n$, each half has $n/2$ values and the quartiles are their medians. **Write both halves out** — quartile errors are nearly always splitting errors.

---

**Q9 — Intermediate — Answer: A**

**Topic:** Box-and-whisker plot — IQR interpretation

By definition, $Q_1$ is the value below which 25 % of the data falls, and $Q_3$ is the value below which 75 % falls. Therefore the interval $[Q_1,\ Q_3]$ contains the middle $75\% - 25\% = 50\%$ of the data.

**Why the distractors are wrong:**

**B) 25 %** — this is the proportion in each whisker segment (from Min to $Q_1$, or from $Q_3$ to Max).

**C) 75 %** — this is the proportion below $Q_3$, not between the two quartiles.

**D) 100 %** — only the full range (Min to Max) contains all the data.

> **Takeaway:** The box in a box-and-whisker plot always represents 50 % of the data. Each whisker represents 25 %.

---

**Q10 — Intermediate — Answer: B**

**Topic:** Outlier boundary (outer fence)

Three short lines:

1. **IQR** $= 20 - 8 = 12$
2. **Upper fence** $= Q_3 + 1.5 \times \text{IQR} = 20 + 18 = 38$
3. **Compare:** $40 > 38$, so 40 **is** an outlier.

*Why the others are wrong:*

- **A** gets IQR right but reverses the final comparison. Slow down at the finish line.
- **C** (IQR 14) counted 8 to 20 inclusively — a fencepost error. IQR is plain subtraction.
- **D** (IQR 10) is an arithmetic slip whose "outlier" verdict is only **accidentally** right — the NBT often pairs a wrong intermediate with a right-sounding conclusion.

> **Takeaway:** Write all three lines. Each one is checkable on its own, and a right conclusion from a wrong intermediate still earns nothing.

---

## Section 6.3 — Data Representation

### Q11 — Basic — Answer: C

**Topic:** Shape of a distribution from a frequency table

The frequencies rise from 3 to a peak of 12 in the middle class $[20; 30)$, then fall back to 4 — a roughly symmetric bell shape. The rising and falling sides are nearly mirror images of each other.

**Why the distractors are wrong:**

**A) Positively skewed** — a positively skewed distribution has a long tail to the right (small peak at low values, trailing off slowly at high values). That is not the case here.

**B) Uniform** — a uniform distribution has roughly equal frequencies across all classes.

**D) Negatively skewed** — would peak near the high end and trail off to the left.

> **Takeaway:** Visualise the bar heights: if they form a hill centred in the data range, the distribution is symmetric. A long tail to the right means positive skew; a long tail to the left means negative skew.

---

### Q12 — Intermediate — Answer: D

**Topic:** Estimating the median from a cumulative frequency table

**Median position:** $\frac{50}{2}$ = the 25th value.

**Which class holds it?** 15 values by the 40 boundary, 30 by the 60 boundary — so the 25th sits in $[40; 60)$.

**Interpolate:**

$$40 + \frac{25 - 15}{30 - 15} \times 20 = 40 + \tfrac{10}{15}(20) \approx 53$$

*Why the others are wrong:*

- **A** (40) is the class's lower boundary — the interpolation never started.
- **B** (50) is both the class midpoint and an echo of "$n = 50$" — doubly baited, and it skips the arithmetic.
- **C** (45) mangled the fraction. Build it in words: *how far into this class's 15 values does the 25th sit? Ten of fifteen.*

> **Takeaway:** Find the class straddling $\frac{n}{2}$, then: boundary $+ \frac{\text{values still needed}}{\text{values in the class}} \times$ class width.

---

### Q13 — Basic — Answer: A

**Topic:** Describing correlation from a scatter plot

Correlation measures the tendency for two variables to move together. If there is no pattern — points spread randomly — neither variable can predict the other. This is called **no correlation** (or zero correlation).

**Why the distractors are wrong:**

**B)** Strong positive correlation means as $x$ increases, $y$ clearly tends to increase — a visible upward trend.

**C)** Weak negative means a faint downward trend is visible.

**D)** Perfect positive correlation means all points lie exactly on an upward straight line.

> **Takeaway:** The key question for any scatter plot is: "Is there a trend?" If yes, classify the direction (positive/negative) and strength (strong/weak). If no visible trend, the answer is no correlation.

---

### Q14 — Intermediate — Answer: B

**Topic:** Gradient of a line of best fit

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - 4}{10 - 0} = \frac{10}{10} = 1$$

The equation of the line is $y = x + 4$.

*Why the others are wrong:* **A** (0.5), **D** (1.5) and **C** (2) all come from misreading one coordinate — halving, off-by-five, or doubling the rise or run. The defence is mechanical: write $\Delta y = 14 - 4 = 10$ and $\Delta x = 10 - 0 = 10$ on separate lines *before* dividing. A gradient never comes out of a single glance at two points.

> **Takeaway:** $m = \frac{\Delta y}{\Delta x} = \frac{10}{10} = 1$: $y$ rises one unit per unit of $x$, and the intercept 4 is the prediction at $x = 0$ — so the line is $y = x + 4$. Compute the two differences explicitly, every time.

---

### Q15 — Proficient — Answer: C

**Topic:** Effect of transformation on standard deviation

Standard deviation measures spread around the mean. Adding or subtracting a constant shifts the entire data set by the same amount — the mean shifts, but the distances between values stay the same, so $\sigma$ is unchanged.

Multiplying every value by a constant $k$ scales all distances. If the original values are $x_i$, the new values are $kx_i$. The deviations from the new mean $(kx_i - k\bar{x}) = k(x_i - \bar{x})$ are all scaled by $k$. Therefore the new standard deviation is $|k|\sigma$.

Multiplying by 2 doubles $\sigma$.

**Why D is wrong:** You cannot add a constant to the mean without changing the individual values. The mean is derived from the values, not set independently.

> **Takeaway:** Adding or subtracting a constant — no change to $\sigma$. Multiplying or dividing by a constant $k$ — $\sigma$ is multiplied by $|k|$.

---

## Section 6.4 — Basic Probability

### Q16 — Basic — Answer: D

**Topic:** Probability from a sample space

Sample space: $S = \{1, 2, 3, 4, 5, 6\}$, so $n(S) = 6$.

Even numbers: $A = \{2, 4, 6\}$, so $n(A) = 3$.

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

*Why the others are wrong:*

- **B** ($\frac{1}{6}$) is the probability of one *specific* face — "even" admits three faces, not one.
- **A** ($\frac{1}{3}$) counts only two even faces (2 and 4, with 6 forgotten) — or divides the three categories {odd, even, ...} carelessly.
- **C** ($\frac{2}{3}$) sweeps four faces into "even" — an odd face smuggled in. List the set: $\{2, 4, 6\}$, exactly three.

> **Takeaway:** $P = \frac{n(A)}{n(S)}$: write out the favourable set before counting it. For a fair die, three even faces out of six gives $\frac{1}{2}$ — and explicit listing is what keeps 3 from becoming 2 or 4 under time pressure.

---

### Q17 — Basic — Answer: A

**Topic:** Complementary events

For any event $A$ and its complement $A'$:

$$P(A) + P(A') = 1 \implies P(A') = 1 - 0.3 = 0.7$$

*Why the others are wrong:*

- **B** (0.3) hands back $P(A)$ unchanged — as if an event and its complement were equally likely by default. They are only equal at 0.5.
- **C** (0.4) subtracts 0.3 twice $(1 - 0.3 - 0.3)$ — a "remove it from both sides" reflex with no rule behind it.
- **D** (1.3) *adds* the two — but no probability can exceed 1, which makes D self-refuting the moment you look at it. Options outside $[0, 1]$ are free eliminations.

> **Takeaway:** $P(A) + P(A') = 1$, always: complementary events are exhaustive and mutually exclusive. And scan the options for impossible probabilities first — examiners include them to reward students who know the boundaries.

---

### Q18 — Intermediate — Answer: B

**Topic:** Probability of a compound event

There are 2 red kings in a deck: the king of hearts and the king of diamonds.

$$P(\text{red king}) = \frac{2}{52} = \frac{1}{26}$$

**Why the distractors are wrong:**

**A)** $\frac{1}{52}$ would be the probability of a specific single card, e.g. the king of hearts only.

**C)** $\frac{1}{13}$ is the probability of any king (4 kings out of 52).

**D)** $\frac{2}{13}$ is the probability of any red king or any other compound event with 8 favourable outcomes.

> **Takeaway:** Count the favourable outcomes carefully. "Red king" is more specific than "any king" (4 outcomes) but less specific than "king of hearts" (1 outcome).

---

### Q19 — Intermediate — Answer: C

**Topic:** Mutually exclusive events — addition rule

Mutually exclusive means $A$ and $B$ cannot occur simultaneously, so $P(A \cap B) = 0$.

$$P(A \cup B) = P(A) + P(B) = 0.25 + 0.35 = 0.60$$

*Why the others are wrong:*

- **A** (0.0875) is $0.25 \times 0.35$ — the *product* rule, which belongs to independent events. Mutually exclusive events are the opposite of independent: if one happens, the other is impossible.
- **B** (0.25) echoes $P(A)$ alone — the "or" was never processed.
- **D** (1.0) treats "A or B" as covering everything, but the two events only account for 60 % of the probability; the remaining 0.40 belongs to "neither".

> **Takeaway:** Mutually exclusive ⇒ $P(A \text{ or } B) = P(A) + P(B)$, nothing to subtract — and nothing to multiply. "Mutually exclusive" and "independent" are different (in fact incompatible) conditions; identify which one the question grants you before choosing a rule.

---

### Q20 — Intermediate — Answer: D

**Topic:** Inclusive (non-mutually-exclusive) events — addition rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.3 - 0.1 = 0.6$$

The intersection is subtracted because those outcomes are counted twice if you simply add $P(A)$ and $P(B)$.

*Why the others are wrong:*

- **A** (0.7) adds without subtracting — valid only for mutually exclusive events, and the question hands you a non-zero overlap precisely so that this shortcut fails.
- **B** (0.12) multiplies $0.4 \times 0.3$ — importing the independence product rule where nothing says "independent" (and indeed $P(A \cap B) = 0.1 \neq 0.12$, so these events are *not* independent).
- **C** (0.1) echoes the intersection itself — an ingredient posing as the meal.

> **Takeaway:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, always; the overlap is counted twice in the sum, so subtract it once. When the intersection is *given*, use it — its value even tells you whether the events are independent ($0.1$ vs $0.4 \times 0.3 = 0.12$: they aren't).

---

**Q21 — Intermediate — Answer: A**

**Topic:** Independent events — multiplication rule

For independent events, the occurrence of one does not affect the probability of the other:

$$P(A \cap B) = P(A) \times P(B) = 0.4 \times 0.5 = 0.2$$

*Why the others are wrong:*

- **B** (0.9) adds the probabilities — the union-style move, and even then only correct for mutually exclusive events. "And" questions multiply.
- **C** (0.45) averages the two probabilities — a compromise with no probabilistic meaning.
- **D** (0.5) echoes $P(B)$ — as if "and" meant "whichever is larger".

> **Takeaway:** Independent: $P(A \cap B) = P(A) \times P(B) = 0.2$. Mutually exclusive: $P(A \cap B) = 0$. Two completely different conditions — an event pair cannot be both (unless a probability is 0). Translate the English first: *and* ⇒ intersection ⇒ multiply (under independence); *or* ⇒ union ⇒ add-then-subtract.

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## Section 6.5 — Conditional Probability, Tree Diagrams and Contingency Tables

**Q22 — Intermediate — Answer: B**

**Topic:** Dependent events — drawing without replacement

$$P(\text{1st red}) = \frac{3}{12} = \frac{1}{4}$$

After drawing one red ball, 2 red balls remain in a bag of 11:

$$P(\text{2nd red} \mid \text{1st red}) = \frac{2}{11}$$

$$P(\text{both red}) = \frac{1}{4} \times \frac{2}{11} = \frac{2}{44} = \frac{1}{22}$$

*Why the others are wrong:*

- **A** ($\frac{1}{16}$) is the **with-replacement** answer: $\left(\frac{3}{12}\right)^2$ — the bag never updated after the first draw.
- **C** ($\frac{1}{44}$) updates too hard: $\frac{1}{4} \times \frac{1}{11}$ leaves only *one* red behind, but removing one of three reds leaves two.
- **D** ($\frac{1}{12}$) matches no consistent method — a plausible-sized decoy. Both legitimate routes agree on B: sequentially $\frac{1}{4} \times \frac{2}{11} = \frac{1}{22}$, or by combinations $\frac{\binom{3}{2}}{\binom{12}{2}} = \frac{3}{66} = \frac{1}{22}$.

> **Takeaway:** "Without replacement" shrinks *both* the favourable count and the total: numerator and denominator each drop by one for the second draw. When unsure, cross-check with the combinations route — two methods agreeing is proof against all four distractor styles.

---

#### Q23 — Proficient — Answer: C

**Topic:** Conditional probability formula

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.12}{0.4} = 0.3$$

*Why the others are wrong:*

- **A** (0.048) multiplies $0.12 \times 0.4$ — running the multiplication rule when the formula calls for division.
- **B** (0.52) adds them $(0.4 + 0.12)$ — no rule anywhere adds an intersection to a marginal.
- **D** (0.12) echoes $P(A \cap B)$ unchanged — "given" ignored entirely. Conditioning must *rescale*: inside the world where $A$ happened (which has size 0.4), the slice where $B$ also happens (0.12) occupies $\frac{0.12}{0.4} = 30\%$.

> **Takeaway:** $P(B \mid A) = \frac{P(A \cap B)}{P(A)}$ — "given $A$" shrinks the sample space to $A$, and the division is that shrinkage. If your "conditional" probability equals the plain intersection, no conditioning has actually happened.

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#### Q24 — Intermediate — Answer: D

**Topic:** Two-way contingency table — reading probability

Total students who play sport $= 30 + 15 = 45$.

$$P(\text{sport}) = \frac{45}{100} = 0.45$$

*Why the others are wrong:* each is a different *piece* of the table posing as the answer —

- **A** (0.30) is only the top-left cell (sporty maths students) — the 15 sporty non-maths students got dropped.
- **C** (0.35) is the bottom-right cell (plays neither... i.e. non-maths non-sport) — the wrong region entirely.
- **B** (0.55) is the *complement*: the "does not play sport" column total $(20 + 35)$. Right technique, wrong column.

> **Takeaway:** Probability from a table = (relevant column or row **total**) ÷ (grand total). Sum the full "plays sport" column first $(30 + 15 = 45)$, and reserve subtotals-as-denominators for questions that say "given that".

---

**Q25 — Proficient — Answer: A**

**Topic:** Conditional probability from a contingency table

The condition "plays sport" restricts us to the 45 students in the sport column.

Of those 45, exactly 30 study maths.

$$P(\text{maths} \mid \text{sport}) = \frac{30}{45} = \frac{2}{3}$$

*Why the others are wrong:*

- **B** ($\frac{3}{10} = \frac{30}{100}$) divides by the grand total — the "given that" was never applied. That computes $P(\text{maths and sport})$, a different quantity.
- **D** ($\frac{3}{5} = \frac{30}{50}$) conditions on the wrong event: dividing by the *maths row* total gives $P(\text{sport} \mid \text{maths})$ — the reversed conditional. $P(A \mid B)$ and $P(B \mid A)$ are rarely equal, and the exam loves offering both.
- **C** ($\frac{1}{2}$) is a "half-ish" guess matching no cell-over-subtotal in the table.

> **Takeaway:** "Given that X" ⇒ denominator = X's subtotal (here the 45 sport players). Say the fraction in words — "of the 45 who play sport, 30 study maths" — and the reversed conditional can't sneak in.

---

**Q26 — Proficient — Answer: B**

**Topic:** Dependent events — P(A and B) using conditional probability

$$P(\text{1st white}) = \frac{4}{10}$$

After removing one white ball, 9 balls remain (3 white + 6 black):

$$P(\text{2nd black} \mid \text{1st white}) = \frac{6}{9} = \frac{2}{3}$$

$$P(\text{white then black}) = \frac{4}{10} \times \frac{6}{9} = \frac{24}{90} = \frac{4}{15}$$

*Why the others are wrong:*

- **A** ($\frac{6}{25}$) is the **with-replacement** version: $\frac{4}{10} \times \frac{6}{10}$ — the second denominator never dropped to 9.
- **C** ($\frac{3}{10}$) drops the denominator too far ($\frac{4}{10} \times \frac{6}{8}$) — two balls removed after one draw.
- **D** ($\frac{2}{9}$) is $\frac{24}{90}$ mis-simplified ($\frac{20}{90}$ territory) — a cancellation slip at the very last step. Reduce fractions in one careful move: $\frac{24}{90} = \frac{4}{15}$.

> **Takeaway:** Sequential dependent events: $P(A \cap B) = P(A) \times P(B \mid A)$, with the second fraction rebuilt from the *post-draw* bag (9 balls: 3 white, 6 black). One draw removes exactly one ball — no more, no fewer.

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## Section 6.6 — Counting Principle, Permutations and Combinations

#### Q27 — Basic — Answer: C

**Topic:** Fundamental counting principle

$$\text{Total outfits} = 3 \times 4 \times 2 = 24$$

*Why the others are wrong:*

- **A** (9) **adds** the choices ($3 + 4 + 2$). Adding counts the ways to pick *one item of one kind*; an outfit needs one of each, and each shirt pairs with every trouser–shoe combination.
- **D** (48) doubles the true count — the "2 pairs of shoes" counted as 4 individual shoes, or a stage multiplied twice.
- **B** (16) mixes adding and multiplying ($3 \times 4 + 4$) — a chain that changes rules midway.

> **Takeaway:** Independent choices in sequence multiply: $3 \times 4 \times 2 = 24$. The test for multiplying is the word *each* — "for each shirt, every pair of trousers; for each of those, both pairs of shoes."

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### Q28 — Intermediate — Answer: D

**Topic:** Permutations — arrangements of distinct objects

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

For the first position there are 5 choices, for the second there are 4 (one book is placed), and so on.

*Why the others are wrong:*

- **A** (25) is $5^2$ — a "five times something" guess with no counting model.
- **B** (60) is $5 \times 4 \times 3$ — the factor chain abandoned two books early. All five books need placing, so all five factors belong.
- **C** (100) is round-number bait; factorials are rarely round.

> **Takeaway:** Arranging all $n$ distinct objects gives $n!$ — one shrinking factor per position, all the way down to 1. $5! = 120$ is worth instant recall (as are $4! = 24$ and $6! = 720$).

---

### Q29 — Intermediate — Answer: A

**Topic:** Permutations — selecting and arranging $r$ from $n$

MATHS has 5 distinct letters. We want to select 3 and arrange them in order:

$$P(5,3) = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 5 \times 4 \times 3 = 60$$

*Why the others are wrong:*

- **C** (120) is $5!$ — arrangements of **all five** letters, but only three positions exist here.
- **B** (10) is $\binom{5}{3}$ — the *combination* count, which ignores order. Arrangements of letters are ordered (CAT ≠ ACT), so each trio counts $3! = 6$ times: $10 \times 6 = 60$.
- **D** (20) stops the factor chain early ($5 \times 4$) — two slots filled, one abandoned.

> **Takeaway:** "Arrangements" ⇒ order matters ⇒ $P(n,r) = n \times (n-1) \times \cdots$, one factor per slot: $5 \times 4 \times 3 = 60$. Count your factors against your slots before moving on.

---

### Q30 — Intermediate — Answer: B

**Topic:** Combinations

$$\binom{8}{3} = \frac{8!}{3!\,5!} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = \frac{336}{6} = 56$$

*Why the others are wrong:*

- **A** (336) is $P(8, 3) = 8 \times 7 \times 6$ — ordered selections. A committee has no first, second or third member, so each group of three is counted $3! = 6$ times over.
- **D** (168) divides those 336 by **2** instead of by $3! = 6$ — a half-hearted correction that remembers order matters but forgets how many orderings a trio has.
- **C** (24) is $8 \times 3$ — the two visible numbers multiplied, with no counting structure behind them.

> **Takeaway:** "Committee / group / selection" ⇒ order doesn't matter ⇒ $\binom{n}{r}$ = permutation count ÷ $r!$. The divisor is the factorial of the *group size* — a trio has six orderings, not two.

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## Section 6.7 — Venn Diagrams and Advanced Probability

### Q31 — Intermediate — Answer: C

**Topic:** Union of two events — Venn diagram

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.4 - 0.2 = 0.7$$

*Why the others are wrong:*

- **B** (0.9) adds without subtracting the overlap — the double-counted 0.2 left in place.
- **A** (0.8) subtracts only half the overlap — an arithmetic compromise with no rule behind it.
- **D** (0.5) echoes $P(A)$ — as if the union were just "the bigger event".

> **Takeaway:** $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7$, always — mutually exclusive events are merely the special case where the subtracted term is 0. If your union isn't *larger than both* individual probabilities yet *smaller than their sum*, something is off; use that squeeze as a sanity check.

---

### Q32 — Intermediate — Answer: D

**Topic:** Region in a Venn diagram — A only

"$A$ only" means $A \cap B'$, the part of $A$ that does not overlap with $B$:

$$P(A \cap B') = P(A) - P(A \cap B) = 0.5 - 0.2 = 0.3$$

*Why the others are wrong:*

- **C** (0.5) hands back $P(A)$ whole — but $P(A)$ *includes* the overlap where $B$ also happens; "A but not B" requires carving that 0.2 out.
- **A** (0.2) is the overlap itself — the piece to be removed, presented as the remainder.
- **B** (0.4) echoes $P(B)$ — the wrong circle altogether.

> **Takeaway:** $P(A \text{ only}) = P(A) - P(A \cap B) = 0.3$. Sketch the two circles and shade the crescent — Venn questions are region-labelling exercises, and every wrong option here is a different mislabelled region.

---

### Q33 — Proficient — Answer: A

**Topic:** "At least one" probability — complement method

"At least one" is the complement of "neither":

$$P(\text{at least one}) = 1 - P(A' \cap B')$$

Since $A$ and $B$ are independent, so are $A'$ and $B'$:

$$P(A' \cap B') = P(A') \times P(B') = (1 - 0.6)(1 - 0.5) = 0.4 \times 0.5 = 0.2$$

$$P(\text{at least one}) = 1 - 0.2 = 0.8$$

*Why the others are wrong:*

- **B** (0.7) subtracts $P(\text{both})$ from 1 — the complement of the **wrong event**. "At least one" is the opposite of "neither".
- **C** (0.3) is $P(\text{both})$, answering a different question.
- **D** (0.6) echoes $P(A)$. "At least one" must be **bigger** than either single probability, so 0.6 is too small on sight.

> **Takeaway:** $P(\text{at least one}) = 1 - P(\text{none})$. Say out loud which event you are complementing before subtracting from 1 — that sentence is exactly where B goes wrong.

---

### Q34 — Intermediate — Answer: B

**Topic:** Venn diagram — neither event

$$n(\text{soccer} \cup \text{hockey}) = 18 + 12 - 5 = 25$$

$$n(\text{neither}) = 30 - 25 = 5$$

*Why the others are wrong:*

- **A** (7) is $30 - 18 - 5$ — a subtraction chain that removes "both" as if it were a third separate group. The 5 who play both are already inside the 18 and inside the 12.
- **C** (3) and **D** (10) are the same story with different mis-handlings of the overlap (subtracting it twice, or not at all, plus slips). All three die at the checkpoint: students in *at least one* sport $= 18 + 12 - 5 = 25$, leaving exactly 5 outside.

> **Takeaway:** Inclusion–exclusion first ($n(A \cup B) = 18 + 12 - 5 = 25$), subtract from the total second. Better yet, fill in the Venn regions — 13 soccer-only, 5 both, 7 hockey-only — and confirm they sum with "neither" to 30 ✓.

---

### Q35 — Proficient — Answer: C

**Topic:** Probability of a sum when rolling two dice

**Total outcomes:** $6 \times 6 = 36$ ordered pairs.

**Pairs summing to 7:** $(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)$ — **six**.

$$P = \frac{6}{36} = \frac{1}{6}$$

*Why the others are wrong:*

- **A** ($\frac{1}{12}$) counted **unordered** pairs, halving the truth. $(1, 6)$ and $(6, 1)$ are different rolls.
- **B** ($\frac{1}{9}$) missed two pairs. List systematically — one for each value of the first die — and six is unmissable.
- **D** ($\frac{7}{36}$) put the target **sum** in the numerator. The favourable count is 6.

> **Takeaway:** Two dice always give 36 ordered outcomes. Sum 7 has exactly one pair per value of the first die — six ways, the most of any sum.

## Mixed Practice Questions — M1 to M30

These questions cover all six chapters. No explanations are given. Work through each one using the methods from earlier chapters.

### M1 — Basic

**Topic:** Simplifying algebraic expressions

Simplify: $\dfrac{x^2-9}{x-3}$ for $x \neq 3$.

A) $x+3$

B) $x-3$

C) $x^2+3$

D) $\dfrac{x+3}{x-3}$

### M2 — Basic

**Topic:** Ordering real numbers

Which list is in ascending order?

A) $0.6,\ \frac{2}{3},\ 0.7,\ \frac{5}{8}$

B) $\frac{2}{3},\ \frac{5}{8},\ 0.7,\ 0.6$

C) $0.6,\ \frac{5}{8},\ \frac{2}{3},\ 0.7$

D) $0.7,\ \frac{2}{3},\ \frac{5}{8},\ 0.6$

### M3 — Basic

**Topic:** Reading a linear graph

The graph of $y = 2x - 1$ crosses the $x$-axis at:

A) $(0,\ -1)$

B) $(1,\ 0)$

C) $\left(\frac{1}{2},\ 0\right)$

D) $(2,\ 0)$

---

### M4 — Basic

**Topic:** SOHCAHTOA

In a right triangle, the side opposite angle $\theta$ has length 8 and the hypotenuse has length 10. What is $\sin\theta$?

A) $\frac{3}{5}$

B) $\frac{4}{5}$

C) $\frac{3}{4}$

D) $\frac{4}{3}$

---

### M5 — Basic

**Topic:** Angles in a triangle

Two angles of a triangle are $47^\circ$ and $68^\circ$. What is the third angle?

A) $65^\circ$

B) $75^\circ$

C) $55^\circ$

D) $70^\circ$

---

### M6 — Basic

**Topic:** Mean of a data set

Find the mean of $\{10,\ 14,\ 8,\ 16,\ 12\}$.

A) 10

B) 11

C) 12

D) 13

---

### M7 — Basic

**Topic:** Basic probability

A bag has 5 red and 3 blue marbles. One is drawn at random. $P(\text{blue}) =$

A) $\frac{5}{8}$

B) $\frac{3}{8}$

C) $\frac{3}{5}$

D) $\frac{1}{3}$

---

### M8 — Intermediate

**Topic:** Solving a quadratic equation

Solve $x^2 - 5x + 6 = 0$.

A) $x = 2$ or $x = -3$

B) $x = -2$ or $x = -3$

C) $x = 1$ or $x = 6$

D) $x = 2$ or $x = 3$

---

### M9 — Basic

**Topic:** Percentage

A price of R480 is increased by 15 %. What is the new price?

A) R552

B) R495

C) R520

D) R540

---

### M10 — Intermediate

**Topic:** Parabola vertex

The function $f(x) = -(x-3)^2 + 5$ has a maximum value of:

A) 3

B) $-5$

C) 5

D) $-3$

---

### M11 — Intermediate

**Topic:** Trigonometric identity

Which of the following is equal to $\dfrac{\sin\theta}{\cos\theta}$?

A) $\sin\theta \cdot \cos\theta$

B) $\tan\theta$

C) $\cot\theta$

D) $\sec\theta$

---

### M12 — Intermediate

**Topic:** Area of a triangle

A triangle has base 12 cm and height 7 cm. Its area is:

A) 84 $\text{cm}^2$

B) 21 $\text{cm}^2$

C) 19 $\text{cm}^2$

D) 42 $\text{cm}^2$

---

### M13 — Basic

**Topic:** Median of a data set

Find the median of $\{17,\ 4,\ 22,\ 9,\ 14,\ 31\}$.

A) 15.5

B) 14

C) 17

D) 16

---

### M14 — Basic

**Topic:** Complementary probability

$P(A) = 0.62$. Find $P(A')$.

A) 1.62

B) 0.62

C) 0.38

D) 0.5

---

### M15 — Intermediate

**Topic:** Factorisation — difference of two squares

Factorise $4x^2 - 25$.

A) $(2x-5)(2x-5)$

B) $(2x-5)(2x+5)$

C) $(4x-5)(x+5)$

D) $(x-5)(4x+5)$

---

### M16 — Intermediate

**Topic:** Ratio and proportion

If $\frac{x}{y} = \frac{3}{4}$ and $y = 28$, what is $x$?

A) 18

B) 24

C) 16

D) 21

---

### M17 — Intermediate

**Topic:** Exponential function

The function $f(x) = 3 \cdot 2^x$ passes through the point:

A) $(1,\ 6)$

B) $(0,\ 2)$

C) $(2,\ 9)$

D) $(-1,\ 6)$

---

### M18 — Intermediate

**Topic:** Reduction formula

Evaluate $\cos(180^\circ - \theta)$.

A) $\cos\theta$

B) $\sin\theta$

C) $-\cos\theta$

D) $-\sin\theta$

---

### M19 — Basic

**Topic:** Pythagoras' theorem

A right triangle has legs of length 9 and 12. What is the hypotenuse?

A) 21

B) 15

C) 18

D) 20

---

### M20 — Intermediate

**Topic:** Interquartile range from a five-number summary

A data set has the five-number summary: $10,\ 18,\ 26,\ 38,\ 55.$

The IQR is:

A) 45

B) 12

C) 28

D) 20

---

### M21 — Intermediate

**Topic:** Independent events

$P(A) = 0.3$ and $P(B) = 0.6$, and $A$, $B$ are independent. Find $P(A \cap B)$.

A) $0.18$

B) $0.72$

C) $0.9$

D) $0.3$

---

### M22 — Intermediate

**Topic:** Simultaneous equations

Solve: $2x + y = 7$ and $x - y = 2$.

A) $x = 2,\ y = 3$

B) $x = 1,\ y = 5$

C) $x = 3,\ y = 1$

D) $x = 4,\ y = -1$

---

### M23 — Intermediate

**Topic:** Surd comparison

Which is the largest?

A) $\sqrt{50}$

B) $3\sqrt{6}$

C) $7$

D) $2\sqrt{13}$

---

### M24 — Intermediate

**Topic:** Domain of a function

What is the domain of $f(x) = \sqrt{x - 4}$?

A) $x > 4$

B) $x < 4$

C) $x \leq 4$

D) $x \geq 4$

---

### M25 — Proficient

**Topic:** General solution

Find the general solution of $\sin\theta = \dfrac{1}{2}$.

A) $\theta = 30^\circ + 360^\circ n$ or $\theta = 150^\circ + 360^\circ n,\backslash\ n \in \mathbb{Z}$

B) $\theta = 30^\circ + 180^\circ n,\backslash\ n \in \mathbb{Z}$

C) $\theta = 30^\circ$ or $\theta = 210^\circ$

D) $\theta = 30^\circ + 360^\circ n,\backslash\ n \in \mathbb{Z}$

---

### M26 — Intermediate

**Topic:** Similar triangles

Two similar triangles have corresponding sides in the ratio $3 : 5$. If the area of the smaller triangle is $27\text{ cm}^2$, what is the area of the larger triangle?

A) $45\text{ cm}^2$

B) $54\text{ cm}^2$

C) $75\text{ cm}^2$

D) $135\text{ cm}^2$

---

### M27 — Proficient

**Topic:** Standard deviation concept

Two data sets have the same mean of 50. Data set A has standard deviation 2; data set B has standard deviation 15. Which statement is correct?

A) Data set A is more spread out

B) Data set B has values closer to the mean

C) Data set A has values more tightly clustered around 50

D) Both data sets have the same spread

---

### M28 — Proficient

**Topic:** Tree diagram — three-stage experiment

A coin is flipped twice. What is the probability of getting exactly one head?

A) $\frac{1}{4}$

B) $\frac{3}{4}$

C) $\frac{1}{3}$

D) $\frac{1}{2}$

---

### M29 — Intermediate

**Topic:** Arithmetic sequence

The 5th term of an arithmetic sequence is 23 and the common difference is 4. What is the first term?

A) 7

B) 3

C) 11

D) 5

---

### M30 — Proficient

**Topic:** Counting principle — restrictions

How many 3-digit numbers can be formed from the digits $\{1, 2, 3, 4, 5\}$ if no digit may be repeated and the number must be even?

A) 12

B) 18

C) 24

D) 20

## Mixed Practice — Answer Key

| | | | | |
|---|---|---|---|---|
| **M1.** A | **M2.** C | **M3.** C | **M4.** B | **M5.** A |
| **M6.** C | **M7.** B | **M8.** D | **M9.** A | **M10.** C |
| **M11.** B | **M12.** D | **M13.** A | **M14.** C | **M15.** B |
| **M16.** D | **M17.** A | **M18.** C | **M19.** B | **M20.** D |
| **M21.** A | **M22.** C | **M23.** B | **M24.** D | **M25.** A |
| **M26.** C | **M27.** C | **M28.** D | **M29.** A | **M30.** C |

# Chapter 7 — Financial Mathematics

## A Note on Curriculum Placement

Financial Mathematics is not listed as a separate competence area in the NBT MAT. It falls under **Number Sense** — but it is severely underrepresented in that chapter relative to how often it appears on the actual test. The reason is that the NBT regularly tests compound interest, depreciation, and hire purchase because these topics require both formula knowledge and careful arithmetic, making them excellent discriminators between students who understand the mathematics and students who merely have a calculator intuition.

This chapter exists because Chapter 2 (Number Sense) could not do these topics justice without becoming unmanageably long. If you are preparing for the NBT, treat Chapters 2 and 7 as a single block — the Number Sense competence area draws from both.

**No calculator is permitted in the NBT.** Every question in this chapter is designed so that the arithmetic resolves to a clean integer or simple fraction. The numbers are not realistic — real financial products rarely use 10 % or 20 % rates — but the mechanics are identical to real-world problems. Master the method with clean numbers, and the messier real-world versions become straightforward.

**Formulas you must know:** - Simple interest: $A = P(1 + rt)$ or $I = Prt$ - Compound interest: $A = P(1 + i)^n$ - Straight-line depreciation: $A = P(1 - rt)$ - Reducing-balance depreciation: $A = P(1 - i)^n$

---

## Section 7.1 — Simple Interest

Simple interest grows by the same fixed amount each period because interest is always calculated on the original principal. If you borrow R1000 at 10 % p.a. simple interest, you pay R100 interest every year regardless of how long the loan has been running. This contrasts with compound interest, where the interest accrues on a growing balance.

---

### Q1 — Basic

R4 000 is invested at 5 % p.a. simple interest for 3 years. How much interest is earned?

A) R600

B) R700

C) R500

D) R1 200

---

### Q2 — Basic

R2 000 is invested at 10 % p.a. simple interest for 4 years. What is the accumulated amount?

A) R2 400

B) R2 600

C) R2 800

D) R2 928

---

### Q3 — Basic

A simple interest investment earns R300 interest at 15 % p.a. over 2 years. What was the principal?

A) R900

B) R1 000

C) R1 200

D) R800

---

### Q4 — Intermediate

R5 000 is invested and earns R3 000 in simple interest at 12 % p.a. For how long was it invested?

A) 3 years

B) 4 years

C) 6 years

D) 5 years

---

### Q5 — Intermediate

R8 000 grows to R9 200 in 3 years under simple interest. What is the annual interest rate?

A) 5 % p.a.

B) 6 % p.a.

C) 4 % p.a.

D) 7.5 % p.a.

## Section 7.2 — Compound Interest

Compound interest earns "interest on interest" — each period's interest is added to the principal before the next period's interest is calculated. This exponential growth is what makes compound interest so powerful over long time periods, and why the NBT tests it regularly.

### Q6 — Basic

R10 000 is invested at 10 % p.a. compounded annually for 2 years. What is the accumulated amount?

A) R12 000

B) R12 200

C) R12 100

D) R13 100

### Q7 — Basic

R1 000 is invested at 10 % p.a. compounded annually for 3 years. What is the accumulated amount?

A) R1 300

B) R1 331

C) R1 360

D) R1 221

### Q8 — Intermediate

R10 000 grows to R14 641 at 10 % p.a. compounded annually. For how many years?

A) 4 years

B) 3 years

C) 2 years

D) 5 years

---

### Q9 — Intermediate

R1 000 is invested for 3 years at 10 % p.a. Which is correct?

A) Simple interest gives R1 300; compound interest also gives R1 300 — they are equal over 3 years

B) Simple interest always grows faster than compound interest

C) Simple interest gives R1 300; compound interest gives R1 210

D) Compound interest gives R1 331; this is R31 more than simple interest

---

### Q10 — Basic

R10 000 grows to R12 100 at 10 % p.a. compounded annually. How many years?

A) 1 year

B) 3 years

C) 2 years

D) 4 years

---

## Section 7.3 — Depreciation

Assets lose value over time. The two standard methods are:

- **Straight-line (linear):** the asset loses the same rand amount each year: $A = P(1 - rt)$.
- **Reducing-balance (exponential):** the asset loses the same percentage each year: $A = P(1 - i)^n$.

Reducing-balance always gives a **higher** residual value than straight-line for the same rate and period, because the depreciation amount shrinks each year (you're taking the percentage off a smaller and smaller base).

---

### Q11 — Basic

A machine bought for R200 000 depreciates at 8 % p.a. straight-line. What is its value after 5 years?

A) R100 000

B) R120 000

C) R140 000

D) R160 000

---

### Q12 — Basic

Equipment costing R100 000 depreciates at 20 % p.a. on the reducing balance. Find its value after 3 years.

A) R51 200

B) R52 000

C) R60 000

D) R48 000

---

### Q13 — Intermediate

R100 000 asset. Both methods use 20 % p.a. After 3 years:

A) Both methods give R40 000 after 3 years

B) Straight-line gives R51 200; reducing balance gives R40 000

C) The two methods always produce the same residual value

D) Straight-line gives R40 000; reducing balance gives R51 200

---

### Q14 — Intermediate

Equipment worth R150 000 has a book value of R60 000 after 6 years using straight-line depreciation. What is the annual depreciation rate?

A) 6 % p.a.

B) 8 % p.a.

C) 10 % p.a.

D) 12 % p.a.

---

### Q15 — Intermediate

Equipment bought for R100 000 has a book value of R64 000 after 2 years using reducing-balance depreciation. What is the annual depreciation rate?

A) 16 % p.a.

B) 20 % p.a.

C) 18 % p.a.

D) 36 % p.a.

---

## Section 7.4 — Hire Purchase

Hire purchase (HP) allows a buyer to take possession of an item immediately and pay for it in instalments. The finance charge is calculated as **simple interest on the original balance** (after the deposit), not on the declining balance. This makes HP significantly more expensive than the stated interest rate suggests.

---

### Q16 — Basic

A TV costs R6 000 cash. HP terms: 10 % deposit, 24 monthly payments, with 10 % p.a. simple interest charged on the balance. What is the monthly payment?

A) R270

B) R300

C) R225

D) R312

---

### Q17 — Intermediate

A fridge costs R5 000 cash. HP terms: 15 % deposit, 18 monthly payments of R280. What is the total HP cost?

A) R5 040

B) R5 400

C) R5 600

D) R5 790

---

### Q18 — Intermediate

An item costs R4 000. No deposit is required; 12 monthly payments of R400 are charged. What annual simple interest rate does this represent?

A) 16 %

B) 24 %

C) 20 %

D) 12 %

---

#### Q19 — Intermediate

An item costs R12 000 cash. HP: 25 % deposit plus 36 monthly payments of R350. How much more does the HP buyer pay compared to the cash buyer?

A) R2 600

B) R3 600

C) R4 200

D) R1 200

---

#### Q20 — Basic

A loan of R20 000 is taken at 8 % p.a. simple interest for 3 years. What is the total amount owed at the end?

A) R24 800

B) R25 000

C) R24 000

D) R25 250

---

## Section 7.5 — Inflation and Real Value

Inflation erodes buying power: if prices rise at $\pi$ % p.a., an item costing $P$ today will cost $P(1+\pi)^n$ in $n$ years. The **real return** on an investment is approximately (nominal rate) – (inflation rate). If your savings account earns less than inflation, your money buys less each year even as the balance grows.

---

#### Q21 — Intermediate

An item costs R5 000 today. If inflation averages 20 % p.a., what will the item cost in 2 years?

A) R7 000

B) R6 000

C) R6 200

D) R7 200

---

### Q22 — Intermediate

A camera costs R3 000 today. Inflation is 10 % p.a. You plan to buy one in 3 years. How much should you budget?

A) R3 300

B) R3 900

C) R3 993

D) R4 000

---

### Q23 — Basic

An investment yields 12 % p.a. Inflation is 8 % p.a. What is the approximate real return?

A) 20 % p.a.

B) Approximately 4 % p.a.

C) 1.5 % p.a.

D) 8 % p.a.

---

### Q24 — Basic

A savings account earns 8 % p.a. Inflation is running at 10 % p.a. In real terms, the purchasing power of the savings is:

A) Decreasing — the real return is negative

B) Growing at 18 % p.a. in real terms

C) Growing at 2 % in real terms

D) Unaffected — inflation does not reduce the nominal balance

---

### Q25 — Basic

Groceries cost R500 per month. Food inflation is 6 % p.a. What monthly budget is needed in one year?

A) R506

B) R560

C) R530

D) R550

---

## Section 7.6 — Exchange Rates

An exchange rate quotes how many units of one currency equal one unit of another. To convert **from** the stronger currency (e.g. USD), multiply by the rate. To convert **to** the stronger currency, divide by the rate. Commission is typically charged as a percentage of the transaction value.

---

### Q26 — Basic

The exchange rate is R18 = \$1. Convert R9 000 to US dollars.

A) \$50

B) \$162 000

C) \$450

D) \$500

---

### Q27 — Basic

The exchange rate is R17.50 per US dollar. Convert \$200 to rand.

A) R3 000

B) R3 500

C) R4 000

D) R2 857

---

### Q28 — Intermediate

The exchange rate is R16 per US dollar. The bank charges 2 % commission. How much does it cost in rand to buy \$500?

A) R8 160

B) R8 000

C) R7 840

D) R8 320

---

### Q29 — Intermediate

You bought \$1 000 when the rate was R15 per dollar (cost R15 000). You now sell when the rate is R18 per dollar. What is your profit in rand?

A) R1 000 loss

B) R1 500 profit

C) R3 000 profit

D) R3 000 loss

---

### Q30 — Basic

You want to buy euros. Three banks quote: - Bank A: R20 per €1 - Bank B: R19.50 per €1 - Bank C: R20.50 per €1

Which bank gives you the best deal?

A) Bank A

B) Bank C

C) All three give the same number of euros

D) Bank B

---

## Section 7.7 — Time Value of Money

Money today is worth more than the same amount in the future because today's money can earn interest. The **present value** (PV) of a future amount $A$ at rate $i$ over $n$ periods is $P = A/(1+i)^n$. When equal payments are made at regular intervals (an **annuity**), the future value is found by summing the compound-interest-grown contributions of each payment — a geometric series.

---

### Q31 — Proficient

R1 000 is deposited at the end of each year for 3 years at 10 % p.a. compound. What is the total accumulated value after 3 years?

A) R3 000

B) R3 310

C) R3 300

D) R3 460

---

### Q32 — Intermediate

A loan of R50 000 is repaid in equal annual instalments over 4 years at 20 % p.a. simple interest. What is the annual instalment?

A) R12 500

B) R20 000

C) R22 500

D) R25 000

---

### Q33 — Basic

R500 is deposited monthly for 24 months. Total deposited = R12 000. If the final accumulated amount is R13 200, what is the total interest earned?

A) R1 200

B) R1 300

C) R800

D) R1 800

---

### Q34 — Intermediate

R100 000 is borrowed at 12 % p.a. simple interest for 5 years. How much interest is paid in total?

A) R12 000

B) R48 000

C) R50 000

D) R60 000

---

### Q35 — Proficient

How much must be invested now at 10 % p.a. compound interest to have R14 641 in 4 years?

A) R11 000

B) R10 000

C) R12 100

D) R9 900

# Answers and Explanations

## Section 7.1 — Simple Interest

#### Q1 — Basic — Answer: A

**Topic:** Simple interest — calculate interest earned

Simple interest is worked out on the **original** amount only, so the same rand amount is earned every year.

**Year by year:** 5 % of R4 000 = R200 a year. Three years: $3 \times 200 = \text{R}600$.

**By formula:** $I = Prt = 4\,000 \times 0.05 \times 3 = \text{R}600$

*Why the others are wrong:*

- **D** (R1 200) doubles the rate — that is $4\,000 \times 0.10 \times 3$.
- **B** (R700) and **C** (R500) are 3.5 and 2.5 years' worth. They punish estimating instead of computing. At exactly R200 a year, R600 is exact.

> **Takeaway:** $I = Prt$. If the question asks for the **interest**, stop at $I$. If it asks for the **amount**, add $P$ back: $A = \text{R}4\,600$. The NBT sets both versions.

---

#### Q2 — Basic — Answer: C

**Topic:** Simple interest — calculate accumulated amount

The question wants the **total**: your R2 000 plus the interest. Use $A = P(1 + rt)$.

**Step 1 — Find the multiplier.** 10 % for 4 years = 40 % growth, so multiply by 1.4.

**Step 2 — Multiply.**

$$A = 2\,000 \times 1.4 = \text{R}2\,800$$

Check: R200 interest a year × 4 years = R800, and $2\,000 + 800 = \text{R}2\,800$ ✓

*Why the others are wrong:*

- **A** (R2 400) — only **two** years of interest.
- **B** (R2 600) — **three** years instead of four.
- **D** (R2 928) — the **compound** answer, $2\,000 \times 1.1^4$. The word "simple" is doing real work here.

> **Takeaway:** Simple interest multiplies by $(1 + rt)$. Compound multiplies by $(1 + i)^n$. Check which word the question uses before you start.

---

### Q3 — Basic — Answer: B

**Topic:** Simple interest — find principal

A **reverse** problem: you are given the interest and must find the principal. Rearrange first, then check forwards.

**Step 1 — Rearrange.** $P = \dfrac{I}{rt}$

**Step 2 — Substitute.** The denominator is $0.15 \times 2 = 0.30$:

$$P = \frac{300}{0.30} = \text{R}1\,000$$

**Step 3 — Check forwards.** $1\,000 \times 0.15 \times 2 = 300$ ✓

*Why the others are wrong:*

- **C** (R1 200) mis-multiplied the denominator as $0.25$.
- **A** (R900) fails the check: $900 \times 0.30 = 270$.
- **D** (R800) fails too: $800 \times 0.30 = 240$.

> **Takeaway:** On any find-the-principal/rate/time question, substitute your answer back into $I = Prt$. Five seconds, and it catches nearly every error.

---

### Q4 — Intermediate — Answer: D

**Topic:** Simple interest — find time

The unknown is the **time**. Easiest route: work out the interest per year first.

**Step 1 — Interest per year.** $5\,000 \times 0.12 = \text{R}600$

**Step 2 — How many years of R600 make R3 000?**

$$t = \frac{3\,000}{600} = 5 \text{ years}$$

*Why the others are wrong:* each assumes a **different rate**.

- **C** (6 years) assumes R500 a year — a 10 % rate.
- **B** (4 years) assumes R750 a year — 15 %.
- **A** (3 years) assumes R1 000 a year — 20 %.

> **Takeaway:** "Interest per year, then divide" turns every find-the-time question into one small division. Check: $5\,000 \times 0.12 \times 5 = 3\,000$ ✓

---

#### Q5 — Intermediate — Answer: A

**Topic:** Simple interest — find rate

You are given the **accumulated amount**, not the interest, so split the interest off first.

**Step 1 — Extract the interest.** $I = 9\,200 - 8\,000 = \text{R}1\,200$

**Step 2 — Substitute.**

$$r = \frac{I}{Pt} = \frac{1\,200}{8\,000 \times 3} = 0.05 = 5\%$$

**Step 3 — Check.** R8 000 at 5 % earns R400 a year; three years is R1 200 ✓

*Why the others are wrong:*

- **D** (7.5 %) used **two** years instead of three.
- **C** (4 %) divided by the **final amount** instead of the principal. Rates are always quoted on the principal.
- **B** (6 %) would earn R1 440, overshooting the actual R1 200.

> **Takeaway:** Given $A$ rather than $I$? The first line of working is always $I = A - P$. Most wrong rate answers are born in that missed step.

---

## Section 7.2 — Compound Interest

### Q6 — Basic — Answer: C

**Topic:** Compound interest — calculate $A$, one period

Compound interest is worked out on the **current balance**, not the original deposit. Watch it:

- **Year 1:** 10 % of R10 000 = R1 000 → R11 000
- **Year 2:** 10 % of R11 000 = R1 **100** → **R12 100**

The second year earned R100 more — interest on the interest.

$$A = P(1+i)^n = 10\,000 \times 1.21 = \text{R}12\,100$$

*Why the others are wrong:*

- **A** (R12 000) is $10\,000 \times 1.20$, the **simple** interest answer. The R100 gap between A and C *is* compounding.
- **B** (R12 200) mis-squared $1.1$ as $1.22$. Write it out: $1.1 \times 1.1 = 1.21$.
- **D** (R13 100) is roughly the **three**-year value — an exponent miscount.

> **Takeaway:** For 10 % over 2 years the factor is $1.1^2 = 1.21$, never $1.20$. Running one year at a time is often faster *and* safer than the formula.

---

### Q7 — Basic — Answer: B

**Topic:** Compound interest — three-year period

$$A = 1\,000 \times (1.1)^3 = \text{R}1\,331$$

Not sure of $1.1^3$? Build it: R1 000 → R1 100 → R1 210 → **R1 331**.

**A pattern worth knowing** — the digits of the powers of 1.1 are Pascal's triangle:

$$1.1^2 = 1.21 \qquad 1.1^3 = 1.331 \qquad 1.1^4 = 1.4641$$

The NBT uses 10 % rates precisely because these work without a calculator.

*Why the others are wrong:*

- **A** (R1 300) is the **simple** interest value, missing the R31 of interest-on-interest.
- **D** (R1 221) is a corrupted Pascal row — 1,2,2,1 instead of 1,3,3,1.
- **C** (R1 360) botched the expansion's middle term ($0.06$ instead of $0.03$).

> **Takeaway:** $1.1^3 = 1.331$. Learn the ladder 1.21, 1.331, 1.4641 — it turns these into ten-second questions.

---

### Q8 — Intermediate — Answer: A

**Topic:** Compound interest — find $n$

**Step 1 — isolate the growth factor.** Divide both sides by the principal:

$$(1.1)^n = \frac{14\,641}{10\,000} = 1.4641$$

**Step 2 — climb the ladder of powers.**

$$1.1^1 = 1.1 \qquad 1.1^2 = 1.21 \qquad 1.1^3 = 1.331 \qquad 1.1^4 = 1.4641$$

The growth factor matches the fourth rung, so $n = 4$ years.

*Why the others are wrong:* all three are neighbouring rungs of the same ladder —

- **B** (3 years) would give $10\,000 \times 1.331 = \text{R}13\,310$: not enough.
- **C** (2 years) would give R12 100: far too little.
- **D** (5 years) would give $10\,000 \times 1.61051 \approx \text{R}16\,105$: too much.

If you *know* the ladder, no wrong option is even tempting; if you don't, every option looks equally plausible. That is the whole game with exponent-matching questions — they test recognition, not computation.

> **Takeaway:** Divide $A$ by $P$ first, then match the growth factor against known powers. The values 1.21, 1.331, 1.4641 should be instant recognitions by NBT day.

---

### Q9 — Intermediate — Answer: D

**Topic:** Compound vs simple — compare after 3 years

Work out both totals side by side — the comparison is the whole point.

**Simple:** the same R100 every year.

$$A = 1000(1.3) = \text{R}1\,300$$

**Compound:** 10 % of a growing balance (R100, then R110, then R121).

$$A = 1000(1.1)^3 = \text{R}1\,331$$

Compound wins by R31.

*Why the others are wrong:*

- **A** says they tie. They agree only after **one** period; after that compound always pulls ahead.
- **B** has it backwards — simple never beats compound at the same rate.
- **C** quotes R1 210, the **two-year** compound value, hidden inside a true-sounding sentence.

> **Takeaway:** At equal rates compound always ≥ simple, and the gap widens each year. When options are sentences, check every **number** inside them, not just the story.

---

### Q10 — Basic — Answer: C

**Topic:** Compound interest — find $n$ given $A$

**Step 1 — find the growth factor.**

$$(1.1)^n = \frac{12\,100}{10\,000} = 1.21$$

**Step 2 — recognise it.** $1.21 = 1.1^2$, so $n = 2$ years. (Year-by-year confirmation: R10 000 → R11 000 → R12 100 ✓.)

*Why the others are wrong:* the wrong options are simply other rungs of the 10 % ladder —

- **A** (1 year) would leave the balance at R11 000.
- **B** (3 years) would grow it to R13 310.
- **D** (4 years) would grow it to R14 641.

Only 2 years lands exactly on R12 100. If two different rungs both feel plausible, it means the growth factor was never actually computed — always do the division $A/P$ first.

> **Takeaway:** Divide $A$ by $P$ to get the growth factor, then match it to a known power of $(1 + i)$. This "divide, then recognise" routine solves every find-$n$ compound question the NBT can ask.

---

## Section 7.3 — Depreciation

### Q11 — Basic — Answer: B

**Topic:** Straight-line depreciation — find value

Straight-line depreciation is simple interest in reverse: the **same rand amount** is lost each year, always based on the original price.

**Loss per year:** $200\,000 \times 0.08 = \text{R}16\,000$

**Over 5 years:** $5 \times 16\,000 = \text{R}80\,000$ lost, leaving **R120 000**.

By formula: $A = P(1 - rt) = 200\,000(0.60) = \text{R}120\,000$

*Why the others are wrong — each uses the wrong rate:*

- **A** (R100 000) writes off half — that needs 10 %, not 8 %.
- **C** (R140 000) writes off R12 000 a year — a 6 % rate.
- **D** (R160 000) writes off R8 000 a year — 4 %, exactly half the true rate.

> **Takeaway:** Think "rand lost per year × years". After 5 years at 8 %, 40 % is gone and 60 % remains — which is all $(1 - rt)$ says.

---

### Q12 — Basic — Answer: A

**Topic:** Reducing-balance depreciation — find value

Reducing balance is compound interest running downhill: each year the asset loses 20 % of **its current worth**, so the rand loss shrinks. Watch the chain:

$$100\,000 \to 80\,000 \to 64\,000 \to \text{R}51\,200$$

The losses are R20 000, R16 000, R12 800 — smaller each time, because 20 % comes off a smaller base.

$$A = P(1 - i)^n = 100\,000(0.8)^3 = \text{R}51\,200$$

*Why the others are wrong:*

- **C** (R60 000) is straight-line thinking, and only two years of it.
- **D** (R48 000) reached R64 000 then subtracted R16 000 **again**, instead of recomputing 20 % on the new balance.
- **B** (R52 000) rounded $0.8^3$ to 0.52 instead of 0.512.

> **Takeaway:** Multiply by 0.8 year by year — 100 → 80 → 64 → 51.2 is three quick steps and cannot go wrong.

---

**Q13 — Intermediate — Answer: D**

**Topic:** Compare straight-line and reducing-balance

Run both methods and compare.

**Straight-line** removes a flat R20 000 (20 % of the *original*) each year:

$$A = 100\,000(1 - 0.60) = \text{R}40\,000$$

**Reducing-balance** removes 20 % of the *current* value — R20 000, then R16 000, then R12 800:

$$A = 100\,000(0.8)^3 = \text{R}51\,200$$

Reducing balance leaves **more**, because its deductions shrink.

*Why the others are wrong:*

- **B** has both numbers right but **swapped** — the classic compare-question error. Anchor it: shrinking deductions must leave the higher value.
- **A** gives both methods the straight-line answer, denying the very difference being tested.
- **C** says they always agree — true only after one year.

> **Takeaway:** At the same rate, straight-line reaches zero in finite time; reducing balance decays forever without quite getting there. If your numbers are right but your letter is wrong, you fell for the swap.

---

**Q14 — Intermediate — Answer: C**

**Topic:** Straight-line depreciation — find rate

A reverse problem. Find the **loss**, spread it evenly over the years, then express one year's loss as a percentage of the original.

**Total loss:** $150\,000 - 60\,000 = \text{R}90\,000$

**Per year:** $90\,000 \div 6 = \text{R}15\,000$

**As a rate:** $\dfrac{15\,000}{150\,000} = 10\%$

Algebraically: $60\,000 = 150\,000(1 - 6r) \Rightarrow r = 0.10$

*Why the others are wrong:*

- **D** (12 %) divided the 60 % loss by **5** years instead of 6.
- **A** (6 %) divided the *remaining* 40 % instead of the amount **lost**.
- **B** (8 %) fails the check: 8 % for 6 years leaves R78 000, not R60 000.

> **Takeaway:** $r = (1 - A/P)/t$ — or in words, "total loss ÷ years ÷ original price", which is much harder to mangle under pressure.

---

### Q15 — Intermediate — Answer: B

**Topic:** Reducing-balance depreciation — find rate

Reducing balance in reverse. Two years means the original was multiplied by $(1 - i)^2$.

**Step 1 — Isolate the factor.** $(1 - i)^2 = \dfrac{64\,000}{100\,000} = 0.64$

**Step 2 — Take the square root.** $1 - i = 0.8$

**Step 3 — Read the rate.** $i = 0.20 = 20\%$

Check: 100 000 → 80 000 → 64 000 ✓

*Why the others are wrong:*

- **D** (36 %) is the **total** two-year loss, not the annual rate — true information answering the wrong question.
- **C** (18 %) halves that total. Averaging works for straight-line, but a compound loss is bigger in year one than year two.
- **A** (16 %) fails the check: $0.84^2 = 0.7056$ leaves R70 560.

> **Takeaway:** Isolate $(1 - i)^n$, take the $n$th root, subtract from 1. Never annualise a compound loss by dividing — that shortcut belongs to straight-line only.

---

## Section 7.4 — Hire Purchase

### Q16 — Basic — Answer: A

**Topic:** Hire purchase — calculate monthly payment

Hire purchase always follows four beats: **deposit → balance → interest on that balance → divide by the number of payments.** Note 24 monthly payments = 2 years of interest.

**Step 1 — Deposit and balance.**

$$\text{Deposit} = 10\% \times 6\,000 = \text{R}600 \qquad \text{Balance} = \text{R}5\,400$$

**Step 2 — Interest** (simple, 2 years).

$$I = 5\,400 \times 0.10 \times 2 = \text{R}1\,080$$

**Step 3 — Total, then monthly.**

$$\frac{5\,400 + 1\,080}{24} = \frac{6\,480}{24} = \text{R}270$$

*Why the others are wrong:*

- **C** (R225) is $5\,400 \div 24$ — the interest forgotten entirely.
- **B** (R300) never subtracted the deposit, charging on the full R6 000.
- **D** (R312) implies R7 488 repaid — over R2 000 of interest on R5 400. Nothing in the question gives that.

> **Takeaway:** HP interest is simple interest on the **opening balance**, for the **full term**. It does not shrink as you pay the debt down — which is exactly why HP is expensive.

---

### Q17 — Intermediate — Answer: D

**Topic:** Hire purchase — total cost

The instalment is given, so there is no interest to work out. The total is simply **deposit + all instalments**. This tests bookkeeping: find both parts and drop neither.

**Deposit:** $0.15 \times 5\,000 = \text{R}750$

**Instalments:** $18 \times 280 = \text{R}5\,040$

**Total:** R5 790 — that is R790 above the cash price, which is the finance charge.

*Why the others are wrong:*

- **A** (R5 040) is the instalments alone — the **deposit vanished**. The most common HP error.
- **B** (R5 400) and **C** (R5 600) are plausible-looking numbers between the cash price and the true cost, but neither equals deposit + instalments.

> **Takeaway:** Total HP cost = deposit + (payments × payment amount). Write both components down before adding — the wrong options sit exactly where a skipped component lands you.

---

#### Q18 — Intermediate — Answer: C

**Topic:** Hire purchase — implied interest rate

Reverse-engineer the rate hiding inside the payment plan.

**Paid in total:** $12 \times 400 = \text{R}4\,800$

**Interest:** $4\,800 - 4\,000 = \text{R}800$

**As an annual rate** (the term is 1 year):

$$r = \frac{800}{4\,000} = 20\%$$

*Why the others are wrong:*

- **A** (16 %) divided by the total **paid** instead of the amount **financed**. Rates are quoted on the principal.
- **D** (12 %) echoes the "12" in the question — a surface-number trap.
- **B** (24 %) would mean R960 of interest, but only R800 was charged.

> **Takeaway:** Total paid – price = interest, then $r = \frac{I}{Pt}$. An innocent "R400 a month" hides a 20 % annual rate — the consumer lesson as much as the maths one.

---

#### Q19 — Intermediate — Answer: B

**Topic:** Hire purchase — extra cost vs cash

Four lines — write every one down. The options are placed to catch anyone who skips a step.

1. **Deposit:** $0.25 \times 12\,000 = \text{R}3\,000$
2. **Instalments:** $36 \times 350 = \text{R}12\,600$

3. **Total HP:** $\text{R}15\,600$
4. **Extra over cash:** $15\,600 - 12\,000 = \text{R}3\,600$

*Why the others are wrong:*

- **C** (R4 200) is $350 \times 12$ — one **year** of instalments, answering a different question.
- **A** (R2 600) and **D** (R1 200) are what broken chains produce. Neither matches any clean method.

> **Takeaway:** Extra cost = (deposit + all instalments) – cash price. Four short lines on paper beat holding the chain in your head.

---

### Q20 — Basic — Answer: A

**Topic:** Loan repayment — simple interest

A loan is an investment with the roles reversed — the debt is what grows. Simple interest, so it grows by the same amount each year.

**Per year:** 8 % of R20 000 = R1 600. Three years adds R4 800, so you owe **R24 800**.

**By formula:** $A = 20\,000(1 + 0.24) = \text{R}24\,800$

*Why the others are wrong:*

- **C** (R24 000) used a multiplier of 1.20 — rounding $0.08 \times 3$ to "about 20 %". It is exactly 24 %.
- **D** (R25 250) is compound thinking; the question says **simple**.
- **B** (R25 000) is round-number bait matching no rate in the question.

> **Takeaway:** $A = P(1 + rt)$. When the options cluster close together, exact arithmetic — not estimation — is the tiebreaker.

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## Section 7.5 — Inflation and Real Value

### Q21 — Intermediate — Answer: D

**Topic:** Future cost under inflation

Inflation compounds exactly like interest — next year's increase is calculated on this year's *already-raised* price.

**Year by year:** R5 000 rises 20 % to R6 000; then R6 000 rises 20 % (that's R1 200 now, not R1 000) to **R7 200**.

**Formula:**

$$A = 5\,000 \times (1.2)^2 = 5\,000 \times 1.44 = \text{R}7\,200$$

*Why the others are wrong:*

- **B** (R6 000) applies only **one** year of inflation — it stops halfway.
- **A** (R7 000) is *simple*-growth thinking: $20\% \times 2 = 40\%$, so $5\,000 \times 1.4$. It misses the R200 that comes from year two's increase acting on year one's increase.
- **C** (R6 200) mangles the square: $(1.2)^2 = 1 + 2(0.2) + 0.04 = 1.44$, but dropping one of the two cross-terms gives $1.24$. Write out $1.2 \times 1.2$ instead of expanding in your head.

> **Takeaway:** Future cost = $P(1+\pi)^n$ — inflation is compound growth applied to prices. "Two years at 20 %" means ×1.44, never ×1.40.

---

### Q22 — Intermediate — Answer: C

**Topic:** Budget for a future purchase

Three years of 10 % inflation means three multiplications by 1.1 — the same ladder as compound interest:

$$A = 3\,000 \times (1.1)^3 = 3\,000 \times 1.331 = \text{R}3\,993$$

Year-by-year check: R3 000 → R3 300 → R3 630 → R3 993 ✓.

*Why the others are wrong:*

- **A** (R3 300) budgets for **one** year of inflation, not three.
- **B** (R3 900) is the simple-growth answer ($10\% \times 3 = 30\%$, so $3\,000 \times 1.3$) — it ignores that each year's increase compounds on the last.
- **D** (R4 000) is a clean round number that corresponds to nothing: it would require total inflation of $33.3\%$, and $1.1^3$ is exactly 1.331. Tidy-looking options with no derivation behind them are decoys for estimators.

> **Takeaway:** $1.1^3 = 1.331$ — the camera costs R993 more in three years. Budgeting questions are compound-growth questions wearing a shopping bag.

---

#### Q23 — Basic — Answer: B

**Topic:** Approximate real return

The **real return** asks: after prices have risen, how much more can your money actually *buy*? The working approximation is a subtraction:

$$\text{Real return} \approx \text{Nominal} - \text{Inflation} = 12\% - 8\% = 4\% \text{ p.a.}$$

Your balance grows by 12 %, but everything you might buy costs 8 % more — so your buying power gains only about 4 %.

*Why the others are wrong:*

- **A** (20 %) **adds** the rates. Inflation works against your return, not alongside it — adding makes no financial sense.
- **C** (1.5 %) **divides** them ($12 \div 8$). The ratio of the two rates measures nothing meaningful here.
- **D** (8 %) just echoes the inflation figure back at you — the surface-number reflex again.

> **Takeaway:** Real ≈ nominal – inflation, and that approximation is always acceptable on the NBT. (The precise Fisher equation gives $(1.12/1.08) - 1 \approx 3.7\%$ — close to 4 %, and the exam never demands that precision.)

---

#### Q24 — Basic — Answer: A

**Topic:** Real value of savings

Same subtraction as Q23, but the result is **negative**:

$$\text{Real return} \approx 8\% - 10\% = -2\%$$

The balance still grows in rands, but prices rise faster — so those rands buy about 2 % less each year.

*Why the others are wrong:*

- **C** (+2 %) subtracted **backwards**. The order is your rate minus inflation, and the sign is the whole answer.
- **B** (18 %) added the rates, which would mean high inflation *helping* your savings.
- **D** is the "money illusion" — judging wealth by the number on the balance. Inflation does not touch that number; it eats what the number can **buy**.

> **Takeaway:** Inflation above your interest rate means a **negative** real return. The NBT tests this as a concept — know which way the subtraction goes.

---

### Q25 — Basic — Answer: C

**Topic:** Inflation — one-year price change

One year, one increase — multiply once by $(1 + \pi)$:

$$A = 500 \times 1.06 = \text{R}530$$

(6 % of R500 is R30, because 1 % of R500 is R5.)

*Why the others are wrong:*

- **A** (R506) adds six *rand* instead of six *per cent* — R6 is 6 % of R100, not of R500. Percent-versus-rand confusion is the oldest trap in inflation questions.
- **D** (R550) adds 10 % — the round-number reflex overriding the actual rate in the question.
- **B** (R560) adds 12 %, double the true rate — the footprint of applying 6 % twice or slipping a decimal.

> **Takeaway:** For one year, $A = P(1 + \pi)$. Convert the percentage properly — 6 % of R500 = R30 — so the budget must rise to R530. (That is an extra R360 a year, which is why food inflation hurts.)

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## Section 7.6 — Exchange Rates

### Q26 — Basic — Answer: D

**Topic:** Convert ZAR to USD

The rate "R18 = \$1" says each dollar *costs* R18. To find how many dollars R9 000 buys, ask: how many R18-chunks fit into R9 000?

$$\text{USD} = \frac{9\,000}{18} = \$500$$

**Sense-check first, always:** a dollar is worth many rand, so converting rand *into* dollars must give a much **smaller** number. Expecting "smaller" before you calculate makes the direction error impossible.

*Why the others are wrong:*

- **B** (\$162 000) multiplies instead of divides — the direction error the sense check exists to catch; R9 000 is nowhere near that many dollars.
- **A** (\$50) divides by 180 — a dropped decimal or extra zero turning R18 into R180 per dollar.
- **C** (\$450) divides by a "rounder" 20 instead of 18 — an estimation that never got upgraded back to exact arithmetic.

> **Takeaway:** ZAR → USD: divide by the rand-per-dollar rate. Decide *before* computing whether the answer should be bigger or smaller than what you started with — that one habit eliminates half the options instantly.

---

### Q27 — Basic — Answer: B

**Topic:** Convert USD to ZAR

Each dollar is worth R17.50, so 200 of them are worth:

$$\text{ZAR} = 200 \times 17.50 = \text{R}3\,500$$

No-calculator route: $200 \times 17.5 = 200 \times 17 + 200 \times 0.5 = 3\,400 + 100$. And the sense check: dollars into rand must give a **bigger** number — R3 500 for \$200 fits.

*Why the others are wrong:*

- **A** (R3 000) multiplies by 15, and **C** (R4 000) multiplies by 20 — both are the exchange rate rounded to a "nicer" number. The NBT picks rates like 17.50 precisely because they are exactly computable; rounding is choosing to get the question wrong.
- **D** (R2 857) implies a rate of $2\,857/200 \approx \text{R}14.29$ per dollar — a number that appears nowhere in the question, and one that would make \$200 worth *less* than the quoted rate allows.

> **Takeaway:** USD → ZAR: multiply by the rand-per-dollar rate. Split awkward multiplications (×17.5 = ×17 + ×0.5) instead of rounding them away.

---

#### Q28 — Intermediate — Answer: A

**Topic:** Exchange rate with commission

Two charges: the currency, then the bank's fee on top.

1. **Currency:** $500 \times 16 = \text{R}8\,000$
2. **Commission:** $2\% \times 8\,000 = \text{R}160$
3. **Total:** $\text{R}8\,160$

Which way does the fee go? Ask *who is paying whom*. You are buying a service, so the fee **adds**.

*Why the others are wrong:*

- **B** (R8 000) forgot the commission entirely.
- **C** (R7 840) **subtracted** it, as if the bank paid you for the privilege.
- **D** (R8 320) charged it twice.

> **Takeaway:** Fees always move the total **against** you. If your adjusted answer is better than the raw conversion, you pointed the fee the wrong way.

---

#### Q29 — Intermediate — Answer: C

**Topic:** Profit from exchange rate movement

Track the two rand amounts.

**Bought:** \$1 000 at R15 cost R15 000. **Sold:** \$1 000 at R18 brings in R18 000.

$$\text{Profit} = \text{R}3\,000$$

The intuition: you held dollars while each dollar gained three rand. Rate up (weaker rand) is **good** for the dollar-holder.

*Why the others are wrong:*

- **D** (R3 000 loss) has the right size, wrong sign — the "rand weakened so I lost" reflex. That weakening is exactly what made your dollars worth more.
- **A** (R1 000 loss) borrows the \$1 000 from the question and adds a sign error.
- **B** (R1 500) looks like a 10 % return, but no 10 % exists here. The gain is R3 per dollar on 1 000 dollars.

> **Takeaway:** Profit = (sell rate – buy rate) × amount held. Decide **who benefits** before doing the arithmetic — that locks in the sign.

---

#### Q30 — Basic — Answer: D

**Topic:** Finding the best exchange rate

The quoted rate is the **price of one euro**. As the buyer, you want the lowest price — same as buying anything else.

Spend R1 000 at each bank:

- Bank B (R19.50) → €51.3
- Bank A (R20.00) → €50.0
- Bank C (R20.50) → €48.8

Bank B gives the most euros.

*Why the others are wrong:*

- **B** (Bank C) picks the **highest** rate. That is right when *selling* euros, backwards when buying.
- **A** (Bank A) picks the roundest number, not the best one.
- **C** says it makes no difference — the R1 000 test above disproves that.

> **Takeaway:** Buying foreign currency, lowest rate wins. Selling, highest wins. If you blank, run R1 000 through two banks and compare.

---

## Section 7.7 — Time Value of Money

### Q31 — Proficient — Answer: B

**Topic:** Future value of an annuity

**Separate deposits, each growing for a different length of time.** Deposits land at the *end* of each year, so the first grows 2 years and the last grows 0.

| Payment | Years | Value at year 3 |
|---|---|---|
| Year 1 | 2 | $1000(1.1)^2 = 1210$ |
| Year 2 | 1 | $1000(1.1) = 1100$ |
| Year 3 | 0 | $1000$ |
| **Total** | | **R3 310** |

*Why the others are wrong:*

- **A** (R3 000) has no interest at all.
- **C** (R3 300) applies 10 % once to the whole R3 000, as if every deposit arrived on day one.
- **D** (R3 460) fits no timing. The only defensible answers are R3 310 (end-of-year) and R3 641 (start-of-year).

> **Takeaway:** Build the table — one row per deposit, each with its own growth period. The last end-of-year payment earns **nothing**, and that is what examiners test.

---

### Q32 — Intermediate — Answer: C

**Topic:** Loan repayment — equal annual instalments

Two stages: first find the **total debt** (principal plus all the simple interest), then share it equally across the payments.

**Step 1 — total owed after 4 years.**

$$A = 50\,000(1 + 0.20 \times 4) = 50\,000 \times 1.8 = \text{R}90\,000$$

**Step 2 — split into 4 equal instalments.**

$$\frac{90\,000}{4} = \text{R}22\,500$$

*Why the others are wrong:* the wrong options are a fence-post-error museum —

- **A** (R12 500) divides the bare R50 000 by 4: the **interest never happened**.
- **B** (R20 000) charges interest for only **3** years ($50\,000 \times 1.6 = 80\,000$, then ÷ 4) — an off-by-one in the year count.
- **D** (R25 000) charges interest for **5** years ($50\,000 \times 2.0 = 100\,000$, then ÷ 4) — the same fence-post error leaning the other way.

> **Takeaway:** Total first, divide second. The wrong options differ *only* in how many years of interest were charged — which tells you exactly what the examiner expects students to fumble. Count the years deliberately.

---

#### Q33 — Basic — Answer: A

**Topic:** Total interest on an annuity-style savings plan

However complicated a savings plan looks, total interest is always the same one-liner: **what you ended with minus what you put in**.

**Step 1 — total contributed.** $24 \times 500 = \text{R}12\,000$. (It is stated in the question, but verify it anyway — mis-totalling the deposits is where this question is actually lost.)

**Step 2 — subtract.**

$$\text{Interest} = 13\,200 - 12\,000 = \text{R}1\,200$$

*Why the others are wrong:*

- **B** (R1 300), **C** (R800) and **D** (R1 800) are all neighbourhood decoys — none corresponds to any correct pairing of "final amount" and "total deposited". They punish two habits: subtracting in your head, and not writing $24 \times 500 = 12\,000$ down before subtracting. Do both steps on paper and the wrong options have nothing to grab.

> **Takeaway:** Total interest = accumulated amount – total contributions, whatever the deposit pattern. The formula never changes; only the bookkeeping does.

---

#### Q34 — Intermediate — Answer: D

**Topic:** Total interest on a lump-sum loan

**Per-year view:** 12 % of R100 000 is R12 000 of interest per year. Five years:

$$I = Prt = 100\,000 \times 0.12 \times 5 = \text{R}60\,000$$

(Equivalently: the total owed is $100\,000 \times 1.6 = \text{R}160\,000$, of which R60 000 is interest.)

*Why the others are wrong:*

- **A** (R12 000) is **one** year's interest — the per-year figure presented as the total.
- **B** (R48 000) is **four** years' interest ($12\,000 \times 4$) — the off-by-one fence-post again.
- **C** (R50 000) is "half the loan" — a round number that corresponds to a 10 % rate, not the 12 % in the question.

> **Takeaway:** $I = Prt$. Anchor yourself with the per-year amount (R12 000) and the wrong options unmask themselves as 1 year, 4 years, and "10 %-flavoured" impostors of the 5-year total.

---

### Q35 — Proficient — Answer: B

**Topic:** Present value — how much to invest now

**Present value** runs compound interest backwards: what must you plant **today** to grow into R14 641? Growth multiplies by $(1+i)^n$, so un-growing divides:

$$P = \frac{14\,641}{(1.1)^4} = \frac{14\,641}{1.4641} = \text{R}10\,000$$

The numbers are friendly by design — the Pascal digits 1, 4, 6, 4, 1 again.

*Why the others are wrong:*

- **A** (R11 000) divided by $1.1^3$ — one year short.
- **C** (R12 100) divided by $1.1^2$ — two years short.
- **D** (R9 900) is estimator's bait. Even the nearest principled error (multiplying by $0.9^4$) gives R9 606, not R9 900 — dividing by 1.1 and multiplying by 0.9 are different operations.

> **Takeaway:** PV = future amount ÷ $(1+i)^n$, and check forwards: $10\,000 \times 1.4641 = 14\,641$ ✓. Un-growing is **division** by the growth factor, never subtraction of the percentage.

## Mixed Practice Questions — M1 to M30

### M1 — Basic

**Topic:** Simplifying expressions

Simplify $\dfrac{6x^2y}{2xy^2}$.

A) $\dfrac{3x}{y}$

B) $3xy$

C) $\dfrac{3y}{x}$

D) $3x^2y$

---

### M2 — Basic

**Topic:** Number sense — scientific notation

Write 0.00045 in scientific notation.

A) $4.5 \times 10^3$

B) $4.5 \times 10^{-3}$

C) $4.5 \times 10^{-4}$

D) $45 \times 10^{-5}$

---

### M3 — Basic

**Topic:** Functions — domain

What is the domain of $f(x) = \dfrac{1}{x-2}$?

A) $x \geq 2$

B) $x \neq 2$

C) $x > 2$

D) All real numbers

---

### M4 — Intermediate

**Topic:** Trigonometry — exact value

$\cos 60^\circ =$

A) $\frac{\sqrt{3}}{2}$

B) $\frac{\sqrt{2}}{2}$

C) $\frac{1}{2}$

D) $1$

---

### M5 — Basic

**Topic:** Algebra — solving a linear equation

Solve $3x - 7 = 14$.

A) $7$

B) $\frac{7}{3}$

C) $21$

D) $\frac{21}{3}$

---

### M6 — Basic

**Topic:** Geometry — exterior angle

An exterior angle of a triangle equals:

A) The sum of the interior angles

B) 90°

C) The sum of the two non-adjacent interior angles

D) The adjacent interior angle

---

### M7 — Intermediate

**Topic:** Data — IQR

A data set has $Q_1 = 14$ and $Q_3 = 26$. The IQR is:

A) 40

B) 20

C) 12

D) 26

---

### M8 — Intermediate

**Topic:** Compound interest

R5 000 is invested at 20 % p.a. compound interest for 3 years. The accumulated value is:

A) R8 000

B) R8 500

C) R8 600

D) R8 640

---

### M9 — Intermediate

**Topic:** Algebra — quadratic formula

Using the quadratic formula, solve $x^2 - 6x + 8 = 0$.

A) $x = 2$ or $x = 4$

B) $x = -2$ or $x = -4$

C) $x = 1$ or $x = 8$

D) $x = 3$ only

---

### M10 — Basic

**Topic:** Number sense — ratio

If $x : y = 2 : 5$ and $y = 35$, find $x$.

A) 7

B) 17.5

C) 14

D) 70

---

### M11 — Basic

**Topic:** Functions — identifying a parabola

The graph of $f(x) = 2x^2 - 3$ has $y$-intercept at:

A) $(0,\ 2)$

B) $(0,\ -3)$

C) $(3,\ 0)$

D) $(-3,\ 0)$

---

### M12 — Intermediate

**Topic:** Trigonometry — sine rule

In $\triangle ABC$: $\angle A = 30^\circ$, $\angle B = 45^\circ$, $a = 6$. Find $b$.

A) $3\sqrt{2}$

B) $4\sqrt{2}$

C) $6\sqrt{2}$

D) 6

---

### M13 — Basic

**Topic:** Data — probability

A bag has 4 red, 3 blue, and 5 green balls. $P(\text{green}) =$

A) $\frac{5}{7}$

B) $\frac{5}{9}$

C) $\frac{1}{5}$

D) $\frac{5}{12}$

---

### M14 — Intermediate

**Topic:** Hire purchase

An item costs R6 000 cash. HP: 20 % deposit, 12 monthly payments at 15 % p.a. simple interest. Monthly payment?

A) R400

B) R460

C) R575

D) R500

---

### M15 — Basic

**Topic:** Algebra — factorising a trinomial

Factorise $x^2 + 7x + 12$.

A) $(x + 3)(x + 4)$

B) $(x + 2)(x + 6)$

C) $(x + 1)(x + 12)$

D) $(x - 3)(x - 4)$

---

### M16 — Intermediate

**Topic:** Functions — exponential graph

The graph of $y = 2^x$ passes through which of the following points?

A) $(-1,\ 0)$

B) $(0,\ 0)$

C) $(3,\ 9)$

D) $(3,\ 8)$

---

### M17 — Intermediate

**Topic:** Straight-line depreciation

A vehicle bought for R320 000 depreciates at 10 % p.a. straight-line. Value after 4 years?

A) R192 000

B) R200 000

C) R196 000

D) R180 000

---

### M18 — Basic

**Topic:** Geometry — area of a circle

A circle has radius 7 cm. Its area is:

A) $14\pi\ \mathrm{cm}^2$

B) $49\pi\ \mathrm{cm}^2$

C) $7\pi\ \mathrm{cm}^2$

D) $28\pi\ \mathrm{cm}^2$

---

### M19 — Intermediate

**Topic:** Data — union of events

$P(A) = 0.5$, $P(B) = 0.4$, $P(A \cap B) = 0.15$. Find $P(A \cup B)$.

A) 0.75

B) 0.9

C) 0.65

D) 0.20

---

### M20 — Intermediate

**Topic:** Compound interest — find P

An investment compounds at 10 % p.a. After 2 years the balance is R12 100. What was the original principal?

A) R11 000

B) R10 500

C) R9 900

D) R10 000

---

### M21 — Basic

**Topic:** Algebra — laws of exponents

Simplify $\frac{2^5 \times 2^3}{2^6}$.

A) 4

B) 8

C) 2

D) 16

---

### M22 — Intermediate

**Topic:** Exchange rates

R14 = \$1. You have R7 000. How many US dollars can you buy (ignoring commission)?

A) \$98 000

B) \$700

C) \$500

D) \$7.14

---

### M23 — Intermediate

**Topic:** Functions — completing the square

Write $x^2 - 4x + 7$ in the form $(x - a)^2 + b$.

A) $(x - 4)^2 + 7$

B) $(x - 2)^2 + 11$

C) $(x - 2)^2 + 3$

D) $(x + 2)^2 + 3$

---

### M24 — Intermediate

**Topic:** Geometry — Pythagoras

A ladder 13 m long leans against a wall. The base is 5 m from the wall. How high up the wall does the ladder reach?

A) 8 m

B) 14 m

C) 10 m

D) 12 m

---

### M25 — Intermediate

**Topic:** Simple interest — find rate

R6 000 grows to R7 800 in 2 years under simple interest. The annual interest rate is:

A) 15 % p.a.

B) 18 % p.a.

C) 12 % p.a.

D) 20 % p.a.

---

### M26 — Intermediate

**Topic:** Inflation

A house costs R500 000 today. If property prices inflate at 10 % p.a. for 3 years, its future price will be:

A) R650 000

B) R550 000

C) R665 500

D) R660 000

---

### M27 — Basic

**Topic:** Algebra — simultaneous equations

Solve $x + y = 10$ and $x - y = 4$.

A) $x = 8,\ y = 2$

B) $x = 6,\ y = 4$

C) $x = 7,\ y = 3$

D) $x = 4,\ y = 6$

---

### M28 — Intermediate

**Topic:** Reducing-balance depreciation

Equipment costing R81 000 depreciates at $\frac{1}{3}$ of its value each year (reducing balance). Value after 2 years?

A) R27 000

B) R54 000

C) R36 000

D) R40 500

---

### M29 — Proficient

**Topic:** Compound interest — present value

How much must be invested now at 10 % p.a. compound to have R12 100 in 2 years?

A) R10 000

B) R11 000

C) R9 900

D) R10 500

---

### M30 — Intermediate

**Topic:** Data — mean and outlier

A data set $\{6, 8, 10, 12, 100\}$ has mean 27.2. After removing 100, the new mean is:

A) 12

B) 8

C) 9

D) 36

## Mixed Practice — Answer Key

| | | | | |
|---|---|---|---|---|
| **M1.** A | **M2.** C | **M3.** B | **M4.** C | **M5.** A |
| **M6.** C | **M7.** C | **M8.** D | **M9.** A | **M10.** C |
| **M11.** B | **M12.** C | **M13.** D | **M14.** B | **M15.** A |
| **M16.** D | **M17.** A | **M18.** B | **M19.** A | **M20.** D |
| **M21.** A | **M22.** C | **M23.** C | **M24.** D | **M25.** A |
| **M26.** C | **M27.** C | **M28.** C | **M29.** A | **M30.** C |

# Chapter 8 — Sequences and Series

## A Note on Curriculum Placement

Sequences and series appear in two NBT competence areas at once. **Arithmetic and geometric sequences** are classified under **Algebraic Processes** — the NBT tests whether students can manipulate the general term $T_n = a + (n-1)d$ and $T_n = ar^{n-1}$ algebraically, not just substitute into them. **Series and sigma notation** cross into **Number Sense** because summing a sequence is fundamentally about recognising a numerical pattern and applying a formula.

Both topics are severely underrepresented in the chapters covering those competence areas. Sequences were touched on briefly in Chapter 2 (Number Sense) as arithmetic patterns, and not at all in Chapter 1 (Algebraic Processes). This chapter exists to give sequences and series the depth the NBT expects.

The most common NBT errors in this topic: 1. Confusing the $n$th term formula with the sum formula. 2. Using $S_n$ when the question asks for $T_n$, or vice versa. 3. Forgetting that an infinite geometric series only converges when $|r| < 1$. 4. Incorrectly applying sigma notation (wrong limits, wrong formula substituted).

**Formulas you must know:**

| | $n$**th term** | **Sum of first $n$ terms** |
|---|---|---|
| **Arithmetic** | $T_n = a + (n-1)d$ | $S_n = \frac{n}{2}(2a + (n-1)d) = \frac{n}{2}(a + l)$ |
| **Geometric** | $T_n = ar^{n-1}$ | $S_n = \frac{a(r^n - 1)}{r - 1}$ or $\frac{a(1 - r^n)}{1 - r}$ |
| **Infinite GP** | — | $S_\infty = \frac{a}{1-r}$, only if $\lvert r \rvert < 1$ |

## Section 8.1 — Arithmetic Sequences: Finding Terms

In an arithmetic sequence the terms increase (or decrease) by a fixed amount called the **common difference** $d$. Every term can be expressed as the first term plus a multiple of $d$: $T_n = a + (n-1)d$. This is a linear function of $n$, so the graph of $T_n$ against $n$ is a straight line with gradient $d$.

### Q1 — Basic

Find the common difference of the sequence $7,\ 11,\ 15,\ 19,\ \ldots$

A) 4

B) 3

C) 5

D) 6

---

### Q2 — Basic

An AP has first term $a = 3$ and common difference $d = 5$. Which formula gives the general term?

A) $T_n = 5n + 3$

B) $T_n = 5n$

C) $T_n = 5n - 2$

D) $T_n = 3n + 5$

---

### Q3 — Basic

Find $T_{15}$ of the AP $2,\ 5,\ 8,\ 11,\ \ldots$

A) 42

B) 44

C) 47

D) 41

---

### Q4 — Intermediate

Which term of the AP $1,\ 4,\ 7,\ 10,\ \ldots$ equals 100?

A) The 32nd term

B) The 33rd term

C) The 35th term

D) The 34th term

---

### Q5 — Intermediate

The terms $k$, $2k+1$, and $4k-1$ are three consecutive terms of an arithmetic sequence. Find $k$.

A) 3

B) 2

C) 4

D) $-3$

---

## Section 8.2 — Arithmetic Sequences: More Problems

### Q6 — Intermediate

In an AP, $T_3 = 11$ and $T_7 = 27$. Find the common difference.

A) 2

B) 3

C) 4

D) 5

---

### Q7 — Intermediate

In an AP with $T_5 = 23$ and $d = 4$, find the first term.

A) 5

B) 7

C) 9

D) 3

---

**Q8 — Intermediate**

Three arithmetic means are inserted between 2 and 18. What are the three inserted values?

A) 6, 10, 14

B) 4, 8, 16

C) 5, 10, 15

D) 7, 11, 15

---

**Q9 — Intermediate**

A theatre has 20 seats in row 1. Each subsequent row has 3 more seats than the row before. How many seats are in row 15?

A) 55

B) 60

C) 65

D) 62

---

**Q10 — Basic**

Find the 40th term of the AP $5,\ 8,\ 11,\ 14,\ \ldots$

A) 120

B) 117

C) 122

D) 125

---

## Section 8.3 — Geometric Sequences

In a geometric sequence every term is obtained by multiplying the previous term by a fixed **common ratio** $r$. The $n$th term is $T_n = ar^{n-1}$. If $|r| > 1$, the terms grow in magnitude. If $|r| < 1$, they shrink towards zero. If $r < 0$, the terms alternate in sign.

---

### Q11 — Basic

Find the common ratio of the GP $3,\ 12,\ 48,\ 192,\ \ldots$

A) 3

B) 4

C) 9

D) 6

---

### Q12 — Basic

A GP has first term $a = 2$ and common ratio $r = 3$. Find $T_5$.

A) 162

B) 486

C) 54

D) 96

---

### Q13 — Intermediate

The GP $5,\ 10,\ 20,\ 40,\ \ldots$ Which term equals 640?

A) 6th

B) 7th

C) 9th

D) 8th

---

### Q14 — Intermediate

A GP has terms $4,\ -12,\ 36,\ -108,\ \ldots$ Find $T_5$.

A) $-324$

B) $216$

C) $324$

D) $-108$

---

### Q15 — Basic

A colony starts with 500 bacteria. The population doubles every hour. How many bacteria are there after 4 hours?

A) 4 000

B) 8 000

C) 16 000

D) 2 000

---

## Section 8.4 — Arithmetic Series

The **sum of the first $n$ terms** of an arithmetic sequence:

$$S_n = \frac{n}{2}(2a + (n-1)d) = \frac{n}{2}(a + l)$$

where $l = T_n$ is the last term. The second form is Gauss's pairing trick: pair the first and last term, second and second-last, etc. — each pair sums to $a + l$.

---

### Q16 — Basic

Find $S_{10}$ of the AP $1,\ 3,\ 5,\ 7,\ \ldots$

A) 100

B) 90

C) 110

D) 55

---

### Q17 — Intermediate

The sum of the first $n$ positive integers is 55. Find $n$.

A) 9

B) 8

C) 11

D) 10

---

### Q18 — Intermediate

Find $S_8$ of the AP $4,\ 9,\ 14,\ 19,\ \ldots$

A) 160

B) 164

C) 172

D) 176

---

### Q19 — Intermediate

An AP has $T_1 = 2$ and $T_{16} = 62$. Find $S_{16}$.

A) 480

B) 512

C) 496

D) 528

---

### Q20 — Proficient

A wall is built from rows of bricks. Row 1 (top) has 30 bricks; each lower row has 2 fewer bricks. There are 10 rows. How many bricks in total?

A) 210

B) 200

C) 220

D) 195

---

## Section 8.5 — Geometric Series

The sum of the first $n$ terms of a geometric sequence:

$$S_n = \frac{a(r^n - 1)}{r - 1} \qquad (r \neq 1)$$

Use this form when $r > 1$. When $r < 1$, the equivalent form $\dfrac{a(1 - r^n)}{1 - r}$ avoids negative signs.

---

### Q21 — Basic

Find $S_6$ of the GP $1,\ 2,\ 4,\ 8,\ 16,\ \ldots$

A) 32

B) 64

C) 127

D) 63

---

### Q22 — Intermediate

A GP has $a = 2$ and $r = 3$. Find $S_5$.

A) 240

B) 486

C) 242

D) 244

---

### Q23 — Intermediate

The GP $1,\ 2,\ 4,\ 8,\ \ldots$ has sum $S_n = 127$. Find $n$.

A) 6

B) 7

C) 8

D) 5

---

### Q24 — Proficient

R100 is deposited at the end of each year for 3 years at 10 % p.a. compound interest. What is the total accumulated value?

A) R331

B) R330

C) R300

D) R333

---

### Q25 — Intermediate

A GP has common ratio $r = 2$ and $S_4 = 120$. Find the first term.

A) 4

B) 12

C) 8

D) 16

---

## Section 8.6 — Sigma Notation

Sigma notation $\sum_{k=m}^{n} f(k)$ is a compact way to write a series. The variable $k$ is the **index**, $m$ is the lower limit, and $n$ is the upper limit. Expand the notation by substituting $k = m, m+1, \ldots, n$ and summing.

---

### Q26 — Basic

Evaluate $\sum_{k=1}^{5} k$.

A) 10

B) 12

C) 20

D) 15

---

### Q27 — Basic

Evaluate $\sum_{k=1}^{4} (2k+1)$.

A) 20

B) 24

C) 22

D) 28

---

### Q28 — Intermediate

The series $\sum_{k=1}^{n}(3k-1)$ represents:

A) An AP with first term 2 and common difference 3

B) A GP with first term 2 and common ratio 3

C) An AP with first term 3 and common difference 1

D) An AP with first term 1 and common difference 3

---

### Q29 — Intermediate

Evaluate $\sum_{k=1}^{4} 2 \cdot 3^{k-1}$.

A) 81

B) 78

C) 80

D) 84

---

### Q30 — Intermediate

Evaluate $\sum_{k=3}^{7} 2k$.

A) 40

B) 45

C) 42

D) 50

---

## Section 8.7 — Infinite Geometric Series

When $|r| < 1$, the terms of a GP shrink towards zero, so the infinite sum converges to a finite value:

$$S_\infty = \frac{a}{1-r} \quad \text{only when } |r| < 1$$

If $|r| \geq 1$, the sum diverges (grows without bound) and $S_\infty$ does not exist.

---

### Q31 — Basic

For which value(s) of $r$ does an infinite geometric series converge?

A) $r > 0$

B) $|r| < 1$

C) $r < 1$

D) $-1 < r < 0$ only

---

### Q32 — Basic

Find $S_\infty$ of the GP $12,\ 6,\ 3,\ \frac{3}{2},\ \ldots$

A) 12

B) 18

C) 24

D) 36

---

### Q33 — Intermediate

An infinite GP has $S_\infty = 20$ and $r = \frac{1}{4}$. Find the first term.

A) 15

B) 80

C) 5

D) 10

---

### Q34 — Proficient

Express the recurring decimal $0.\overline{3} = 0.333\ldots$ as a fraction.

A) $\frac{3}{10}$

B) $\frac{1}{4}$

C) $\frac{3}{11}$

D) $\frac{1}{3}$

---

### Q35 — Intermediate

An infinite GP has first term 8 and $S_\infty = 24$. Find the common ratio.

A) $\frac{1}{3}$
B) $\frac{2}{3}$
C) $\frac{1}{2}$
D) $\frac{3}{4}$

---

## Exam-Bank Extras — Question Types Confirmed in Recent Papers

The NBT MAT reuses question types from a stable bank year after year. The three questions below are modelled directly on types repeatedly confirmed in recent papers and not yet represented in this chapter. Every answer has been independently machine-verified.

---

### Q36 — Intermediate

Which expression equals $\sum_{n=1}^{p}(2n-1)$?

A) $p(p+1)$

B) $p^2$

C) $2p^2 - p$

D) $p(p-1)$

---

### Q37 — Intermediate

In the sequence

$$34,\ 21,\ 13,\ 8,\ 5,\ 3,\ w,\ r$$

each term is the **sum of the two terms immediately to its right**. Find $w + r$.

A) 2

B) 5

C) 3

D) 4

### Q38 — Proficient

Solve for $x$:

$$199 + 195 + 191 + \cdots + 7 + 3 \ = \ x + 1 + 5 + 9 + \cdots + 193 + 197$$

A) 50

B) 200

C) 2

D) 100

# Answers and Explanations

## Section 8.1 — Arithmetic Sequences: Finding Terms

**Q1 — Basic — Answer: A**

**Topic:** Arithmetic sequence — find the common difference

The **common difference** is the fixed amount added each time. Subtract any term from the next:

$$d = T_2 - T_1 = 11 - 7 = 4$$

Confirm on another pair: $15 - 11 = 4$ ✓ and $19 - 15 = 4$ ✓

*Why the others are wrong:*

- **D** (6) divided the span by the wrong gap count: $(19 - 7)/2$. Four terms have **three** gaps, so $(19 - 7)/3 = 4$.
- **B** (3) is the gap **count**, not the gap **size**.
- **C** (5) counted numbers instead of steps: $7, 8, 9, 10, 11$ is five numbers but four steps.

> **Takeaway:** $d = T_{n+1} - T_n$, checked on two pairs. Never confuse "how many gaps" with "how big is each gap".

---

**Q2 — Basic — Answer: C**

**Topic:** Arithmetic sequence — general term formula

Substitute into $T_n = a + (n - 1)d$ and **expand fully**:

$$T_n = 3 + (n - 1)(5) = 3 + 5n - 5 = 5n - 2$$

**The one-second test:** substitute $n = 1$. The formula must give the first term, 3. Option C gives $5(1) - 2 = 3$ ✓

*Why the others are wrong — each fails that test:*

- **A** $(5n + 3)$ gives $T_1 = 8$. This is $a + dn$ — the "forgot the $-1$" version.
- **B** $(5n)$ gives $T_1 = 5$. The constant vanished.
- **D** $(3n + 5)$ gives $T_1 = 8$. The roles of $a$ and $d$ are swapped — the coefficient of $n$ must be $d$.

> **Takeaway:** Every AP expands to $T_n = dn + (a - d)$. Whatever you derive, test it at $n = 1$. Wrong formulas rarely survive one substitution.

---

### Q3 — Basic — Answer: B

**Topic:** Arithmetic sequence — find a specific term

Read off $a = 2$ and $d = 3$, then think of the journey: to reach the 15th term you start at 2 and take **fourteen** steps of 3 (the first term needs no step).

$$T_{15} = 2 + (15 - 1)(3) = 2 + 42 = 44$$

*Why the others are wrong:* the options are a map of every way to fumble $a + (n - 1)d$ —

- **A** (42) is $(15 - 1) \times 3$ alone — the steps were taken but the **starting value was never added**.
- **C** (47) is $2 + 15 \times 3$ — fifteen steps instead of fourteen: the "$n$ instead of $n - 1$" slip.
- **D** (41) is $2 + 13 \times 3$ — an over-correction to thirteen steps ($n - 2$), usually from "subtracting 1" twice.

> **Takeaway:** $T_n = a + (n - 1)d$: the $n$th term is reached after $n - 1$ steps, never $n$. Say it as a story — "start at $a$, step $n - 1$ times" — and the off-by-one options lose their pull.

---

### Q4 — Intermediate — Answer: D

**Topic:** Arithmetic sequence — find which term equals a given value

The value is known; the **position** is wanted. Set the general term equal to 100:

$$1 + (n - 1)(3) = 100 \implies 3n = 102 \implies n = 34$$

Check forwards: $T_{34} = 1 + 33(3) = 100$ ✓

*Why the others are wrong:*

- **B** (33rd) solved to $n - 1 = 33$ and stopped. That is the number of **steps**, not the position.
- **C** (35th) added 1 to a finished answer.
- **A** (32nd) counted from $T_2 = 4$ instead of the first term.

> **Takeaway:** $n - 1$ and $n$ are different numbers, and the wrong options wait for whichever you confuse. The forward check settles it in five seconds.

---

### Q5 — Intermediate — Answer: A

**Topic:** Arithmetic sequence — three consecutive terms

In three consecutive AP terms, the middle one is the **average** of its neighbours. So double the middle term equals the sum of the outer two:

$$2(2k+1) = k + (4k-1) \implies 4k+2 = 5k-1 \implies k = 3$$

Check: the terms are $3, 7, 11$ — differences 4 and 4 ✓

*Why the others are wrong:* substitute each and the equal-differences test fails.

- **B** ($k = 2$): $2, 5, 7$ — gaps 3 and 2 ✗
- **C** ($k = 4$): $4, 9, 15$ — gaps 5 and 6 ✗
- **D** ($k = -3$): $-3, -5, -13$ — gaps $-2$ and $-8$ ✗

> **Takeaway:** Three consecutive AP terms satisfy $2q = p + r$. Solve, then substitute back — if the differences are not equal, it is not the answer.

---

## Section 8.2 — Arithmetic Sequences: More Problems

### Q6 — Intermediate — Answer: C

**Topic:** Arithmetic sequence — find $d$ from non-consecutive terms

Two non-consecutive terms still pin down $d$, because the distance between them is a known number of equal jumps. Position 3 to position 7 is $7 - 3 = 4$ jumps:

$$4d = 27 - 11 = 16 \implies d = 4$$

Check: $11 \to 15 \to 19 \to 23 \to 27$ ✓ — four hops of 4.

*Why the others are wrong:* each fails the same forward check.

- **A** ($d = 2$): $11 + 4(2) = 19 \neq 27$
- **B** ($d = 3$): $11 + 4(3) = 23 \neq 27$
- **D** ($d = 5$): $11 + 4(5) = 31 \neq 27$

> **Takeaway:** $T_m - T_n = (m-n)d$. Get the position gap by **subtracting**, never by counting terms inclusively — 3→4→5→6→7 is four jumps, not five.

---

### Q7 — Intermediate — Answer: B

**Topic:** Arithmetic sequence — find the first term

The fifth term sits **four** steps above the first ($T_5 = a + 4d$), so walk those four steps backwards from 23:

$$T_5 = a + 4d = a + 16 = 23 \implies a = 7$$

Or step down explicitly: $23 \to 19 \to 15 \to 11 \to 7$ — four subtractions of 4.

*Why the others are wrong:*

- **D** (3) subtracts **five** steps ($23 - 5 \times 4$) — the "$n$ instead of $n - 1$" slip running in reverse.
- **A** (5) and **C** (9) both fail the forward check instantly: $5 + 16 = 21 \neq 23$ and $9 + 16 = 25 \neq 23$. They bracket the true answer to punish students who "walk back" hastily and land a step or two off.

> **Takeaway:** $a = T_n - (n-1)d$. Whether you use the formula or literally count backwards, the number of steps between $T_5$ and $T_1$ is four — and a forward check ($7 + 16 = 23$ ✓) certifies the landing.

---

### Q8 — Intermediate — Answer: A

**Topic:** Inserting arithmetic means

"Insert 3 means between 2 and 18" means build a **five-term** AP from 2 to 18 and report the three middle terms. Three insertions make **four gaps**:

$$d = \frac{18 - 2}{4} = 4$$

The run is $2,\ 6,\ 10,\ 14,\ 18$, so the means are **6, 10, 14**. Do not report the endpoints.

*Why the others are wrong:* write each run out and check the gaps are equal.

- **C** (5,10,15): $2, 5, 10, 15, 18$ has gaps $3, 5, 5, 3$ ✗
- **D** (7,11,15): spacing of 4 inside, but anchored wrong — gaps $5, 4, 4, 3$ ✗
- **B** (4,8,16): doubling, a *geometric* instinct. Gaps $2, 4, 8, 2$ ✗

> **Takeaway:** Inserting $k$ means makes $k + 2$ terms and $k + 1$ gaps, so $d = \frac{\text{last} - \text{first}}{k+1}$. Write the run out — unequal spacing is instantly visible.

---

### Q9 — Intermediate — Answer: D

**Topic:** Arithmetic sequence — real-world application

Strip the story: it starts at 20 and grows by 3 each row. An AP with $a = 20$, $d = 3$.

$$T_{15} = 20 + 14(3) = 62$$

Why 14 and not 15? Row 1 already has its 20 seats with no increase. The "+3" happens on each of the 14 steps from row 1 to row 15.

*Why the others are wrong:*

- **C** (65) is $20 + 15(3)$ — charging the increase to row 1 as well. The most common word-problem AP error.
- **B** (60) and **A** (55) are arithmetic slips: $14 \times 3 = 42$, not 40 or 35.

> **Takeaway:** Starting value is $a$, per-step change is $d$, and the step count is (row number − 1). Translate the story into $T_n = a + (n-1)d$ before touching numbers.

---

### Q10 — Basic — Answer: C

**Topic:** Arithmetic sequence — large term

$a = 5$, $d = 3$, and a large $n$ — exactly the situation the formula exists for (nobody lists forty terms):

$$T_{40} = 5 + 39(3) = 5 + 117 = 122$$

*Why the others are wrong:* the three classic fumbles, all present and accounted for —

- **B** (117) is $39 \times 3$ alone — the first term $a = 5$ was never added back.
- **D** (125) is $5 + 40 \times 3$ — forty steps instead of thirty-nine ($n$ instead of $n-1$).
- **A** (120) treats the sequence as pure multiples of 3 ($3 \times 40$) — plausible-looking here only because $a$ and $d$ are small and close.

> **Takeaway:** Big-$n$ term questions are two operations: $(n-1) \times d$, **then** $+a$. Each wrong option drops or distorts exactly one of the two — do both deliberately.

---

## Section 8.3 — Geometric Sequences

#### Q11 — Basic — Answer: B

**Topic:** Geometric sequence — find the common ratio

In a geometric sequence the question is never "what is added?" but "**what is multiplied?**". Divide any term by the one before it:

$$r = \frac{T_2}{T_1} = \frac{12}{3} = 4$$

Confirm: $48/12 = 4$ and $192/48 = 4$ ✓ — the ratio is constant, which is what "geometric" means.

*Why the others are wrong:*

- **C** (9) is $12 - 3$ — **subtracting** instead of dividing, the arithmetic-sequence reflex applied to a geometric one.
- **A** (3) just echoes the first term — a surface grab requiring no calculation.
- **D** (6) is $\sqrt{3 \times 12}$, the *geometric mean* of the first two terms — a related idea, but the mean of two terms is not the ratio between them.

> **Takeaway:** $r = T_{n+1}/T_n$, verified on a second pair. The first question to ask of any sequence: is it add-the-same (AP) or multiply-by-the-same (GP)? Every later formula depends on getting that classification right.

---

#### Q12 — Basic — Answer: A

**Topic:** Geometric sequence — find a specific term

Same journey logic as the AP, but the steps are multiplications: to reach the 5th term you start at 2 and multiply by 3 exactly **four** times.

$$T_5 = ar^{5-1} = 2 \times 3^4 = 2 \times 81 = 162$$

Walk it: $2 \to 6 \to 18 \to 54 \to 162$ — four hops ✓.

*Why the others are wrong:*

- **B** (486) is $2 \times 3^5$ — five multiplications instead of four: the GP version of the "$n$ vs $n-1$" slip.
- **C** (54) is $2 \times 3^3$ — one multiplication short ($T_4$, in fact).
- **D** (96) is $3 \times 2^5$ — the roles of $a$ and $r$ swapped (starting at 3 and doubling). Read carefully which number is the *first term* and which is the *ratio*.

> **Takeaway:** $T_n = ar^{n-1}$ — the exponent counts the jumps, and there are always $n-1$ of them. When the numbers are small, walking the chain term by term is the perfect anti-slip check.

---

#### Q13 — Intermediate — Answer: D

**Topic:** Geometric sequence — find which term equals a value

$a = 5$, $r = 2$. Set the general term to 640 and divide out the first term:

$$5 \times 2^{n-1} = 640 \implies 2^{n-1} = 128$$

List the powers: $2, 4, 8, 16, 32, 64, 128$ — that is $2^7$. So $n - 1 = 7$ and $n = 8$.

Check: $5 \times 2^7 = 640$ ✓

*Why the others are wrong:*

- **B** (7th) reported $n-1$ and forgot the final "+1".
- **C** (9th) added 1 to a finished answer.
- **A** (6th) miscounted the ladder, taking 128 as $2^5$.

> **Takeaway:** Isolate $r^{n-1}$, write it as a power of $r$, then remember that exponent is $n-1$, **not** $n$. Listing the powers takes ten seconds and makes miscounting impossible.

---

#### Q14 — Intermediate — Answer: C

**Topic:** Geometric sequence — negative common ratio

Find the ratio **including its sign**: $r = -12/4 = -3$. Alternating signs are the fingerprint of a negative ratio.

$$T_5 = 4 \times (-3)^4 = 4 \times 81 = 324$$

An **even** number of negatives multiplies to a positive. Or just extend the pattern: the signs run $+, -, +, -, +$, so the 5th term is positive.

*Why the others are wrong:*

- **A** ($-324$) right size, wrong sign — treated $(-3)^4$ as $-81$.
- **D** ($-108$) is just $T_4$ repeated.
- **B** (216) used a made-up ratio of $-2$. The ratio comes from dividing, not from guessing.

> **Takeaway:** With $r < 0$, decide the **sign** first (even power → positive, odd → negative), then the size. Two separate decisions.

---

#### Q15 — Basic — Answer: B

**Topic:** Geometric sequence — real-world (doubling)

"Doubles every hour" is a GP with $r = 2$ — and the only real danger here is counting the doublings. **After 4 hours, exactly 4 doublings have happened:**

$$500 \;\rightarrow\; 1\,000 \;\rightarrow\; 2\,000 \;\rightarrow\; 4\,000 \;\rightarrow\; 8\,000$$

$$500 \times 2^4 = 500 \times 16 = 8\,000$$

(In sequence language this is $T_5$, because $T_1$ is "0 hours" — which is exactly why $a \times 2^{\text{hours}}$ is safer than $T_n$ bookkeeping here.)

*Why the others are wrong:*

- **A** (4 000) used only **3** doublings — that is 3 hours.
- **C** (16 000) used **5** doublings — 5 hours.
- **D** (2 000) multiplied by 4 — linear thinking about exponential growth.

> **Takeaway:** After $n$ doublings, multiply by $2^n$. Count **doublings**, not terms — the hour-by-hour chain settles it in seconds.

---

## Section 8.4 — Arithmetic Series

#### Q16 — Basic — Answer: A

**Topic:** Arithmetic series — find $S_n$ directly

This is the sum of the first 10 **odd numbers**. With $a = 1, d = 2$:

$$S_{10} = \frac{10}{2}(2(1) + 9(2)) = 5(2 + 18) = 5 \times 20 = 100$$

Or use Gauss's pairing: the 10th odd number is $T_{10} = 1 + 9(2) = 19$, so $S_{10} = \frac{10}{2}(1 + 19) = 5 \times 20 = 100$.

*Why the others are wrong:*

- **D** (55) is the sum of the first 10 *integers* $1 + 2 + \cdots + 10$ — it ignores that this sequence skips the evens.
- **B** (90) is the pairing form with the last term taken as **17** ($T_9$, the 9th odd number) — an off-by-one in the final term: $5(1 + 17) = 90$.
- **C** (110) makes the opposite slip, pairing with **21** ($T_{11}$): $5(1 + 21) = 110$.

> **Takeaway:** The sum of the first $n$ odd numbers is always $n^2$ — here $10^2 = 100$. It is one of the most re-used facts in the NBT's sequence questions (see Q36), so own it outright.

---

### Q17 — Intermediate — Answer: D

**Topic:** Arithmetic series — find $n$ given the sum

The sum of the first $n$ positive integers has the famous closed form $S_n = \dfrac{n(n+1)}{2}$ (Gauss's formula — pair 1 with $n$, 2 with $n - 1$, and so on).

$$\frac{n(n+1)}{2} = 55 \implies n(n+1) = 110$$

Two *consecutive* integers multiplying to 110: $10 \times 11$ ✓. So $n = 10$.

*Why the others are wrong:* all three are neighbours on the triangular-number ladder —

- **A** ($n = 9$): $\frac{9 \times 10}{2} = 45 \neq 55$.
- **B** ($n = 8$): $\frac{8 \times 9}{2} = 36 \neq 55$.
- **C** ($n = 11$): $\frac{11 \times 12}{2} = 66 \neq 55$.

If 55 isn't instantly recognisable as the 10th triangular number, the ten-second test above (does $n(n + 1)$ hit 110?) separates the candidates without any guessing.

> **Takeaway:** $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$. Memorise the small triangular numbers — 15, 55, 210 for $n = 5, 10, 20$ — the NBT re-uses them constantly.

---

### Q18 — Intermediate — Answer: C

**Topic:** Arithmetic series — sum of a given AP

$a = 4, d = 5, n = 8$:

$$S_8 = \frac{8}{2}(8 + 35) = 4(43) = 172$$

**Cross-check with the pairing form.** Last term $T_8 = 4 + 7(5) = 39$, so $S_8 = 4(4 + 39) = 172$ ✓

*Why the others are wrong:* **A** (160), **B** (164) and **D** (176) all come from the bracket $8 + 35 = 43$ becoming 40, 41 or 44 — slips like using $8 \times 5$ instead of $7 \times 5$. None survives the two-route check.

> **Takeaway:** Work out $2a$ and $(n-1)d$ as two separate products before adding, then confirm with $\frac{n}{2}(a + l)$. Two routes agreeing is the strongest check you have without a calculator.

---

### Q19 — Intermediate — Answer: B

**Topic:** Arithmetic series — use first and last term

With the first and last terms both known, use the pairing form — you never need $d$:

$$S_{16} = \frac{16}{2}(2 + 62) = 8 \times 64 = 512$$

The picture: pair $T_1$ with $T_{16}$, $T_2$ with $T_{15}$, and so on — eight pairs, each summing to 64.

*Why the others are wrong — each has the wrong bracket:*

- **C** (496) is $8 \times 62$ — the first term never joined its pair.
- **A** (480) is $8 \times 60$ — the bracket taken as a **difference**. Pairs are added.
- **D** (528) is $8 \times 66$ — the first term counted twice.

> **Takeaway:** $S_n = \frac{n}{2}(a + l)$. Say the bracket aloud — "first plus last" — and the wrong-bracket options lose their pull.

---

### Q20 — Proficient — Answer: A

**Topic:** Arithmetic series — real-world

A shrinking stack is still an AP, with a negative step: $a = 30$, $d = -2$.

**Step 1 — Bottom row.** Nine steps down:

$$T_{10} = 30 + 9(-2) = 12 \text{ bricks}$$

**Step 2 — Pair top with bottom.**

$$S_{10} = \frac{10}{2}(30 + 12) = 210$$

*Why the others are wrong — each used a wrong last term:*

- **B** (200) took ten steps instead of nine, giving a bottom row of 10.
- **C** (220) miscounted the other way, giving 14.
- **D** (195) fits no clean method — the sort of answer you get juggling both steps mentally.

> **Takeaway:** Find the last term **first**, on its own line, then pair. This problem is won or lost on the bottom row.

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## Section 8.5 — Geometric Series

### Q21 — Basic — Answer: D

**Topic:** Geometric series — find $S_n$ directly

$a = 1, r = 2$, six terms:

$$S_6 = \frac{1(2^6 - 1)}{2 - 1} = \frac{64 - 1}{1} = 63$$

Sanity check by brute force — the terms are small enough: $1 + 2 + 4 + 8 + 16 + 32 = 63$ ✓. Notice the pattern: each doubling sum lands one short of the next power of 2.

*Why the others are wrong:*

- **A** (32) is $2^5$ — the sixth **term**, not the sum of six terms. ($T_n$ vs $S_n$ confusion is error #2 on this chapter's opening list.)
- **B** (64) is $2^6$ — the "one short" fact forgotten: the sum is $2^6 - 1$, not $2^6$ itself.
- **C** (127) is $2^7 - 1 = S_7$ — one term too many.

> **Takeaway:** For the doubling GP starting at 1, $S_n = 2^n - 1$ ("all the previous powers sum to one less than the next power"). And always ask first: is the question requesting a *term* or a *sum*?

---

### Q22 — Intermediate — Answer: C

**Topic:** Geometric series — larger ratio

$$S_5 = \frac{2(3^5 - 1)}{3 - 1} = \frac{2 \times 242}{2} = 242$$

($3^5 = 243$: walk the powers $3, 9, 27, 81, 243$.) Brute-force check: the five terms are $2 + 6 + 18 + 54 + 162 = 242$ ✓.

*Why the others are wrong:*

- **B** (486) is $2 \times 3^5$ — a **term** ($T_6$), not a sum. The formula's $r^n$ lives inside $\frac{a(r^n-1)}{r-1}$; on its own it computes terms.
- **A** (240) is the sum with the first term missing: $6 + 18 + 54 + 162 = 240$. Every term got multiplied by $r$ once too often — the series must *start at* $a$, not at $ar$.
- **D** (244) is $242 + 2$ — the first term counted twice, the mirror image of A's error.

> **Takeaway:** $S_5 = \frac{a(r^5-1)}{r-1}$, and with numbers this small, adding the five terms directly is a fast, decisive check. Options that differ from the truth by exactly $\pm a$ are telling you someone mislaid the first term.

---

### Q23 — Intermediate — Answer: B

**Topic:** Geometric series — find $n$ given the sum

For the doubling GP starting at 1, the sum has the clean closed form $S_n = 2^n - 1$ (Q21). Set it equal to 127:

$$2^n - 1 = 127 \implies 2^n = 128$$

List the powers: $2, 4, 8, 16, 32, 64, 128$ — seven of them. So $n = 7$.

*Why the others are wrong:* the wrong options are neighbouring rungs of the $2^n - 1$ ladder —

- **A** ($n = 6$): $S_6 = 63$, not 127.
- **D** ($n = 5$): $S_5 = 31$ — far short.
- **C** ($n = 8$): $S_8 = 255$ — overshoot.

As with Q8's interest ladder, these questions test *recognition*: if $63, 127, 255$ ring bells as "one less than a power of 2", no wrong option can touch you.

> **Takeaway:** Add 1 to the target sum and ask "which power of $r$ is this?" — then that exponent is $n$ itself (because the series started at $a = 1$). The values $2^7 = 128$ and $S_7 = 127$ are worth owning outright.

---

### Q24 — Proficient — Answer: A

**Topic:** Geometric series — future value of an annuity

Chapter 7's annuity seen through sequence eyes. Each deposit grows for a different number of years — 2, then 1, then 0:

$$100(1.1)^2 + 100(1.1) + 100 = 100(1.21 + 1.10 + 1.00) = \text{R}331$$

The amounts $100, 110, 121$ form a GP with $r = 1.1$ — an annuity's future value **is** a geometric series.

*Why the others are wrong:*

- **C** (R300) ignores interest entirely.
- **B** (R330) applies 10 % once to the whole R300, as if every deposit arrived together.
- **D** (R333) is careless addition — the bracket is exactly 3.31.

> **Takeaway:** Equal end-of-period deposits form a GP with ratio $(1 + i)$, and the last deposit earns nothing. Write one line per payment and the series appears by itself.

---

### Q25 — Intermediate — Answer: C

**Topic:** Geometric series — find first term given sum

A reverse sum problem. With $r = 2$ and $n = 4$, the total is always **15 times the first term**, since $1 + 2 + 4 + 8 = 15$:

$$15a = 120 \implies a = 8$$

Check: $8 + 16 + 32 + 64 = 120$ ✓

*Why the others are wrong:*

- **A** (4) divided by 30 — summing from $ar$ instead of $a$ $(2 + 4 + 8 + 16)$.
- **D** (16) divided by 7.5 — the arithmetic habit of halving, sneaking into a geometric formula.
- **B** (12) fails the check: $12 + 24 + 48 + 96 = 180$.

> **Takeaway:** $a = \frac{S_n(r-1)}{r^n-1}$. Reverse problems give you a free check — rebuild the sequence from your $a$ and add it up.

---

## Section 8.6 — Sigma Notation

### Q26 — Basic — Answer: D

**Topic:** Sigma notation — evaluate a simple sum

Sigma notation is an instruction, not a mystery: "substitute $k = 1$ through 5 into the expression after the $\Sigma$, and add the results."

$$\sum_{k=1}^{5} k = 1 + 2 + 3 + 4 + 5 = 15$$

*Why the others are wrong:*

- **A** (10) stops at $k = 4$ — the upper limit is inclusive; $k = 5$ belongs in the sum.
- **C** (20) is $5 \times 4$, a corrupted use of the formula ($n(n-1)$ instead of $n(n+1)/2$).
- **B** (12) matches no correct reading — with only five tiny terms, expanding and adding leaves no room for such an option to feel plausible.

> **Takeaway:** When limits are small, expand — always. For big limits, $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$; for $n = 5$ : $\frac{5 \times 6}{2} = 15$ ✓ (the formula and the expansion must agree).

---

### Q27 — Basic — Answer: B

**Topic:** Sigma notation — evaluate with a formula

Substitute each $k$ and add:

$$k = 1:\ 3 \qquad k = 2:\ 5 \qquad k = 3:\ 7 \qquad k = 4:\ 9$$
$$\text{Sum} = 3 + 5 + 7 + 9 = 24$$

Formula check: this is an AP with $a = 3$, $l = 9$, $n = 4$: $S_4 = \frac{4}{2}(3 + 9) = 24$ ✓.

*Why the others are wrong:*

- **A** (20) sums $2k$ without the $+1$'s: $2 + 4 + 6 + 8 = 20$. Four dropped $+1$'s are exactly the four marks between A and the right answer.
- **D** (28) reads $2k + 1$ as $2(k + 1)$: $4 + 6 + 8 + 10 = 28$. Brackets change everything.
- **C** (22) is an addition slip in the four-term sum — the pairing check $(3 + 9) + (5 + 7) = 12 + 12$ makes the true total hard to miss.

> **Takeaway:** Substitute-and-add is the base method; the AP sum formula is the check. Watch two things like a hawk: the exact expression being summed, and every $+$constant it carries.

---

### Q28 — Intermediate — Answer: A

**Topic:** Sigma notation — identify the sequence type

Expand the first few terms — that is how you identify any sequence hiding in sigma notation.

$$T_1 = 3(1) - 1 = 2, \qquad T_2 = 5, \qquad T_3 = 8$$

Differences of 3 and 3 — constant. So it is an **AP with** $a = 2$**,** $d = 3$.

*Why the others are wrong:*

- **D** took the "$-1$" as the first term. The first term comes from substituting $k = 1$, giving 2.
- **C** swapped the roles — the 3 is the coefficient of $k$, so it is $d$, not $a$.
- **B** calls it a GP. A $k$ **multiplied** by a constant means "add that much each step"; only $k$ in an **exponent** multiplies.

> **Takeaway:** Linear in $k \rightarrow$ AP (difference = the coefficient). Exponential in $k \rightarrow$ GP (ratio = the base). In doubt, expand three terms.

---

### Q29 — Intermediate — Answer: C

**Topic:** Sigma notation — geometric series

The $k$ is in the **exponent**, so this is geometric. Expand:

$$2 + 6 + 18 + 54 = 80$$

Formula check ($a = 2$, $r = 3$, $n = 4$): $S_4 = \frac{2(81-1)}{2} = 80$ ✓

*Why the others are wrong:*

- **A** (81) is just $3^4$ — the factor 2 and the summing both ignored.
- **B** (78) dropped the $k = 1$ term. The exponent $k - 1$ makes it $2 \cdot 3^0 = 2$, and $3^0 = 1$ catches people.
- **D** (84) is an addition slip; expanding *and* formula-checking leaves it no room.

> **Takeaway:** $k$ in the exponent $\rightarrow$ GP. Watch the $k = 1$ term (anything to the power 0 is 1), expand when $n$ is small, and let the formula confirm.

#### Q30 — Intermediate — Answer: D

**Topic:** Sigma notation — shifted limits

The lower limit is **3**, not 1 — the most-missed detail in sigma questions. Substitute $k = 3$ to $7$ (five terms):

$$6 + 8 + 10 + 12 + 14 = 50$$

Structure check: five terms with middle term 10, so $5 \times 10 = 50$ ✓

*Why the others are wrong:*

- **A** (40) lost the middle term — pairing $6 + 14$ and $8 + 12$ leaves the unpaired 10 behind.
- **C** (42) misread both limits, summing $k = 1$ to $6$.
- **B** (45) misread the upper limit as 6.

> **Takeaway:** Read **both** limits first, count the terms as $(\text{top} - \text{bottom}) + 1$, then expand. With an odd number of AP terms, "middle × count" is a fast check.

## Section 8.7 — Infinite Geometric Series

#### Q31 — Basic — Answer: B

**Topic:** Infinite geometric series — convergence condition

An infinite sum only settles at a finite value if the terms **shrink towards zero**. That happens exactly when $|r| < 1$, meaning $-1 < r < 1$.

*Why the others are wrong — each admits or excludes the wrong ratios:*

- **C** ($r < 1$) allows $r = -5$: the terms $a, -5a, 25a, \ldots$ explode. The absolute value is the condition, not decoration.
- **A** ($r > 0$) allows $r = 2$, where the terms double forever.
- **D** keeps only the negative half, wrongly excluding $r = \frac{1}{2}$ — the commonest convergent case of all.

> **Takeaway:** $S_\infty$ exists only when $|r| < 1$. Test any proposed condition by hunting for a ratio it wrongly lets in or leaves out.

### Q32 — Basic — Answer: C

**Topic:** Infinite geometric series — calculate $S_\infty$

$a = 12, r = \frac{1}{2}$, and $|r| < 1$ so the sum exists:

$$S_\infty = \frac{12}{1 - \frac{1}{2}} = 24$$

Watch it converge: $12, 18, 21, 22.5, 23.25, \ldots$ — halving the remaining gap each time, leaning on 24.

*Why the others are wrong:*

- **A** (12) is only the first term.
- **B** (18) stops after two terms; the infinite tail adds another 6.
- **D** (36) mangles the denominator — and the running total never passes 24, so it is impossible.

> **Takeaway:** $S_\infty = \frac{a}{1-r}$, and dividing by a fraction means multiplying by its reciprocal. Adding three or four terms tells you roughly where the answer must land — a useful sanity rail.

---

### Q33 — Intermediate — Answer: A

**Topic:** Infinite geometric series — find first term given $S_\infty$ and $r$

Reverse problem: the sum is known, the first term isn't. Rearranging $S_\infty = a/(1-r)$ gives $a = S_\infty(1-r)$:

$$a = 20\left(1 - \frac{1}{4}\right) = 20 \times \frac{3}{4} = 15$$

**Check forwards:** $S_\infty = \dfrac{15}{1 - \frac{1}{4}} = \dfrac{15}{\frac{3}{4}} = 20$ ✓. (The series $15 + 3.75 + 0.9375 + \cdots$ indeed leans on 20.)

*Why the others are wrong:*

- **B** (80) divides where it should multiply: $20 \div \frac{1}{4} = 80$ — running the formula in the wrong direction. Its forward check explodes: $80/(3/4) \approx 107$, not 20.
- **C** (5) multiplies by $r$ instead of by $(1-r)$: $20 \times \frac{1}{4} = 5$.
- **D** (10) multiplies by $\frac{1}{2}$ — a "take half" reflex with no basis in the given $r = \frac{1}{4}$.

> **Takeaway:** $a = S_\infty(1-r)$: the first term is always *smaller* than the infinite sum (the tail contributes the rest) — so any candidate bigger than 20, like option B, is disqualified before you compute anything.

---

### Q34 — Proficient — Answer: D

**Topic:** Infinite geometric series — recurring decimal

A recurring decimal is an infinite geometric series hiding in plain sight — each repeat is the previous one shifted a decimal place:

$$0.333\ldots = 0.3 + 0.03 + 0.003 + \cdots$$

$a = 0.3$, $r = 0.1$ (each term is a tenth of the one before), and $|r| < 1$:

$$S_\infty = \frac{0.3}{1 - 0.1} = \frac{0.3}{0.9} = \frac{3}{9} = \frac{1}{3}$$

*Why the others are wrong:*

- **A** ($\frac{3}{10}$) is $0.3$ exactly — the first term alone. Truncating a recurring decimal always undershoots.
- **C** ($\frac{3}{11}$) is what the formula produces if you **add** $r$ instead of subtracting: $0.3/1.1 = \frac{3}{11} = 0.2727\ldots$ — visibly the *wrong* recurring decimal.
- **B** ($\frac{1}{4}$) is $0.25$, a familiar-fraction guess. Divide it out: $0.25 \neq 0.333\ldots$ Candidate fractions can always be tested by division.

> **Takeaway:** Recurring decimal → infinite GP with $r = 0.1^{(\text{repeating digits})} \to a/(1-r)$. And $0.\overline{3} = \frac{1}{3}$ deserves to be instant recall — it anchors the whole family ($0.\overline{6} = \frac{2}{3}$, $0.\overline{9} = 1$).

---

### Q35 — Intermediate — Answer: B

**Topic:** Infinite geometric series — find $r$ given $a$ and $S_\infty$

Solve the sum formula for $r$ — note that the fraction $a/S_\infty$ gives you $1 - r$, not $r$ itself:

$$24 = \frac{8}{1-r} \implies 1 - r = \frac{8}{24} = \frac{1}{3} \implies r = \frac{2}{3}$$

**Check:** $|r| = \frac{2}{3} < 1$ ✓ (convergent), and forwards: $\dfrac{8}{1 - \frac{2}{3}} = \dfrac{8}{\frac{1}{3}} = 24$ ✓.

*Why the others are wrong:*

- **A** ($\frac{1}{3}$) is the value of $1 - r$ reported as $r$ — the solution abandoned one line before the finish. This near-miss is the whole reason the question exists.
- **C** ($\frac{1}{2}$) gives $S_\infty = 8/\frac{1}{2} = 16 \neq 24$.
- **D** ($\frac{3}{4}$) gives $S_\infty = 8/\frac{1}{4} = 32 \neq 24$. Both fail the same five-second forward check.

> **Takeaway:** $r = 1 - a/S_\infty$. Whenever a rearrangement produces "$1 - r = \ldots$", underline it — the examiner is betting you'll stop there and forget the final subtraction.

---

## Exam-Bank Extras — Question Types Confirmed in Recent Papers

#### Q36 — Intermediate — Answer: B

**Topic:** Sigma notation — sum of the first $p$ odd numbers

The rule $2n - 1$ generates the **odd numbers**: $1, 3, 5, \ldots, 2p - 1$. So this is an AP with $a = 1$, $d = 2$, and $p$ terms.

**By formula:**

$$S_p = \frac{p}{2}\big(2 + (p-1)2\big) = \frac{p}{2}(2p) = p^2$$

**By picture:** $1 = 1^2$, $1 + 3 = 2^2$, $1 + 3 + 5 = 3^2$, $1 + 3 + 5 + 7 = 4^2$ — each new odd number wraps an L around the square, completing the next one.

*Why the others are wrong:*

- **A** $p(p+1)$ is the sum of the first $p$ **even** numbers.
- **C** $2p^2 - p$ equals $p(2p-1)$ — the *last term* times $p$, a mis-expansion.
- **D** $p(p-1)$ is the even-number sum one step short.

> **Takeaway:** $\sum_{n=1}^{p}(2n-1) = p^2$ — worth knowing as a single fact. "$2n \pm 1$ summed from 1" should trigger "odd numbers → perfect square".

---

#### Q37 — Intermediate — Answer: C

**Topic:** A sequence defined by a hidden additive rule (Fibonacci-type)

**Verify the rule first** — never trust a stated pattern blindly: $34 = 21 + 13$ ✓, $21 = 13 + 8$ ✓, $13 = 8 + 5$ ✓, $8 = 5 + 3$ ✓. (Fibonacci, running backwards.)

Now apply it to the terms containing the unknowns:

**The term 5** has right-neighbours 3 and $w$: $5 = 3 + w \Rightarrow w = 2$

**The term 3** has right-neighbours $w$ and $r$: $3 = 2 + r \Rightarrow r = 1$

$$w + r = 3$$

*Why the others are wrong:*

- **A** (2) is $w$ alone — the question asks for $w + r$.
- **D** (4) is $w$ counted twice; the equation for $r$ was never solved.
- **B** (5) just echoes a visible term.

> **Takeaway:** If a sequence fails both the AP test (constant differences) and the GP test (constant ratios), try an **additive rule**. Once one unknown falls, the rest follow one equation at a time.

---

### Q38 — Proficient — Answer: D

**Topic:** Equating two arithmetic series (pairing strategy)

Both sides are arithmetic series with difference 4 and 50 terms each — plus the lone $x$ on the right.

**The insight: pair the terms across the equation.**

$$199 \leftrightarrow 197, \quad 195 \leftrightarrow 193, \quad \ldots, \quad 3 \leftrightarrow 1$$

Every pair differs by 2, and there are 50 pairs. So the left side exceeds the right-side series by $2 \times 50 = 100$, and $x$ must supply exactly that:

$$x = 100$$

Formula check: left $= \frac{50}{2}(3 + 199) = 5050$; right series $= \frac{50}{2}(1 + 197) = 4950$; difference $= 100$ ✓

*Why the others are wrong:*

- **A** (50) is the number of pairs, not the surplus.
- **C** (2) is one pair's difference; there are fifty.
- **B** (200) used the *within-series* difference of 4 as the per-pair gap. The gap across the equation is 2.

> **Takeaway:** Two long series across an equals sign? Do not compute either side — **pair the terms and multiply the per-pair difference by the number of pairs.**

## Mixed Practice Questions — M1 to M30

### M1 — Intermediate

**Topic:** Algebra — factorising with a common factor

Factorise completely: $(x+2)^2 - (x+2)(x-1)$.

A) $3(x+2)$

B) $(x+2)(2x+1)$

C) $3x+2$

D) $(x-1)(x+3)$

---

### M2 — Basic

**Topic:** Number sense — percentage of an amount

What is 20 % of 350?

A) 35

B) 17.5

C) 70

D) 175

---

### M3 — Intermediate

**Topic:** Functions — vertex of a parabola

The function $f(x) = x^2 - 6x + 5$ has its turning point at:

A) $(3,\ 4)$

B) $(3,\ -4)$

C) $(-3,\ 4)$

D) $(-3,\ -4)$

---

### M4 — Basic

**Topic:** Trigonometry — exact value

$\sin 30^\circ + \cos 60^\circ =$

A) $0$

B) $\frac{\sqrt{2}}{2}$

C) $\frac{\sqrt{3}}{2}$

D) $1$

---

### M5 — Basic

**Topic:** Algebra — linear inequality

Solve $-2x + 6 > 0$.

A) $x < 3$

B) $x > 3$

C) $x > -3$

D) $x < -3$

---

### M6 — Basic

**Topic:** Geometry — co-interior angles

Co-interior angles formed by a transversal cutting two parallel lines:

A) Are equal

B) Sum to 360°

C) Sum to 180°

D) Are supplementary only when acute

---

### M7 — Basic

**Topic:** Data — median

The median of $\{3,\ 7,\ 7,\ 10,\ 12\}$ is:

A) 10

B) 7

C) 7.8

D) 8

---

### M8 — Intermediate

**Topic:** Financial maths — compound interest

R5 000 is invested at 20 % p.a. compound interest for 2 years. The accumulated value is:

A) R6 000

B) R7 000

C) R6 500

D) R7 200

---

### M9 — Basic

**Topic:** Arithmetic sequence — find a term

Find the 10th term of the AP $3,\ 7,\ 11,\ 15,\ \ldots$

A) 39

B) 41

C) 43

D) 37

---

### M10 — Basic

**Topic:** Number sense — simplifying surds

Simplify $\sqrt{75}$.

A) $3\sqrt{5}$

B) 15

C) $5\sqrt{3}$

D) $25\sqrt{3}$

---

### M11 — Basic

**Topic:** Functions — hyperbola

Which statement about $y = \dfrac{3}{x}$ is correct?

A) It has an $x$-intercept at $x = 3$

B) It has no $x$-intercept and no $y$-intercept

C) It has a $y$-intercept at $y = 3$

D) It has a minimum at $(1,\ 3)$

---

### M12 — Basic

**Topic:** Trigonometry — solve a basic equation

Solve $\sin\theta = -1$ for $\theta \in [0^\circ,\ 360^\circ]$.

A) $90^\circ$

B) $180^\circ$

C) $0^\circ$

D) $270^\circ$

---

#### M13 — Basic

**Topic:** Geometry — area of a parallelogram

A parallelogram has base 8 cm and perpendicular height 5 cm. Its area is:

A) 40 $cm^2$

B) 20 $cm^2$

C) 80 $cm^2$

D) 26 $cm^2$

---

#### M14 — Intermediate

**Topic:** Data — independent events

$P(A) = 0.6$ and $P(B) = 0.4$; $A$ and $B$ are independent. Find $P(A \cap B)$.

A) 1.0

B) 0.76

C) 0.24

D) 0.2

---

#### M15 — Basic

**Topic:** Geometric sequence — find a term

A GP has first term 3 and common ratio 4. Find $T_4$.

A) 48

B) 192

C) 64

D) 768

---

### M16 — Intermediate

**Topic:** Financial maths — straight-line depreciation

A machine costs R50 000. It depreciates at 5 % p.a. straight-line for 6 years. Its book value after 6 years is:

A) R40 000

B) R15 000

C) R45 000

D) R35 000

---

### M17 — Basic

**Topic:** Algebra — fully factorise

Factorise completely: $3x^2 - 12$.

A) $3(x-2)(x+2)$

B) $(3x-4)(x+3)$

C) $3(x-2)^2$

D) $3(x+2)^2$

---

### M18 — Basic

**Topic:** Functions — exponential evaluation

If $f(x) = 2^x$, find $f(3) - f(1)$.

A) 2

B) 4

C) 6

D) 16

---

### M19 — Intermediate

**Topic:** Arithmetic series — sum to 20 terms

Find the sum of the AP $5,\ 10,\ 15,\ \ldots$ to 20 terms.

A) 1 000

B) 1 050

C) 1 100

D) 950

---

### M20 — Basic

**Topic:** Number sense — laws of exponents

Simplify $\dfrac{2^3 \times 2^4}{2^5}$.

A) $2^{12}$

B) 8

C) 16

D) 4

---

### M21 — Intermediate

**Topic:** Geometry — arc length

A circle has radius 6 cm. What is the arc length subtended by a 60° central angle?

A) $2\pi$ cm

B) $6\pi$ cm

C) $3\pi$ cm

D) $12\pi$ cm

---

### M22 — Basic

**Topic:** Data — probability of a prime

A fair die is rolled. What is the probability of rolling a prime number?

A) $\frac{1}{3}$
B) $\frac{1}{6}$
C) $\frac{1}{2}$
D) $\frac{2}{3}$

---

### M23 — Intermediate

**Topic:** Infinite geometric series — calculate $S_\infty$

An infinite GP has first term 6 and common ratio $\frac{1}{2}$. Find $S_\infty$.

A) 6

B) 12

C) 18

D) 24

---

### M24 — Intermediate

**Topic:** Financial maths — accumulated amount

R3 000 is invested at 15 % p.a. simple interest for 4 years. What is the total accumulated amount?

A) R3 450

B) R4 500

C) R5 250

D) R4 800

---

### M25 — Basic

**Topic:** Algebra — solving $x^2 = k$

Solve $x^2 = 25$.

A) $x = \pm 5$

B) $x = 5$

C) $x = -5$

D) $x = \sqrt{25}$ only

---

### M26 — Basic

**Topic:** Functions — direction of parabola

The graph of $y = -x^2 + 3$ opens in which direction?

A) Right

B) Left

C) Downwards

D) Upwards

---

### M27 — Intermediate

**Topic:** Sigma notation — writing a series

Which sigma expression represents $2 + 5 + 8 + 11 + 14$?

A) $\sum_{k=1}^{5} (3k + 1)$

B) $\sum_{k=1}^{5} (3k - 1)$

C) $\sum_{k=1}^{5} 3k$

D) $\sum_{k=1}^{4} (3k - 1)$

---

### M28 — Intermediate

**Topic:** Trigonometry — angle of elevation

A flagpole casts a shadow of 12 m when the sun's angle of elevation is 30°. The height of the flagpole is:

A) 6 m

B) $12\sqrt{3}$ m

C) $6\sqrt{3}$ m

D) $4\sqrt{3}$ m

---

### M29 — Intermediate

**Topic:** Arithmetic sequence — find $n$

In the AP $5,\ 8,\ 11,\ \ldots$, which term equals 77?

A) 25th

B) 24th

C) 26th

D) 27th

---

### M30 — Proficient

**Topic:** Financial maths — annuity future value

R1 000 is deposited at the end of each year for 4 years at 10 % p.a. compound. The total accumulated value after 4 years is:

A) R4 000

B) R4 400

C) R4 641

D) R5 000

## Mixed Practice — Answer Key

| | | | | |
|---|---|---|---|---|
| **M1.** A | **M2.** C | **M3.** B | **M4.** D | **M5.** A |
| **M6.** C | **M7.** B | **M8.** D | **M9.** A | **M10.** C |
| **M11.** B | **M12.** D | **M13.** A | **M14.** C | **M15.** B |
| **M16.** D | **M17.** A | **M18.** C | **M19.** B | **M20.** D |
| **M21.** A | **M22.** C | **M23.** B | **M24.** D | **M25.** A |
| **M26.** C | **M27.** B | **M28.** D | **M29.** A | **M30.** C |

# Chapter 9 — Full Mock Examinations

## How to use these mock exams

Each exam contains **60 questions** in NBT MAT format: four options (A–D), one correct answer. No calculator is allowed. Allow **3 hours** per exam. Work through the entire paper before checking the answer key at the end.

The 5 NBT competence areas appear in every exam in the same proportion as the real test:

| Competence area | Questions per exam | Chapters covered |
|---|---|---|
| Algebraic Processes | ~15 | Ch1, Ch8 (sequences) |
| Functions & Graphs | ~12 | Ch3 |
| Number Sense | ~12 | Ch2, Ch7 (financial), Ch8 (series) |
| Geometry & Trigonometry | ~12 | Ch4, Ch5 |
| Data Handling & Probability | ~9 | Ch6 |

Questions are arranged by difficulty within each competence block (not labelled — mirroring the real NBT, which does not indicate difficulty).

# Mock Exam 1

### Exam 1 — Q1

Simplify $\dfrac{x^2-9}{x^2-x-6}$.

A) $\dfrac{x-3}{x-3}$
B) $\dfrac{x+3}{x+2}$
C) $\dfrac{x-3}{x+2}$
D) $\dfrac{x+3}{x-3}$

---

### Exam 1 — Q2

Solve for $x$: $2x^2-8=0$.

A) $x=4$

B) $x=\pm 4$

C) $x=\pm 2$

D) $x=2$

---

### Exam 1 — Q3

Which value of $x$ satisfies $\dfrac{3}{x-1}=6$?

A) $x=\dfrac{1}{2}$
B) $x=\dfrac{3}{2}$
C) $x=2$

D) $x=3$

---

**Exam 1 — Q4**

Factorise completely: $2x^2 + 5x - 3$.

A) $(2x - 1)(x + 3)$

B) $(2x + 1)(x - 3)$

C) $(x + 3)(2x - 1)$

D) $(2x - 3)(x + 1)$

---

**Exam 1 — Q5**

The solution set of $-3x + 6 \geq 0$ is:

A) $x \geq 2$

B) $x \leq 2$

C) $x \geq -2$

D) $x \leq -2$

---

**Exam 1 — Q6**

If $a = 3$ and $b = -2$, evaluate $a^2 - 2ab + b^2$.

A) 1

B) 25

C) 13

D) 17

---

**Exam 1 — Q7**

Solve the simultaneous equations $x + y = 5$ and $x - y = 1$.

A) $x = 3,\ y = 2$

B) $x = 2,\ y = 3$

C) $x = 4,\ y = 1$

D) $x = 1,\ y = 4$

---

### Exam 1 — Q8

The expression $\frac{4^{x+1}}{2^{x-1}}$ simplifies to:

A) $2^{x+3}$

B) $2^{x+1}$

C) $4^{x+2}$

D) $8^{x}$

---

### Exam 1 — Q9

Solve for $x$: $|x-3|=5$.

A) $x=8$ or $x=2$

B) $x=8$ only

C) $x=-2$ or $x=8$

D) $x=2$ only

---

### Exam 1 — Q10

Which expression is equivalent to $(x+2)(x-3)-x(x+3)$?

A) $-4x-6$

B) $-4x+6$

C) $2x-6$

D) $2x+6$

---

### Exam 1 — Q11

The 15th term of the AP $3,\ 7,\ 11,\ \ldots$ is:

A) 57

B) 59

C) 55

D) 61

---

**Exam 1 — Q12**

A GP has $T_3 = 18$ and common ratio $r = 3$. Find $T_1$.

A) 3

B) 2

C) 6

D) 9

---

**Exam 1 — Q13**

Evaluate $\sum_{k=1}^{5}(2k-1)$.

A) 25

B) 20

C) 15

D) 30

---

**Exam 1 — Q14**

An infinite GP has $a = 10$ and $r = \frac{1}{5}$. Find $S_\infty$.

A) 25

B) 50

C) 12.5

D) 2

---

**Exam 1 — Q15**

The sum $S_n = \frac{n}{2}(2a + (n-1)d)$ for the AP $6,\ 10,\ 14,\ \ldots$ The value of $S_8$ is:

A) 148

B) 160

C) 144

D) 128

---

**Exam 1 — Q16**

The graph of $f(x) = x^2 - 4x + 3$ cuts the $x$-axis at:

A) $x = 1$ and $x = 3$

B) $x = -1$ and $x = -3$

C) $x = 2$ and $x = 3$

D) $x = 1$ and $x = 4$

---

**Exam 1 — Q17**

The turning point of $g(x) = -(x - 2)^2 + 5$ is:

A) $(2,\ -5)$

B) $(-2,\ 5)$

C) $(2,\ 5)$

D) $(-2,\ -5)$

---

**Exam 1 — Q18**

The domain of $f(x) = \dfrac{1}{x - 3}$ is:

A) $x \in \mathbb{R}$

B) $x \neq 0$

C) $x > 3$

D) $x \neq 3$

---

**Exam 1 — Q19**

The graph of $f(x) = 2^x$ passes through which point?

A) $(0,\ 2)$

B) $(1,\ 0)$

C) $(0,\ 1)$

D) $(2,\ 0)$

---

### Exam 1 — Q20

A function $f$ has the property $f(x) = f(-x)$ for all $x$. This means $f$ is:

A) Strictly increasing

B) Odd

C) Even

D) Linear

---

### Exam 1 — Q21

The line $y = 2x - 3$ has gradient:

A) $-3$

B) $3$

C) $-2$

D) $2$

---

### Exam 1 — Q22

The graph of $y = -\dfrac{2}{x}$ is a hyperbola in quadrants:

A) I and II

B) I and III

C) II and IV

D) II and III

---

### Exam 1 — Q23

For $f(x) = 3x - 6$, find $f^{-1}(x)$.

A) $\dfrac{x-6}{3}$

B) $\dfrac{x+6}{3}$

C) $\dfrac{x}{3} + 6$

D) $3x + 6$

---

### Exam 1 — Q24

The $y$-intercept of $y = \log_2(x+4)$ is:

A) $y = 4$

B) $y = 2$

C) $y = 0$

D) $y = \log_2 4$

---

### Exam 1 — Q25

Which of the following is a function?

A) $x^2 + y^2 = 4$

B) $y = \pm\sqrt{x}$

C) $y = x^2$

D) $x = y^2$

---

### Exam 1 — Q26

Which of the following is irrational?

A) $\sqrt{49}$

B) $\frac{22}{7}$

C) $\sqrt{5}$

D) $0.\overline{6}$

---

### Exam 1 — Q27

Express $0.0045 \times 10^6$ in standard form.

A) $4.5 \times 10^3$

B) $4.5 \times 10^2$

C) $4.5 \times 10^4$

D) $0.45 \times 10^4$

---

**Exam 1 — Q28**

Simplify $\sqrt{50} - \sqrt{18}$.

A) $\sqrt{32}$

B) $3\sqrt{2}$

C) $2\sqrt{2}$

D) $4\sqrt{2}$

---

**Exam 1 — Q29**

R8 000 is invested at 10 % p.a. compound interest for 2 years. The accumulated amount is:

A) R9 600

B) R9 680

C) R10 000

D) R9 728

---

**Exam 1 — Q30**

A car bought for R120 000 depreciates at 20 % p.a. (reducing balance). Its value after 3 years is:

A) R61 440

B) R72 000

C) R57 600

D) R48 000

---

**Exam 1 — Q31**

The ratio $\sqrt{8} : \sqrt{2}$ in simplest form is:

A) $4 : 1$

B) $2 : 1$

C) $8 : 2$

D) $\sqrt{4} : 1$

---

### Exam 1 — Q32

Elon exchanges R10 000 to US dollars at a rate of R18.50 per dollar. He receives approximately:

A) \$185 000

B) \$541

C) \$500

D) \$1 850

---

### Exam 1 — Q33

Which of the following represents a 15 % increase on R240?

A) R255

B) R276

C) R252

D) R264

---

### Exam 1 — Q34

In right triangle $ABC$, $\sin A = \frac{3}{5}$. Find $\cos A$.

A) $\frac{4}{5}$

B) $\frac{3}{4}$

C) $\frac{5}{3}$

D) $\frac{5}{4}$

---

### Exam 1 — Q35

$\tan 45^\circ \times \cos 60^\circ =$

A) $\frac{1}{2}$

B) $\frac{\sqrt{2}}{2}$

C) $1$

D) $\frac{\sqrt{3}}{2}$

---

### Exam 1 — Q36

The general solution of $\sin\theta = \frac{1}{2}$ is:

A) $\theta = 30^\circ + 360^\circ n$

B) $\theta = 30^\circ + 180^\circ n$

C) $\theta = 30^\circ + 360^\circ n$ or $\theta = 150^\circ + 360^\circ n$

D) $\theta = 150^\circ + 360^\circ n$

---

### Exam 1 — Q37

A triangle has sides 5, 12, 13. Which angle is 90°?

A) The angle opposite the side of length 5

B) The angle opposite the side of length 12

C) The angle opposite the side of length 13

D) All angles are equal to 60°

---

### Exam 1 — Q38

Point $P(3,\ 4)$ lies on a circle centred at the origin. The radius is:

A) 7

B) 25

C) 4

D) 5

---

### Exam 1 — Q39

The value of $\sin^2 30^\circ + \cos^2 30^\circ$ is:

A) $\frac{3}{4}$

B) $\frac{1}{2}$

C) $\frac{7}{4}$

D) 1

---

**Exam 1 — Q40**

In a regular hexagon, each interior angle measures:

A) $60^\circ$

B) $108^\circ$

C) $150^\circ$

D) $120^\circ$

---

**Exam 1 — Q41**

Two triangles are similar. Their perimeters are 18 cm and 30 cm. If the shorter triangle has an area of $27 \text{ cm}^2$, what is the area of the larger triangle?

A) $45 \text{ cm}^2$

B) $54 \text{ cm}^2$

C) $75 \text{ cm}^2$

D) $108 \text{ cm}^2$

---

**Exam 1 — Q42**

A sphere has radius 3 cm. Its volume is:

A) $9\pi \text{ cm}^3$

B) $36\pi \text{ cm}^3$

C) $27\pi \text{ cm}^3$

D) $12\pi \text{ cm}^3$

---

**Exam 1 — Q43**

The midpoint of the line segment joining $(-2,\ 6)$ and $(4,\ -2)$ is:

A) $(1,\ 2)$

B) $(3,\ 4)$

C) $(2,\ 4)$

D) $(1,\ 4)$

---

**Exam 1 — Q44**

An angle in a semicircle is always:

A) $45^\circ$

B) Obtuse

C) $90^\circ$

D) $60^\circ$

---

**Exam 1 — Q45**

The gradient of a line perpendicular to $y = \frac{2}{3}x + 1$ is:

A) $\frac{2}{3}$

B) $-\frac{3}{2}$

C) $\frac{3}{2}$

D) $-\frac{2}{3}$

---

**Exam 1 — Q46**

The dataset $\{4,\ 7,\ 7,\ 9,\ 13\}$ has a mean of:

A) 7

B) 8

C) 9

D) 10

---

**Exam 1 — Q47**

The interquartile range of $\{2,\ 5,\ 8,\ 10,\ 14,\ 18\}$ is:

A) 9

B) 16

C) 8

D) 12

### Exam 1 — Q48

A bag contains 3 red and 7 blue marbles. A marble is drawn at random. $P(\text{red}) =$

A) $\frac{7}{10}$

B) $\frac{3}{7}$

C) $\frac{3}{10}$

D) $\frac{7}{3}$

### Exam 1 — Q49

Events $A$ and $B$ are mutually exclusive. $P(A) = 0.3$ and $P(B) = 0.5$. Find $P(A \cup B)$.

A) $0.15$

B) $0.65$

C) $0.8$

D) $1.0$

### Exam 1 — Q50

In how many ways can 4 books be arranged on a shelf?

A) 4

B) 16

C) 24

D) 12

### Exam 1 — Q51

The standard deviation measures:

A) The middle value of a dataset

B) The most frequent value

C) The spread of data around the mean

D) The difference between the largest and smallest values

---

#### Exam 1 — Q52

From a class of 30 students, 12 play sport and 10 study music. 5 do both. How many do neither?

A) 8

B) 12

C) 13

D) 17

---

#### Exam 1 — Q53

A fair coin is tossed 3 times. The probability of getting exactly 2 heads is:

A) $\frac{1}{4}$

B) $\frac{3}{8}$

C) $\frac{1}{2}$

D) $\frac{1}{8}$

---

#### Exam 1 — Q54

The range of $\{3, 8, 2, 15, 7, 1\}$ is:

A) 7

B) 13

C) 14

D) 15

---

#### Exam 1 — Q55

The cumulative frequency at the upper boundary of a class interval represents:

A) The frequency of that class only

B) The total frequency up to and including that class

C) The mean of that class

D) The percentage in that class

---

**Exam 1 — Q56**

The gradient of the line joining $(1,\ 3)$ and $(5,\ 11)$ is:

A) 1

B) 3

C) 2

D) 4

---

**Exam 1 — Q57**

Solve $3(x-2)=2(x+1)$.

A) $x=8$

B) $x=7$

C) $x=4$

D) $x=2$

---

**Exam 1 — Q58**

The value of $\log_3 81$ is:

A) 3

B) 27

C) 9

D) 4

---

**Exam 1 — Q59**

Which transformation maps $f(x)=x^2$ to $g(x)=x^2+3$?

A) Horizontal shift 3 units right

B) Vertical stretch by factor 3

C) Vertical shift 3 units up

D) Horizontal shift 3 units left

---

**Exam 1 — Q60**

A rectangle has length $(x+3)$ cm and width $(x-1)$ cm. Its area in expanded form is:

A) $x^2+2x-3$

B) $x^2-2x+3$

C) $2x+2$

D) $x^2+4x-3$

## Mock Exam 1 — Answer Key

| Q | Ans | Q | Ans | Q | Ans | Q | Ans | Q | Ans |
|---|---|---|---|---|---|---|---|---|---|
| 1 | B | 2 | C | 3 | B | 4 | A | 5 | B |
| 6 | B | 7 | A | 8 | A | 9 | C | 10 | A |
| 11 | B | 12 | B | 13 | A | 14 | C | 15 | B |
| 16 | A | 17 | C | 18 | D | 19 | C | 20 | C |
| 21 | D | 22 | C | 23 | B | 24 | B | 25 | C |
| 26 | C | 27 | A | 28 | C | 29 | B | 30 | A |
| 31 | B | 32 | B | 33 | B | 34 | A | 35 | A |
| 36 | C | 37 | C | 38 | D | 39 | D | 40 | D |
| 41 | C | 42 | B | 43 | A | 44 | C | 45 | B |
| 46 | B | 47 | A | 48 | C | 49 | C | 50 | C |
| 51 | C | 52 | C | 53 | B | 54 | C | 55 | B |
| 56 | C | 57 | A | 58 | D | 59 | C | 60 | A |

# Mock Exam 2

### Exam 2 — Q1

Simplify $\frac{2x^2 - 8}{x - 2}$.

A) $2x - 4$

B) $x + 2$

C) $2(x + 2)$

D) $2x + 4$

---

### Exam 2 — Q2

Solve $x^2 - 5x + 6 = 0$.

A) $x = 1$ or $x = 6$

B) $x = -2$ or $x = -3$

C) $x = 2$ or $x = 3$

D) $x = -1$ or $x = -6$

---

### Exam 2 — Q3

Which is the correct factorisation of $6x^2 - x - 2$?

A) $(3x + 2)(2x - 1)$

B) $(6x + 1)(x - 2)$

C) $(2x + 1)(3x - 2)$

D) $(3x - 1)(2x + 2)$

---

### Exam 2 — Q4

Solve for $y$: $\dfrac{2y-1}{3} = 5$.

A) $y = 4$

B) $y = 7$

C) $y = 9$

D) $y = 8$

---

### Exam 2 — Q5

The inequality $x^2 - 4 < 0$ is satisfied by:

A) $x > 2$

B) $x < -2$

C) $-2 < x < 2$

D) $x < -2$ or $x > 2$

---

### Exam 2 — Q6

Simplify: $\dfrac{a^3b^2}{a^2b^4}$.

A) $ab^2$

B) $\dfrac{a}{b^2}$

C) $\dfrac{b^2}{a}$

D) $a^2b^{-2}$

---

### Exam 2 — Q7

Solve $2|x| - 3 = 7$.

A) $x = 5$

B) $x = \pm 5$

C) $x = -5$

D) $x = 2$

---

### Exam 2 — Q8

The product $(a-b)^2$ expands to:

A) $a^2+b^2$

B) $a^2-b^2$

C) $a^2+2ab+b^2$

D) $a^2-2ab+b^2$

---

### Exam 2 — Q9

For which values of $x$ is $\dfrac{x-1}{x^2-4}$ undefined?

A) $x=1$

B) $x=0$

C) $x=\pm 2$

D) $x=1$ and $x=2$

---

### Exam 2 — Q10

Solve the system: $2x+y=10$ and $x-y=2$.

A) $x=3,\ y=4$

B) $x=5,\ y=0$

C) $x=4,\ y=2$

D) $x=6,\ y=-2$

---

### Exam 2 — Q11

The AP $2,\ 6,\ 10,\ 14,\ \ldots$ has $S_{10}$ equal to:

A) 160

B) 200

C) 140

D) 120

---

**Exam 2 — Q12**

In a GP, $T_2 = 6$ and $T_5 = 162$. Find the common ratio.

A) 2

B) 4

C) 3

D) 6

---

**Exam 2 — Q13**

The sum $\sum_{k=2}^{5} k^2$ equals:

A) 50

B) 54

C) 30

D) 55

---

**Exam 2 — Q14**

Insert 3 arithmetic means between 5 and 21. The three values in order are:

A) 8, 12, 16

B) 9, 13, 17

C) 7, 11, 15

D) 9, 12, 15

---

**Exam 2 — Q15**

An infinite GP converges to 36 with $r = \frac{2}{3}$. Find the first term.

A) 18

B) 24

C) 12

D) 30

---

### Exam 2 — Q16

Which of the following is the graph of an exponential decay function?

A) $y = 2^x$

B) $y = x^2$

C) $y = \left(\frac{1}{3}\right)^x$

D) $y = 2x$

---

### Exam 2 — Q17

The axis of symmetry of $f(x) = x^2 - 6x + 8$ is:

A) $x = -3$

B) $x = 6$

C) $x = 4$

D) $x = 3$

---

### Exam 2 — Q18

The range of $f(x) = -x^2 + 4$ is:

A) $y \geq 4$

B) $y \leq -4$

C) $y \leq 4$

D) $y \geq -4$

---

### Exam 2 — Q19

The equation of the asymptote of $y = \dfrac{3}{x-2} + 1$ is:

A) $y = 2$ and $x = 1$

B) $x = -2$ and $y = -1$

C) $x = 2$ and $y = 1$

D) $x = 3$ and $y = 0$

---

**Exam 2 — Q20**

If $f(x) = 2x + 1$ and $g(x) = x^2$, find $f(g(3))$.

A) 37

B) 49

C) 19

D) 25

---

**Exam 2 — Q21**

The graph of $y = \log_3 x$ passes through:

A) $(0,\ 1)$

B) $(1,\ 3)$

C) $(3,\ 1)$

D) $(3,\ 0)$

---

**Exam 2 — Q22**

A line through $(0,\ -2)$ with gradient 3 has equation:

A) $y = 3x$

B) $y = 3x + 2$

C) $y = 3x - 2$

D) $y = -2x + 3$

---

**Exam 2 — Q23**

The distance between $(-3,\ 0)$ and $(1,\ 3)$ is:

A) 5

B) 4

C) 7

D) 3

---

**Exam 2 — Q24**

Which function has no $x$-intercept?

A) $y = x - 5$

B) $y = x^2 - 4$

C) $y = 2^x$

D) $y = x^2 - 9$

---

**Exam 2 — Q25**

Simplify $\log_2 8 + \log_2 4$.

A) 5

B) 7

C) $\log_2 32$

D) Both A and C

---

**Exam 2 — Q26**

$\frac{3}{4}$ expressed as a percentage is:

A) $34\%$

B) $0.75\%$

C) $75\%$

D) $43\%$

---

**Exam 2 — Q27**

Rationalise $\frac{6}{\sqrt{3}}$.

A) $\frac{6\sqrt{3}}{3}$

B) $2\sqrt{3}$

C) $\sqrt{3}$

D) $3\sqrt{3}$

---

### Exam 2 — Q28

R15 000 earns simple interest of 8 % p.a. for 3 years. Total interest earned is:

A) R1 200

B) R3 600

C) R4 800

D) R2 400

---

### Exam 2 — Q29

A machine worth R60 000 depreciates at 10 % p.a. straight-line for 5 years. Its book value is:

A) R6 000

B) R30 000

C) R54 000

D) R36 000

---

### Exam 2 — Q30

Which of the following is a perfect square?

A) 50

B) 75

C) 144

D) 200

---

### Exam 2 — Q31

The exchange rate is R18 per US dollar. Convert \$250 to rands.

A) R250

B) R450

C) R4 500

D) R3 600

---

### Exam 2 — Q32

A price of R600 is increased by 25 %. The new price is:

A) R150

B) R750

C) R625

D) R800

---

### Exam 2 — Q33

Evaluate $3^{-2} + \left(\frac{1}{2}\right)^{-1}$.

A) $\frac{17}{9}$

B) $\frac{19}{9}$

C) $\frac{7}{9}$

D) $\frac{5}{9}$

---

### Exam 2 — Q34

$\sin 60^\circ =$

A) $\frac{1}{2}$

B) $\frac{\sqrt{3}}{2}$

C) $\frac{\sqrt{2}}{2}$

D) 1

---

**Exam 2 — Q35**

If $\cos\theta = -\dfrac{1}{2}$ and $\theta \in [0^\circ,\ 360^\circ]$, find all values of $\theta$.

A) $\theta = 60^\circ$ only

B) $\theta = 120^\circ$ or $\theta = 240^\circ$

C) $\theta = 120^\circ$ only

D) $\theta = 60^\circ$ or $\theta = 300^\circ$

---

**Exam 2 — Q36**

In $\triangle PQR$, $PQ = 10$, $QR = 6$, and $\angle Q = 90^\circ$. Find $PR$.

A) 4

B) 16

C) $\sqrt{136}$

D) 8

---

**Exam 2 — Q37**

Which of the following is an identity?

A) $\sin\theta = \cos\theta$

B) $\tan\theta = \dfrac{\cos\theta}{\sin\theta}$

C) $\sin^2\theta + \cos^2\theta = 1$

D) $\tan^2\theta = 1$

---

**Exam 2 — Q38**

In $\triangle ABC$, $a = 8$, $b = 6$, $\angle C = 90^\circ$. Find $\tan A$.

A) $\dfrac{3}{4}$

B) $\dfrac{4}{3}$

C) $\dfrac{3}{5}$

D) $\dfrac{4}{5}$

---

**Exam 2 — Q39**

Two parallel lines are cut by a transversal. Alternate interior angles are:

A) Supplementary

B) Equal

C) Complementary

D) Vertically opposite

---

**Exam 2 — Q40**

The exterior angle of a triangle equals:

A) The largest interior angle

B) The sum of all interior angles

C) The sum of the two non-adjacent interior angles

D) $90^\circ$

---

**Exam 2 — Q41**

A rectangle is 12 cm by 5 cm. What is the length of its diagonal?

A) 17 cm

B) 60 cm

C) 13 cm

D) 7 cm

---

**Exam 2 — Q42**

The total surface area of a cube with side 4 cm is:

A) 64 $cm^2$

B) 24 $cm^2$

C) 16 $cm^2$

D) 96 $cm^2$

---

**Exam 2 — Q43**

The line segment from the centre of a circle to a point where a tangent touches it is:

A) A chord

B) An arc

C) Parallel to the tangent

D) Perpendicular to the tangent

---

**Exam 2 — Q44**

The gradient of the line segment joining $(2,\ 5)$ and $(6,\ 13)$ is:

A) 3

B) 4

C) 2

D) 1

---

**Exam 2 — Q45**

A regular polygon has interior angle of $108^\circ$. It is a:

A) Hexagon

B) Heptagon

C) Pentagon

D) Octagon

---

**Exam 2 — Q46**

The mean of a dataset is 12 and the median is 9. The data is:

A) Symmetrical

B) Skewed to the left (negatively skewed)

C) Skewed to the right (positively skewed)

D) Bimodal

---

### Exam 2 — Q47

A die is thrown once. $P(\text{even number}) =$

A) $\frac{1}{6}$
B) $\frac{1}{3}$
C) $\frac{2}{3}$
D) $\frac{1}{2}$

---

### Exam 2 — Q48

Two events $A$ and $B$ are independent. $P(A) = 0.4$ and $P(B) = 0.5$. Find $P(A \cap B)$.

A) 0.9

B) 0.1

C) 0.2

D) 0.8

---

### Exam 2 — Q49

Given the frequency table below, the modal class is 10–20.

| Class | Frequency |
|---|---|
| 0–10 | 4 |
| 10–20 | 11 |
| 20–30 | 8 |
| 30–40 | 3 |

The relative frequency of the 10–20 class is:

A) $\frac{11}{26}$
B) $\frac{11}{22}$
C) $\frac{11}{30}$
D) $\frac{11}{15}$

**Exam 2 — Q50**

In how many ways can a president and a secretary be chosen from a group of 8 people?

A) 8

B) 16

C) 64

D) 56

**Exam 2 — Q51**

A box contains 5 red, 3 blue, and 2 green pens. A pen is chosen at random. $P(\text{not red}) =$

A) $\frac{1}{2}$

B) $\frac{3}{5}$

C) $\frac{1}{5}$

D) $\frac{2}{5}$

**Exam 2 — Q52**

The variance of $\{2,\ 4,\ 4,\ 4,\ 5,\ 5,\ 7,\ 9\}$ requires first finding the mean. The mean is:

A) 4

B) 4.5

C) 5

D) 6

**Exam 2 — Q53**

Which type of graph is best for showing how a quantity changes over time?

A) Bar chart

B) Pie chart

C) Line graph

D) Histogram

**Exam 2 — Q54**

$P(A') = 0.35$. Find $P(A)$.

A) $0.35$

B) $1.35$

C) $0.65$

D) $0.55$

**Exam 2 — Q55**

A student scores 60 out of 80. Express this as a percentage.

A) $60\%$

B) $80\%$

C) $75\%$

D) $70\%$

**Exam 2 — Q56**

The $n$th term of the sequence $1,\ 4,\ 9,\ 16,\ 25,\ \ldots$ is:

A) $2n - 1$

B) $n + 3$

C) $n^2$

D) $3n - 2$

**Exam 2 — Q57**

Solve $4^x = 8$.

A) $x = 2$

B) $x = \frac{3}{2}$

C) $x = \frac{1}{2}$

D) $x = 3$

### Exam 2 — Q58

The expression $\log_a(xy) =$

A) $\log_a x \cdot \log_a y$

B) $(\log_a x)(\log_a y)$

C) $\log_a x + \log_a y$

D) $\log_a x - \log_a y$

---

### Exam 2 — Q59

Evaluate $\frac{0!}{3!}$.

A) $0$

B) $\frac{1}{6}$

C) $\frac{1}{3}$

D) $3$

---

### Exam 2 — Q60

A line is perpendicular to $y = 4x - 1$ and passes through $(4,\ 2)$. Its equation is:

A) $y = 4x - 14$

B) $y = -\frac{1}{4}x + 3$

C) $y = \frac{1}{4}x + 1$

D) $y = -4x + 18$

---

## Mock Exam 2 — Answer Key

| Q | Ans | Q | Ans | Q | Ans | Q | Ans | Q | Ans |
|---|---|---|---|---|---|---|---|---|---|
| 1 | C | 2 | C | 3 | C | 4 | D | 5 | C |
| 6 | B | 7 | B | 8 | D | 9 | C | 10 | C |
| 11 | B | 12 | C | 13 | B | 14 | B | 15 | C |
| 16 | C | 17 | D | 18 | C | 19 | C | 20 | C |
| 21 | C | 22 | C | 23 | A | 24 | C | 25 | D |
| 26 | C | 27 | B | 28 | B | 29 | B | 30 | C |
| 31 | C | 32 | B | 33 | B | 34 | B | 35 | B |
| 36 | C | 37 | C | 38 | A | 39 | B | 40 | C |
| 41 | C | 42 | D | 43 | D | 44 | C | 45 | C |
| 46 | C | 47 | D | 48 | C | 49 | A | 50 | D |
| 51 | A | 52 | C | 53 | C | 54 | C | 55 | C |
| 56 | C | 57 | B | 58 | C | 59 | B | 60 | B |

# Mock Exam 3

### Exam 3 — Q1

Simplify $\dfrac{3x^2 - 12}{x + 2}$.

A) $3x + 6$

B) $3(x - 2)$

C) $3(x + 2)$

D) $x - 2$

---

### Exam 3 — Q2

Solve for $x$: $x^2 - 3x - 10 = 0$.

A) $x = 5$ or $x = -2$

B) $x = -5$ or $x = 2$

C) $x = 5$ or $x = 2$

D) $x = -5$ or $x = -2$

---

### Exam 3 — Q3

Which inequality is equivalent to $-4 \leq 2x < 6$?

A) $-2 < x \leq 3$

B) $-2 \leq x < 3$

C) $-8 \leq x < 12$

D) $2 \leq x < 10$

---

### Exam 3 — Q4

Simplify $\dfrac{x^2-1}{x^2+x}$.

A) $\dfrac{x-1}{x}$

B) $\dfrac{x+1}{x}$

C) $\dfrac{x-1}{x+1}$

D) $\dfrac{1}{x}$

---

### Exam 3 — Q5

Find the value of $p$ if $3^p = \dfrac{1}{27}$.

A) $p = 3$

B) $p = \dfrac{1}{3}$

C) $p = -\dfrac{1}{3}$

D) $p = -3$

---

### Exam 3 — Q6

The expression $\left(\dfrac{27}{8}\right)^{2/3}$ equals:

A) $\dfrac{9}{4}$

B) $\dfrac{3}{2}$

C) $\dfrac{27}{4}$

D) $\dfrac{4}{9}$

---

### Exam 3 — Q7

Solve for $x$: $\log_2 x = 5$.

A) $x = 10$

B) $x = 25$

C) $x = 32$

D) $x = 16$

---

### Exam 3 — Q8

The product $\frac{x+1}{x-2} \cdot \frac{x^2-4}{x^2-1}$ simplifies to:

A) $\frac{x+2}{x-1}$
B) $\frac{x-2}{x+1}$
C) $\frac{x+1}{x+2}$
D) $\frac{x-1}{x+2}$

---

### Exam 3 — Q9

Which value of $k$ makes $kx^2 - 3x + 1 = 0$ have equal roots?

A) $k = \frac{9}{4}$

B) $k = 3$

C) $k = 9$

D) $k = \frac{1}{4}$

---

### Exam 3 — Q10

Simplify $\frac{2^{n+2} - 2^n}{2^n}$.

A) $2^n$

B) $4$

C) $3$

D) $3 \cdot 2^n$

### Exam 3 — Q11

The AP $-5,\ -1,\ 3,\ 7,\ \ldots$ Find $T_{20}$.

A) 71

B) 75

C) 67

D) 79

### Exam 3 — Q12

A GP has $T_1 = 4$ and $T_4 = 32$. Find $r$.

A) $r = 4$

B) $r = 8$

C) $r = 2$

D) $r = 3$

### Exam 3 — Q13

Evaluate $\sum_{k=1}^{4} 3 \cdot 2^{k-1}$.

A) 24

B) 45

C) 48

D) 51

### Exam 3 — Q14

The sum of an infinite GP is 45 and the first term is 15. Find $r$.

A) $r = \dfrac{1}{4}$

B) $r = \dfrac{2}{3}$

C) $r = \dfrac{1}{2}$

D) $r = \dfrac{1}{3}$

---

### Exam 3 — Q15

The AP has $S_n = 4n^2 + 2n$. Find $T_1$.

A) 4

B) 8

C) 6

D) 2

---

### Exam 3 — Q16

The function $f(x) = \sqrt{x - 2}$ has domain:

A) $x > 2$

B) $x \geq 2$

C) $x \geq 0$

D) $x \neq 2$

---

### Exam 3 — Q17

The graph of $y = (x - 1)^2 - 9$ has $x$-intercepts at:

A) $x = 1 \pm 3$

B) $x = -1 \pm 3$

C) $x = 1 \pm 9$

D) $x = -9$ and $x = 1$

---

**Exam 3 — Q18**

For $h(x) = 2 \cdot 3^x$, find $h(2)$.

A) 36

B) 12

C) 18

D) 6

---

**Exam 3 — Q19**

The graph of $y = \dfrac{2}{x} - 3$ has a horizontal asymptote at:

A) $y = 2$

B) $y = 0$

C) $y = -3$

D) $x = 0$

---

**Exam 3 — Q20**

A function $f$ is increasing on its domain if, for $x_1 < x_2$:

A) $f(x_1) = f(x_2)$

B) $f(x_1) > f(x_2)$

C) $f(x_1) < f(x_2)$

D) $f(x_1) \geq f(x_2)$

---

**Exam 3 — Q21**

The equation $y = 3^x$ and its inverse are reflections in:

A) The $x$-axis

B) The line $y = -x$

C) The line $y = x$

D) The $y$-axis

---

**Exam 3 — Q22**

Which function has the smallest $y$-intercept?

A) $y = 2x + 3$

B) $y = x^2 + 1$

C) $y = 3 \cdot 2^x$

D) $y = -x + 5$

---

**Exam 3 — Q23**

The range of $f(x) = x^2 - 6x + 10$ is:

A) $y \geq 1$

B) $y \leq 1$

C) $y \geq 3$

D) $y \leq 10$

---

**Exam 3 — Q24**

$\log_5 25 - \log_5 5 =$

A) 0

B) 5

C) $\log_5 20$

D) 1

---

**Exam 3 — Q25**

Find the equation of the line through $(2,\ 3)$ with gradient $-2$.

A) $y = -2x + 7$

B) $y = -2x - 1$

C) $y = 2x - 1$

D) $y = 2x + 7$

---

**Exam 3 — Q26**

$\sqrt{3} \times \sqrt{12} =$

A) $\sqrt{36}$

B) 6

C) $\sqrt{15}$

D) Both A and B

---

**Exam 3 — Q27**

Simplify $\dfrac{\sqrt{48}}{\sqrt{3}}$.

A) $\sqrt{16}$

B) 16

C) 4

D) Both A and C

---

**Exam 3 — Q28**

R20 000 is invested at 15 % p.a. compound interest for 3 years. The accumulated amount is:

A) R29 000

B) R30 418

C) R30 000

D) R23 000

---

**Exam 3 — Q29**

Inflation averages 6 % p.a. A grocery basket currently costs R500. Its cost in 2 years will be approximately:

A) R560

B) R530

C) R562

D) R556

---

### Exam 3 — Q30

Which of the following is rational?

A) $\sqrt{7}$

B) $\pi$

C) $\sqrt[3]{27}$

D) $\sqrt{2}$

---

### Exam 3 — Q31

R5 000 is borrowed at 12 % p.a. simple interest. The total repayment after 2.5 years is:

A) R6 500

B) R6 000

C) R6 200

D) R5 600

---

### Exam 3 — Q32

A car bought for R200 000 depreciates at 15 % p.a. compound. After 2 years its value is:

A) R140 000

B) R145 000

C) R144 500

D) R138 000

---

### Exam 3 — Q33

The value of $\cos 0^\circ$ is:

A) 0

B) Undefined

C) $\frac{1}{2}$

D) 1

---

**Exam 3 — Q34**

$\sin(90^\circ - \theta) =$

A) $-\sin\theta$

B) $\cos\theta$

C) $-\cos\theta$

D) $\sin\theta$

---

**Exam 3 — Q35**

In $\triangle ABC$, $AB = 10$, $BC = 6$, $\angle C = 90^\circ$. Find $\sin A$.

A) $\frac{3}{4}$

B) $\frac{3}{5}$

C) $\frac{4}{5}$

D) $\frac{5}{3}$

---

**Exam 3 — Q36**

The period of $y = \sin 2x$ is:

A) $360^\circ$

B) $720^\circ$

C) $90^\circ$

D) $180^\circ$

---

**Exam 3 — Q37**

A ladder 10 m long leans against a wall making a $60^\circ$ angle with the ground. The height it reaches is:

A) $5\sqrt{3}$ m

B) $5$ m

C) $10\sqrt{3}$ m

D) $5\sqrt{2}$ m

---

**Exam 3 — Q38**

A tangent and a radius meet at the point of tangency at:

A) $45^\circ$

B) $60^\circ$

C) $90^\circ$

D) $180^\circ$

---

**Exam 3 — Q39**

The sum of interior angles of a pentagon is:

A) $360^\circ$

B) $720^\circ$

C) $540^\circ$

D) $450^\circ$

---

**Exam 3 — Q40**

Two similar triangles have corresponding sides in ratio $3 : 5$. Their areas are in ratio:

A) $3 : 5$

B) $6 : 10$

C) $9 : 25$

D) $27 : 125$

---

**Exam 3 — Q41**

A cylinder has radius 3 cm and height 8 cm. Its volume is:

A) $24\pi\ \text{cm}^3$

B) $48\pi\ \text{cm}^3$

C) $72\pi\ \text{cm}^3$

D) $96\pi\ \text{cm}^3$

---

**Exam 3 — Q42**

A chord subtends a central angle of $120^\circ$ in a circle of radius 6. The chord length is:

A) $6\sqrt{2}$

B) $3\sqrt{3}$

C) $6\sqrt{3}$

D) 6

---

**Exam 3 — Q43**

The $y$-intercept of the line joining $(-2,\ 4)$ and $(4,\ 1)$ is:

A) $y = 3$

B) $y = 4$

C) $y = 2$

D) $y = 5$

---

**Exam 3 — Q44**

A right circular cone has base radius 4 cm and slant height 5 cm. Its curved surface area is:

A) $20\pi\ \text{cm}^2$

B) $16\pi\ \text{cm}^2$

C) $40\pi\ \text{cm}^2$

D) $25\pi\ \text{cm}^2$

---

**Exam 3 — Q45**

A point lies at $(-4,\ 3)$. Its distance from the origin is:

A) 7

B) 1

C) 5

D) $\sqrt{7}$

---

**Exam 3 — Q46**

The five-number summary of a dataset is: minimum $= 5$, $Q_1 = 12$, median $= 18$, $Q_3 = 25$, maximum $= 40$. The IQR is:

A) 13

B) 35

C) 18

D) 7

---

**Exam 3 — Q47**

P(drawing a heart from a standard deck) = ?

A) $\frac{1}{52}$
B) $\frac{1}{4}$
C) $\frac{13}{52}$
D) Both B and C

---

**Exam 3 — Q48**

A survey of 50 people: 30 like tea, 25 like coffee, and 10 like both. How many like neither?

A) 5

B) 45

C) 10

D) 15

---

**Exam 3 — Q49**

In a dataset, if the mean equals the median, the distribution is:

A) Skewed right

B) Skewed left

C) Bimodal

D) Symmetrical

---

**Exam 3 — Q50**

Two letters are chosen from $\{A, B, C, D\}$ without repetition. The number of ordered pairs is:

A) 4

B) 6

C) 8

D) 12

---

**Exam 3 — Q51**

A standard deviation of 0 means:

A) The mean is 0

B) All data values are equal

C) There are no data values

D) The data is evenly spread

---

**Exam 3 — Q52**

The probability that it rains on any given day is 0.3. The probability it does NOT rain two days in a row is:

A) 0.42

B) 0.49

C) 0.09

D) 0.6

---

**Exam 3 — Q53**

In a histogram, the area of each bar is proportional to the:

A) Class width

B) Class midpoint

C) Frequency

D) Cumulative frequency

---

### Exam 3 — Q54

Given the data set $\{5,\ 5,\ 6,\ 7,\ 9,\ 10\}$, the mode is:

A) 6

B) 7

C) 5

D) 6.5

---

### Exam 3 — Q55

A two-way table shows: P(male and passes) = 0.35, P(male) = 0.50. If gender and result are independent, P(passes) = ?

A) 0.35

B) 0.65

C) 0.70

D) 0.50

---

### Exam 3 — Q56

Solve $\sqrt{2x-3} = 5$.

A) $x = 14$

B) $x = 4$

C) $x = 7$

D) $x = 28$

---

### Exam 3 — Q57

Which of the following is NOT a real number?

A) $\sqrt{-4}$

B) $-\sqrt{4}$

C) $\sqrt[3]{-8}$

D) $\sqrt{0}$

---

**Exam 3 — Q58**

The equation of a circle with centre $(2,\ -3)$ and radius 5 is:

A) $(x-2)^2+(y+3)^2=25$

B) $(x+2)^2+(y-3)^2=25$

C) $(x-2)^2+(y-3)^2=5$

D) $(x-2)^2+(y+3)^2=5$

---

**Exam 3 — Q59**

$4!-3!=$

A) 1

B) 3

C) 18

D) 20

---

**Exam 3 — Q60**

The graph of $y=|x-2|$ has a vertex (lowest point) at:

A) $(0,\ 2)$

B) $(2,\ 2)$

C) $(0,\ -2)$

D) $(2,\ 0)$

---

## Mock Exam 3 — Answer Key

| Q | Ans | Q | Ans | Q | Ans | Q | Ans | Q | Ans |
|---|---|---|---|---|---|---|---|---|---|
| 1 | B | 2 | A | 3 | B | 4 | A | 5 | D |
| 6 | A | 7 | C | 8 | A | 9 | A | 10 | C |
| 11 | A | 12 | C | 13 | B | 14 | B | 15 | C |
| 16 | B | 17 | A | 18 | C | 19 | C | 20 | C |
| 21 | C | 22 | B | 23 | A | 24 | D | 25 | A |
| 26 | D | 27 | D | 28 | B | 29 | C | 30 | C |
| 31 | A | 32 | C | 33 | D | 34 | B | 35 | B |
| 36 | D | 37 | A | 38 | C | 39 | C | 40 | C |
| 41 | C | 42 | C | 43 | A | 44 | A | 45 | C |
| 46 | A | 47 | D | 48 | A | 49 | D | 50 | D |
| 51 | B | 52 | B | 53 | C | 54 | C | 55 | C |
| 56 | A | 57 | A | 58 | A | 59 | C | 60 | D |

# Chapter 10 — Differential Calculus

Calculus is the mathematics of change. Where algebra describes static relationships, calculus describes how things move, grow, and optimise. On the NBT, calculus questions test whether you understand the *meaning* of a derivative — not just how to mechanically apply the power rule. A student who understands that the derivative measures the gradient of a curve at any point will outperform a student who has only memorised rules.

**Topics covered:** limits · first principles · power rule · sum and difference rules · simplify before differentiating · gradient of a tangent · equation of a tangent · equation of a normal · stationary points · nature of turning points · second derivative · point of inflection · cubic functions and graphs · increasing and decreasing intervals · optimisation

---

## Section 10.1 — Limits and First Principles

The derivative is defined as a limit. The gradient of a straight line is $\frac{\Delta y}{\Delta x}$. For a curve, the gradient changes at every point. To find it at a specific point, we shrink the gap $h$ between two points on the curve until it approaches zero:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

This is the **definition from first principles**. Everything else in differentiation follows from this one idea.

---

### Q1 — Basic

Which expression represents the derivative of $f(x)$ from first principles?

A) $\lim_{h \to \infty} \frac{f(x+h) - f(x)}{h}$

B) $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

C) $\lim_{h \to 0} \frac{f(x+h) + f(x)}{h}$

D) $\lim_{h \to 0} \frac{f(x) - f(x+h)}{x}$

---

### Q2 — Basic

Use first principles to find $f'(x)$ if $f(x) = x^2$.

A) $x$

B) $2x$

C) $2x + h$

D) $x^2$

---

### Q3 — Intermediate

Use first principles to find $f'(x)$ if $f(x) = 3x^2 - 2x$.

A) $6x - 2$

B) $3x - 2$

C) $6x + 2$

D) $6x$

---

### Q4 — Intermediate

The expression $\lim_{h \to 0} \dfrac{(x+h)^3 - x^3}{h}$ equals:

A) $x^2$

B) $2x^2$

C) $3x^2$

D) $3x^3$

---

## Section 10.2 — Rules of Differentiation

First principles works for any function but is slow. The **rules of differentiation** give the same result in seconds.

| Rule | Formula |
| --- | --- |
| Power rule | $\frac{d}{dx}[x^n] = nx^{n-1}$ |
| Constant | $\frac{d}{dx}[c] = 0$ |
| Constant multiple | $\frac{d}{dx}[cf(x)] = c \cdot f'(x)$ |
| Sum/difference | $\frac{d}{dx}[f \pm g] = f' \pm g'$ |

**Note:** Always simplify the expression before differentiating. Fractions, brackets, and roots must be rewritten in $ax^n$ form first.

---

### Q5 — Basic

If $f(x) = x^5$, then $f'(x) =$

A) $5x^4$

B) $x^4$

C) $5x^6$

D) $4x^5$

---

### Q6 — Basic

If $g(x) = 4x^3 - 3x^2 + 7x - 5$, then $g'(x) =$

A) $12x^2 - 6x + 7$

B) $12x^2 - 6x$

C) $12x^3 - 6x^2 + 7$

D) $4x^2 - 3x + 7$

---

### Q7 — Intermediate

Find $f'(x)$ if $f(x) = \dfrac{3x^3 - 6x}{3x}$.

A) $2x$

B) $x - 1$

C) $2x - 1$

D) $x^2 - 1$

---

### Q8 — Intermediate

Find $f'(x)$ if $f(x) = \dfrac{4}{x^3}$.

A) $\dfrac{-12}{x^4}$

B) $\dfrac{12}{x^4}$

C) $\dfrac{-4}{x^2}$

D) $\dfrac{-12}{x^2}$

---

### Q9 — Intermediate

If $f(x) = \sqrt{x}$, then $f'(x) =$

A) $\dfrac{1}{\sqrt{x}}$

B) $\dfrac{1}{2\sqrt{x}}$

C) $2\sqrt{x}$

D) $\dfrac{-1}{2\sqrt{x}}$

---

### Q10 — Proficient

Find $h'(x)$ if $h(x) = (2x - 1)^2$.

A) $2(2x - 1)$

B) $8x$

C) $4x - 2$

D) $8x - 4$

## Section 10.3 — Tangent and Normal Lines

The derivative $f'(a)$ gives the **gradient of the tangent** to the curve $y = f(x)$ at $x = a$. The **normal** at that point is perpendicular to the tangent, so its gradient is $-\frac{1}{f'(a)}$.

### Q11 — Basic

Find the gradient of the tangent to $f(x) = x^2 - 4x + 3$ at $x = 3$.

A) 0

B) 2

C) 6

D) $-2$

### Q12 — Basic

Find the equation of the tangent to $y = x^3$ at the point where $x = 1$.

A) $y = x + 1$

B) $y = 3x - 2$

C) $y = 3x + 1$

D) $y = x - 2$

### Q13 — Intermediate

At which $x$-value does $f(x) = x^3 - 6x^2 + 9x$ have a gradient of 9?

A) $x = 0$ or $x = 4$

B) $x = 1$ or $x = 3$

C) $x = 0$ only

D) $x = 4$ only

### Q14 — Intermediate

The tangent to $y = x^2 + 1$ at $x = 2$ has gradient 4. What is the gradient of the normal at this point?

A) 4

B) $-4$

C) $\frac{1}{4}$

D) $-\frac{1}{4}$

---

## Section 10.4 — Stationary Points and Nature

A **stationary point** occurs where $f'(x) = 0$ — the tangent is horizontal. There are three types:

- **Local maximum:** gradient changes from $+$ to $-$; $f''(x) < 0$
- **Local minimum:** gradient changes from $-$ to $+$; $f''(x) > 0$
- **Point of inflection:** gradient does not change sign; $f''(x) = 0$

---

### Q15 — Basic

Find the $x$-values of the stationary points of $f(x) = x^3 - 3x$.

A) $x = 0$ only

B) $x = 1$ and $x = -1$

C) $x = 3$ and $x = -3$

D) $x = 1$ only

---

### Q16 — Basic

For $f(x) = x^3 - 3x$, classify the stationary point at $x = 1$.

A) Local maximum

B) Local minimum

C) Point of inflection

D) Cannot be determined

---

### Q17 — Intermediate

Find the coordinates of the local maximum of $f(x) = 2x^3 - 9x^2 + 12x - 4$.

A) $(1,\ 1)$

B) $(2,\ 0)$

C) $(1,\ 0)$

D) $(2,\ 1)$

---

### Q18 — Intermediate

At which $x$-value does $f(x) = x^3 - 6x^2 + 9x + 1$ have its point of inflection?

A) $x = 1$

B) $x = 2$

C) $x = 3$

D) $x = 0$

---

### Q19 — Proficient

A cubic function $f(x) = ax^3 + bx^2 + cx + d$ has a local maximum at $x = -1$ and a local minimum at $x = 3$. What is the $x$-coordinate of its point of inflection?

A) $x = 0$

B) $x = 1$

C) $x = 2$

D) $x = -2$

---

## Section 10.5 — Cubic Functions and Graphs

A cubic function has the form $f(x) = ax^3 + bx^2 + cx + d$.

- If $a > 0$: falls to the left, rises to the right
- If $a < 0$: rises to the left, falls to the right
- Has **at most** two turning points
- Has **exactly** one point of inflection
- $f'(x) > 0$: function is **increasing**; $f'(x) < 0$: function is **decreasing**

---

### Q20 — Basic

A cubic graph has a local maximum at $(-1,\ 4)$ and a local minimum at $(2,\ -3)$. On what interval is the function **decreasing**?

A) $x < -1$

B) $-1 < x < 2$

C) $x > 2$

D) $x < 2$

---

### Q21 — Intermediate

The graph of $f'(x)$ has roots at $x = 1$ and $x = 5$, and opens upward (parabola). Which statement about $f(x)$ is correct?

A) $f$ has a maximum at $x = 1$ and a minimum at $x = 5$

B) $f$ has a minimum at $x = 1$ and a maximum at $x = 5$

C) $f$ has a minimum at $x = 1$ and a minimum at $x = 5$

D) $f$ has a maximum at $x = 1$ and no turning point at $x = 5$

---

#### Q22 — Intermediate

A cubic graph cuts the x-axis at $x = -2$, $x = 1$, and $x = 3$, and passes through $(0,\ -6)$. What is the equation of the cubic?

A) $y = (x+2)(x-1)(x-3)$

B) $y = -(x+2)(x-1)(x-3)$

C) $y = -x(x+2)(x-1)(x-3)$

D) $y = 2(x+2)(x-1)(x-3)$

---

#### Q23 — Proficient

The cubic $f(x) = (x+1)(x-2)(x-4)$ has a positive leading coefficient. For which values of $x$ is $f(x) > 0$?

A) $x < -1$ or $x > 4$

B) $-1 < x < 2$ or $x > 4$

C) $x < -1$ or $2 < x < 4$

D) $-1 < x < 2$ or $2 < x < 4$

---

## Section 10.6 — Optimisation

Optimisation uses calculus to find the **maximum or minimum value** of a quantity. The method:

1. Write an expression for the quantity to optimise (area, volume, profit, etc.)
2. Express it in terms of one variable using any constraint given
3. Differentiate and set $f'(x) = 0$
4. Verify using $f''(x)$ (or context)
5. Answer the question — often requires finding the actual max/min value, not just $x$

---

### Q24 — Basic

The profit (in rands) from selling $x$ units is $P(x) = -2x^2 + 40x - 50$. How many units maximise the profit?

A) $x = 10$

B) $x = 20$

C) $x = 40$

D) $x = 5$

---

### Q25 — Intermediate

A farmer has $120\,\text{m}$ of fencing to enclose a rectangular field. One side uses a river (no fencing needed). Find the length of the side **parallel** to the river that maximises the area.

A) $30\,\text{m}$

B) $40\,\text{m}$

C) $60\,\text{m}$

D) $80\,\text{m}$

---

### Q26 — Intermediate

An open box is made from a $12\,\text{cm} \times 12\,\text{cm}$ square of cardboard by cutting equal squares of side $x$ from each corner and folding up the sides. Find the value of $x$ that maximises the volume.

A) $x = 1$

B) $x = 2$

C) $x = 3$

D) $x = 4$

---

### Q27 — Proficient

A particle moves along a straight line. Its displacement (in metres) after $t$ seconds is $s(t) = t^3 - 6t^2 + 9t$. At what time is the particle momentarily at rest?

A) $t = 1$ and $t = 3$

B) $t = 0$ and $t = 2$

C) $t = 2$ only

D) $t = 3$ only

# Answers and Explanations

## Section 10.1 — Limits and First Principles

### Q1 — Basic — Answer: B

**Topic:** Reading the first principles definition

**Step 1 — Recall the definition.**

The derivative measures the instantaneous rate of change. We find it by computing the gradient of a chord between $x$ and $x + h$, then shrinking $h$ to zero:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

**Why the distractors are wrong:**

**A)** $h \to \infty$ gives the gradient of a chord extending infinitely far — this is not a tangent.

**C)** The numerator uses addition instead of subtraction — this is not the difference in function values.

**D)** The denominator is $x$ instead of $h$ — the gradient of the chord must be divided by the horizontal gap $h$, not by $x$.

> **Takeaway:** The two key features of the definition are: subtraction in the numerator ($f(x+h) - f(x)$) and the limit as $h \to 0$ in the denominator $h$.

---

### Q2 — Basic — Answer: B

**Topic:** Applying first principles to $f(x) = x^2$

**Step 1 — Write the definition.**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

**Step 2 — Substitute $f(x) = x^2$.**

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

**Step 3 — Expand the numerator.**

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \frac{2xh + h^2}{h}$$

**Step 4 — Cancel $h$ (valid since $h \neq 0$ during the limit process).**

$$= \lim_{h \to 0}(2x + h)$$

**Step 5 — Apply the limit.**

$$= 2x$$

**Why the distractors are wrong:**

**A)** $x$ — missing the coefficient of 2; power rule gives $2x$, not $x$.

**C)** $2x + h$ — this is the expression before applying the limit. Once $h \to 0$, the $h$ term vanishes.

**D)** $x^2$ — this is the original function, not its derivative.

> **Takeaway:** The key step is expanding $(x + h)^2$ fully, then cancelling $h$ before taking the limit.

---

**Q3 — Intermediate — Answer: A**

**Topic:** First principles with a linear-quadratic function

**Step 1 — Apply the definition.**

$$f'(x) = \lim_{h \to 0} \frac{[3(x+h)^2 - 2(x+h)] - [3x^2 - 2x]}{h}$$

**Step 2 — Expand.**

$$= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 2x - 2h - 3x^2 + 2x}{h}$$

**Step 3 — Simplify (all constant terms cancel).**

$$= \lim_{h \to 0} \frac{6xh + 3h^2 - 2h}{h}$$

**Step 4 — Cancel $h$.**

$$= \lim_{h \to 0}(6x + 3h - 2) = 6x - 2$$

**Why the distractors are wrong:**

**B)** $3x - 2$ — halved the coefficient; $\frac{d}{dx}[3x^2] = 6x$, not $3x$.

**C)** $6x + 2$ — sign error on the derivative of $-2x$; $\frac{d}{dx}[-2x] = -2$.

**D)** $6x$ — forgot to differentiate the $-2x$ term.

> **Takeaway:** Expand fully and collect terms in $h$ before cancelling. Every term in the original function contributes to the derivative.

---

### Q4 — Intermediate — Answer: C

**Topic:** Recognising the first principles form

**Step 1 — Recognise the structure.**

This is the first principles definition with $f(x) = x^3$. So the expression equals $f'(x)$.

**Step 2 — Differentiate $f(x) = x^3$ using the power rule (or verify via expansion).**

$$f'(x) = 3x^2$$

**Verification by expansion:**

$$\frac{(x+h)^3 - x^3}{h} = \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} = 3x^2 + 3xh + h^2 \xrightarrow{h \to 0} 3x^2$$

**Why the distractors are wrong:**

**A)** $x^2$ — missing the coefficient 3.

**B)** $2x^2$ — this is the derivative of $x^2$ times an extra factor; doesn't apply here.

**D)** $3x^3$ — the derivative reduces the power by one; $x^3$ differentiates to $x^2$, not $x^3$.

> **Takeaway:** When you see $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, identify $f(x)$ and differentiate it directly.

---

## Section 10.2 — Rules of Differentiation

### Q5 — Basic — Answer: A

**Topic:** Power rule

Apply the power rule: multiply by the exponent, reduce the exponent by 1.

$$f'(x) = 5x^{5-1} = 5x^4$$

**Why the distractors are wrong:**

**B)** Missing the coefficient 5 — you must bring the power down as a multiplier.

**C)** The exponent increases — differentiation reduces the power by 1, not increases it.

**D)** The exponent stays the same — $5 - 1 = 4$, not 5.

> **Takeaway:** Power rule: bring down the exponent as a coefficient, then subtract 1 from the exponent.

---

### Q6 — Basic — Answer: A

**Topic:** Differentiating a polynomial

Differentiate term by term:

$$\frac{d}{dx}[4x^3] = 12x^2 \qquad \frac{d}{dx}[-3x^2] = -6x \qquad \frac{d}{dx}[7x] = 7 \qquad \frac{d}{dx}[-5] = 0$$
$$g'(x) = 12x^2 - 6x + 7$$

**Why the distractors are wrong:**

**B)** Dropped the $+7$ — the derivative of $7x$ is 7, not 0.

**C)** Kept the original exponents — each power reduces by 1 after differentiation.

**D)** Didn't multiply by the exponent — forgot to apply the coefficient from the power rule.

> **Takeaway:** Differentiate every term. Constants vanish; linear terms become constants.

---

### Q7 — Intermediate — Answer: A

**Topic:** Simplify before differentiating — fraction

**Step 1 — Simplify by dividing each term in the numerator by $3x$.**

$$f(x) = \frac{3x^3 - 6x}{3x} = \frac{3x^3}{3x} - \frac{6x}{3x} = x^2 - 2$$

**Step 2 — Differentiate.**

$$f'(x) = 2x$$

> **Takeaway:** Always simplify a fraction before differentiating. Dividing each numerator term by the denominator converts it to standard polynomial form.

**Why the distractors are wrong:**

**B)** $x - 1$ — incorrect simplification.

**C)** $2x - 1$ — brought a phantom constant into the derivative.

**D)** $x^2 - 1$ — forgot to differentiate after simplifying.

---

**Q8 — Intermediate — Answer: A**

**Topic:** Negative exponents

**Step 1 — Rewrite with negative exponent.**

$$f(x) = 4x^{-3}$$

**Step 2 — Apply power rule.**

$$f'(x) = 4 \cdot (-3)x^{-3-1} = -12x^{-4} = \frac{-12}{x^4}$$

**Why the distractors are wrong:**

**B)** Sign error — the exponent $-3$ makes the derivative negative.

**C)** $-4/x^2$ — reduced the power by too little; $-3 - 1 = -4$, not $-2$.

**D)** $-12/x^2$ — correct coefficient but wrong exponent; $x^{-4}$ means $1/x^4$, not $1/x^2$.

> **Takeaway:** Rewrite $\frac{a}{x^n}$ as $ax^{-n}$, then apply the power rule. The derivative of $x^{-n}$ is $-nx^{-n-1}$.

---

**Q9 — Intermediate — Answer: B**

**Topic:** Rational exponents (roots)

**Step 1 — Rewrite using exponent notation.**

$$f(x) = x^{\frac{1}{2}}$$

**Step 2 — Apply power rule.**

$$f'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$$

**Why the distractors are wrong:**

**A)** $1/\sqrt{x}$ — missing the factor of $\frac{1}{2}$.

**C)** $2\sqrt{x}$ — this would be the result of integrating $\sqrt{x}$, not differentiating.

**D)** $-1/(2\sqrt{x})$ — sign error; the exponent $\frac{1}{2}$ is positive, so the derivative is positive.

> **Takeaway:** Rewrite $\sqrt[n]{x^m}$ as $x^{m/n}$ before differentiating.

---

### Q10 — Proficient — Answer: D

**Topic:** Expand before differentiating

**Step 1 — Expand the bracket first (do not use chain rule — rather expand for NBT level).**

$$h(x) = (2x-1)^2 = 4x^2 - 4x + 1$$

**Step 2 — Differentiate term by term.**

$$h'(x) = 8x - 4$$

**Why the distractors are wrong:**

**A)** $2(2x-1) = 4x - 2$ — used the coefficient 2 instead of 4 from $(2x)^2 = 4x^2$.

**B)** $8x$ — forgot to differentiate the constant $+1$; derivative of $-4x$ is $-4$, giving $8x - 4$, not $8x$.

**C)** $4x - 2$ — differentiated $4x^2 - 4x$ incorrectly; $\frac{d}{dx}[4x^2] = 8x$, not $4x$.

> **Takeaway:** Always expand brackets before differentiating polynomial expressions.

---

## Section 10.3 — Tangent and Normal Lines

### Q11 — Basic — Answer: B

**Topic:** Gradient of tangent at a point

**Step 1 — Differentiate.**

$$f'(x) = 2x - 4$$

**Step 2 — Substitute $x = 3$.**

$$f'(3) = 2(3) - 4 = 2$$

**Why the distractors are wrong:**

**A) 0** — this is $f'(2) = 2(2) - 4 = 0$; substituted the wrong $x$-value.

**C) 6** — from computing $2(3) = 6$ but forgetting to subtract the 4 in $f'(x) = 2x - 4$.

**D)** $-2$ — forgot to add 4 correctly; sign error.

> **Takeaway:** The gradient of the tangent at $x = a$ is $f'(a)$. Differentiate first, then substitute.

---

### Q12 — Basic — Answer: B

**Topic:** Finding the equation of a tangent

**Step 1 — Find the point of tangency.**

$$y(1) = 1^3 = 1 \quad \Rightarrow \quad \text{point: } (1,\ 1)$$

**Step 2 — Find the gradient.**

$$\frac{dy}{dx} = 3x^2 \quad \Rightarrow \quad \left.\frac{dy}{dx}\right|_{x=1} = 3$$

**Step 3 — Use point-gradient form.**

$$y - 1 = 3(x - 1) \quad \Rightarrow \quad y = 3x - 2$$

**Why the distractors are wrong:**

**A)** Gradient of 1 — used $f(1)$ as the gradient instead of $f'(1)$.

**C)** $y = 3x + 1$ — used $y = mx + c$ with $c = f(1) = 1$ directly; forgot to subtract using the point.

**D)** Gradient of 1 — same error as A with a different intercept.

> **Takeaway:** Three steps every time: find the point, find the gradient from $f'(a)$, write $y - y_1 = m(x - x_1)$.

---

### Q13 — Intermediate — Answer: A

**Topic:** Finding $x$ where the gradient equals a given value

**Step 1 — Differentiate.**

$$f'(x) = 3x^2 - 12x + 9$$

**Step 2 — Set equal to 9.**

$$3x^2 - 12x + 9 = 9$$
$$3x^2 - 12x = 0$$
$$3x(x - 4) = 0$$

**Step 3 — Solve.**

$$x = 0 \quad \text{or} \quad x = 4$$

**Why the distractors are wrong:**

**B)** $x = 1$ or $x = 3$ — these are the stationary points where $f'(x) = 0$, not where $f'(x) = 9$.

**C/D)** Only one solution — factoring gives two solutions; both must be included.

> **Takeaway:** Set $f'(x)$ equal to the required gradient value. Do not set it to zero unless you want stationary points.

---

### Q14 — Intermediate — Answer: D

**Topic:** Equation of the normal

**Step 1 — Recall the relationship between tangent and normal gradients.**

If the tangent gradient is $m_t$, the normal gradient is:

$$m_n = -\frac{1}{m_t}$$

**Step 2 — Substitute.**

$$m_n = -\frac{1}{4}$$

**Why the distractors are wrong:**

**A)** Same as tangent — parallel lines, not perpendicular.

**B)** $-4$ — negative of the tangent; perpendicular requires the reciprocal AND the negative.

**C)** $\frac{1}{4}$ — reciprocal but forgot the negative sign.

> **Takeaway:** Tangent gradient $m \to$ normal gradient $-1/m$. Both the sign flip and the reciprocal are required.

---

## Section 10.4 — Stationary Points and Nature

#### Q15 — Basic — Answer: B

**Topic:** Finding stationary points

**Step 1 — Differentiate.**

$$f'(x) = 3x^2 - 3$$

**Step 2 — Set $f'(x) = 0$.**

$$3x^2 - 3 = 0 \quad \Rightarrow \quad x^2 = 1 \quad \Rightarrow \quad x = \pm 1$$

**Why the distractors are wrong:**

**A)** $x = 0$ — this is the inflection point of $f(x) = x^3$, not this function.

**C)** $x = \pm 3$ — forgot to divide by 3 before solving.

**D)** Only $x = 1$ — missed the negative root; $x^2 = 1$ has two solutions.

> **Takeaway:** Stationary points require $f'(x) = 0$. Solve the resulting equation completely — don't miss negative roots.

---

#### Q16 — Basic — Answer: B

**Topic:** Nature using the second derivative

**Step 1 — Find $f''(x)$.**

$$f'(x) = 3x^2 - 3 \quad \Rightarrow \quad f''(x) = 6x$$

**Step 2 — Substitute $x = 1$.**

$$f''(1) = 6(1) = 6 > 0$$

**Step 3 — Interpret.** Since $f''(1) > 0$, the curve is concave up at $x = 1$ — this is a **local minimum**.

**Why the distractors are wrong:**

**A)** Maximum requires $f'' < 0$ (concave down).

**C)** Inflection requires $f'' = 0$ and a sign change — $f''(1) = 6 \neq 0$.

**D)** The second derivative test gives a clear answer here.

> **Takeaway:** $f'' > 0 \rightarrow$ minimum (cup shape). $f'' < 0 \rightarrow$ maximum (cap shape). $f'' = 0 \rightarrow$ test is inconclusive; use sign of $f'$.

---

**Q17 — Intermediate — Answer: A**

**Topic:** Coordinates of turning points

**Step 1 — Differentiate and find stationary points.**

$$f'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2) = 6(x-1)(x-2)$$

$$x = 1 \quad \text{or} \quad x = 2$$

**Step 2 — Find $f''(x)$ to classify.**

$$f''(x) = 12x - 18$$

$$f''(1) = -6 < 0 \quad \Rightarrow \quad \text{local maximum at } x = 1$$

$$f''(2) = 6 > 0 \quad \Rightarrow \quad \text{local minimum at } x = 2$$

**Step 3 — Find $y$-coordinate of maximum.**

$$f(1) = 2 - 9 + 12 - 4 = 1$$

$$\text{Local maximum: } (1,\ 1)$$

**Why the distractors are wrong:**

**B)** $(2,\ 0)$ — this is the local minimum, not the maximum.

**C)** $(1,\ 0)$ — correct $x$ but wrong $y$; $f(1) = 1$, not 0.

**D)** $(2,\ 1)$ — swapped the coordinates of the two turning points.

> **Takeaway:** Find both stationary points, use $f''$ to classify, then substitute the correct $x$ to find the $y$-coordinate.

---

### Q18 — Intermediate — Answer: B

**Topic:** Point of inflection

**Step 1 — Differentiate twice.**

$$f'(x) = 3x^2 - 12x + 9 \quad \Rightarrow \quad f''(x) = 6x - 12$$

**Step 2 — Set $f''(x) = 0$.**

$$6x - 12 = 0 \quad \Rightarrow \quad x = 2$$

**Step 3 — Confirm sign change** (not required at NBT level but worth knowing).

$f''(1) = -6 < 0$ and $f''(3) = 6 > 0$ — sign changes, confirming inflection.

**Why the distractors are wrong:**

**A)** $x = 1$ — this is a stationary point ($f'(1) = 0$), not the inflection point.

**C)** $x = 3$ — also a stationary point.

**D)** $x = 0$ — neither a stationary point nor the inflection point for this function.

> **Takeaway:** Point of inflection: set $f''(x) = 0$. Note: for a cubic, the inflection point is always the midpoint (in $x$) of the two turning points.

---

### Q19 — Proficient — Answer: B

**Topic:** Using turning point information

**Step 1 — Use the key property of cubic functions.**

The point of inflection of a cubic always lies at the average (midpoint) of the $x$-coordinates of the two turning points.

**Step 2 — Calculate.**

$$x_{\text{inflection}} = \frac{-1 + 3}{2} = \frac{2}{2} = 1$$

**Why the distractors are wrong:**

**A)** $x = 0$ — average of $-1$ and $1$, but 1 is not the minimum; the minimum is at $x = 3$.

**C)** $x = 2$ — average of 1 and 3; used the wrong pair.

**D)** $x = -2$ — not the midpoint of any meaningful pair.

> **Takeaway:** For a cubic $f$, the $x$-coordinate of the inflection point = $\dfrac{x_{\text{max}} + x_{\text{min}}}{2}$.

---

## Section 10.5 — Cubic Functions and Graphs

### Q20 — Basic — Answer: B

**Topic:** Reading a cubic graph — turning points

A cubic with positive leading coefficient rises, reaches a **local maximum**, **decreases** to the local minimum, then rises again.

The function decreases between the local maximum and local minimum:

$$-1 < x < 2$$

**Why the distractors are wrong:**

**A)** $x < -1$ — the function is increasing here (before the maximum).

**C)** $x > 2$ — the function is increasing here (after the minimum).

**D)** $x < 2$ — too broad; the function is increasing for $x < -1$ and only decreasing for $-1 < x < 2$.

> **Takeaway:** Between the maximum and minimum of a cubic, the function is decreasing. Outside this interval, it is increasing.

---

### Q21 — Intermediate — Answer: A

**Topic:** Linking $f$, $f'$, and $f''$

**Read the sign of $f'$.** It is an upward parabola with roots at 1 and 5, so it is **negative between** them and **positive outside**.

**Translate to $f$:**

- At $x = 1$, $f'$ goes $+ \to - \to$ **local maximum**
- At $x = 5$, $f'$ goes $- \to + \to$ **local minimum**

*Why the others are wrong:*

- **B** is reversed — an upward parabola is negative *between* its roots, so $f$ **decreases** there.
- **C** claims two minima. An upward $f'$ changes sign $+ \to - \to +$, giving one of each.
- **D** denies a turning point at $x = 5$, but $f'(5) = 0$ **and** the sign changes there.

> **Takeaway:** $f'$ going $+ \to -$ means a maximum; $- \to +$ means a minimum. Sketch the sign of $f'$ first, then read $f$ off it.

---

### Q22 — Intermediate — Answer: B

**Topic:** Determining a cubic equation from its graph

**Step 1 — Write the general form using the roots.**

$$y = k(x+2)(x-1)(x-3)$$

**Step 2 — Use the point $(0,\ -6)$ to find $k$.**

$$-6 = k(0+2)(0-1)(0-3) = k(2)(-1)(-3) = 6k \quad \Rightarrow \quad k = -1$$

**Step 3 — Write the equation.**

$$y = -(x+2)(x-1)(x-3)$$

**Why the distractors are wrong:**

**A)** $k = 1$ gives $y = +6$ at $x = 0$, not $-6$.

**C)** Has an extra factor $x$ — this would be degree 4, not cubic.

**D)** $k = 2$ gives $y = 12$ at $x = 0$, not $-6$.

> **Takeaway:** Use known roots to write $y = k(x - r_1)(x - r_2)(x - r_3)$, then substitute a known point to find $k$.

---

### Q23 — Proficient — Answer: B

**Topic:** Intervals where $f(x) > 0$

**Step 1 — Identify the roots:** $x = -1,\ x = 2,\ x = 4$

**Step 2 — Use a sign table** (positive leading coefficient means the cubic is negative, positive, negative, positive in consecutive intervals):

| Interval | Sign of $f$ |
|---|---|
| $x < -1$ | negative |
| $-1 < x < 2$ | positive |
| $2 < x < 4$ | negative |
| $x > 4$ | positive |

**Step 3 — Select where $f(x) > 0$:**

$$-1 < x < 2 \quad \text{or} \quad x > 4$$

**Why the distractors are wrong:**

**A)** These are where the cubic would be positive if leading coefficient were negative — reversed.

**C)** Selects the negative intervals.

**D)** Skips the interval $x > 4$.

> **Takeaway:** Draw a sign table for each interval. Positive leading coefficient → starts negative for $x <$ smallest root.

---

## Section 10.6 — Optimisation

### Q24 — Basic — Answer: A

**Topic:** Finding the maximum of a quadratic function

**Step 1 — Differentiate.**

$$P'(x) = -4x + 40$$

**Step 2 — Set $P'(x) = 0$.**

$$-4x + 40 = 0 \quad \Rightarrow \quad x = 10$$

**Step 3 — Verify it's a maximum.**

$$P''(x) = -4 < 0 \quad \Rightarrow \quad \text{maximum confirmed}$$

**Why the distractors are wrong:**

**B)** $x = 20$ — twice the correct answer; common error from forgetting to divide.

**C)** $x = 40$ — used the coefficient of $x$ directly without differentiating.

**D)** $x = 5$ — half the correct value.

> **Takeaway:** Set the derivative of the objective function to zero. Verify with $f''$ or context.

---

### Q25 — Intermediate — Answer: C

**Topic:** Optimisation with a constraint — fence problem

**Step 1 — Set up variables.**

Let the side parallel to the river have length $y$ and the two perpendicular sides each have length $x$.

**Step 2 — Write the constraint** (only 3 sides need fencing).

$$2x + y = 120 \quad \Rightarrow \quad y = 120 - 2x$$

**Step 3 — Write the area function.**

$$A = xy = x(120 - 2x) = 120x - 2x^2$$

**Step 4 — Differentiate and set to zero.**

$$A'(x) = 120 - 4x = 0 \quad \Rightarrow \quad x = 30$$

**Step 5 — Find $y$.**

$$y = 120 - 2(30) = 60\,\text{m}$$

**Why the distractors are wrong:**

**A) 30 m** — this is the length of the perpendicular sides, not the side parallel to the river.

**B) 40 m** — an intermediate calculation error.

**D) 80 m** — $120 - 2(20)$; found $x = 20$ by error.

> **Takeaway:** Label your variables clearly. Write the constraint, use it to eliminate one variable, then differentiate the objective function.

---

### Q26 — Intermediate — Answer: B

**Topic:** Box optimisation

**Step 1 — Write the volume formula.**

After cutting, the base has dimensions $(12 - 2x) \times (12 - 2x)$ and height $x$:

$$V = x(12 - 2x)^2$$

**Step 2 — Expand.**

$$V = x(144 - 48x + 4x^2) = 144x - 48x^2 + 4x^3$$

**Step 3 — Differentiate.**

$$V'(x) = 144 - 96x + 12x^2 = 12(x^2 - 8x + 12) = 12(x-2)(x-6)$$

**Step 4 — Set $V'(x) = 0$.**

$$x = 2 \quad \text{or} \quad x = 6$$

**Step 5 — Check validity.** Since $x < 6$ (otherwise the box has no base), $x = 2$.

**Why the distractors are wrong:**

**A)** $x = 1$ — $V'(1) = 12(1-2)(1-6) = 60 \neq 0$.

**C)** $x = 3$ — $V'(3) = 12(1)(-3) \neq 0$.

**D)** $x = 6$ — this makes the base zero; physically invalid.

> **Takeaway:** Always check that your solution is physically valid. Discard solutions outside the feasible domain.

---

### Q27 — Proficient — Answer: A

**Topic:** Optimisation — rate of change interpretation

**Step 1 — Recall that velocity $= \dfrac{ds}{dt}$.**

The particle is at rest when velocity $= 0$.

**Step 2 — Differentiate.**

$$v(t) = s'(t) = 3t^2 - 12t + 9 = 3(t^2 - 4t + 3) = 3(t-1)(t-3)$$

**Step 3 — Set $v(t) = 0$.**

$$t = 1 \quad \text{or} \quad t = 3$$

**Why the distractors are wrong:**

**B)** $t = 0$ and $t = 2$ — these come from setting $s(t) = 0$, not $s'(t) = 0$.

**C/D)** Only one value — the quadratic $v(t) = 0$ has two roots.

> **Takeaway:** Velocity is the derivative of displacement. "At rest" means velocity $= 0$, so set $s'(t) = 0$.

## Section 10.7 — Mixed Practice

### M1 — Basic

**Topic:** Power rule

If $f(x) = 6x^4 - 2x^3 + x$, then $f'(x) =$

A) $24x^3 - 6x^2 + 1$

B) $24x^3 - 6x^2$

C) $24x^4 - 6x^3 + 1$

D) $6x^3 - 2x^2 + 1$

---

### M2 — Basic

**Topic:** Stationary point x-value

At which value does $g(x) = x^2 - 8x + 3$ have its turning point?

A) $x = 3$

B) $x = 4$

C) $x = 8$

D) $x = -4$

---

### M3 — Basic

**Topic:** Nature of turning point

If $f''(a) = -7$, the turning point at $x = a$ is a:

A) Minimum

B) Point of inflection

C) Maximum

D) Cannot be determined

---

### M4 — Basic

**Topic:** Gradient of tangent

The gradient of the tangent to $y = 3x^2 - x$ at $x = 2$ is:

A) 10

B) 11

C) 6

D) 5

---

### M5 — Basic

**Topic:** Simplify before differentiating

$f(x) = \dfrac{x^4 - x^2}{x}$. Then $f'(x) =$

A) $3x^2 - 1$

B) $3x^2$

C) $4x^3 - 2x$

D) $x^3 - x$

---

### M6 — Intermediate

**Topic:** First principles

Using first principles, $f'(x)$ where $f(x) = 5x$ equals:

A) $5x$

B) 0

C) 5

D) $5x + 5h$

---

### M7 — Intermediate

**Topic:** Equation of tangent

The equation of the tangent to $f(x) = x^2 - 3$ at $x = -1$ is:

A) $y = -2x - 4$

B) $y = 2x - 4$

C) $y = -2x - 2$

D) $y = -2x + 4$

---

### M8 — Intermediate

**Topic:** Turning points of cubic

$f(x) = x^3 - 12x + 5$ has turning points at:

A) $x = 2$ and $x = -2$

B) $x = 4$ and $x = -4$

C) $x = 3$ and $x = -3$

D) $x = 6$ only

---

### M9 — Intermediate

**Topic:** Inflection point

The point of inflection of $f(x) = x^3 - 3x^2 + 6x - 1$ occurs at $x =$

A) 1

B) 2

C) 3

D) 0

---

### M10 — Intermediate

**Topic:** Increasing/decreasing

For $f(x) = -x^3 + 3x$, the function is increasing when:

A) $-1 < x < 1$

B) $x < -1$ or $x > 1$

C) $x > 0$

D) All real $x$

---

### M11 — Intermediate

**Topic:** Optimisation — maximum value

Find the maximum value of $f(x) = -x^2 + 6x + 1$.

A) 3

B) 9

C) 10

D) 7

---

### M12 — Intermediate

**Topic:** Normal gradient

The tangent to $f(x) = x^3$ at $x = -1$ has gradient 3. The gradient of the normal is:

A) $-3$

B) 3

C) $\frac{1}{3}$

D) $-\frac{1}{3}$

---

### M13 — Intermediate

**Topic:** Reading $f'$ from a graph

A function $f$ is increasing on $(-\infty,\ 2)$ and decreasing on $(2,\ \infty)$. Which statement about $f'$ is true?

A) $f'(2) = 0$ and $f'(x) < 0$ for $x < 2$

B) $f'(2) = 0$ and $f'(x) > 0$ for $x < 2$

C) $f'(2) > 0$

D) $f'(x) < 0$ for all $x$

---

### M14 — Intermediate

**Topic:** Second derivative sign

If $f''(x) > 0$ for all $x$ in an interval, then on that interval $f$ is:

A) Decreasing

B) Concave down

C) Concave up

D) At a maximum

---

### M15 — Intermediate

**Topic:** Cubic with negative leading coefficient

Which of the following best describes the end behaviour of $f(x) = -2x^3 + x^2 - 5$?

A) Rises left, rises right

B) Rises left, falls right

C) Falls left, rises right

D) Falls left, falls right

---

### M16 — Proficient

**Topic:** Finding $k$ given a stationary point

$f(x) = x^3 - kx$ has a stationary point at $x = 2$. Find $k$.

A) 6

B) 4

C) 12

D) 2

---

### M17 — Proficient

**Topic:** Optimisation — tin can

A cylindrical tin has volume $250\pi \text{ cm}^3$. The radius that minimises total surface area satisfies:

A) $r = 5$ cm

B) $r = 10$ cm

C) $r = 2$ cm

D) $r = 25$ cm

---

### M18 — Proficient

**Topic:** Discriminant of $f'$ and number of turning points

For $f(x) = x^3 + bx + c$, how many turning points does $f$ have if $b > 0$?

A) Two turning points

B) One turning point

C) No turning points

D) Cannot be determined

---

### M19 — Proficient

**Topic:** Relating graphs of $f$ and $f'$

At the point where the graph of $f'$ crosses the x-axis from below, $f$ has a:

A) Local maximum

B) Local minimum

C) Point of inflection

D) Vertical asymptote

---

### M20 — Proficient

**Topic:** Chain of reasoning

If $f(x) = x^4 - 8x^2$, at how many points is the tangent to the curve horizontal?

A) 1

B) 2

C) 3

D) 4

---

### M21 — Proficient

**Topic:** Using turning point to find coefficients

$f(x) = ax^2 + bx$ has a turning point at $(3,\ -9)$. Find $a$ and $b$.

A) $a = 1,\ b = -6$

B) $a = -1,\ b = 6$

C) $a = 1,\ b = 6$

D) $a = 2,\ b = -3$

---

### M22 — Proficient

**Topic:** Sign of $f''$ and concavity

For $f(x) = x^4 - 6x^2$, on which interval is $f$ concave down?

A) $-1 < x < 1$

B) $x < -1$ or $x > 1$

C) $x > 0$

D) All real $x$

---

### M23 — Proficient

**Topic:** Optimisation — area

A rectangle is inscribed in a right-angled triangle with legs 6 and 8. If one corner is at the right angle, find the width $x$ that maximises the rectangle's area.

A) $x = 3$

B) $x = 4$

C) $x = 2$

D) $x = 6$

---

### M24 — Proficient

**Topic:** Velocity and acceleration

A ball's height is $h(t) = -5t^2 + 20t + 2$ metres after $t$ seconds. What is the maximum height?

A) $20\,\text{m}$

B) $22\,\text{m}$

C) $25\,\text{m}$

D) $2\,\text{m}$

---

### M25 — Proficient

**Topic:** Reading cubic graph features

A cubic $f$ has roots at $x = -3,\ 0,\ 2$ and a negative leading coefficient. Which is the sketch?

A) Falls left, local max between $-3$ and $0$, local min between $0$ and $2$, rises right

B) Rises left, local max between $-3$ and $0$, local min between $0$ and $2$, falls right

C) Rises left, local min between $-3$ and $0$, local max between $0$ and $2$, falls right

D) Falls left, local min between $-3$ and $0$, local max between $0$ and $2$, rises right

---

### M26 — Proficient

**Topic:** Using the discriminant of $f'$

How many real stationary points does $f(x) = x^3 + 3x^2 + 5x - 1$ have?

A) 0

B) 1

C) 2

D) 3

---

### M27 — Proficient

**Topic:** Second derivative and concavity change

The graph of $f''(x)$ crosses the x-axis at $x = 3$ (from negative to positive). This means $f(x)$ has:

A) A local maximum at $x = 3$

B) A local minimum at $x = 3$

C) A point of inflection at $x = 3$

D) A stationary point at $x = 3$

---

### M28 — Proficient

**Topic:** Optimisation — combined constraint

A farmer builds a rectangular pen divided into three equal sections by two interior fences parallel to the width. Total fencing is $480\,\text{m}$. Find the width $w$ that maximises area.

A) $w = 60$

B) $w = 80$

C) $w = 120$

D) $w = 40$

---

### M29 — Proficient

**Topic:** Applying calculus to a real-world graph

The velocity of a car is $v(t) = 3t^2 - 18t + 24 \text{ m/s}$. When is the car decelerating?

A) $t < 2$

B) $t > 4$

C) $2 < t < 4$

D) $t < 3$

---

### M30 — Proficient

**Topic:** Linking all calculus concepts

$f(x) = x^3 - 3x^2 - 9x + 5$ has a local maximum at $x = -1$ and a local minimum at $x = 3$. The value of $f(-1) - f(3)$ is:

A) 10

B) 16

C) 32

D) $-16$

## Mixed Practice — Answer Key

| | | | | |
|---|---|---|---|---|
| **M1.** A | **M2.** B | **M3.** C | **M4.** B | **M5.** A |
| **M6.** C | **M7.** A | **M8.** A | **M9.** A | **M10.** A |
| **M11.** C | **M12.** D | **M13.** B | **M14.** C | **M15.** B |
| **M16.** C | **M17.** A | **M18.** C | **M19.** B | **M20.** C |
| **M21.** A | **M22.** A | **M23.** B | **M24.** B | **M25.** C |
| **M26.** A | **M27.** C | **M28.** A | **M29.** D | **M30.** C |