The Complete SAT Mathematics Practice Book

Eben Roux

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Every question in this book was written from scratch and solved independently by computer algebra before it was published — including the wrong options, so the explanation of a distractor is never hand-waved.

The book's structure comes from the eight official College Board practice tests: what is examined, in what proportion, and where the questions with no options sit. No question from those tests is reproduced here.

Read it online at mathbench.academy/sat/

Licence and disclaimer

Confirm test dates, fees and registration arrangements with the College Board. This book cannot do that for you.

Contents

SAT Mathematics — Complete Study Guide

Read this first. It is written for you.

If maths has never quite clicked, this guide is built for exactly that. Nothing here assumes you already understand it. Every rule is explained in plain English *before* the formula appears, every symbol is spelled out in words, and every method comes with a worked example where **every step is shown** — nothing is left for you to "just know".

You do not have to be naturally good at maths to do well on this test. The SAT Math section examines a **small and fixed** list of ideas. It is not trying to surprise you with new mathematics; it is checking whether you can use familiar mathematics accurately, under time pressure, on a problem worded in an unfamiliar way. That is a skill, and skills are built by practice.

What the Test Actually Is

The SAT has two sections: **Reading and Writing**, and **Math**. This book covers Math only.

Everything in this section is measured, not remembered. It comes from the eight official College Board practice tests (numbers 4 to 11) and their scoring guides. Where this book says "in every module we checked", it means all sixteen Math modules across those eight papers.

The shape of the Math section

The Math section comes in **two modules**, done one after the other. On the official paper practice tests, each module has **27 questions**, so the section has **54** in total.

The digital test you sit is **adaptive**: how you do on the first module decides which second module you get. That is why the practice papers are slightly longer than the live test — they have to cover the same ground without adapting. The mathematics is identical either way, and so is everything in this book.

A calculator is allowed on every question

This is the single most important thing to know, and it is printed on the directions page of every Math module:

> Use of a calculator is permitted for all questions.

There is a **Desmos graphing calculator** built into the test app, and you may bring your own approved one. **You will never be asked to do arithmetic a calculator cannot handle.**

That built-in graphing calculator is worth learning properly, and most students never do. Section 5 is about it.

That has a consequence worth sitting with. If the calculator can do the arithmetic, then the arithmetic is not what is being examined. **The marks are in setting the problem up correctly** — choosing the right equation, reading the question accurately, knowing what the answer is supposed to mean. A calculator will give you a beautifully precise answer to the wrong equation.

This is why almost every wrong option in this book is a *correct calculation of something the question did not ask*. Those are the mistakes that survive having a calculator.

The two kinds of question

Kind	How many	What you do
Multiple choice	about 74%	Choose from four options, A to D
Student-produced response	about 26%	Work it out and enter the answer yourself

Student-produced responses are often called **grid-ins** or **fill-ins**. There are no options at all. Chapter 8 is devoted to them, because they need a different technique — but there is one thing to learn right now, in Section 4 below, because it is worth marks and takes ten minutes to learn.

Where the grid-ins sit — and they do not move

In **all sixteen** official modules examined, the student-produced questions were at exactly these positions:

6, 7, 13, 14, 20, 21, 27

Every module. No exceptions. Seven of the 27 questions, which is where the 26% comes from. Two things follow:

- When you reach question 6, expect no options. It is not a printing error.
- The last question of each module, number 27, is **always** a grid-in.

There is no penalty for a wrong answer

Nothing is subtracted for being wrong. A blank and a wrong answer score exactly the same: zero. So **answer every question**, including ones you have not read. On a multiple-choice question a blind guess is worth a 1-in-4 chance; a blank is worth nothing at all.

This applies to grid-ins too. If you have any number at all, enter it.

Section 1 — What the Test Examines

Every SAT Math question belongs to one of four areas. The share each one takes is set by the test's own blueprint.

Area	Share	Where in this book
Algebra	~35%	Chapters 1 and 2
Advanced Math	~35%	Chapters 3, 4 and 5
Problem-Solving and Data Analysis	~15%	Chapter 6
Geometry and Trigonometry	~15%	Chapter 7

Notice what that means: **70% of the test is algebra of one kind or another.** Geometry is the smallest area, not the largest — which is the opposite of what most people expect, and the opposite of how most people spend their study time.

If you have limited time, that table tells you where to spend it.

What is *not* on the test

It is worth knowing what you can stop worrying about. The SAT Math section does not examine formal proof, matrices, vectors, logarithms beyond the most basic manipulation, calculus of any kind, or three-dimensional trigonometry.

Section 2 — The Reference Sheet You Are Given

Every Math module begins with a reference sheet. **You do not have to memorise anything on it.** It is on the screen throughout.

Here is what it gives you.

Areas and circumference

$$A = \pi r^2 \qquad C = 2\pi r \qquad A = \ell w \qquad A = \tfrac{1}{2}bh$$

Pythagoras

$$c^2 = a^2 + b^2$$

Volumes

$$V = \ell wh \qquad V = \pi r^2 h \qquad V = \tfrac{4}{3}\pi r^3 \qquad V = \tfrac{1}{3}\pi r^2 h \qquad V = \tfrac{1}{3}\ell wh$$

Special right triangles

The 30–60–90 triangle, with sides in the ratio $x : x\sqrt{3} : 2x$, and the 45–45–90 triangle, with sides in the ratio $s : s : s\sqrt{2}$.

Three facts stated in words

- A circle has 360 degrees of arc.
- A circle has 2π radians of arc.
- The angles of a triangle add to 180 degrees.

What it does *not* give you

This is the more useful list, because these are the things you must actually know. None of the following appears on the reference sheet:

- The quadratic formula
- Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
- The equation of a line in any form
- The equation of a circle
- SOHCAHTOA, or any trigonometric identity
- The distance or midpoint formulas
- Anything at all about mean, median, standard deviation or probability
- The exponent rules

The last one catches people out. Learn the list above; ignore the reference sheet until you need a volume.

Section 3 — The Language of the Test

Most people who struggle with SAT maths are not bad at maths. They lose marks on **words**. Here is the vocabulary the test uses, in plain English.

Everyday maths words

Word	What it means
Sum	The answer when you add
Difference	The answer when you subtract
Product	The answer when you multiply
Quotient	The answer when you divide
Term	One piece of an expression, separated by $+$ or $-$ signs
Coefficient	The number multiplying a variable. In $7x$, the coefficient is 7
Constant	A plain number that does not change
Expression	Maths with no equals sign, like $3x + 5$. You simplify it
Equation	Two expressions joined by $=$. You solve it
Integer	A whole number, positive, negative, or zero
Consecutive	Following on: $n, n + 1, n + 2$

Phrases that decide the answer

These are worth more marks than any formula, because misreading one makes every subsequent step wrong.

The phrase	What it means
5 more than x	$x + 5$
5 less than x	$x - 5$ — note the order reverses
5 fewer than x	$x - 5$
the product of 5 and x	$5x$
x increased by 20%	$1.20x$
x decreased by 20%	$0.80x$
at least	$\geq$
at most, no more than, maximum	$\leq$
a is directly proportional to b	$a = kb$ for some constant k
a is inversely proportional to b	$a = \frac{k}{b}$

Words that tell you which formula

The question says	It wants
rate of change, per	a slope, or a rate
initial, starting, before any	the value at $x = 0$
the value of the expression	substitute, do not solve
in terms of y	rearrange until x is alone
must be true	true for *every* case, not just one
could be true	true for *at least one* case

That last pair costs people marks constantly. "Must be true" means you have to rule out every counterexample; "could be true" means one example is enough.

Section 4 — How to Enter a Grid-In Answer

There are no options, so the way you write the answer *is* the answer. These rules come straight from the test's instructions page, and every one of them throws marks away if broken.

The rules

- Your answer may be up to **5 characters** if positive, **6** if negative (the minus sign counts as one).
- If a fraction is too long to fit, write the decimal instead.
- If a decimal is too long to fit, **truncate it or round it at the fourth digit**. Either is accepted.
- A mixed number like $3\frac{1}{2}$ must be entered as $7/2$ or 3.5. **Writing `3 1/2` scores zero** — it gets read as thirty-one halves.
- Do not enter a percent sign, a comma, or a dollar sign. The answer one thousand two hundred and fifty is `1250`, never `1,250`.

Several forms are usually accepted

This is the part almost nobody is taught. If the answer is $\frac{1}{4}$, then `1/4`, `0.25` and `.25` are all accepted. You do not have to guess which form the test wants — any correct form scores.

Here are real accepted answers, taken from official answer keys, and what each one is teaching:

The key accepts	What it shows
`0.25` or `1/4`	Fraction or decimal, your choice
`29/3`, `9.666` or `9.667`	You may truncate **or** round — both are accepted
`361/8`, `45.12` or `45.13`	The fraction is too long, so the decimals are the way in
`15` or `-5`	Two genuinely **different** correct answers. Enter either one

That last row deserves attention. Some questions have two valid answers — a quadratic with two roots, for instance. You only enter one. Do not waste time deciding which is "the" answer; enter whichever you found first.

The safest habit

When your answer is a fraction that fits in five characters, **enter the fraction**. It cannot be wrong through rounding. Only convert to a decimal when the fraction is too long, and then give four digits.

Section 5 — The Built-In Graphing Calculator

The digital SAT gives you a **Desmos graphing calculator** inside the testing app, on every Math question. It is not a plain calculator with a graph button — it is a proper graphing tool, and knowing how to use it is a skill in its own right.

This section is worth more marks per minute of study than anything else in this guide. A calculator that can graph turns a large class of algebra questions into questions you can *see*.

The one idea behind almost every use

Anything you can write as an equation, Desmos can graph. Anything you can graph, you can read the answer off.

That single move covers most of what follows.

The four things to practise

1. Solving any equation. Put the left-hand side into one line as $y = \ldots$ and the right-hand side into another. Where the two graphs cross, the equation is true — so the x-coordinate of the crossing point is your solution.

If the equation is already "something $= 0$", just graph the something and read off where it crosses the x-axis. Desmos labels those points for you: click one and it shows the coordinates.

2. Systems of two equations. Type both in. The intersection is the solution. This is often faster than elimination even when you know elimination perfectly, because you were going to reach for the calculator for the arithmetic anyway.

3. Questions with an unknown constant. This is the least-known technique and the most useful. When a question says something like "*where a is a positive constant*", type the equation in **with the letter still in it**. Desmos will offer to add a **slider** for that letter. Always accept.

Now try values on the slider. If the question is well posed, the answer will be the same for every value the question allows — and that *is* the answer. You have replaced the algebra with an experiment.

4. Checking a candidate answer. On a multiple-choice question you can type an option in and see whether it fits. On a grid-in, you can substitute your own answer back and confirm the two sides agree.

When not to use it

The temptation, once you can graph, is to graph everything. Do not.

- **Simple arithmetic and one-step algebra are faster by hand.** Typing $3x + 5 = 20$ into a graphing tool costs more time than solving it.
- **Interpretation questions have nothing to graph.** "What does the 25 represent?" is answered by thinking about units, not by drawing anything.
- **Setting up is still your job.** A graph of the wrong equation is a precise answer to a question nobody asked. On a word problem, write the equation down first; only then decide whether to graph it.

A reasonable rule: if you can see the answer within about twenty seconds by hand, do it by hand. If you cannot, or if the algebra looks long or error-prone, graph it.

Practise on the real thing

The testing app is called **Bluebook**, and its Desmos is a specific version. Practise in the same place you will sit the test, so nothing about the interface is new on the day. Free official practice tests run inside Bluebook.

> **The short version:** learn to find an intersection, find a zero, and add a slider. Those three things cover most of what the calculator is for, and most students never learn the third one at all.

Section 6 — Strategy on the Day

Order

The questions get harder as the module goes on, roughly. They are not in strict difficulty order, but question 25 is usually harder than question 3.

You are not required to work in order. If a question is not moving after about 90 seconds, mark it, put down *something*, and move on. There are no bonus marks for the hard ones — every question is worth exactly the same.

Timing

Two modules, 27 questions each. That works out at a little over a minute per question, which sounds tight and is actually comfortable if you do not get stuck. The single biggest time loss is refusing to abandon a question.

When you are stuck on a multiple-choice question

Three techniques, in order of usefulness:

1. **Work backwards from the options.** Put each option into the question and see which one fits. On many algebra questions this is faster than solving. It is unavailable on grid-ins, which is one reason they need separate practice.
2. **Pick a number.** If a question is entirely in letters — "which expression is equivalent to..." — choose a value like $x = 2$, work out what the question gives, then test each option with the same value. Anything that does not match is out. Avoid 0 and 1: they make too many things look equal.
3. **Estimate.** Especially on geometry with a figure, a rough answer often eliminates two options immediately. Figures are drawn to scale unless the question says otherwise — the directions page says so explicitly.

Read the last line twice

More marks are lost to answering the wrong question than to any mathematical error. The question asks for $2x$, you find x and stop. It asks for the perimeter, you find the area. It asks how many were *left*, you find how many were *taken*.

Before you enter an answer, look at the final sentence of the question again and check you have produced the thing it named.

Section 7 — How Your Score Is Worked Out

The Math section is scored **200 to 800**. Your Reading and Writing score is also 200 to 800, and the two add to give a total out of 1600.

The number of questions you get right — your **raw score** — is converted to the 200–800 scale by a table published with each test. Here is what that conversion looks like, from official practice test 10:

Questions right (out of 54)	Math section score
54	800
50	730–770
44	620–680
38	550–610
27	440–480
14	330–370

Two things are worth reading off that table.

Every question is worth roughly the same. There is no bonus for hard ones. So the fastest way to raise a score is to stop losing the easy ones — which is mostly about reading carefully, not about learning more mathematics.

Nothing is worth as much as the middle. Going from 27 right to 38 right is about 120 points. Going from 44 to 50 is about the same again, but takes far more work. If your score is in the middle, the gains available to you are large.

The conversion is slightly different for every test, because papers are equated against each other for difficulty. The mock exams in Chapter 9 use these real tables, so the score they report is a genuine estimate rather than a guess.

Section 8 — How to Use This Book

Each chapter has the same shape:

1. **Teaching** — the idea in plain English, with the formulas you need.
2. **Questions** — attempt these on paper before reading on.
3. **Full solutions** — every question re-taught from the beginning, including **why each wrong option is wrong**. Each wrong option is a real mistake that real students make; learning to recognise them is most of the work.
4. **A takeaway** for each question — the transferable part, worth more than the answer.

The routine that makes this book work: attempt the question honestly first, even if you only get partway. Then read the full solution **even when you got it right** — the solutions carry the shortcuts and connections that turn one question into ten marks' worth of understanding. When you get one wrong, find *your* mistake among the wrong-option notes. Recognising your own mistake pattern is the fastest improvement available to you.

A study plan

If you have **eight weeks**:

Weeks	What to do
1–2	Chapters 1 and 2 — algebra. This is 35% of the test
3–4	Chapters 3, 4 and 5 — advanced math. Another 35%
5	Chapter 6 — data and problem solving
6	Chapter 7 — geometry and trigonometry
7	Chapter 8 — grid-ins, and revisit whatever went worst
8	Chapter 9 — the two mock papers, timed, one week apart

If you have **two weeks**, do chapters 1 and 2, then chapter 8, then one mock. That is the highest-value slice of the book.

> **If you remember only seven things from this guide:** 1. Answer **every** question. A blank scores zero; a guess might not. 2. **70% of the test is algebra.** Study accordingly. 3. You have a calculator, so the marks are in the **set-up**, not the sums. 4. Learn the built-

in graphing calculator: **intersection, zero, slider.** 5. Questions 6, 7, 13, 14, 20, 21 and 27 have **no options**. 6. On a grid-in, `3 1/2` scores zero. Write `7/2` or `3.5`. 7. Re-read the last line of the question before you answer it.

Chapter 1 — Linear Equations and Inequalities

Algebra is the largest thing on the SAT Math section: about **35%** of the questions, and the single most common instruction in the whole test is *"What is the value of x?"* — it appeared 32 times across the eight official practice tests. This chapter is that instruction, taken apart.

Almost every question here is a chain of the same two moves: **undo what was done to the variable, in the reverse order it was done**, and **keep both sides equal by doing the same thing to each**. If those two ideas are solid, a surprising amount of the test becomes routine.

A note about the calculator. You are allowed one on every SAT Math question, and there is a graphing calculator built into the test app. That changes the advice you will read in most books. It does *not* mean arithmetic stops mattering — it means the marks move to **setting the problem up correctly**. A calculator will happily give you a precise answer to the wrong equation. Most of the wrong options in this chapter are exactly that: a correct calculation of something the question did not ask.

Topics covered: solving linear equations · brackets and distribution · equations with fractions · variables on both sides · no solution and infinitely many solutions · translating words into equations · linear inequalities · reading a linear model

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Section 1 — Undoing a Linear Equation

A linear equation is one where the variable appears by itself — no x^2, no $\sqrt{x}$, no x in a denominator. To solve it, strip away everything that has been done to x, in reverse.

If the equation says $3x + 5 = 20$, then x was **multiplied by 3** and then **had 5 added**. Undo in reverse: subtract 5 first, then divide by 3. The order matters, and getting it backwards is the most common wrong answer in this section.

Q1 — Basic

A printing service charges a fixed setup fee of \$5 for an order and a further \$3 for each poster printed. One customer's order came to \$20 in total, so the order satisfies $3x + 5 = 20$, where x is the number of posters. What is the value of x?

A) $\frac{20}{3}$

B) 5

C) $\frac{25}{3}$

D) 15

Q2 — Basic

Two workshops quote for the same job using different formulas. The first workshop's charge in dollars is $7(x - 2)$ and the second's is $3x + 6$, where x is the number of items ordered. If $7(x - 2) = 3x + 6$, what is the value of x?

A) 2

B) 5

C) -2

D) 20

Q3 — Medium

A rod of length x centimetres is cut into several pieces, one of which is a third of the whole rod and another a quarter of it. Those two pieces together measure 7 centimetres. If $\dfrac{x}{3} + \dfrac{x}{4} = 7$, what is the value of x?

A) 49

B) 12

C) 84

D) $\frac{49}{2}$

Q4 — Medium

If $5(2x-3)=3(x+4)+7$, what is the value of x?

A) $\frac{4}{7}$

B) 2

C) $\frac{20}{7}$

D) $\frac{34}{7}$

Section 2 — No Solution, or Every Solution

Some SAT questions do not want a number. They give you an equation with an unknown coefficient and tell you it has **no solution** or **infinitely many solutions**, and ask what that coefficient must be. These look strange the first time and are formulaic once you see the idea.

Strip both sides down to the form $(\text{number})x + (\text{number})$. Then:

The two sides are...	The equation has...
Same x coefficient, **same** constant	infinitely many solutions — it is always true
Same x coefficient, **different** constant	no solution — it is never true
Different x coefficients	exactly one solution

That table is the entire topic. The reasoning behind it: if the x terms are identical they cancel, and you are left with a statement about numbers only — either a true one ($15=15$, always true) or a false one ($15=19$, never true).

Q5 — Medium

The equation $3(2x+5)=ax+15$, in which a is a constant, has infinitely many solutions. What is the value of a?

A) 2

B) 3

C) 15

D) 6

Q6 — Hard

In the equation $4x + 9 = kx - 3$, k is a constant. The equation has no solution. What is the value of k ?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Section 3 — Turning Words into an Equation

Some questions never ask you to solve anything. They describe a situation and ask which equation represents it. The whole question is the translation.

Translate one phrase at a time, left to right, and be careful with two traps the test uses repeatedly:

- **"3 more than 8 times a number"** is $8x + 3$, not $8(x + 3)$. The "8 times" happens first; the 3 is added to the result.
- **"5 less than x"** is $x - 5$, not $5 - x$. The order reverses when you write it down.

Q7 — Basic

Three more than eight times a number x is equal to 83. Which equation represents this situation?

A) $8x + 24 = 83$

B) $8x + 3 = 83$

C) $3x + 8 = 83$

D) $8x - 3 = 83$

Q8 — Medium

A technician charges a fixed call-out fee of $45 plus $28 for each hour worked. The technician charged $185 for one job. Which equation can be used to find h, the number of hours worked?

A) $28h - 45 = 185$

B) $45h + 28 = 185$

C) $28h + 1260 = 185$

D) $28h + 45 = 185$

Q9 — Medium

Hana deposited a fixed amount into her bank account each month. The total in the account after t months is modelled by $f(t) = 100 + 25t$, where $f(t)$ is in dollars. What does the 25 represent?

A) The amount in the account before any deposits

B) The amount she deposits each month

C) The number of months she saved for

D) The total amount in the account after 25 months

Section 4 — Linear Inequalities

An inequality is solved exactly like an equation, with **one** extra rule:

> **When you multiply or divide both sides by a negative number, the inequality sign flips.**

$\leq$ becomes $\geq$, $<$ becomes $>$. Nothing else changes. Adding or subtracting a negative number does *not* flip anything — only multiplying or dividing by one does.

The reason is worth seeing once. Start with something true: $2 < 6$. Multiply both sides by -1: on the number line -2 sits to the *right* of -6, so $-2 > -6$. The order reversed.

Q10 — Basic

A machine setting x must be chosen so that the quantity $-4x + 7$ stays at or above the safety threshold of 19. Which of the following gives all values of x that satisfy $-4x + 7 \geq 19$?

A) $x \leq -3$

B) $x \geq -3$

C) $x \leq 3$

D) $x \geq 3$

Q11 — Medium

A lift can carry a maximum load of 600 kilograms. A crate weighs 45 kilograms and an operator weighs 80 kilograms. Which inequality gives n, the greatest number of crates the operator can take up in one trip?

A) $45n \leq 600$

B) $45n + 80 \geq 600$

C) $45n - 80 \leq 600$

D) $45n + 80 \leq 600$

Q12 — Hard

What is the least integer value of x that satisfies $9 - 2x < 4x - 15$?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q13 — Hard

A control system holds only while the quantity $-3(x - 4)$ stays strictly greater than $2x + 1$, where x is an input the operator can vary. Which of the following gives all values of x for which $-3(x - 4) > 2x + 1$?

A) $x < \frac{11}{5}$

B) $x > \frac{11}{5}$

C) $x < -\frac{13}{5}$

D) $x > -\frac{13}{5}$

Section 5 — Rearranging, and Reading a Model

Two things get asked constantly and neither requires you to find a number.

Rearranging — "solve for x in terms of y" means get x alone on one side. The other letters just come along for the ride; treat them exactly like numbers.

Reading a model — you are given a formula describing a situation and asked what one of its numbers *means*. The answer is always about the **units**.

Q14 — Medium

A budget is described by the equation $4x - 3y = 12$, where x and y are two quantities being planned. Which of the following expresses x in terms of y?

A) $\frac{3y}{4} - 3$

B) $3 - \frac{3y}{4}$

C) $3y + 12$

D) $\frac{3y}{4} + 3$

Q15 — Medium

Three seats in a row of a theatre are numbered consecutively, and the three numbers add up to 84. What is the largest of the three?

A) 27

B) 28

C) 29

D) 42

Q16 — Medium

A tank holds 400 litres of water and is being drained at a constant 15 litres per minute. Which function gives V, the number of litres left after t minutes?

A) $15t + 400$

B) $400 - 15t$

C) $15t - 400$

D) $15 - 400t$

Q17 — Hard

A phone plan charges a fixed \$18 a month plus \$0.06 for each text message. In one month the bill was \$33.60. How many text messages were sent?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Section 6 — Absolute Value

$|x|$ means **how far x is from zero**, ignoring direction. So $|5| = 5$ and $|-5| = 5$. Distance is never negative.

That one idea drives everything here. $|x-3| = 7$ says "x is 7 away from 3", and there are always **two** places 7 away from 3: one above and one below. This is why absolute-value equations usually have two answers, and why forgetting the second one is the mistake the test is looking for.

The method. To solve $|\text{something}| = k$ where k is positive, write two equations and solve each:

$$\text{something} = k \qquad \text{and} \qquad \text{something} = -k$$

Two warnings worth having in advance:

- $|\text{something}| = k$ **has no solution when k is negative.** Nothing is -4 units away from anywhere.
- **Isolate the bars first.** In $2|x-1| + 3 = 11$, subtract the 3 and divide by the 2 *before* splitting into two cases.

Q18 — Basic

A reading x lies exactly 7 units away from the target value 3 on a number line. If $|x-3| = 7$, what is the sum of all possible values of x?

A) 10

B) 6

C) 14

D) -4

Q19 — Medium

If $|2x+1| = 9$, what is the least possible value of x?

A) -5

B) 4

C) -4

D) -10

Q20 — Medium

A component's measurement x satisfies $2|x-1|+3=11$. What is the greatest possible value of x?

A) 4

B) 9

C) $\frac{7}{2}$

D) 5

Q21 — Hard

In the equation $|4x-7|=k-2$, the constant k is chosen by the person setting the question. For which value of k does the equation have **no** solution?

A) 1

B) 2

C) 5

D) 9

Section 7 — Ranges and Compound Inequalities

Two inequalities at once describe a **range**. $2<x<8$ says x is bigger than 2 *and* smaller than 8.

The method. Whatever you do, do it to **all three parts**:

$$1<x+4<9 \quad \Longrightarrow \quad -3<x<5$$

(subtract 4 from each part). And the flip rule still applies: multiplying or dividing all three parts by a negative reverses **both** signs.

Absolute value gives ranges too, and there are exactly two shapes:

Statement	Means	Picture
$\lvert x-a\rvert<k$	within k of a	one interval, $a-k<x<a+k$
$\lvert x-a\rvert>k$	further than k from a	two pieces, going outwards

The first is "less than, and it's *between*". The second is "greater than, and it's *outside*". Mixing those two up is most of what goes wrong here.

Q22 — Medium

A process is running correctly only while the quantity $3x + 4$ stays above -5 and at the same time does not exceed 13, where x is a measured input. If $-5 < 3x + 4 \leq 13$, which of the following gives all possible values of x?

A) $-3 < x < 3$

B) $-3 < x \leq 3$

C) $-9 < x \leq 3$

D) $-3 \leq x < 3$

Q23 — Medium

A machined part is acceptable when its length x lies within 3 millimetres of the target length of 5 millimetres, in either direction. That condition is written $|x - 5| < 3$. Which of the following gives all values of x for which the part is acceptable?

A) $x < 2$ or $x > 8$

B) $2 \leq x \leq 8$

C) $2 < x < 8$

D) $-3 < x < 3$

Q24 — Hard

The acceptable diameter of a machined bolt is within 0.04 mm of 12.00 mm. A bolt whose diameter is d millimetres is **rejected**. Which inequality describes d?

A) $|d - 12| < 0.04$

B) $|d - 0.04| > 12$

C) $|d - 12| \leq 0.04$

D) $|d - 12| > 0.04$

Section 8 — When You Do Not Need to Find x

Some questions look like they want you to solve, and do not. They give you an equation and ask for the value of an **expression** — $2x + 6$, say, rather than x.

You can always find x and substitute. But often the expression is just a multiple of something you already have, and spotting that turns a two-minute question into a five-second one. The SAT sets these deliberately, and the wrong options are built from the values you pass through on the long route.

Q25 — Medium

A quantity satisfies $5x - 3 = 12$, and a second quantity is defined as $10x - 6$. What is the value of $10x - 6$?

A) 3

B) 15

C) 24

D) 30

Q26 — Medium

The equation $3(x + 2) = 3x + k$ is true for all values of x. What is the value of k?

A) 0

B) 2

C) 6

D) 3

Q27 — Basic

Two departments account for the same total in different ways. If $2(3x - 4) = 5x + 7$, what is the value of x?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q28 — Medium

A meter reading is modelled by $0.4x + 1.2$, and on one occasion the reading came to 3.6. If $0.4x + 1.2 = 3.6$, what is the value of x?

A) 12

B) 6

C) 2.4

D) 0.96

Q29 — Hard

Two numbers x and y have a sum of 9 and a difference of 3. What is the value of $x^2 - y^2$?

A) 12

B) 6

C) 36

D) 27

Section 9 — Mixed Practice

Everything above, shuffled and unlabelled, the way the real test presents it. No section heading tells you which method to reach for — deciding that *is* the skill being practised here.

The solutions are shorter from here on. The reasoning has been laid out in full above; what these are for is fluency and speed. Give yourself about ninety seconds a question.

Q30 — Medium

A supplier's charge is $4(x - 3)$ dollars and a rival's is $2x + 8$ dollars for the same order of x units. If $4(x - 3) = 2x + 8$, what is the value of x?

A) $\frac{11}{2}$

B) -2

C) 10

D) 20

Q31 — Medium

A tolerance rule requires the quantity $-2(x+5)$ to remain at or above 4, where x is a setting chosen by the operator. Which of the following gives all values of x for which $-2(x+5) \geq 4$?

A) $x \leq -7$

B) $x \geq -7$

C) $x \leq \frac{1}{2}$

D) $x \geq \frac{1}{2}$

Q32 — Medium

If $7 - 2(x-4) = 3$, what is the value of x?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q33 — Medium

Two numbers are recorded during an experiment. Their sum is 40 and the amount by which the first exceeds the second is 8. What is the larger number?

A) 16

B) 24

C) 32

D) 20

Q34 — Basic

Three quarters of a length x measures 9 centimetres, so $\dfrac{3}{4}x = 9$. What is the value of x?

A) 12

B) $\frac{27}{4}$

C) 36

D) 3

Q35 — Hard

A student's first three test scores are 75, 82 and 78. What is the least score the student can get on a fourth test and still have an average of at least 80 across all four?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q36 — Basic

Two quantities are linked by the equation $2x + 3y = 12$, and in one particular case the second quantity is known to be $y = 2$. What is the value of x?

A) 3

B) 6

C) 9

D) $\frac{9}{2}$

Q37 — Medium

Which expression is equivalent to $5(2x - 3) - 2(x + 4)$?

A) $8x - 23$

B) $8x - 7$

C) $12x - 23$

D) $8x + 23$

Q38 — Basic

A reading is acceptable only while the quantity $5 - 3x$ stays strictly below 20, where x is an adjustable input. Which of the following gives all values of x for which $5 - 3x < 20$?

A) $x < -5$

B) $x > 5$

C) $x > -5$

D) $x < 5$

Q39 — Medium

A measurement x satisfies $|2x - 6| = 10$. What is the product of all possible values of x?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q40 — Medium

A courier charges a fixed \$25 for accepting a job and then a further \$0.15 for every kilometre driven. One delivery was invoiced at \$100 in total. How many kilometres was that delivery?

A) 50

B) 75

C) 11.25

D) 500

Answers and Explanations

Section 1 — Undoing a Linear Equation

Q1 — Basic — Answer: B

Topic: Solving a two-step linear equation

Two things were done to x: it was multiplied by 3, then 5 was added. Undo them in the opposite order.

Subtract 5 from **both** sides:

$$3x + 5 - 5 = 20 - 5 \quad \Longrightarrow \quad 3x = 15$$

Now divide both sides by 3:

$$\frac{3x}{3} = \frac{15}{3} \quad \Longrightarrow \quad x = 5$$

Check it, which takes three seconds: $3(5) + 5 = 15 + 5 = 20$. ✓

Why each wrong option is wrong:

- **A,** $\frac{20}{3}$ — divided by 3 first and ignored the $+5$ entirely. This is the "wrong order" mistake: you cannot divide by 3 until the $+5$ is gone, because the 3 is multiplying only the x, not the whole left-hand side.
- **C,** $\frac{25}{3}$ — *added* 5 to both sides instead of subtracting, giving $3x = 25$. Undoing means doing the **opposite**.
- **D,** 15 — subtracted the 5 correctly to get $3x = 15$, then stopped and wrote down 15. It is the value of $3x$, not of x. Finishing the last step is worth more marks on this test than any clever trick.

> **Takeaway:** Undo in reverse order — additions and subtractions first, multiplication and division last. Then check by substituting back; on a multiple-choice test a three-second check is nearly free.

Q2 — Basic — Answer: B

Topic: Distributing over brackets

Clear the bracket first. The 7 multiplies **everything** inside:

$$7(x-2) = 7x - 14$$

So the equation is $7x - 14 = 3x + 6$. Now gather the x terms on one side and the numbers on the other. Subtract $3x$ from both sides:

$$4x - 14 = 6$$

Add 14 to both sides:

$$4x = 20$$

Divide by 4:

$$x = 5$$

Check: $7(5-2) = 7 \times 3 = 21$, and $3(5) + 6 = 21$. ✓

Why each wrong option is wrong:

- **A,** 2 — distributed the 7 to the x but not to the -2, giving $7x - 2 = 3x + 6$. This is the single most common algebra slip on the test. The 7 sits outside the bracket, so it multiplies **both** terms inside.
- **C,** -2 — moved the -14 across as a -14 instead of a $+14$. When a term crosses the equals sign its sign flips.
- **D,** 20 — did everything right down to $4x = 20$ and then wrote 20. The same unfinished-last-step mistake as Q1's option D, and it is deliberately offered again because it is that common.

> **Takeaway:** The number outside a bracket multiplies every term inside it. Write the expanded form on your paper rather than doing it in your head — this one slip accounts for more lost algebra marks than any other.

Q3 — Medium — Answer: B

Topic: A linear equation containing fractions

Add the two fractions. The lowest common denominator of 3 and 4 is 12:

$$\frac{x}{3} + \frac{x}{4} = \frac{4x}{12} + \frac{3x}{12} = \frac{7x}{12}$$

So $\dfrac{7x}{12} = 7$. Multiply both sides by 12:

$$7x = 84$$

Divide by 7:

$$x = 12$$

Check: $\frac{12}{3} + \frac{12}{4} = 4 + 3 = 7$. ✓

There is a faster route worth knowing: multiply **every term** by 12 at the very start, and the fractions vanish before you ever add them — $4x + 3x = 84$, so $7x = 84$ and $x = 12$.

Why each wrong option is wrong:

- **A,** 49 — added the denominators, treating the sum as $\frac{x}{7} = 7$. Denominators are never added. $\frac{1}{2} + \frac{1}{2}$ is 1, not $\frac{2}{4}$.
- **C,** 84 — reached $7x = 84$ correctly, then stopped. The unfinished last step again.
- **D,** $\frac{49}{2}$ — added numerators *and* denominators, giving $\frac{2x}{7} = 7$. This is the same mistake as A wearing a different hat.

> **Takeaway:** Multiply every term by the common denominator immediately. It turns a fraction problem into an ordinary one, and it is the step a calculator cannot do for you.

Q4 — Medium — Answer: D

Topic: Variables on both sides with brackets

Expand both sides first:

$$5(2x - 3) = 10x - 15 \qquad 3(x + 4) + 7 = 3x + 12 + 7 = 3x + 19$$

So $10x - 15 = 3x + 19$. Subtract $3x$:

$$7x - 15 = 19$$

Add 15:

$$7x = 34$$

$$x = \frac{34}{7}$$

An ugly fraction is not a sign you went wrong. The SAT allows a calculator and often accepts fractions, so do not "fix" a correct answer into a rounder one.

Why each wrong option is wrong:

- **A,** $\frac{4}{7}$ — moved the -15 across as a -15 instead of a $+15$, giving $10x = 3x + 19 - 15$ and so $7x = 4$. When a term crosses the equals sign, its sign flips.
- **B,** 2 — distributed each bracket to its first term only: $10x - 3 = 3x + 4 + 7$, which solves to $7x = 14$. Two distribution slips in one line, and they land on a tidy whole number — which is exactly why a tidy answer is not evidence of a correct one.
- **C,** $\frac{20}{7}$ — subtracted the 7 instead of adding it, treating $3(x + 4) + 7$ as $3x + 12 - 7$. Read the sign in front of a term, not the sign you expect.

> **Takeaway:** Expand every bracket before moving anything. And do not distrust an answer for being an awkward fraction — on this test, roughly a quarter of answers are typed in by hand, and fractions are accepted.

Section 2 — No Solution, or Every Solution

Q5 — Medium — Answer: D

Topic: Infinitely many solutions

Expand the left-hand side:

$$3(2x+5) = 6x + 15$$

So the equation reads $6x + 15 = ax + 15$.

For this to be true for **every** value of x, the two sides must be the same expression. The constants already match (15 on both sides), so the x coefficients must match too:

$$a = 6$$

Sanity check with a number: put $a = 6$ and try $x = 100$. Left: $3(200+5) = 615$. Right: $600 + 15 = 615$. Equal — and it would be equal for any x you tried, which is what "infinitely many solutions" means.

Why each wrong option is wrong:

- **A,** 2 — read the coefficient of x *inside* the bracket and stopped there. The bracket has not been expanded yet; the 3 still has to get in.
- **B,** 3 — took the number outside the bracket instead of the product of the two.
- **C,** 15 — matched the constant term rather than the x coefficient. The constants do have to match, but they already do; a multiplies x.

> **Takeaway:** "Infinitely many solutions" means the two sides are the *same expression*. Expand, then match the x coefficients and the constants separately.

Q6 — Hard — Answer: 4

Topic: No solution

Both sides are already in the form $(\text{number})x + (\text{number})$:

$$4x + 9 \qquad \text{and} \qquad kx - 3$$

"No solution" means there is no value of x that makes the two sides equal. That happens when the x terms are identical — so they cancel — but the constants are not:

$$k = 4$$

Then the equation becomes $4x + 9 = 4x - 3$. Subtract $4x$ from both sides and you are left with

$$9 = -3$$

which is false, no matter what x is. So there is no solution. ✓

Note the contrast with Q5. Same setup, opposite conclusion, and the *only* difference is whether the constants match. Here they cannot: $9 \neq -3$ regardless of k, so matching the coefficients guarantees no solution.

This is a student-produced response — one of the seven questions per module with no options to choose from. You work out the answer and type it in. Note what that removes: you cannot test the four choices, and a blank scores the same as a wrong guess, so always enter something.

> **Takeaway:** No solution ⇒ same x coefficient, different constants. Infinitely many ⇒ same coefficient *and* same constant. Set the coefficients equal and then check what the constants are doing.

Section 3 — Turning Words into an Equation

Q7 — Basic — Answer: B

Topic: Translating a sentence into an equation

Take the sentence in pieces:

The words	The maths
eight times a number x	$8x$
three more than [that]	$8x + 3$
is equal to 83	$8x + 3 = 83$

So the answer is $8x + 3 = 83$. (If you want to check it, solving gives $8x = 80$, so $x = 10$ — and eight tens plus three really is 83.)

Why each wrong option is wrong:

- **A,** $8x + 24 = 83$ — this says "eight times *three more than* the number". The bracket makes the addition happen first, which reverses the order the sentence gives.
- **C,** $3x + 8 = 83$ — swapped the 3 and the 8. Reading the numbers in the order they appear in the sentence is not the same as reading the *operations* in order.
- **D,** $8x - 3 = 83$ — "more than" means add. This option exists because under time pressure people pattern-match on "than" and reach for subtraction.

> **Takeaway:** Translate phrase by phrase and write each piece down before joining them. "More than" adds; "less than" subtracts **and** reverses the order.

Q8 — Medium — Answer: D

Topic: Building an equation from a described situation

Sort the two numbers by what they do:

- $45 is **fixed** — it is charged once, whatever happens. It is not multiplied by anything.
- $28 is **per hour** — it is multiplied by the number of hours, h.

Total charge = hourly part + fixed part:

$$28h + 45 = 185$$

(Solving is not required here, but it gives $28h = 140$, so $h = 5$ hours.)

Why each wrong option is wrong:

- **B,** $45h + 28 = 185$ — swapped the roles. This charges $45 an hour and adds a $28 fee. The words "for each hour worked" attach to 28, not 45.
- **C,** $28h + 1260 = 185$ — puts the fixed fee inside the bracket, so it gets multiplied by 28 as well. A fixed fee is added once, at the end.
- **A,** $28h - 45 = 185$ — subtracts the call-out fee. The technician is charging it, not discounting it.

> **Takeaway:** In any "fixed amount plus a rate" question, find which number is multiplied by the variable and which stands alone. The word "per" or "each" marks the one that gets multiplied.

Q9 — Medium — Answer: B

Topic: Interpreting the parts of a linear model

In a linear model $f(t) = b + mt$, the two numbers have fixed jobs:

- b**, the constant** — the value when $t = 0$, the **starting** amount.
- m**, the number multiplied by** t — how much the total changes **per unit of** t, here per month.

The 25 is multiplied by t, so it is the rate: $25 added each month.

Test it. After 1 month: $f(1) = 100 + 25 = 125$. After 2 months: $f(2) = 100 + 50 = 150$. The total goes up by $25 each month. That is what the 25 means.

Why each wrong option is wrong:

- **A** — that is the 100, the value at $t = 0$, before any monthly deposit.
- **C** — t is the number of months; 25 is what multiplies it. Confusing a variable with its coefficient is one of the test's favourite traps.
- **D** — nothing here is "after 25 months". The 25 is a rate in dollars per month, not a count of months. Watch the **units**: the answer must be in dollars per month, and this option is in dollars.

> **Takeaway:** In $f(t) = b + mt$, b is where it starts and m is how fast it changes. When two options are close, check the units — a rate and a total cannot both be right.

Section 4 — Linear Inequalities

Q10 — Basic — Answer: A

Topic: Solving an inequality that requires a sign flip

Subtract 7 from both sides. Subtraction never flips the sign:

$$-4x \geq 12$$

Now divide both sides by -4. This is a division by a negative number, so the sign **flips** from $\geq$ to $\leq$:

$$x \leq -3$$

Check with a number that should work: $x = -10$ satisfies $x \leq -3$. Put it in the original: $-4(-10) + 7 = 40 + 7 = 47$, and $47 \geq 19$. ✓

Now check one that should fail: $x = 0$. Then $-4(0) + 7 = 7$, and $7 \geq 19$ is false. ✓ The boundary is in the right place and the inequality points the right way.

Why each wrong option is wrong:

- **B,** $x \geq -3$ — the arithmetic is right and the flip was forgotten. This is *the* mistake this topic tests, and it is why the option is offered.

- **C,** $x \leq 3$ — flipped the sign but dropped the minus on the 3. Both the sign of the inequality and the sign of the number come from that division.
- **D,** $x \geq 3$ — neither the flip nor the minus sign. Substituting $x = 4$ gives -9, which is not ≥ 19, so this fails the check immediately.

> **Takeaway:** Divide by a negative, flip the sign. Then test one number from your answer in the *original* inequality — it catches this mistake every time, and it takes five seconds.

Q11 — Medium — Answer: D

Topic: Writing an inequality from a constraint

Work out what is in the lift, then say what the limit does to it.

The load is the crates plus the operator, who rides up with them:

$$45n + 80$$

"A maximum load of 600" means the load must be **600 or less**:

$$45n + 80 \leq 600$$

That is the answer. (Solving it — not asked for here — gives $45n \leq 520$, so $n \leq 11.6$, meaning 11 crates. You cannot take 0.6 of a crate, so the answer to *that* question would round **down**, never up, no matter what the decimal is.)

Why each wrong option is wrong:

- **A,** $45n \leq 600$ — forgot that the operator goes up with the crates. The sign is right and the crate weight is right, but the person standing in the lift counts towards the load. This option allows 13 crates, and 13 crates plus an operator is 665 kg.
- **B,** $45n + 80 \geq 600$ — the correct left-hand side with the sign reversed. "Maximum" means at most, so the sign is $\leq$.
- **C,** $45n - 80 \leq 600$ — subtracts the operator, as though the person reduces the load. They are standing in the lift; they add to it.

> **Takeaway:** "Maximum", "at most", "no more than" all mean $\leq$. "Minimum", "at least", "no fewer than" all mean $\geq$. Write down which one the sentence uses before you write the inequality — and when a count of physical objects comes out as a decimal, round **down**.

Q12 — Hard — Answer: 5

Topic: An inequality with the variable on both sides

Gather the x terms on the side that keeps them positive — it avoids the sign flip altogether. Add $2x$ to both sides:

$$9 < 6x - 15$$

Add 15 to both sides:

$$24 < 6x$$

Divide by 6. Six is positive, so **nothing flips**:

$$4 < x$$

So x is strictly greater than 4. The question asks for the least **integer**, and the inequality is strict — $x = 4$ does not satisfy it, because $4 < 4$ is false. The least integer greater than 4 is

$$x = 5$$

Check both sides of the boundary. At $x = 5$: left is $9 - 10 = -1$, right is $20 - 15 = 5$, and $-1 < 5$. ✓ At $x = 4$: left is 1, right is 1, and $1 < 1$ is false. ✓ So 5 is the first integer that works.

Two things this question is testing at once, and you need both:

1. Solving the inequality without a sign error.
2. Noticing that **strict** ($<$) excludes the boundary, so the answer is the next integer up, not 4.

> **Takeaway:** Move the variable to the side that keeps its coefficient positive and you never have to flip anything. Then read the sign again: $<$ excludes the boundary value, $\leq$ includes it — and that difference is usually the whole question.

Q13 — Hard — Answer: A

Topic: An inequality where the bracket hides a sign

Expand the bracket. The -3 multiplies both terms, and $-3 \times -4 = +12$:

$$-3x + 12 > 2x + 1$$

That plus sign is the whole question. Now move the x terms to the side that keeps them positive — add $3x$ to both sides:

$$12 > 5x + 1$$

Subtract 1:

$$11 > 5x$$

Divide by 5, which is positive, so nothing flips:

$$\frac{11}{5} > x \qquad \text{that is} \qquad x < \frac{11}{5}$$

Check a value that should work, $x = 0$: left is $-3(-4) = 12$, right is 1, and $12 > 1$. ✓ Check one that should not, $x = 3$: left is $-3(-1) = 3$, right is 7, and $3 > 7$ is false. ✓

Why each wrong option is wrong:

- **B,** $x > \frac{11}{5}$ — right boundary, wrong direction. This comes from gathering the x terms on the left instead, getting $-5x > -11$, and then dividing by -5 without flipping.
- **C,** $x < -\frac{13}{5}$ — distributed $-3(x-4)$ as $-3x - 12$. Multiplying two negatives gives a positive, so the 12 is positive.
- **D,** $x > -\frac{13}{5}$ — both mistakes at once. Substituting $x = 0$, which satisfies this option, gives $12 > 1$ — true, so this option is not obviously wrong from one test. Test the *boundary*, not a convenient number.

> **Takeaway:** $-3 \times -4 = +12$. When a negative multiplies a bracket, write the expansion down rather than doing it in your head, then move the variable to whichever side keeps it positive.

Section 5 — Rearranging, and Reading a Model

Q14 — Medium — Answer: D

Topic: Solving for one variable in terms of another

Get the x term on its own. The $-3y$ moves across and becomes $+3y$:

$$4x = 3y + 12$$

Now divide **everything** on the right by 4:

$$x = \frac{3y + 12}{4}$$

Check it by picking a value. Let $y = 4$. The original says $4x - 12 = 12$, so $4x = 24$ and $x = 6$. The answer gives $\frac{12+12}{4} = 6$. ✓

That substitution trick is worth having: it turns a question in letters into a question in numbers, and it works on every option.

Why each wrong option is wrong:

- **A,** $\frac{3y}{4} - 3$ — moved the 12 across as if it needed a sign change. It was already on the right-hand side; nothing moved, so nothing flips.
- **B,** $3 - \frac{3y}{4}$ — moved the $-3y$ across but kept it negative. A term that crosses the equals sign changes sign.
- **C,** $3y + 12$ — divided nothing by 4. Test it with $y = 4$: it gives 24, not 6.

> **Takeaway:** Treat the other letters exactly as you would treat numbers. Then check by substituting one convenient value into both the original and your answer — it catches every sign slip.

Q15 — Medium — Answer: C

Topic: Consecutive integers

"Consecutive" means each is one more than the last. Call the smallest n; then the three are n, $n + 1$ and $n + 2$. Their sum:

$$n + (n + 1) + (n + 2) = 3n + 3$$

Set that equal to 84:

$$3n + 3 = 84 \quad \Longrightarrow \quad 3n = 81 \quad \Longrightarrow \quad n = 27$$

So the three integers are $27, 28, 29$, and the **largest** is 29.

Check: $27 + 28 + 29 = 84$. ✓

Why each wrong option is wrong:

- **A,** 27 — the smallest, not the largest. Everything was solved correctly and the last line of the question was not read.
- **B,** 28 — the middle one. Worth noticing that the middle of three consecutive integers is always the sum divided by 3, which is a genuinely useful shortcut: $84 \div 3 = 28$, so the three are $27, 28, 29$ immediately.
- **D,** 42 — halved 84. There are three numbers, not two.

> **Takeaway:** With three consecutive integers, the middle one is the sum divided by 3 — that gets you there in one step. Then read the last line again and check *which* of the three the question wanted.

Q16 — Medium — Answer: B

Topic: What a coefficient means in a model

Sort the two numbers by the job they do:

- **400** is the amount at the start, before any time passes. It stands alone.
- **15** is a rate — litres **per minute** — so it is multiplied by t.

The water is being *removed*, so the 15 per minute is subtracted:

$$V = 400 - 15t$$

Check at a couple of times. At $t = 0$: $V = 400$, the tank is full. ✓ At $t = 10$: $V = 400 - 150 = 250$ litres. ✓

Why each wrong option is wrong:

- **A,** $15t + 400$ — this fills the tank rather than draining it. After ten minutes it claims 550 litres, in a tank that holds 400.
- **C,** $15t - 400$ — the right two numbers, subtracted the wrong way round. At $t = 0$ it gives -400 litres, which is not a quantity of water.
- **D,** $15 - 400t$ — swapped the roles, so it starts with 15 litres and drains 400 a minute. Checking $t = 0$ eliminates this in two seconds.

> **Takeaway:** In any "starts at, changes by" model, the standalone number is the starting value and the number multiplied by the variable is the rate. Then substitute $t = 0$: it instantly kills every option with the two swapped.

Q17 — Hard — Answer: 260

Topic: Building and solving in one question

Let m be the number of messages. The bill is the fixed part plus the message part:

$$18 + 0.06m = 33.60$$

Subtract the fixed charge:

$$0.06m = 15.60$$

Divide:

$$m = \frac{15.60}{0.06} = 260$$

Check: $260 \times 0.06 = 15.60$, and $15.60 + 18 = 33.60$. ✓

A word about the division, since a calculator is allowed and people still get this one wrong. $15.60 \div 0.06$ is not 2.6. Multiply both numbers by 100 first — $1560 \div 6 = 260$ — and the decimal stops being frightening. If your answer to a "how many messages" question comes out as 2.6, the size of it is telling you something is wrong.

This is a grid-in. Enter `260`. Not `260 messages`, not `$260`.

> **Takeaway:** Subtract the fixed charge before dividing, and sanity-check the *size* of your answer against the situation. A phone bill of $33.60 buying 2.6 text messages should stop you immediately.

Section 6 — Absolute Value

Q18 — Basic — Answer: B

Topic: Both solutions of an absolute-value equation

$|x-3| = 7$ says x sits 7 away from 3. Split it into the two cases:

$$\begin{aligned} x - 3 = 7 &\quad\Longrightarrow\quad x = 10 \\ x - 3 = -7 &\quad\Longrightarrow\quad x = -4 \end{aligned}$$

Check both. $|10-3| = |7| = 7\,\checkmark$. $|-4-3| = |-7| = 7\,\checkmark$. Both work, which is the point — there are genuinely two answers.

The question asks for their **sum**:

$$10 + (-4) = 6$$

Why each wrong option is wrong:

- **A,** 10 — solved only the first case and stopped. This is the mistake the whole question exists to catch: the bars always give you two cases.
- **C,** 14 — treated the answers as $+7$ and -7 and added the sizes. The 7 is the *distance*, not either answer; you still have to undo the -3.
- **D,** -4 — solved only the second case. The mirror image of A.

There is a shortcut worth knowing. The two solutions of $|x-a| = k$ are always $a+k$ and $a-k$, so they are symmetric about a — and their sum is always $2a$. Here $2 \times 3 = 6$, in one step and with no cases at all.

> **Takeaway:** Absolute value gives **two** cases, always. And the two solutions of $|x - a| = k$ sum to $2a$ no matter what k is, which answers a whole family of questions instantly.

Q19 — Medium — Answer: A

Topic: The least solution of an absolute-value equation

Split into two cases. The bars come off, and the right-hand side takes each sign in turn:

$$2x + 1 = 9 \implies 2x = 8 \implies x = 4$$
$$2x + 1 = -9 \implies 2x = -10 \implies x = -5$$

Check the second one, since it is the one being asked for: $|2(-5) + 1| = |-10 + 1| = |-9| = 9$ ✓

The two solutions are 4 and -5, and the **least** is -5.

Why each wrong option is wrong:

- **B,** 4 — the *greatest* solution. Both cases were solved correctly and the last line of the question was not read. On a test this well-behaved, that is the most expensive habit there is.
- **C,** -4 — in the second case, moved the $+1$ across without changing its sign: $2x = -9 + 1$ instead of $-9 - 1$.
- **D,** -10 — got to $2x = -10$ and wrote down -10. The unfinished last step.

> **Takeaway:** Solve both cases, then go back and read whether the question wants the greatest, the least, the sum, or the product. The maths is usually the easy half.

Q20 — Medium — Answer: D

Topic: Isolating the bars before splitting

The bars are wrapped in other operations, so undo those **first**. Do not split into cases while there is still a $\times 2$ and a $+3$ attached.

Subtract 3:

$$2|x - 1| = 8$$

Divide by 2:

$$|x - 1| = 4$$

Now split:

$$x - 1 = 4 \implies x = 5$$
$$x - 1 = -4 \implies x = -3$$

The greatest is $x = 5$. Check: $2|5 - 1| + 3 = 2(4) + 3 = 11$ ✓

Why each wrong option is wrong:

- **A,** 4 — reached $|x - 1| = 4$ and stopped. That is the *distance* from 1, not the value of x; the -1 still has to be undone.
- **B,** 9 — subtracted the 3 but never divided by 2, so the working used $|x - 1| = 8$ instead of 4. Both operations outside the bars have to come off.
- **C,** $\frac{7}{2}$ — divided by 2 before subtracting the 3, so the 3 got halved too, giving $|x - 1| + 3 = \frac{11}{2}$. Undo additions before multiplications, exactly as in Section 1.

> **Takeaway:** Get the absolute value alone on one side before you split into cases. Everything outside the bars is undone first, in the usual reverse order.

Q21 — Hard — Answer: A

Topic: An absolute-value equation with no solution

Everything turns on one fact: **an absolute value is never negative.** $|4x - 7|$ is a distance, so whatever x is, the left-hand side is zero or more.

So the equation has no solution exactly when the right-hand side is **negative**:

$$k - 2 < 0 \implies k < 2$$

Of the options, only $k = 1$ satisfies that. With $k = 1$ the equation reads $|4x - 7| = -1$, and nothing is -1 units away from anything.

It is worth seeing the full picture, because the test asks about all three cases:

$k - 2$	Number of solutions
negative	**none**
zero	exactly **one** — where $4x - 7 = 0$
positive	**two**

Why each wrong option is wrong:

- **B,** 2 — gives $|4x - 7| = 0$, which has exactly **one** solution, $x = \frac{7}{4}$. One is not none, and this is the option that catches people who half-remember the rule.

- **C,** 5 — gives $|4x - 7| = 3$: two solutions.
- **D,** 9 — gives $|4x - 7| = 7$: two solutions.

> **Takeaway:** An absolute value is never negative. Negative right-hand side means no solution; zero means exactly one; positive means two. The zero case is the one the test uses to separate people who know the rule from people who nearly do.

Section 7 — Ranges and Compound Inequalities

Q22 — Medium — Answer: B

Topic: Solving a three-part inequality

A three-part inequality is solved by doing the same thing to **all three parts** at once. Start with

$$-5 < 3x + 4 \leq 13$$

Subtract 4 from every part:

$$-9 < 3x \leq 9$$

Divide every part by 3. Three is positive, so no sign flips:

$$-3 < x \leq 3$$

Notice that the two ends are **different**. The left stayed strict ($<$), the right stayed inclusive ($\leq$) — each end keeps whatever sign it started with. That asymmetry is the whole question, and it is why three of the four options have the right numbers.

Check both ends. At $x = 3$: the middle is $3(3) + 4 = 13$, and $13 \leq 13$ is true, so 3 **is** included. ✓ At $x = -3$: the middle is $-9 + 4 = -5$, and $-5 < -5$ is false, so -3 is **not** included. ✓

Why each wrong option is wrong:

- **A,** $-3 < x < 3$ — excluded 3 as well. But $x = 3$ gives exactly 13, and the original allows 13.
- **C,** $-9 < x \leq 3$ — subtracted the 4 but never divided by 3 at the lower end, so the -9 was left behind.
- **D,** $-3 \leq x < 3$ — the two ends swapped. It includes -3, which fails, and excludes 3, which works — wrong at both ends at once.

> **Takeaway:** Do every step to all three parts, and carry each end's sign through unchanged. When the options differ only in $<$ versus $\leq$, test the two endpoints in the original — it settles the question in ten seconds.

Q23 — Medium — Answer: C

Topic: An absolute value that means "within"

Read the statement in words: $|x - 5| < 3$ says x **is less than 3 away from 5**. Everything within 3 of 5 — so the answer is one interval, sitting around 5.

Go 3 below 5 and 3 above 5:

$$5 - 3 = 2 \qquad 5 + 3 = 8$$

The sign is strict ($<$), so the endpoints themselves are excluded — a point exactly 3 away is not *less* than 3 away:

$$2 < x < 8$$

If you prefer the mechanical route, "less than" unwraps into a three-part inequality directly:

$$-3 < x - 5 < 3 \quad \Longrightarrow \quad 2 < x < 8$$

Check: $x = 5$ gives $|0| = 0 < 3$ ✓. $x = 2$ gives $|-3| = 3$, and $3 < 3$ is false ✓. $x = 9$ gives $|4| = 4$, not less than 3 ✓.

Why each wrong option is wrong:

- **A,** $x < 2$ **or** $x > 8$ — this is the answer to $|x - 5| > 3$, the "further than" version. It has the right boundaries pointing the wrong way. **Less than gives you the inside; greater than gives you the outside.**
- **B,** $2 \leq x \leq 8$ — right interval, but includes the endpoints. $x = 2$ is exactly 3 away, and the question asked for *less* than 3.
- **D,** $-3 < x < 3$ — used the 3 as the boundaries and ignored the 5. The 3 is the distance; the 5 is where you measure from.

> **Takeaway:** $|x - a| < k$ is "within k of a" — one interval, centred on a, reaching out k each way. $|x - a| > k$ is "further than k from a" — two pieces heading outwards. Say the statement in words before you write anything.

Q24 — Hard — Answer: D

Topic: An absolute value that means "further than"

Work out what is being asked before reaching for notation. The bolt is **acceptable** when its diameter is within 0.04 of 12.00:

$$|d - 12| \leq 0.04$$

But the question asks when the bolt is **rejected** — the opposite. A bolt is rejected when it is *further* than 0.04 from 12:

$$|d - 12| > 0.04$$

That is the whole question: spotting that "rejected" means you want the outside, not the inside.

Test it. A bolt at 12.05 mm: $|12.05 - 12| = 0.05$, and $0.05 > 0.04$, so it is rejected ✓. A bolt at 12.03: $|0.03| = 0.03$, not greater than 0.04, so it is kept ✓. A bolt at 11.90: $|-0.10| = 0.10 > 0.04$, rejected ✓ — and notice the absolute value handles the *undersized* bolt without any extra work. That is exactly why the notation is used here.

Why each wrong option is wrong:

- **A,** $|d - 12| < 0.04$ — describes the bolts that are **accepted**, not rejected. The arithmetic is right and the question was answered backwards.
- **B,** $|d - 0.04| > 12$ — swapped the target and the tolerance, asking whether d is more than 12 away from 0.04. Every bolt near 12 mm satisfies it, so this rejects the entire batch.
- **C,** $|d - 12| \leq 0.04$ — the accepted set again, this time including the boundary. Two ways of answering the opposite question are offered because answering the opposite question is the mistake being tested.

> **Takeaway:** Write the condition the words give you, then check whether the question wants that set or its opposite. "Within" is $\leq$ and gives the inside; "further than", "outside tolerance" and "rejected" are $>$ and give the outside.

Section 8 — When You Do Not Need to Find x

Q25 — Medium — Answer: C

Topic: Evaluating an expression rather than solving for the variable

Look at the two expressions side by side before doing anything:

$$5x - 3 \qquad \text{and} \qquad 10x - 6$$

The second is exactly **twice** the first. So if $5x - 3 = 12$, then

$$10x - 6 = 2(5x - 3) = 2 \times 12 = 24$$

No solving required at all.

The long way works too and is worth checking against. $5x - 3 = 12$ gives $5x = 15$, so $x = 3$. Then $10(3) - 6 = 30 - 6 = 24$ ✓ — same answer, more steps.

Why each wrong option is wrong:

- **A,** 3 — the value of x. Correct as far as it goes, and not what was asked. Every value you pass through on the long route is offered as an option here; that is not an accident.
- **B,** 15 — the value of $5x$.
- **D,** 30 — the value of $10x$, with the -6 forgotten at the last moment.

> **Takeaway:** Before solving, check whether the expression you want is a simple multiple of the one you were given. Look for the doubling — it is the most common version by far. And when the options include the intermediate values, that is a sign the question is testing whether you read the last line.

Q26 — Medium — Answer: C

Topic: An identity, and what makes it one

"True for all values of x" means the two sides are the **same expression**. Expand the left:

$$3(x + 2) = 3x + 6$$

So the equation reads

$$3x + 6 = 3x + k$$

The $3x$ matches on both sides already, so the constants must match too:

$$k = 6$$

There is a quicker route that works on every question of this type. If it is true for *all* x, it is true for the most convenient x you can think of. Put $x = 0$:

$$3(0+2) = 3(0) + k \implies 6 = k$$

One line, no expanding.

Why each wrong option is wrong:

- **A,** 0 — assumed everything has to cancel to $0 = 0$. Not so: an identity needs the two sides to be *equal*, not zero. With $k = 0$ the equation becomes $3x + 6 = 3x$, which is never true.
- **B,** 2 — read the 2 from inside the bracket, before the 3 was distributed.
- **D,** 3 — read the 3 from outside the bracket. k is a constant term, and the 3 is a coefficient; they are not doing the same job.

> **Takeaway:** "True for all x" means the two sides are identical, so match coefficients and constants separately — or just substitute $x = 0$, which is faster and harder to get wrong.

Q27 — Basic — Answer: 15

Topic: Variables on both sides, no options

Expand the bracket:

$$6x - 8 = 5x + 7$$

Subtract $5x$ from both sides, which keeps the x term positive:

$$x - 8 = 7$$

Add 8:

$$x = 15$$

Check: $2(3 \times 15 - 4) = 2(45 - 4) = 2 \times 41 = 82$, and $5 \times 15 + 7 = 75 + 7 = 82$ ✓

This is a grid-in. Enter `15`.

Notice what the check bought you here. There are no options to work backwards from, so substituting your answer into the original is the *only* way to catch a sign error — and it takes ten seconds. On grid-ins, always spend them.

> **Takeaway:** With no options available, substituting your answer back into the original equation is your entire safety net. Build the habit on the easy ones so it is automatic on the hard ones.

Q28 — Medium — Answer: B

Topic: A linear equation with decimals

Decimals change nothing about the method. Subtract 1.2 from both sides:

$$0.4x = 2.4$$

Now divide by 0.4:

$$x = \frac{2.4}{0.4} = 6$$

If dividing by a decimal makes you uneasy, multiply the top and bottom by 10 first — $\frac{24}{4} = 6$ — which is the same number with the decimal point gone. You do have a calculator here, but knowing the answer should be a small whole number tells you instantly whether you typed it in correctly.

Check: $0.4(6) + 1.2 = 2.4 + 1.2 = 3.6$ ✓

Why each wrong option is wrong:

- **A,** 12 — added the 1.2 instead of subtracting it, giving $0.4x = 4.8$. Undoing means the opposite operation.
- **C,** 2.4 — reached $0.4x = 2.4$ and wrote down 2.4. The unfinished last step, now wearing decimals.
- **D,** 0.96 — multiplied by 0.4 rather than dividing: $2.4 \times 0.4 = 0.96$. Worth a sanity check — if x were less than 1, then $0.4x$ would be well under 1, and it needs to reach 2.4.

> **Takeaway:** Treat decimals exactly as you would whole numbers, and clear the decimal point by multiplying top and bottom by 10 when a division looks awkward. Then ask whether the size of your answer makes sense.

Q29 — Hard — Answer: D

Topic: An expression built from two given equations

You could solve for x and y — and it is worth doing once to see that it works. Adding the two equations gives $2x = 12$, so $x = 6$; then $y = 3$. And $36 - 9 = 27$.

But there is a much better route, and it is the one the question is built around. The **difference of two squares** factorises:

$$x^2 - y^2 = (x + y)(x - y)$$

Both of those brackets were handed to you:

$$x^2 - y^2 = 9 \times 3 = 27$$

One multiplication. No solving, no substitution.

This pattern is worth committing to memory because the SAT reuses it constantly: whenever you see $x^2 - y^2$ alongside $x + y$ and $x - y$, the question is about the factorisation and nothing else.

Why each wrong option is wrong:

- **A,** 12 — added the two given values instead of multiplying them.
- **B,** 6 — subtracted them.
- **C,** 36 — found $x = 6$ correctly and then squared only x, forgetting the $-y^2$.

> **Takeaway:** $x^2 - y^2 = (x + y)(x - y)$, always. When a question hands you $x + y$ and $x - y$ and asks for $x^2 - y^2$, it is testing that one identity — multiply and move on.

Section 9 — Mixed Practice

Q30 — Medium — Answer: C

Topic: Brackets with variables on both sides

Expand: $4x - 12 = 2x + 8$. Subtract $2x$: $2x - 12 = 8$. Add 12: $2x = 20$, so $x = 10$. Check: $4(7) = 28$ and $2(10) + 8 = 28$ ✓

A did not distribute the 4 to the -3. **B** carried -12 across without flipping its sign. **D** stopped at $2x = 20$.

> **Takeaway:** Distribute, gather, finish. The three wrong options here are the three places people stop or slip.

Q31 — Medium — Answer: A

Topic: A negative multiplier in an inequality

Expand: $-2x - 10 \geq 4$. Add 10: $-2x \geq 14$. Divide by -2 — a negative, so the sign **flips**: $x \leq -7$. Check $x = -7$: $-2(-2) = 4 \geq 4$ ✓

B forgot the flip. **C** distributed the -2 to the x but not to the 5. **D** made both mistakes.

> **Takeaway:** Distribute first, then flip on dividing by a negative. Test the endpoint in the original to confirm which way it points.

Q32 — Medium — Answer: 6

Topic: A negative multiplier in an equation

The -2 multiplies both terms, and $-2 \times -4 = +8$:

$$7 - 2x + 8 = 3 \quad \Longrightarrow \quad 15 - 2x = 3 \quad \Longrightarrow \quad 2x = 12 \quad \Longrightarrow \quad x = 6$$

Check: $7 - 2(6 - 4) = 7 - 4 = 3$ ✓

This is a grid-in. Enter 6.

> **Takeaway:** A minus in front of a bracket changes the sign of *everything* inside it. Writing the expanded line out is worth the four seconds.

Q33 — Medium — Answer: B

Topic: Two conditions on two numbers

Add the two statements: $(x + y) + (x - y) = 40 + 8$, so $2x = 48$ and $x = 24$. Then $y = 16$. Check: $24 + 16 = 40$ ✓ and $24 - 16 = 8$ ✓

A is the smaller number. **C** subtracted the two given values. **D** halved the sum, which would be right only if the difference were zero.

> **Takeaway:** Sum and difference given? Add the equations and the smaller variable disappears. The larger number is always $\frac{\text{sum}+\text{difference}}{2}$.

Q34 — Basic — Answer: A

Topic: A fractional coefficient

Dividing by $\frac{3}{4}$ is multiplying by $\frac{4}{3}$:

$$x = 9 \times \frac{4}{3} = 12$$

Check: $\frac{3}{4} \times 12 = 9$ ✓

B multiplied by $\frac{3}{4}$ rather than dividing. **C** did only the $\times 4$; **D** did only the $\div 3$.

> **Takeaway:** To undo a fraction, multiply by its reciprocal — both parts of it. Doing half the job is what options C and D are for.

Q35 — Hard — Answer: 85

Topic: An average constraint

An average of at least 80 over four tests means the four scores must total at least $4 \times 80 = 320$. The first three total $75 + 82 + 78 = 235$, so the fourth must be at least

$$320 - 235 = 85$$

Check: $\frac{75+82+78+85}{4} = \frac{320}{4} = 80$ ✓ — exactly 80, and "at least 80" allows exactly 80, so 85 is permitted and is the least such score.

This is a grid-in. Enter 85.

> **Takeaway:** Turn "average of at least A over n things" into "total of at least nA" straight away. Averages are much easier to reason about as totals.

Q36 — Basic — Answer: A

Topic: Substituting a known value

Substitute $y = 2$ first: $2x + 6 = 12$, so $2x = 6$ and $x = 3$.

B is the value of $2x$. **C** and **D** both subtracted 3 rather than $3y = 6$ — C then stopped, D finished.

> **Takeaway:** Substitute the known value before doing anything else, and be careful that $3y$ is 6 here, not 3.

Q37 — Medium — Answer: A

Topic: Expanding and collecting two brackets

$$5(2x - 3) = 10x - 15 \qquad -2(x + 4) = -2x - 8$$

Collect: $10x - 2x = 8x$ and $-15 - 8 = -23$, giving $8x - 23$.

B left the $+4$ positive. **C** added the second bracket instead of subtracting. **D** made the constant positive.

> **Takeaway:** A minus before a bracket flips **every** sign inside it. Expand on one line, collect on the next, and never do both at once.

Q38 — Basic — Answer: C

Topic: An inequality with the variable behind a minus sign

Subtract 5: $-3x < 15$. Divide by -3 and **flip**: $x > -5$. Check $x = 0$: $5 < 20$ ✓. Check $x = -5$: $5 + 15 = 20$, and $20 < 20$ is false ✓

A forgot the flip. **B** lost the minus sign. **D** lost both.

> **Takeaway:** Divide by a negative, flip the sign. Then test one value from your answer and one from outside it.

Q39 — Medium — Answer: -16

Topic: The product of two absolute-value solutions

Split into two cases:

$$2x - 6 = 10 \Longrightarrow x = 8 \qquad 2x - 6 = -10 \Longrightarrow x = -2$$

Their product is $8 \times (-2) = -16$.

This is a grid-in, and the answer is negative. Enter `-16`. Three characters, well inside the six the grid allows for a negative answer — the minus sign counts as one of them.

> **Takeaway:** Solve both cases, then do what the question asked with them. And do not be alarmed by a negative grid-in answer; the grid accepts them.

Q40 — Medium — Answer: D

Topic: Solving a linear model for the input

$$25 + 0.15m = 100 \quad \Longrightarrow \quad 0.15m = 75 \quad \Longrightarrow \quad m = \frac{75}{0.15} = 500$$

Check: $500 \times 0.15 = 75$, and $75 + 25 = 100$ ✓

A used 1.5 in place of 0.15. **B** stopped one step early. **C** multiplied by 0.15 instead of dividing.

> **Takeaway:** Subtract the fixed charge, then divide by the rate. If the division looks awkward, scale both numbers by 100 first: $7500 \div 15 = 500$.

Chapter 2 — Lines, Linear Functions and Systems

This is the other half of the SAT's algebra, and between them Chapters 1 and 2 account for roughly **35%** of the Math section.

Everything here rests on one picture. A **linear** relationship is one that changes by the same amount every step: add 1 to x and y always moves by the same fixed number. That fixed number is the **slope**, and almost every question in this chapter is really asking about it — sometimes by name, more often disguised as "rate of change", "per hour", "for each additional", or a pair of points on a graph.

A note on the calculator. The built-in graphing calculator is genuinely useful here, more than anywhere else on the test. If a question gives you two equations and asks where they meet, you can type both in and read the intersection off the screen. Learn to do it algebraically as well, because a question can hand you letters instead of numbers — but know that the graph is there.

Topics covered: slope from two points · the three forms of a line · parallel and perpendicular lines · interpreting a linear model · solving systems by substitution and elimination · systems with no solution or infinitely many

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Section 1 — Slope

The slope of a line is how much y changes for each 1 that x increases:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

This formula is not on the reference sheet. Learn it.

Two things to hold on to:

- **Subtract in the same order on top and bottom.** If you start with the second point's y, start with the second point's x as well. Reversing one but not the other flips the sign, and a sign-flipped slope is offered as an option every time.
- **A positive slope rises left to right; a negative slope falls.** Before you trust an answer, look at whether it should be going up or down.

Two special cases the test likes:

Line	Slope
Horizontal ($y = 4$)	0
Vertical ($x = 4$)	undefined

Q1 — Basic

A seedling's height was measured on two occasions during an experiment, and the two measurements were recorded in the table below.

Day, x	2	6
Height in centimetres, y	3	11

A) $\frac{1}{2}$

B) 2

C) -2

D) $\frac{7}{4}$

Q2 — Medium

A tree's height was 140 cm after 2 years and 260 cm after 8 years. If the growth is linear, by how many centimetres does the tree grow each year?

A) 60

B) 12

C) $\frac{65}{2}$

D) 20

Q3 — Medium

A sensor recorded the same reading at two different positions along a bench, and those readings are plotted as the points $(-3, 5)$ and $(4, 5)$. What is its slope?

A) 1

B) 7

C) $\frac{5}{7}$

D) 0

Section 2 — The Forms of a Line

The same line can be written three ways. The test moves between them constantly, and each form is convenient for a different question.

Form	Looks like	Read off it immediately
Slope-intercept	$y = mx + b$	slope m, y-intercept b
Point-slope	$y - y_1 = m(x - x_1)$	slope m, a point (x_1, y_1)
Standard	$Ax + By = C$	both intercepts, quickly

Slope-intercept is the one to convert into. If a question gives you $3x + 4y = 12$ and asks for the slope, rearrange to $y = -\frac{3}{4}x + 3$ and read it off. Guessing the slope from the standard form's coefficients is where marks disappear — the slope of $Ax + By = C$ is $-\frac{A}{B}$, not $\frac{A}{B}$ and not A.

Finding intercepts is worth its own line, because it is quick and the test asks often:

- **y-intercept:** set $x = 0$.
- **x-intercept:** set $y = 0$.

Q4 — Medium

A production constraint is written in the form $3x + 4y = 12$, with the two variables on the same side of the equation. What is the slope of the line $3x + 4y = 12$?

A) $-\frac{4}{3}$

B) $\frac{3}{4}$

C) -3

D) $-\frac{3}{4}$

Q5 — Medium

A subscription's total cost was recorded after one month and again after three months, giving the two readings in the table below.

Months elapsed, x	**1**	**3**
Total paid in dollars, y	4	10

A) 4

B) 1

C) 3

D) -2

Q6 — Medium

A supply constraint is graphed in the xy-plane as the line $2x - 5y = 20$. The line crosses the x-axis at the point $(k, 0)$. What is the value of k?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Section 3 — Parallel and Perpendicular

Two rules, and the second is the one that gets forgotten.

Lines	Their slopes
Parallel	equal: $m_2 = m_1$
Perpendicular	negative reciprocals: $m_2 = -\dfrac{1}{m_1}$

"Negative reciprocal" means do **two** things: flip the fraction over, and change the sign. If one slope is $\frac{2}{3}$, the perpendicular slope is $-\frac{3}{2}$. Doing only one of the two is what the wrong options are made of.

A useful check: two perpendicular slopes always multiply to -1. $\frac{2}{3} \times -\frac{3}{2} = -1$ ✓

Q7 — Medium

A surveyor is setting out a path that must meet an existing boundary at a right angle. Line ℓ represents that boundary and has equation $y = \dfrac{2}{5}x + 7$. What is the slope of a line perpendicular to ℓ ?

A) $\frac{2}{5}$

B) $\frac{5}{2}$

C) $-\frac{2}{5}$

D) $-\frac{5}{2}$

Q8 — Hard

Line p has equation $4x + 3y = 9$. Line q is parallel to p and passes through $(6, -1)$. What is the y-intercept of line q?

A) -9

B) 7

C) 3

D) $-\frac{4}{3}$

Section 4 — Systems of Two Equations

A system is two equations that must both be true at once. Its solution is the point (x, y) where the two lines **cross**.

There are two methods, and it is worth being fluent in both because questions are built to favour one or the other.

Substitution — best when one equation already has a variable alone. Rearrange one equation to " $y = \ldots$", put that into the other, solve, then substitute back.

Elimination — best when the equations are lined up as $Ax + By = C$. Add or subtract the two equations so that one variable cancels. Multiply one or both equations first if nothing cancels straight away.

Always find both values, and always check both equations. Half the wrong options on system questions are the *other* variable's value.

Q9 — Basic

Two conditions describe the same pair of quantities at the same moment: the first is $y = 2x - 1$ and the second is $3x + y = 14$. What is the value of x?

A) 2

B) 3

C) 5

D) $\frac{13}{5}$

Q10 — Medium

A pair of measurements (x, y) satisfies both of the conditions $\begin{cases} 2x + 3y = 16 \\ 2x - y = 4 \end{cases}$ at once. What is the value of y?

A) 20

B) $\frac{7}{2}$

C) 5

D) 3

Q11 — Medium

At a stall, 3 coffees and 2 teas cost \$16, and 1 coffee and 4 teas cost \$12. What is the cost, in dollars, of one coffee?

A) 2

B) $\frac{14}{5}$

C) 4

D) 28

Q12 — Hard

The system $\begin{cases} 6x - 2y = 10 \\ 9x + ky = 3 \end{cases}$, in which k is a constant, has **no** solution. What is the value of k?

A) -3

B) 3

C) $-\frac{4}{3}$

D) -2

Q13 — Medium

The system $\begin{cases} 4x + 6y = 14 \\ 2x + 3y = c \end{cases}$, in which c is a constant, has infinitely many solutions. What is the value of c?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q14 — Medium

Two lines are graphed in the xy-plane and intersect at the point $(4, -2)$. Which of the following must be true?

A) Both lines have a slope of 4

B) $x = 4$ and $y = -2$ satisfy both equations

C) Both lines pass through the origin

D) The two lines have the same y-intercept

Section 5 — Reading a Linear Model

The SAT dresses slope and intercept up as a story and asks what one of the numbers *means*. The answer is always about **units**.

In $y = mx + b$ describing a real situation:

- b is the value **at the start**, when $x = 0$. Its units are the units of y.
- m is **how fast y changes per unit of x**. Its units are y-units *per* x-unit.

That unit test settles most of these questions on its own. If the model is dollars against hours, a rate must be **dollars per hour**, so any option measured in plain dollars or plain hours is out before you think about the story.

Q15 — Medium

A gym advertises its membership with the formula $C = 30 + 12w$, where C is the total cost in dollars and w is the number of weeks a member stays. A prospective member wants to know what the two numbers in the formula stand for. What does the 30 represent?

A) The cost of each week of membership

B) The number of weeks in the membership

C) A one-off joining fee, charged before any weeks

D) The total cost of a 30-week membership

Q16 — Medium

The number of bacteria in a culture is modelled by $P = 500 + 80h$, where h is hours since the start. Which statement is best supported by the model?

A) The population increases by 80 each hour

B) The population increases by 500 each hour

C) The population doubles every 80 hours

D) There were 80 bacteria at the start

Section 6 — Mixed Practice

Slopes, forms, parallels and systems, shuffled.

Q17 — Basic

Two readings taken during a cooling experiment were recorded in the table below.

x	-2	$\mathbf{3}$
y	7	-3

What is the slope of the line through those two points?

A) 2

B) $-\frac{1}{2}$

C) $\frac{4}{5}$

D) -2

Q18 — Medium

A new access road must cross an existing pipeline at a right angle. The pipeline is modelled by line ℓ, whose equation is $2x - 5y = 15$. What is the slope of a line perpendicular to ℓ?

A) $\frac{5}{2}$

B) $-\frac{5}{2}$

C) $\frac{2}{5}$

D) $-\frac{2}{5}$

Q19 — Medium

Two orders placed with the same supplier satisfy $3x + 2y = 19$ and $5x - 2y = 13$, where x and y are the quantities of two items. What is the value of x?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q20 — Medium

A line on a scale drawing falls at a steady rate, dropping one unit for every two units travelled across. The line is known to pass through the point $(6, 1)$. What is its y-intercept?

A) 1

B) -2

C) 4

D) 7

Q21 — Hard

Two printing firms quote for n copies. Firm A charges $C = 40 + 0.20n$ and firm B charges $C = 10 + 0.35n$. For how many copies do the two firms charge the same amount?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q22 — Hard

Two conditions on the same pair of quantities are recorded as $2x + 5y = 24$ and $3x - 2y = -2$. What is the value of y?

A) 2

B) 76

C) 4

D) -2

Q23 — Medium

A second road is to run parallel to an existing one, which is modelled by $y = 3x - 2$. The new road passes through the point $(2, 11)$. What is the y-intercept of the new road?

A) 5

B) -2

C) 11

D) 17

Q24 — Medium

A cable is to be strung perpendicular to an existing line modelled by $y = -2x + 1$, and it must pass through the anchor point $(4, 3)$. What is the y-intercept of the new line?

A) 1

B) 11

C) $\frac{1}{2}$

D) -2

Q25 — Basic

A constraint is graphed in the xy-plane as the line $5x + 2y = 30$. The graph crosses the x-axis at the moment the second quantity reaches zero. At what value of x does the graph cross the x-axis?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q26 — Hard

A theatre sold a total of 250 tickets for one performance and took $2400 at the box office. Adult tickets were priced at $12 each and child tickets at $7 each, and every ticket sold was one or the other. How many **adult** tickets were sold?

A) 120

B) 130

C) 250

D) 200

Q27 — Medium

A quantity changes at a constant rate, so it is modelled by a linear function f. Two of its values were recorded in the table below.

x	**2**	**6**
$f(x)$	5	13

The starting value of the model is the value of the function at an input of zero. What is $f(0)$?

A) 1

B) 5

C) 2

D) 9

Q28 — Medium

A constraint on two quantities is graphed as the line $3x - 4y = 12$. Which of the following points lies on the line $3x - 4y = 12$?

A) $(0,\ 3)$

B) $(8,\ 3)$

C) $(4,\ 3)$

D) $(3,\ 8)$

Q29 — Medium

A line drawn through the points $(1, 2)$ and $(5, k)$ is known to rise three units for every one unit it runs. The second coordinate of the second point has been lost. What is the value of k?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q30 — Hard

In the equation $4x + ay = 8$, the letter a is a constant chosen so that the graph has a particular steepness. The graph is required to have slope -2. What is the value of a?

A) 2

B) -2

C) 8

D) $\frac{1}{2}$

Q31 — Medium

Which of the following systems has no solution?

A) $y = 2x + 1$ and $y = 3x + 1$

B) $y = 2x + 1$ and $y = 2x + 1$

C) $y = 2x + 1$ and $y = 2x - 4$

D) $y = 2x + 1$ and $y = -2x + 1$

Q32 — Medium

Four lines are described below, some by an equation and some by two points they pass through. Which of the four has the greatest slope?

A) $y = -5x + 2$

B) the line through $(0, 0)$ and $(4, 8)$

C) $3x - y = 1$

D) the line through $(1, 1)$ and $(3, 9)$

Answers and Explanations

Section 1 — Slope

Q1 — Basic — Answer: B

Topic: Slope from two points

Label the points and subtract in a consistent order. Take $(2, 3)$ as the first and $(6, 11)$ as the second:

$$m = \frac{11 - 3}{6 - 2} = \frac{8}{4} = 2$$

Check it makes sense: as x goes from 2 to 6 the point rises from 3 to 11, so the line goes **up** — and the slope is positive. ✓

It does not matter which point you call first, as long as you are consistent. Starting from $(6, 11)$ instead:

$$m = \frac{3 - 11}{2 - 6} = \frac{-8}{-4} = 2$$

Same answer. The two minus signs cancel, which is exactly why consistency is the only rule.

Why each wrong option is wrong:

- **A,** $\frac{1}{2}$ — the change in x over the change in y, upside down. Slope is rise over run: y on top.
- **B is the answer.**
- **C,** -2 — subtracted the y values in one order and the x values in the other. That is the inconsistency warned about above, and it always produces the right number with the wrong sign.
- **D,** $\frac{7}{4}$ — added the coordinates instead of subtracting them. Slope is about *change*, so it is always a subtraction.

> **Takeaway:** $m = \frac{y_2 - y_1}{x_2 - x_1}$, with y on top and the same order on both. Then check the sign against whether the line should be rising or falling.

Q2 — Medium — Answer: D

Topic: Slope as a rate of change

"How much per year" is a slope. The two points are $(2, 140)$ and $(8, 260)$, where x is years and y is height:

$$m = \frac{260 - 140}{8 - 2} = \frac{120}{6} = 20$$

The tree grows **20 cm a year**.

Check it: from year 2, six years pass to reach year 8, and $6 \times 20 = 120$ cm of growth — which takes 140 cm up to 260 cm ✓

Why each wrong option is wrong:

- **B,** 12 — added the two years instead of subtracting them. The **gap** is 6 years, not 10.
- **C,** $\frac{65}{2}$ — divided a height by a year, which is what you would do if the tree had been 0 cm tall at year 0. It was not; it was 140 cm at year 2.
- **A,** 60 — divided the growth by 2 rather than by the 6-year gap.

> **Takeaway:** "Per year", "per hour", "for each" all mean slope. Use the *gap* between the two x values, never one of the values on its own.

Q3 — Medium — Answer: D

Topic: Slope of a horizontal line

Both points have the same y-coordinate, 5. So there is no rise at all:

$$m = \frac{5 - 5}{4 - (-3)} = \frac{0}{7} = 0$$

The line is **horizontal** — it is the line $y = 5$. Walk along it and you never go up or down, so the slope is zero.

Be careful with the pair this question sits next to. Zero on top gives slope **0**; zero on the *bottom* gives an **undefined** slope, and that is a vertical line like $x = 5$. They sound similar and they are opposites.

y values equal $\rightarrow \frac{0}{\text{something}}$	horizontal, slope 0
x values equal $\rightarrow \frac{\text{something}}{0}$	vertical, slope **undefined**

Why each wrong option is wrong:

- **A,** 1 — there is no default slope; it has to be computed.
- **B,** 7 — the horizontal distance between the points, which is the denominator, not the slope.

- **C,** $\frac{5}{7}$ — used the y-coordinate 5 as though it were the rise. The rise is the *change* in y, which here is zero.

> **Takeaway:** Same y means horizontal and slope 0. Same x means vertical and slope undefined. Check whether the coordinates repeat before reaching for the formula.

Section 2 — The Forms of a Line

Q4 — Medium — Answer: D

Topic: Reading the slope out of standard form

Rearrange into $y = mx + b$. Subtract $3x$ from both sides:

$$4y = -3x + 12$$

Divide **everything** by 4:

$$y = -\frac{3}{4}x + 3$$

The slope is the number multiplying x:

$$m = -\frac{3}{4}$$

Check the sign by thinking about the picture. When x grows, $3x$ grows, so $4y$ must shrink to keep the total at 12. The line falls, so the slope is negative ✓

Why each wrong option is wrong:

- **B,** $\frac{3}{4}$ — the right fraction with the minus dropped. The minus appears when $3x$ moves across the equals sign, and it is the single most common slip in this topic.
- **C,** -3 — took the coefficient of x and never divided by 4.
- **A,** $-\frac{4}{3}$ — inverted the fraction. That is the slope of a **perpendicular** line, which is the next section, and it is offered here on purpose.

> **Takeaway:** Rearrange into $y = mx + b$ rather than guessing from $Ax + By = C$. If you want the shortcut, the slope is $-\frac{A}{B}$ — but rearranging is safer and takes one extra line.

Q5 — Medium — Answer: B

Topic: The equation of a line through two points

Find the slope first:

$$m = \frac{10-4}{3-1} = \frac{6}{2} = 3$$

So the line is $y = 3x + b$. Put one of the points in to find b — use $(1, 4)$:

$$4 = 3(1) + b \implies b = 1$$

The line is $y = 3x + 1$, and the y-intercept is 1.

Check with the *other* point, which is the whole value of having two: $3(3) + 1 = 10$ ✓

Why each wrong option is wrong:

- **A,** 4 — read a y-coordinate straight off one of the given points. The y-intercept is where the line crosses at $x = 0$, and neither given point is there.
- **C,** 3 — the slope, not the intercept. Both numbers appear in $y = 3x + 1$; the question asked for the second one.
- **D,** -2 — substituted the point's coordinates into the wrong slots.

> **Takeaway:** Slope first, then substitute **either** point to find b — and check with the other one. Two points give you a free check; use it.

Q6 — Medium — Answer: 10

Topic: Finding an intercept

Crossing the x-axis means $y = 0$ there. That is the whole question — once you know it, the algebra is one line.

Set $y = 0$:

$$2x - 5(0) = 20 \implies 2x = 20 \implies x = 10$$

So the line crosses at $(10, 0)$, and $k = 10$.

Check: $2(10) - 5(0) = 20$ ✓

The pair worth memorising, because it is easy to swap them under pressure:

- On the x-**axis**, $y = 0$.
- On the y-**axis**, $x = 0$.

Each axis is where the *other* coordinate vanishes. If you had set $x = 0$ here you would have got $-5y = 20$, so $y = -4$ — a real point on the line, and the wrong one.

This is a grid-in. Enter `10`.

> **Takeaway:** To find where a line meets an axis, set the *other* variable to zero. Say which one out loud before you write, because swapping them gives a plausible wrong answer rather than an obviously wrong one.

Section 3 — Parallel and Perpendicular

Q7 — Medium — Answer: D

Topic: The slope of a perpendicular line

The slope of ℓ is $\frac{2}{5}$. A perpendicular slope is the **negative reciprocal** — flip it *and* change its sign:

$$\text{flip: } \frac{5}{2} \qquad \text{change sign: } -\frac{5}{2}$$

Check with the multiplication test:

$$\frac{2}{5} \times \left(-\frac{5}{2}\right) = -\frac{10}{10} = -1 \checkmark$$

Why each wrong option is wrong:

- **A,** $\frac{2}{5}$ — the same slope, which gives a **parallel** line, not a perpendicular one.
- **B,** $\frac{5}{2}$ — flipped the fraction and forgot the sign. $\frac{2}{5} \times \frac{5}{2} = +1$, not -1.
- **C,** $-\frac{2}{5}$ — changed the sign and forgot to flip. $\frac{2}{5} \times -\frac{2}{5} = -\frac{4}{25}$, not -1.

Options B and C are the two halves of the job, offered separately. Doing both is the entire question.

> **Takeaway:** Perpendicular means flip **and** negate. Multiply the two slopes together as a check — if you do not get exactly -1, you have done one half of the job.

Q8 — Hard — Answer: B

Topic: Parallel lines in standard form

Parallel lines have the **same slope**, so start by finding the slope of p. Rearrange $4x + 3y = 9$:

$$3y = -4x + 9 \quad \Longrightarrow \quad y = -\frac{4}{3}x + 3$$

So $m = -\frac{4}{3}$, and line q is

$$y = -\frac{4}{3}x + b$$

Now use the point $(6, -1)$ that q passes through:

$$-1 = -\frac{4}{3}(6) + b \quad \Longrightarrow \quad -1 = -8 + b \quad \Longrightarrow \quad b = 7$$

Check: at $x = 6$, $-\frac{4}{3}(6) + 7 = -8 + 7 = -1$ ✓

Why each wrong option is wrong:

- **A,** -9 — used $+\frac{4}{3}$ as the slope, losing the minus sign when rearranging.
- **C,** 3 — the y-intercept of line p. Parallel lines share a slope, not an intercept; if they shared both they would be the same line.
- **D,** $-\frac{4}{3}$ — the slope, where the question asked for the intercept.

> **Takeaway:** Parallel means copy the slope, then use the given point to find the new intercept. The old intercept is never the answer — that is what makes the two lines different lines.

Section 4 — Systems of Two Equations

Q9 — Basic — Answer: B

Topic: Solving a system by substitution

The first equation already gives y on its own, so substitute it into the second:

$$3x + (2x - 1) = 14$$

Collect:

$$5x - 1 = 14 \quad \Longrightarrow \quad 5x = 15 \quad \Longrightarrow \quad x = 3$$

Then $y = 2(3) - 1 = 5$, so the crossing point is $(3, 5)$.

Check in **both** equations: $y = 2(3) - 1 = 5$ ✓ and $3(3) + 5 = 14$ ✓

Why each wrong option is wrong:

- **A,** 2 — dropped the -1 when substituting, solving $5x = 10$.
- **C,** 5 — the value of y. Both 3 and 5 come out of this question, and the last line asked for x.
- **D,** $\frac{13}{5}$ — moved the -1 to the wrong side, solving $5x = 13$.

> **Takeaway:** When one equation is already "$y = \ldots$", substitute. Then check in both equations — an answer that satisfies only one of them is not a solution to the system.

Q10 — Medium — Answer: D

Topic: Solving a system by elimination

Both equations have $2x$, so **subtracting** them removes x in one step:

$$(2x + 3y) - (2x - y) = 16 - 4$$

On the left, $2x - 2x = 0$ and $3y - (-y) = 4y$:

$$4y = 12 \quad \Longrightarrow \quad y = 3$$

Then from the second equation, $2x - 3 = 4$, so $2x = 7$ and $x = \frac{7}{2}$.

Check both: $2(\frac{7}{2}) + 3(3) = 7 + 9 = 16$ ✓ and $2(\frac{7}{2}) - 3 = 4$ ✓

The step people get wrong is $3y - (-y)$. Subtracting a negative adds, so it is $4y$, not $2y$. Writing the subtraction out with brackets, as above, prevents it.

Why each wrong option is wrong:

- **B,** $\frac{7}{2}$ — the value of x. Again, the question named one of the two.
- **C,** 5 — added the equations instead of subtracting. That gives $4x + 2y = 20$, which eliminates nothing.
- **A,** 20 — a stray total rather than a value of y; if $4y = 12$ then y is 3, and 20 never enters a correct solution.

> **Takeaway:** Line the equations up and look for a variable with matching coefficients. Same sign, subtract; opposite signs, add. And bracket the second equation before subtracting it, so the double negative does not bite.

Q11 — Medium — Answer: C

Topic: Building a system from words

Let c be the price of a coffee and t the price of a tea. The two sentences become two equations:

$$3c + 2t = 16 \qquad c + 4t = 12$$

Eliminate t. Multiply the first equation by 2 so both have $4t$:

$$6c + 4t = 32$$

Now subtract the second equation:

$$(6c + 4t) - (c + 4t) = 32 - 12 \implies 5c = 20 \implies c = 4$$

Then $4 + 4t = 12$, so $t = 2$.

Check both sentences in words: 3 coffees and 2 teas is $12 + 4 = 16$ ✓, and 1 coffee and 4 teas is $4 + 8 = 12$ ✓

Why each wrong option is wrong:

- **A,** 2 — the price of a **tea**. Define your letters in writing before you start; it is the cheapest insurance on the test.
- **B,** $\frac{14}{5}$ — divided all the money, $28, by all the drinks, 10, giving an average price per drink. That would be the answer only if coffee and tea cost the same, and the question exists because they do not.
- **D,** 28 — added the two totals. That is the cost of everything on both orders, not the cost of one coffee.

> **Takeaway:** Write down what each letter means before writing equations, and answer in those terms at the end. Then read both original sentences back with your numbers in them.

Q12 — Hard — Answer: A

Topic: A system with no solution

Two lines fail to have a crossing point exactly when they are **parallel** — same slope, different intercept. So put both into $y = mx + b$ form.

First equation:

$$6x - 2y = 10 \implies -2y = -6x + 10 \implies y = 3x - 5$$

Slope 3, intercept -5.

Second equation, keeping k:

$$9x + ky = 3 \implies ky = -9x + 3 \implies y = -\frac{9}{k}x + \frac{3}{k}$$

For the slopes to match:

$$-\frac{9}{k} = 3 \implies -9 = 3k \implies k = -3$$

Now confirm it is really "no solution" and not "infinitely many". With $k = -3$ the second line is $y = 3x - 1$. Same slope as $y = 3x - 5$, **different** intercept — so the lines are parallel and never meet ✓

That last check matters. Matching the slopes is only half the condition; if the intercepts had matched too, the two equations would describe the same line and the system would have infinitely many solutions instead.

Why each wrong option is wrong:

- **B,** 3 — matched $\frac{9}{k} = 3$ and lost the minus sign that comes from moving $9x$ across.
- **C,** $-\frac{4}{3}$ — divided the wrong pair of coefficients.
- **D,** -2 — copied the -2 out of the first equation. The coefficients do have to be proportional, but $6 \to 9$ is a factor of $\frac{3}{2}$, so -2 would have to become -3.

> **Takeaway:** No solution means parallel: equal slopes, different intercepts. Infinitely many means the *same line*: equal slopes **and** equal intercepts. Always check the intercept after matching the slope — that is what separates the two cases.

Q13 — Medium — Answer: 7

Topic: Infinitely many solutions

Infinitely many solutions means the two equations describe the **same line** — every point on it solves both.

Look at the left-hand sides. $4x + 6y$ is exactly twice $2x + 3y$. So the first equation is the second one doubled, and for them to be the same line the right-hand sides must be in that same ratio:

$$14 = 2c \quad \Longrightarrow \quad c = 7$$

Check by halving the first equation throughout:

$$4x + 6y = 14 \quad \Longrightarrow \quad 2x + 3y = 7$$

which is precisely the second equation with $c = 7$ ✓

Contrast this with Q12. There the coefficients were proportional but the constants were not, giving parallel lines and no solution. Here **everything** is in the same ratio, so it is one line drawn twice.

This is a grid-in. Enter 7.

> **Takeaway:** Infinitely many solutions means one equation is a multiple of the other — coefficients *and* constant, all scaled by the same factor. Find the factor from the x terms, then apply it to the constant.

Q14 — Medium — Answer: B

Topic: What the solution of a system means on a graph

The point where two lines cross is the point that lies on **both** of them. So its coordinates satisfy both equations at once — and that is exactly what "the solution of the system" means.

So $x = 4$ and $y = -2$ make both equations true. That is the whole content of an intersection point, and nothing else is forced.

To see that nothing else follows, here are two actual lines meeting at $(4, -2)$:

$$y = x - 6 \qquad \text{and} \qquad y = -x + 2$$

Check: at $x = 4$ the first gives $4 - 6 = -2$ ✓ and the second gives $-4 + 2 = -2$ ✓ They do cross there. Now test the other options against them.

Why each wrong option is wrong:

- **A, "Both lines have a slope of 4"** — the example lines have slopes 1 and -1. The 4 is an x-coordinate, not a slope. Two lines that cross must have *different* slopes, so they certainly cannot both be 4.
- **C, "Both lines pass through the origin"** — the example lines cross the y-axis at -6 and 2. If both lines went through the origin they would cross *there*, not at $(4, -2)$.
- **D, "The two lines have the same y-intercept"** — again -6 and 2. Two distinct lines meet at only one point; if they shared a y-intercept they would meet on the y-axis.

> **Takeaway:** The solution of a system is the intersection point, and it means one thing only: those coordinates satisfy both equations. When a question asks what **must** be true, try to build an example that breaks each option — one counterexample is enough to eliminate it.

Section 5 — Reading a Linear Model

Q15 — Medium — Answer: C

Topic: Interpreting the intercept

Set $w = 0$ — no weeks used yet:

$$C = 30 + 12(0) = 30$$

So $30 is owed before any weeks are counted. That is a **joining fee**.

A is what the 12 means: each extra week adds $12, since $C(w+1) - C(w) = 12$. **B** confuses a cost with a count — the 30 is in dollars. **D** would be $30 + 12(30) = 390$.

> **Takeaway:** Substitute $x = 0$ to see what the constant means, and check the units. A dollar amount cannot be a number of weeks.

Q16 — Medium — Answer: A

Topic: Interpreting the slope with units

The number multiplied by h is the rate. Between any hour and the next:

$$P(h+1) - P(h) = 80$$

So the population rises by **80 each hour**, and it starts at 500.

B swaps the two numbers. **C** describes exponential growth, which this model is not — a linear model adds a fixed amount, it does not multiply. (After 80 hours the population is $500 + 6400 = 6900$, which is far more than double.) **D** reads the rate as the starting value.

> **Takeaway:** Linear means *adds* the same amount each step. "Doubles every" is exponential and belongs to a different kind of model entirely — Chapter 5.

Section 6 — Mixed Practice

Q17 — Basic — Answer: D

Topic: Slope between two points with negatives

$$m = \frac{-3 - 7}{3 - (-2)} = \frac{-10}{5} = -2$$

The line falls from $y = 7$ to $y = -3$, so a negative slope is right ✓

A has the sign flipped. **B** is upside down. **C** added the y values.

> **Takeaway:** Subtracting a negative adds: $3 - (-2) = 5$. Then sanity-check the sign against whether the line rises or falls.

Q18 — Medium — Answer: B

Topic: Perpendicular slope from standard form

Rearrange: $-5y = -2x + 15$, so $y = \frac{2}{5}x - 3$ and $m = \frac{2}{5}$.

Perpendicular means flip **and** negate: $-\frac{5}{2}$. Check: $\frac{2}{5} \times -\frac{5}{2} = -1$ ✓

A flipped only. **C** is the parallel slope. **D** negated only.

> **Takeaway:** Rearrange to $y = mx + b$ first, then do both halves of "negative reciprocal" and confirm the product is -1.

Q19 — Medium — Answer: 4

Topic: A system solved by elimination with scaling

The y terms are $+2y$ and $-2y$ — opposite signs, so **adding** the equations cancels them:

$$(3x + 2y) + (5x - 2y) = 19 + 13 \quad \Longrightarrow \quad 8x = 32 \quad \Longrightarrow \quad x = 4$$

Then $3(4) + 2y = 19$ gives $2y = 7$, so $y = \frac{7}{2}$.

Check both: $12 + 7 = 19$ ✓ and $20 - 7 = 13$ ✓

This is a grid-in. Enter 4.

> **Takeaway:** Opposite signs on a variable, add the equations. Matching signs, subtract. Spotting which before you start saves the whole scaling step.

Q20 — Medium — Answer: C

Topic: The equation of a line from a graph description

$$y = -\tfrac{1}{2}x + b \quad \Longrightarrow \quad 1 = -\tfrac{1}{2}(6) + b = -3 + b \quad \Longrightarrow \quad b = 4$$

Check: at $x = 6$, $-\frac{1}{2}(6) + 4 = -3 + 4 = 1$ ✓

A used the point's y-coordinate. **B** lost the minus on the slope. **D** added the coordinates together.

> **Takeaway:** Substitute the point into $y = mx + b$ and solve for b. Carry the minus sign through the multiplication before doing anything else.

Q21 — Hard — Answer: 200

Topic: Comparing two linear models

"Charge the same" means the two expressions are equal — the point where the two lines cross:

$$40 + 0.20n = 10 + 0.35n$$

Gather the n terms on the side that keeps them positive, by subtracting $0.20n$:

$$40 = 10 + 0.15n$$

Subtract 10:

$$30 = 0.15n \quad \Longrightarrow \quad n = \frac{30}{0.15} = 200$$

Check both quotes at 200 copies: firm A charges $40 + 40 = 80$, firm B charges $10 + 70 = 80$ ✓

Worth noticing which firm is cheaper on each side, because the test often asks that as a follow-up. Firm B starts cheaper (a \$10 setup against \$40) but charges more per copy, so it wins on small orders and loses on large ones. At 100 copies B costs \$45 against A's \$60; at 300 copies A costs \$100 against B's \$115.

This is a grid-in. Enter `200`.

> **Takeaway:** "The same cost", "break even", "when are they equal" all mean set the two expressions equal to each other. The one with the bigger rate always wins eventually; the crossing point is where the lead changes hands.

Q22 — Hard — Answer: C

Topic: A system needing both equations scaled

Nothing cancels as the equations stand, so scale them until something does. To clear x, make both x terms $6x$ — multiply the first by 3 and the second by 2:

$$6x + 15y = 72 \qquad 6x - 4y = -4$$

Multiply **every** term, including the right-hand sides: $3 \times 24 = 72$ and $2 \times (-2) = -4$. Forgetting the right-hand side is the usual way this goes wrong.

Now subtract the second from the first. Bracket it, so the double negative is visible:

$$(6x + 15y) - (6x - 4y) = 72 - (-4)$$

$$19y = 76 \quad \Longrightarrow \quad y = 4$$

Then $2x + 5(4) = 24$ gives $2x = 4$, so $x = 2$.

Check both original equations: $2(2) + 5(4) = 4 + 20 = 24$ ✓ and $3(2) - 2(4) = 6 - 8 = -2$ ✓

Why each wrong option is wrong:

- **A,** 2 — the value of x. Both 2 and 4 fall out of this question, and the last line named y.
- **B,** 76 — reached $19y = 76$ and wrote 76 down. The unfinished last step.
- **D,** -2 — copied the constant from the second equation.

> **Takeaway:** Scale until one variable's coefficients match, multiplying **every** term including the constant. Same sign, subtract; opposite signs, add. Then find the second variable and check both original equations.

Q23 — Medium — Answer: A

Topic: A parallel line through a given point

Parallel means the same slope, 3, so the new line is $y = 3x + b$. Substitute $(2, 11)$:

$$11 = 3(2) + b = 6 + b \quad \Longrightarrow \quad b = 5$$

Check: at $x = 2$, $3(2) + 5 = 11$ ✓

B is the original line's intercept — parallel lines share a slope, never an intercept. **C** is the point's y-value. **D** added the 6 instead of subtracting it.

> **Takeaway:** Copy the slope, substitute the point, solve for b. If you get the original intercept back, you have written down the same line.

Q24 — Medium — Answer: A

Topic: A perpendicular line through a given point

The given slope is -2, which is $-\frac{2}{1}$. Flip and negate: $m = \frac{1}{2}$. Check: $-2 \times \frac{1}{2} = -1$ ✓

So the line is $y = \frac{1}{2}x + b$. Substitute $(4, 3)$:

$$3 = \frac{1}{2}(4) + b = 2 + b \quad \Longrightarrow \quad b = 1$$

B used -2 as the new slope, so it built a parallel line. **C** gives the slope. **D** gives the original slope.

> **Takeaway:** A whole number -2 is the fraction $-\frac{2}{1}$; flipping it gives $\frac{1}{2}$. Writing the whole number as a fraction first makes the flip obvious.

Q25 — Basic — Answer: 6

Topic: Finding an x-intercept

On the x-axis, $y = 0$:

$$5x + 2(0) = 30 \quad \Longrightarrow \quad 5x = 30 \quad \Longrightarrow \quad x = 6$$

Check: $5(6) + 0 = 30$ ✓

Setting $x = 0$ instead would give $y = 15$ — a real point on the line, and the answer to the other question.

This is a grid-in. Enter 6.

> **Takeaway:** x-axis means $y = 0$. Each axis is where the *other* coordinate is zero.

Q26 — Hard — Answer: B

Topic: A system from a ticket problem

Let a be adult tickets and c child tickets. Two facts, two equations:

$$a + c = 250 \qquad 12a + 7c = 2400$$

Substitute $c = 250 - a$ into the money equation:

$$12a + 7(250 - a) = 2400$$

$$12a + 1750 - 7a = 2400 \quad \Longrightarrow \quad 5a = 650 \quad \Longrightarrow \quad a = 130$$

So 130 adult and 120 child tickets. Check: $130 + 120 = 250$ ✓ and $12(130) + 7(120) = 1560 + 840 = 2400$ ✓

A is the child count — both numbers come out, and the question named adults. **C** is the total. **D** assumed every ticket was an adult ticket.

> **Takeaway:** Two unknowns need two equations: one counting the items, one counting the money. Then check both, and read which of the two the question wanted.

Q27 — Medium — Answer: A

Topic: A linear function given at two inputs

"Linear" means constant slope, so treat $f(2) = 5$ and $f(6) = 13$ as the points $(2, 5)$ and $(6, 13)$:

$$m = \frac{13-5}{6-2} = \frac{8}{4} = 2$$

So $f(x) = 2x + b$, and $5 = 2(2) + b$ gives $b = 1$. Since $f(0)$ *is* the y-intercept, $f(0) = 1$.

Check with the other point: $f(6) = 2(6) + 1 = 13$ ✓

There is a quicker way. Going from $x = 2$ down to $x = 0$ is two steps back, and each step is worth 2, so $f(0) = 5 - 2(2) = 1$.

B is $f(2)$. **C** is the slope. **D** averaged the two given outputs, which would be $f(4)$, not $f(0)$.

> **Takeaway:** $f(0)$ is the y-intercept. With a linear function you can walk backwards from a known point in steps of the slope — often faster than solving for b.

Q28 — Medium — Answer: B

Topic: Testing whether a point is on a line

There is nothing to solve. Substitute each point and see which one works — and with a calculator available this takes seconds.

$$\begin{gathered}\textbf{A}\ 3(0) - 4(3) = -12 \quad \text{no} \\ \textbf{B}\ 3(8) - 4(3) = 24 - 12 = 12 \quad \checkmark \\ \textbf{C}\ 3(4) - 4(3) = 12 - 12 = 0 \quad \text{no} \\ \textbf{D}\ 3(3) - 4(8) = 9 - 32 = -23 \quad \text{no}\end{gathered}$$

Option D is worth a second look: it is B's coordinates swapped. The order of a coordinate pair is (x, y), always, and a question offering both orders is checking that you know which is which.

> **Takeaway:** "Which point lies on..." is a substitution question, not a solving question. Test the options; it is faster and it cannot go wrong.

Q29 — Medium — Answer: 14

Topic: Finding a missing coordinate from a slope

Put the two points into the slope formula and set it equal to 3:

$$\frac{k-2}{5-1} = 3 \quad \Longrightarrow \quad \frac{k-2}{4} = 3$$

Multiply both sides by 4:

$$k - 2 = 12 \quad \Longrightarrow \quad k = 14$$

Check: $\frac{14-2}{5-1} = \frac{12}{4} = 3$ ✓

Or walk it: from $x = 1$ to $x = 5$ is 4 steps, each worth 3, so y climbs by 12, from 2 to 14.

This is a grid-in. Enter 14.

> **Takeaway:** A missing coordinate with a known slope is one equation. Set the slope formula equal to the given slope and solve — or count the steps and multiply.

Q30 — Hard — Answer: A

Topic: Finding a coefficient from a required slope

Rearrange, keeping a as a letter:

$$ay = -4x + 8 \quad \Longrightarrow \quad y = -\frac{4}{a}x + \frac{8}{a}$$

The slope is $-\frac{4}{a}$, and it has to be -2:

$$-\frac{4}{a} = -2 \quad \Longrightarrow \quad 4 = 2a \quad \Longrightarrow \quad a = 2$$

Check by substituting back: $4x + 2y = 8$ becomes $y = -2x + 4$, slope -2 ✓

B copied the slope itself. **C** read the 8. **D** inverted the relationship, giving $a = \frac{1}{2}$, which would make the slope -8.

> **Takeaway:** Rearrange with the unknown constant still in place, then set the slope expression equal to the required value. Always substitute back — it costs one line and catches an inverted fraction immediately.

Q31 — Medium — Answer: C

Topic: A system with no solution, recognised quickly

No solution means the lines are **parallel but not identical** — same slope, different intercept. Both equations are already in $y = mx + b$ form, so read them off:

Option	Slopes	Intercepts	Result
A	2 and 3	1 and 1	different slopes → they cross once
B	2 and 2	1 and 1	the same line → infinitely many
C	2 and 2	1 and -4	**parallel, distinct → none**
D	2 and -2	1 and 1	different slopes → they cross once

Only C has matching slopes with different intercepts.

Option B is the one to be careful with: identical equations look like they should be a special case of "no solution", and they are the opposite — every point on the line solves both.

> **Takeaway:** Compare slopes first. Different slopes always cross. Same slope splits into two cases, and it is the **intercept** that decides which: same intercept means infinitely many, different means none.

Q32 — Medium — Answer: D

Topic: Which line is steeper

Work out all four slopes and compare.

$$\textbf{A}\ m = -5 \qquad \textbf{B}\ m = \frac{8-0}{4-0} = 2$$
$$\textbf{C}\ 3x - y = 1 \Rightarrow y = 3x - 1,\ m = 3 \qquad \textbf{D}\ m = \frac{9-1}{3-1} = \frac{8}{2} = 4$$

The greatest is **4**, option D.

The trap is option A. Its slope has the largest *size*, and -5 is the **smallest** of the four numbers. "Greatest slope" means greatest value, so a negative slope can never win — unless every option is negative.

> **Takeaway:** Convert everything to a single number, then compare as numbers. "Greatest slope" is not "steepest"; a slope of -5 is steeper than 4 but far smaller.

Chapter 3 — Exponents, Polynomials and Equivalent Expressions

This chapter opens **Advanced Math**, which is about 35% of the SAT Math section — tied with Algebra as the largest area on the test. Chapters 3, 4 and 5 cover it between them.

The name is misleading. "Advanced" here does not mean harder ideas; it means expressions that are no longer straight lines. Almost everything in this chapter is one of two skills:

1. **Rewriting an expression** into a different but equal form.
2. **Recognising a pattern** that lets you skip most of the work.

The instruction that signals this chapter is *"Which expression is equivalent to…"*. It appeared in every one of the eight official practice tests examined. "Equivalent" means equal for **every** value of the variable — not just one — and that gives you a powerful checking tool, which Section 5 is entirely about.

Topics covered: exponent rules · negative and fractional exponents · radicals · expanding brackets · factoring · the difference of two squares · rational expressions · checking equivalence by substitution

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Section 1 — The Exponent Rules

None of these is on the reference sheet. They have to be known.

Rule	Why it works
$x^a \cdot x^b = x^{a+b}$	$x^2 \cdot x^3$ is $(xx)(xxx)$ — five x's
$\frac{x^a}{x^b} = x^{a-b}$	the ones on the bottom cancel ones on top
$(x^a)^b = x^{ab}$	$(x^2)^3$ is $x^2 \cdot x^2 \cdot x^2$
$(xy)^a = x^a y^a$	the power reaches every factor inside
$x^0 = 1$	because $\frac{x^a}{x^a} = x^{a-a} = x^0$, and it is also 1
$x^{-a} = \frac{1}{x^a}$	a negative power means "on the other side of the line"
$x^{1/n} = \sqrt[n]{x}$	a fractional power is a root
$x^{m/n} = \sqrt[n]{x^m}$	bottom is the root, top is the power

The single most common error is **adding exponents when the bases differ**. $2^3 \cdot 5^3$ is not 10^6. The rules above only apply when the base is the same — apart from $(xy)^a$, which goes the other way.

Two more worth knowing cold:

- **A negative exponent never makes a number negative.** $2^{-3} = \frac{1}{8}$, which is positive and small. The minus moves it, it does not flip its sign.
- $x^{m/n}$**: the denominator is the root.** In $8^{2/3}$, the 3 is the cube root and the 2 is the square, so $8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$. Take the root first — the numbers stay small.

Q1 — Basic

Which expression is equivalent to $x^5 \cdot x^3$?

A) x^2

B) x^{15}

C) $2x^8$

D) x^8

Q2 — Medium

Which expression is equivalent to $(3x^2)^3$?

A) $3x^6$

B) $27x^6$

C) $9x^6$

D) $27x^5$

Q3 — Medium

What is the value of 4^{-2}?

A) -16

B) -8

C) 8

D) $\frac{1}{16}$

Q4 — Medium

What is the value of $8^{2/3}$?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Section 2 — Expanding

Expanding is multiplying out until no brackets remain. Every term in the first bracket multiplies every term in the second.

$$(a+b)(c+d) = ac + ad + bc + bd$$

Two patterns come up so often they are worth recognising on sight rather than expanding each time:

$$(x+a)(x-a) = x^2 - a^2 \qquad \textbf{difference of two squares}$$
$$(x+a)^2 = x^2 + 2ax + a^2 \qquad \textbf{perfect square}$$

$(x+a)^2$ is not $x^2 + a^2$. That is the most common error in the whole chapter. There is a middle term, and it is $2ax$. Test it with numbers if you ever doubt it: $(3+4)^2 = 49$, while $3^2 + 4^2 = 25$.

Q5 — Basic

Which expression is equivalent to $(x+3)(x-5)$?

A) $x^2 - 15$

B) $x^2 - 2x - 15$

C) $x^2 + 2x - 15$

D) $x^2 - 8x - 15$

Q6 — Medium

Which expression is equivalent to $(2x-3)^2$?

A) $4x^2 + 9$

B) $4x^2 - 9$

C) $4x^2 - 6x + 9$

D) $4x^2 - 12x + 9$

Q7 — Medium

Which expression is equivalent to $(3x+4)(3x-4)$?

A) $9x^2 - 16$

B) $9x^2 + 16$

C) $9x^2 - 24x - 16$

D) $3x^2 - 16$

Section 3 — Factoring

Factoring is expanding run backwards: turning a sum into a product. It is how you solve quadratics (Chapter 4) and how you simplify fractions (Section 4), so it earns its place twice.

Always look for a common factor first. $2x^2 + 8x$ is $2x(x+4)$, and pulling the $2x$ out makes everything after it easier.

To factor $x^2 + bx + c$, find two numbers that **multiply to c and add to b**. For $x^2 + 7x + 12$: which pairs multiply to 12? 1×12, 2×6, 3×4. Which adds to 7? $3 + 4$. So it is $(x+3)(x+4)$.

The signs tell you a lot before you start:

c	b	**The two numbers are**
positive	positive	both positive
positive	negative	both negative
negative	either	one of each; the bigger takes b's sign

Q8 — Basic

Which of the following is a factor of $x^2 - 7x + 12$?

A) $x + 3$

B) $x - 3$

C) $x - 12$

D) $x + 4$

Q9 — Medium

Which expression is equivalent to $3x^2 - 27$?

A) $(x-3)(3x+9)$

B) $3(x-3)^2$

C) $(x-9)(x+3)$

D) $3x^2 + 27$

Q10 — Hard

Which of the following is a factor of $2x^2 + 5x - 3$?

A) $x - 3$

B) $2x + 1$

C) $2x - 1$

D) $x + 1$

Section 4 — Rational Expressions

A rational expression is a fraction with algebra in it. Simplifying means factoring the top and the bottom and cancelling anything they share.

You may only cancel factors, never terms. In $\dfrac{x+3}{x}$ you cannot cancel the x's, because the top is a *sum*, not a product. Factor first, and then cancelling is always safe.

Adding fractions needs a common denominator, exactly as with numbers:

$$\frac{1}{x}+\frac{1}{y}=\frac{y}{xy}+\frac{x}{xy}=\frac{x+y}{xy}$$

Q11 — Medium

Which expression is equivalent to $\dfrac{x^2-9}{x^2+7x+12}$ for all values of x where it is defined?

A) $\frac{x-3}{x+4}$

B) $\frac{x+3}{x+4}$

C) $\frac{x-3}{x-4}$

D) $-\frac{3}{4}$

Q12 — Medium

Which expression is equivalent to $\dfrac{2}{x}+\dfrac{3}{x+1}$?

A) $\frac{5}{2x+1}$

B) $\frac{2x+3}{x(x+1)}$

C) $\frac{5x+2}{x(x+1)}$

D) $\frac{5}{x(x+1)}$

Section 5 — Checking by Substitution

"Equivalent" means equal for **every** value of the variable. That gives you a technique that turns algebra questions into arithmetic, and it is one of the most valuable habits on this test.

Pick a number, put it in the original, put it in each option, and keep only the options that match.

Rules for choosing well:

- **Avoid 0 and 1.** They make too many different expressions agree. x^2 and x^3 and x are all equal at both.
- **Avoid numbers already in the problem**, which can create accidental matches.
- **2, 3 and 5 are usually ideal.**
- **If two options survive, test a second number.** One survivor is the answer.

This is not a fallback for when you are stuck. On "which expression is equivalent" questions with messy algebra it is often the *fastest* correct route, and with a calculator permitted it is very hard to get wrong.

Q13 — Medium

Which expression is equivalent to $(x+2)(x-3)-(x^2-6)$?

A) $2x$

B) $-x$

C) $-x-12$

D) x

Q14 — Hard

If $a \neq 0$, which expression is equivalent to $\dfrac{a^{-1}+b^{-1}}{(ab)^{-1}}$?

A) 1

B) ab

C) $\frac{a+b}{ab}$

D) $a+b$

Section 6 — Mixed Practice

Exponents, expanding, factoring and fractions, shuffled.

Q15 — Basic

Which expression is equivalent to $\dfrac{12x^7}{3x^4}$?

A) $4x^{\frac{7}{4}}$

B) $4x^3$

C) $9x^3$

D) $4x^{11}$

Q16 — Medium

Which expression is equivalent to $\left(\dfrac{2x^3}{y^2}\right)^2$?

A) $\frac{2x^6}{y^4}$

B) $\frac{4x^5}{y^4}$

C) $\frac{4x^6}{y^4}$

D) $\frac{4x^6}{y^2}$

Q17 — Medium

What is the value of $3^0 + 2^{-1}$?

A) 0

B) $\frac{1}{2}$

C) 1

D) $\frac{3}{2}$

Q18 — Medium

Which expression is equivalent to $2x(x+4)(x-1)$?

A) $2x^3+6x^2-8x$

B) $2x^3+6x^2+8x$

C) $2x^3+3x^2-4x$

D) x^3+3x^2-4x

Q19 — Medium

Which of the following is a factor of $x^3+2x^2-9x-18$?

A) $x-2$

B) $x+2$

C) $x+9$

D) $x-18$

Q20 — Hard

If $x^2+kx+49$ is a perfect square for some positive constant k, what is the value of k?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q21 — Medium

Which expression is equivalent to $\dfrac{2x^2+6x}{x^2-9}$ where it is defined?

A) $\frac{2x}{x-3}$

B) $\frac{2x}{x+3}$

C) $\frac{2x+6}{x-3}$

D) $2x-3$

Q22 — Hard

The expression $\dfrac{6x+11}{2x+3}$ can be written as $3+\dfrac{c}{2x+3}$, where c is a constant. What is the value of c?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q23 — Medium

Which expression is equivalent to $\sqrt{18}+\sqrt{8}$?

A) $\sqrt{26}$

B) $2\sqrt{26}$

C) $5\sqrt{2}$

D) $6\sqrt{2}$

Q24 — Medium

If $(x+p)(x+4)=x^2+9x+q$ for all values of x, what is the value of q?

A) 4

B) 9

C) 13

D) 20

Q25 — Medium

Which expression is equivalent to $\dfrac{2^{10}}{4^3}$?

A) 128

B) 16

C) $\frac{1}{128}$

D) 65536

Q26 — Medium

Which expression is equivalent to $-2x^2+10x$?

A) $2x\left(x-5\right)$

B) $-2x\left(x+5\right)$

C) $-2x\left(x-10\right)$

D) $-2x\left(x-5\right)$

Q27 — Hard

Which expression is equivalent to $\dfrac{\frac{1}{x} - \frac{1}{3}}{x - 3}$ for $x \neq 0$ and $x \neq 3$?

A) $-\frac{1}{3x}$

B) $\frac{1}{3x}$

C) $\frac{1}{x-3}$

D) $\frac{3-x}{3x}$

Q28 — Medium

If $x + y = 12$ and $xy = 20$, what is the value of $x^2 + 2xy + y^2$?

A) 164

B) 32

C) 144

D) 184

Q29 — Hard

The expression $\dfrac{x^2 + 5x + 6}{x + 2}$ is equivalent to $x + k$ for $x \neq -2$. What is the value of k?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q30 — Medium

Which expression is equivalent to $\dfrac{6}{\sqrt{3}}$?

A) 2

B) $2\sqrt{3}$

C) $6\sqrt{3}$

D) $\sqrt{2}$

Q31 — Medium

Which expression is equivalent to $(x+2)(x^2-3x+5)$?

A) x^3-x^2+x+10

B) x^3+x^2-x+10

C) x^3-x^2-x+10

D) $x^3-3x^2+5x+10$

Q32 — Hard

If $9^x=27^4$, what is the value of x?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Answers and Explanations

Section 1 — The Exponent Rules

Q1 — Basic — Answer: D

Topic: Multiplying powers with the same base

Multiplying powers of the same base **adds** the exponents:

$$x^5 \cdot x^3 = x^{5+3} = x^8$$

If you ever doubt it, count. x^5 is five x's multiplied together and x^3 is three, so together there are eight.

Why each wrong option is wrong:

- **B,** x^{15} — multiplied the exponents. $5 \times 3 = 15$ is the rule for $(x^5)^3$, a power of a power, not a product.
- **C,** $2x^8$ — added the exponents correctly and then also added the two x terms as though this were $x^5 + x^3$. Multiplying does not produce a coefficient of 2.
- **A,** x^2 — subtracted, which is the rule for division.

> **Takeaway:** Multiply → add exponents. Divide → subtract. Power of a power → multiply. When they blur together, count the x's on a small case.

Q2 — Medium — Answer: B

Topic: A power of a product

The outside power reaches **everything** inside the bracket — the 3 as well as the x^2:

$$(3x^2)^3 = 3^3 \cdot (x^2)^3 = 27 \cdot x^{2\times 3} = 27x^6$$

Two separate steps: cube the coefficient, and multiply the exponents.

Why each wrong option is wrong:

- **A,** $3x^6$ — handled the x correctly and left the 3 untouched. The coefficient is inside the bracket, so it gets cubed too.
- **C,** $9x^6$ — squared the 3 rather than cubing it. The outer power is 3.

- **D,** $27x^5$ — added $2+3$ instead of multiplying. A power raised to a power multiplies.

> **Takeaway:** The outside exponent applies to every factor inside, coefficient included. Deal with the number and the letter separately so neither gets missed.

Q3 — Medium — Answer: D

Topic: A negative exponent

A negative exponent means **reciprocal**, not negative:

$$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

Say it as "one over 4^2". The minus sign moves the power to the bottom of a fraction; it never changes the sign of the answer.

Why each wrong option is wrong:

- **A,** -16 — computed 4^2 and made it negative. A positive base raised to **any** power is positive.
- **B,** -8 — multiplied 4×2 and negated.
- **C,** 8 — multiplied instead of raising to a power.

> **Takeaway:** $x^{-a} = \frac{1}{x^a}$. Negative exponent means *small and positive*, never negative. If your answer to a negative-exponent question is negative, something has gone wrong.

Q4 — Medium — Answer: 4

Topic: A fractional exponent

In $x^{m/n}$, the **bottom** number is the root and the **top** is the power. So $8^{2/3}$ is the cube root of 8, squared:

$$8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$$

You can do it the other way — square first, then take the cube root:

$$8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$$

Same answer, bigger numbers. **Take the root first** and the arithmetic stays small, which matters when a question is written to be done without reaching for the calculator every time.

This is a grid-in. Enter 4.

> **Takeaway:** Bottom is the root, top is the power. Root first, then power.

Section 2 — Expanding

Q5 — Basic — Answer: B

Topic: Expanding two brackets

Multiply every term by every term:

$$x \cdot x = x^2 \qquad x \cdot (-5) = -5x$$
$$3 \cdot x = 3x \qquad 3 \cdot (-5) = -15$$

Collect the two middle terms: $-5x + 3x = -2x$. So

$$(x+3)(x-5) = x^2 - 2x - 15$$

Check with a number. At $x = 1$: the original is $(4)(-4) = -16$, and the answer is $1 - 2 - 15 = -16$ ✓

Why each wrong option is wrong:

- **A,** $x^2 - 15$ — multiplied the first terms and the last terms and skipped the middle two. Four products, not two.
- **C,** $x^2 + 2x - 15$ — the middle term has the wrong sign. $-5x + 3x$ is $-2x$: the bigger number is the negative one.
- **D,** $x^2 - 8x - 15$ — multiplied the middle terms (-5×3) instead of adding them.

> **Takeaway:** Four products, then collect the middle two. Check by substituting $x = 1$: it takes seconds and catches every sign error.

Q6 — Medium — Answer: D

Topic: Squaring a binomial

$(2x-3)^2$ means $(2x-3)(2x-3)$. Multiply it out:

$$2x \cdot 2x = 4x^2 \qquad 2x \cdot (-3) = -6x$$
$$-3 \cdot 2x = -6x \qquad -3 \cdot (-3) = +9$$

The two middle terms are **both** $-6x$, so together they are $-12x$:

$$(2x-3)^2 = 4x^2 - 12x + 9$$

Check at $x = 1$: the original is $(-1)^2 = 1$, and the answer is $4 - 12 + 9 = 1$ ✓

Why each wrong option is wrong:

- **A,** $4x^2 + 9$ — squared each term separately. $(a - b)^2 \neq a^2 + b^2$; there is always a middle term.
- **B,** $4x^2 - 9$ — used the difference-of-squares pattern, which is for $(2x - 3)(2x + 3)$ — brackets with **opposite** signs. Here the signs match, so the middle terms reinforce rather than cancel.
- **C,** $4x^2 - 6x + 9$ — found $-6x$ once and forgot it appears twice.

> **Takeaway:** $(a \pm b)^2 = a^2 \pm 2ab + b^2$ — the middle term is doubled. Matching signs give a middle term; opposite signs cancel it.

Q7 — Medium — Answer: A

Topic: Recognising the difference of two squares

The brackets are identical apart from the sign, which is the difference-of-two- squares pattern:

$$(a + b)(a - b) = a^2 - b^2$$

Here $a = 3x$ and $b = 4$:

$$(3x)^2 - 4^2 = 9x^2 - 16$$

Expanding the long way shows why the middle disappears: $-12x + 12x = 0$.

Check at $x = 1$: $(7)(-1) = -7$, and $9 - 16 = -7$ ✓

Why each wrong option is wrong:

- **B,** $9x^2 + 16$ — the pattern subtracts. $-4 \times 4 = -16$.
- **C,** $9x^2 - 24x - 16$ — kept a middle term. With opposite signs the middle terms are equal and opposite, so they always cancel.
- **D,** $3x^2 - 16$ — squared the x but not the 3. $(3x)^2 = 9x^2$; the whole factor gets squared.

> **Takeaway:** Same two terms, opposite signs → $a^2 - b^2$, with no middle term. Square the **whole** first term, coefficient included.

Section 3 — Factoring

Q8 — Basic — Answer: B

Topic: Factoring a simple quadratic

Find two numbers that multiply to $+12$ and add to -7.

The constant is positive and the middle term is negative, so both numbers are negative. Pairs multiplying to 12: -1 and -12 (adds to -13), -2 and -6 (adds to -8), -3 and -4 (adds to -7 ✓).

$$x^2 - 7x + 12 = (x - 3)(x - 4)$$

So $x - 3$ is a factor. Check by expanding: $x^2 - 4x - 3x + 12 = x^2 - 7x + 12$ ✓

Why each wrong option is wrong:

- **A,** $x + 3$ — right number, wrong sign. $(x + 3)$ would need a $+3$ in the pair, and $+3$ and $+4$ give a middle term of $+7$.
- **C,** $x - 12$ — took the constant 12 as a factor. The numbers in the brackets multiply to give 12; they are not 12 itself.
- **D,** $x + 4$ — the same sign error as A, on the other factor.

> **Takeaway:** Two numbers multiplying to c and adding to b. Read the signs off the table before you hunt: positive c with negative b means both are negative.

Q9 — Medium — Answer: A

Topic: Factoring out a common factor first

Take the common factor of 3 out first:

$$3x^2 - 27 = 3(x^2 - 9)$$

Now $x^2 - 9$ is a difference of two squares, since $9 = 3^2$:

$$x^2 - 9 = (x + 3)(x - 3)$$

So the fully factored form is

$$3(x + 3)(x - 3)$$

Check at $x = 0$: the original is -27, and $3(3)(-3) = -27$ ✓

Why each wrong option is wrong:

- **B,** $3\left(x-3\right)^2$ — a perfect square needs a middle term. This expands to $3x^2-18x+27$, not $3x^2-27$. Test it at $x=0$: it gives 27, not -27.
- **C,** $\left(x-9\right)\left(x+3\right)$ — never pulled the 3 out, then found numbers multiplying to -27 and adding to -6. That factors $x^2-6x-27$, a different expression.
- **D,** $3x^2+27$ — lost the minus sign. x^2+9 is a **sum** of squares, and a sum of squares does not factor at all — which is worth knowing, because the test offers it whenever the difference-of-squares pattern is in play.

> **Takeaway:** Common factor first, then look for a pattern in what is left. a^2-b^2 factors; a^2+b^2 does not. And substituting $x=0$ eliminates wrong options here in one step.

Q10 — Hard — Answer: C

Topic: Factoring with a leading coefficient

With a leading coefficient the pair of numbers must multiply to $2\times(-3)=-6$ and add to $+5$. That pair is $+6$ and -1.

Split the middle term using them:

$$2x^2+6x-x-3$$

Now factor in pairs:

$$2x(x+3)-1(x+3)=(x+3)(2x-1)$$

So the factors are $x+3$ and $2x-1$. Of the options, $2x-1$ is there.

Check by expanding: $(x+3)(2x-1)=2x^2-x+6x-3=2x^2+5x-3$ ✓

There is a faster check when you only need to *test* an option. A factor $2x-1$ means $x=\frac{1}{2}$ makes the expression zero:

$$2\left(\tfrac{1}{4}\right)+5\left(\tfrac{1}{2}\right)-3=\tfrac{1}{2}+\tfrac{5}{2}-3=0 ✓$$

With a calculator, testing four options this way is often quicker than factoring at all.

Why each wrong option is wrong:

- **A,** $x-3$ — would need $x=3$ to give zero: $18+15-3=30$, not 0.
- **B,** $2x+1$ — needs $x=-\frac{1}{2}$: $\frac{1}{2}-\frac{5}{2}-3=-5$, not 0.
- **D,** $x+1$ — needs $x=-1$: $2-5-3=-6$, not 0.

> **Takeaway:** With a leading coefficient, find numbers multiplying to $a \times c$ and adding to b, then split the middle term. Or skip all of it: test each option by finding the x that makes it zero and substituting.

Section 4 — Rational Expressions

Q11 — Medium — Answer: A

Topic: Simplifying by factoring

Factor both parts. The top is a difference of two squares:

$$x^2 - 9 = (x+3)(x-3)$$

For the bottom, two numbers multiplying to 12 and adding to 7 — that is 3 and 4:

$$x^2 + 7x + 12 = (x+3)(x+4)$$

So

$$\frac{(x+3)(x-3)}{(x+3)(x+4)}$$

The $(x+3)$ is a factor of both, so it cancels:

$$\frac{x-3}{x+4}$$

Check at $x = 0$: the original is $\frac{-9}{12} = -\frac{3}{4}$, and the answer is $\frac{-3}{4} = -\frac{3}{4}$ ✓

Why each wrong option is wrong:

- **B,** $\frac{x+3}{x+4}$ — cancelled $(x-3)$ from the top against $(x+3)$ on the bottom. Only identical factors cancel.
- **C,** $\frac{x-3}{x-4}$ — factored the bottom as $(x+3)(x-4)$, which expands to $x^2 - x - 12$, not what was given.
- **D,** $-\frac{3}{4}$ — cancelled the x^2 terms. They are **terms** inside a sum, not factors of the whole expression, so they cannot be cancelled. This is exactly the mistake the section warns about — and note it happens to be the value at $x = 0$, which is why the substitution check has to use more than one number.

> **Takeaway:** Factor top and bottom completely, then cancel whole brackets. Never cancel across a $+$ or $-$ sign.

Q12 — Medium — Answer: C

Topic: Adding algebraic fractions

The denominators share nothing, so the common denominator is their product, $x(x+1)$. Scale each fraction to it:

$$\frac{2}{x} = \frac{2(x+1)}{x(x+1)} \qquad \frac{3}{x+1} = \frac{3x}{x(x+1)}$$

Now add the tops:

$$\frac{2(x+1)+3x}{x(x+1)} = \frac{2x+2+3x}{x(x+1)} = \frac{5x+2}{x(x+1)}$$

Check at $x = 1$: the original is $\frac{2}{1} + \frac{3}{2} = 3.5$, and the answer is $\frac{7}{2} = 3.5$ ✓

Why each wrong option is wrong:

- **A,** $\frac{5}{2x+1}$ — added the tops and the bottoms separately. That is never how fractions add: $\frac{1}{2} + \frac{1}{2}$ is 1, not $\frac{2}{4}$.
- **B,** $\frac{2x+3}{x(x+1)}$ — found the right denominator but forgot to scale the numerators when moving them onto it.
- **D,** $\frac{5}{x(x+1)}$ — the same mistake, having just added 2 and 3.

> **Takeaway:** A common denominator means **rescaling the tops too**. Whatever you multiplied the bottom by, multiply that fraction's top by as well.

Section 5 — Checking by Substitution

Q13 — Medium — Answer: B

Topic: Using substitution to identify an equivalent expression

Expand the first product:

$$(x+2)(x-3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$$

Now subtract the bracket, flipping **both** its signs:

$$(x^2 - x - 6) - (x^2 - 6) = x^2 - x - 6 - x^2 + 6 = -x$$

Check by substitution, which is the point of this section. At $x = 2$:

- Original: $(4)(-1) - (4-6) = -4 - (-2) = -2$
- Answer $-x$: -2 ✓

Test the others at $x = 2$ as well: $2x$ gives 4, $-x - 12$ gives -14, x gives 2. Only $-x$ matches, and it took ten seconds without any expanding at all.

Why each wrong option is wrong:

- **A,** $2x$ — mishandled the subtraction so the x^2 terms added rather than cancelled.
- **C,** $-x - 12$ — subtracted the x^2 but not the -6, leaving $-6 - 6$.
- **D,** x — the right size with the wrong sign.

> **Takeaway:** A minus in front of a bracket flips **every** sign inside it. And when four options differ only in signs, substituting one number settles it faster than expanding.

Q14 — Hard — Answer: D

Topic: Substitution when the algebra is messy

Rewrite every negative exponent as a fraction:

$$\frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{ab}}$$

Add the two fractions on top, over the common denominator ab:

$$\frac{1}{a} + \frac{1}{b} = \frac{b+a}{ab}$$

So the whole thing is

$$\frac{\frac{a+b}{ab}}{\frac{1}{ab}} = \frac{a+b}{ab} \times \frac{ab}{1} = a + b$$

Dividing by $\frac{1}{ab}$ is multiplying by ab, and the ab cancels.

Or substitute, and skip all of it. Take $a = 2$ and $b = 3$:

$$\frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{6}} = \frac{\frac{5}{6}}{\frac{1}{6}} = 5$$

Now test the options at $a = 2, b = 3$: option A gives 1, B gives 6, C gives $\frac{5}{6}$, D gives 5. Only D matches ✓

On a question like this, where the algebra is fiddly and the options are simple, substitution is not the backup plan — it is the better method.

Why each wrong option is wrong:

- **A,** 1 — cancelled the negative exponents against each other as though the top and bottom were the same expression.
- **B,** ab — multiplied a and b rather than adding.
- **C,** $\frac{a+b}{ab}$ — stopped after combining the numerator and never divided by $\frac{1}{ab}$.

> **Takeaway:** Turn negative exponents into fractions before doing anything else. And when the options are simple but the expression is not, substitute two small numbers — it is faster and more reliable than the algebra.

Section 6 — Mixed Practice

Q15 — Basic — Answer: B

Topic: Dividing powers

Handle the numbers and the letters separately. $12 \div 3 = 4$, and dividing powers **subtracts** exponents: $x^{7-4} = x^3$.

$$\frac{12x^7}{3x^4} = 4x^3$$

A divided the exponents. **C** subtracted the coefficients instead of dividing them. **D** added the exponents.

> **Takeaway:** Divide the coefficients, subtract the exponents. The coefficients and the powers follow different rules — do them in separate steps.

Q16 — Medium — Answer: C

Topic: A power of a quotient

The outside power reaches every factor, top and bottom:

$$\left(\frac{2x^3}{y^2}\right)^2 = \frac{2^2(x^3)^2}{(y^2)^2} = \frac{4x^6}{y^4}$$

A left the 2 alone. **B** added $3 + 2$ instead of multiplying. **D** left the denominator unsquared.

> **Takeaway:** Squaring a fraction squares the top **and** the bottom, including every coefficient. Three separate pieces here — check you did all three.

Q17 — Medium — Answer: D

Topic: Zero and negative exponents together

$3^0 = 1$ — **anything** non-zero to the power 0 is 1. And $2^{-1} = \frac{1}{2}$.

$$1 + \frac{1}{2} = \frac{3}{2}$$

A and **B** treated 3^0 as 0. **C** treated 2^{-1} as 0.

> **Takeaway:** $x^0 = 1$, not 0. It is the answer to "how many times have you multiplied by x" — none, so you are left with the 1 you started from.

Q18 — Medium — Answer: A

Topic: Expanding a product of three factors

Do the two brackets first, then multiply through by $2x$:

$$(x+4)(x-1) = x^2 - x + 4x - 4 = x^2 + 3x - 4$$
$$2x(x^2 + 3x - 4) = 2x^3 + 6x^2 - 8x$$

Check at $x = 1$: the original is $2(1)(5)(0) = 0$, and $2 + 6 - 8 = 0$ ✓

B has the last sign wrong. **C** multiplied only the first term by $2x$. **D** lost the 2.

> **Takeaway:** Multiply two brackets first, then distribute the outside factor across **every** term of the result.

Q19 — Medium — Answer: B

Topic: Factoring by grouping

Four terms with no common factor overall — group them in pairs:

$$(x^3 + 2x^2) + (-9x - 18) = x^2(x+2) - 9(x+2)$$

Both pairs now contain $(x+2)$, so pull it out:

$$(x+2)(x^2 - 9) = (x+2)(x+3)(x-3)$$

So $x + 2$ is a factor. Confirm it directly: a factor $x + 2$ means $x = -2$ gives zero.

$$(-8) + 2(4) - 9(-2) - 18 = -8 + 8 + 18 - 18 = 0 \checkmark$$

A needs $x = 2$: $8 + 8 - 18 - 18 = -20$, not 0. **C** needs $x = -9$, which is nowhere near. **D** needs $x = 18$.

> **Takeaway:** Four terms with no common factor → group in pairs and look for a shared bracket. To test a proposed factor, set it to zero and substitute — one line, and it settles the question.

Q20 — Hard — Answer: 14

Topic: A perfect-square trinomial

A perfect square looks like

$$(x+a)^2 = x^2 + 2ax + a^2$$

Match it against $x^2 + kx + 49$. The constant tells you $a^2 = 49$, so $a = 7$ (taking the positive value, since k is positive). Then the middle term is

$$k = 2a = 14$$

Check: $(x+7)^2 = x^2 + 14x + 49$ ✓

This is a grid-in. Enter 14.

> **Takeaway:** In a perfect square the middle coefficient is **twice the square root of the constant**. Take the root of the constant, then double it.

Q21 — Medium — Answer: A

Topic: Simplifying a rational expression

Factor both parts:

$$2x^2 + 6x = 2x(x+3) \qquad x^2 - 9 = (x+3)(x-3)$$

$$\frac{2x(x+3)}{(x+3)(x-3)} = \frac{2x}{x-3}$$

Check at $x = 1$: the original is $\frac{2+6}{1-9} = \frac{8}{-8} = -1$, and the answer is $\frac{2}{-2} = -1$ ✓

B cancelled $(x+3)$ against $(x-3)$. **C** did not take the $2x$ out properly. **D** cancelled terms across a sum.

> **Takeaway:** Factor first, cancel whole brackets only. A common factor of $2x$ on top comes out in one piece.

Q22 — Hard — Answer: 2

Topic: Rewriting to reveal a required form

The aim is to make the top contain a copy of the bottom. The bottom is $2x+3$; three of those give $6x+9$, which is nearly the top:

$$6x + 11 = 3(2x+3) + 2$$

Now split the fraction:

$$\frac{6x+11}{2x+3} = \frac{3(2x+3)}{2x+3} + \frac{2}{2x+3} = 3 + \frac{2}{2x+3}$$

So $c = 2$.

Check at $x = 0$: the original is $\frac{11}{3}$, and the rewritten form is $3 + \frac{2}{3} = \frac{11}{3}$ ✓

There is a quicker way when you only need c. The two forms must be equal for every x, so pick a convenient one. At $x = 0$:

$$\frac{11}{3} = 3 + \frac{c}{3} \quad \Longrightarrow \quad 11 = 9 + c \quad \Longrightarrow \quad c = 2$$

This is a grid-in. Enter 2.

> **Takeaway:** To split a fraction into "whole part plus remainder", work out how many copies of the bottom fit into the top. Or, since the two forms are equal for every x, substitute $x = 0$ and solve for the constant in one line.

Q23 — Medium — Answer: C

Topic: Multiplying expressions with radicals

Simplify each root by pulling out square factors:

$$\sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2} \qquad \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$$

Both are now multiples of $\sqrt{2}$, so they add like terms:

$$3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$$

Check numerically: $\sqrt{18} \approx 4.243$ and $\sqrt{8} \approx 2.828$, summing to about 7.071. And $5\sqrt{2} \approx 7.071$ ✓

A and **B** added the numbers under the roots. $\sqrt{a} + \sqrt{b}$ is **never** $\sqrt{a+b}$ — test it: $\sqrt{9} + \sqrt{16} = 7$, but $\sqrt{25} = 5$. **D** multiplied the coefficients instead of adding.

> **Takeaway:** Simplify each radical first, then add the ones with matching roots the way you would add like terms. Never add under the root sign.

Q24 — Medium — Answer: D

Topic: An expression that must hold for all values

Expand the left-hand side, keeping p as a letter:

$$(x+p)(x+4) = x^2 + 4x + px + 4p = x^2 + (p+4)x + 4p$$

"For all values of x" means the two sides are identical, so match the pieces. The x coefficients:

$$p + 4 = 9 \quad \Longrightarrow \quad p = 5$$

Then the constants:

$$q = 4p = 20$$

Check: $(x+5)(x+4) = x^2 + 9x + 20$ ✓

A and **B** copied numbers straight off the question. **C** added where a multiplication was needed.

> **Takeaway:** "For all values of x" means match coefficients term by term. Find the unknown from the x terms first, then use it in the constant.

Q25 — Medium — Answer: B

Topic: Exponents with different bases

The bases differ, so no exponent rule applies yet. **Rewrite them to match**:

$$4^3 = (2^2)^3 = 2^6$$

Now they share a base and dividing subtracts:

$$\frac{2^{10}}{2^6} = 2^4 = 16$$

Check numerically: $\frac{1024}{64} = 16$ ✓

A subtracted $10 - 3$ without converting the 4 to a power of 2 — the most common error in this whole topic.

> **Takeaway:** Exponent rules need a **common base**. Rewrite 4, 8, 9, 27 and so on as powers of a smaller base first, then apply the rule.

Q26 — Medium — Answer: D

Topic: Factoring out a negative

The common factor is $-2x$. Taking it out flips the sign of every term inside:

$$-2x^2 + 10x = -2x(x - 5)$$

Check by expanding: $-2x \cdot x = -2x^2$ ✓ and $-2x \cdot (-5) = +10x$ ✓

Substituting is quicker still. At $x = 1$ the original is $-2 + 10 = 8$, and $-2(1)(1-5) = -2(-4) = 8$ ✓

A kept the factor positive, giving $2x^2 - 10x$ — the whole expression negated. **B** got the inside sign wrong. **C** divided the $10x$ by -2 rather than by $-2x$, leaving a 10 where a 5 belongs.

One more thing worth knowing: $2x(5 - x)$ is **also** a correct factorisation of this expression, because $5 - x = -(x - 5)$ and the two minus signs cancel. Both forms are right; $-2x(x - 5)$ is the conventional one, with the common factor taken out in front.

> **Takeaway:** Pulling out a negative flips every sign inside the bracket. Expand it back, or test $x = 1$, before moving on — and remember $a - b$ and $b - a$ differ only by a sign, so two factorisations can look different and be equal.

Q27 — Hard — Answer: A

Topic: Simplifying a compound fraction

Combine the top over a common denominator of $3x$:

$$\frac{1}{x} - \frac{1}{3} = \frac{3 - x}{3x}$$

Now the whole expression is

$$\frac{3 - x}{3x} \div (x - 3) = \frac{3 - x}{3x(x - 3)}$$

The key step: $3 - x$ is the **negative** of $x - 3$, so $3 - x = -(x - 3)$ and the brackets cancel, leaving the minus behind:

$$\frac{-(x - 3)}{3x(x - 3)} = -\frac{1}{3x}$$

Check at $x = 1$: the original is $\frac{1 - \frac{1}{3}}{1 - 3} = \frac{2/3}{-2} = -\frac{1}{3}$, and $-\frac{1}{3(1)} = -\frac{1}{3}$ ✓

B lost that minus. **D** stopped one step early.

> **Takeaway:** $a - b$ and $b - a$ differ only by a minus sign, so they cancel and leave -1. Combine the top before dividing, then substitute a number to check the sign.

Q28 — Medium — Answer: C

Topic: An expression with a shared factor

Recognise the pattern before doing anything:

$$x^2 + 2xy + y^2 = (x + y)^2$$

So the answer is simply

$$12^2 = 144$$

The $xy = 20$ is there to be ignored — it is already inside the square, and adding it again is what options A and D do.

You could solve for x and y instead, but they turn out to be irrational, and the question is designed so that recognising the identity avoids all of that.

> **Takeaway:** $x^2 + 2xy + y^2$ is $(x + y)^2$, always. When a question hands you $x + y$ and asks for that expression, the identity **is** the question — and extra information may be a distraction.

Q29 — Hard — Answer: 3

Topic: Rewriting to a required form

Factor the top:

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

The $(x + 2)$ cancels:

$$\frac{(x + 2)(x + 3)}{x + 2} = x + 3$$

So $k = 3$.

The condition $x \neq -2$ is there because the original fraction is undefined at $x = -2$, even though $x + 3$ is perfectly happy there. Cancelling removes the problem from the expression but not from the original, which is why the question states the restriction.

This is a grid-in. Enter 3.

> **Takeaway:** Factor the numerator and look for the denominator inside it. The stated restriction tells you which factor is about to cancel.

Q30 — Medium — Answer: B

Topic: A radical in the denominator

Multiply top and bottom by $\sqrt{3}$, which is multiplying by 1 and so changes nothing but the appearance:

$$\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

Check numerically: $\sqrt{3} \approx 1.732$, so $\frac{6}{1.732} \approx 3.46$, and $2\sqrt{3} \approx 3.46$ ✓

A treated $\sqrt{3}$ as 3. **D** divided under the root, which is not a legal move here.

> **Takeaway:** To clear a root from the bottom, multiply top and bottom by it. Then check numerically — a decimal comparison catches every slip in this topic.

Q31 — Medium — Answer: C

Topic: Multiplying a binomial by a trinomial

Every term in the first bracket multiplies every term in the second, so there are $2 \times 3 = 6$ products, not four. Take the x through first:

$$x(x^2 - 3x + 5) = x^3 - 3x^2 + 5x$$

Then the $+2$, which must reach **all three** terms:

$$2(x^2 - 3x + 5) = 2x^2 - 6x + 10$$

Add the two lines and collect like terms:

$$x^3 + (-3x^2 + 2x^2) + (5x - 6x) + 10 = x^3 - x^2 - x + 10$$

Check at $x = 1$: the original is $(3)(1 - 3 + 5) = 3 \times 3 = 9$, and the answer is $1 - 1 - 1 + 10 = 9$ ✓

Why each wrong option is wrong:

- **A,** $x^3 - x^2 + x + 10$ — collected the two x terms as $+x$ instead of $-x$. $5x - 6x$ is negative, because the $-6x$ is larger.
- **B,** $x^3 + x^2 - x + 10$ — collected the two x^2 terms as $+x^2$. $-3x^2 + 2x^2 = -x^2$.
- **D,** $x^3 - 3x^2 + 5x + 10$ — distributed the x across all three terms but not the 2, multiplying it by the constant alone. Every term in the second bracket has to be reached by every term in the first.

> **Takeaway:** count the products before you start — a binomial times a trinomial gives six. Distribute each term of the first bracket across the **whole** second bracket, then collect.

Q32 — Hard — Answer: 6

Topic: An exponent equation with matched bases

The bases differ, so rewrite both as powers of 3:

$$9^x = (3^2)^x = 3^{2x} \qquad 27^4 = (3^3)^4 = 3^{12}$$

With the same base, the exponents must match:

$$2x = 12 \quad \Longrightarrow \quad x = 6$$

Check: $9^6 = 531{,}441$ and $27^4 = 531{,}441$ ✓

This is a grid-in. Enter 6.

> **Takeaway:** Rewrite both sides over a common base, then set the exponents equal. Powers of 2, 3 and 5 are the ones the test reuses.

Chapter 4 — Quadratic and Nonlinear Equations

A quadratic is an equation with an x^2 in it and no higher power. It is the second-largest single topic on the SAT Math section, and it is where the test stops being about straight lines.

The one structural fact to hold on to: **a quadratic usually has two solutions.** Sometimes one, occasionally none, but two is the normal case. Half the wrong options in this chapter come from finding one solution and stopping.

You have three ways to solve one, and knowing *which* to reach for is most of the skill:

Method	Use it when
Factoring	the numbers are friendly — always try this first
The quadratic formula	factoring does not come quickly. Always works
Square roots	there is no x term, like $x^2 = 49$ or $(x-2)^2 = 9$

The quadratic formula is not on the reference sheet. Neither is anything else in this chapter. Learn it:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Topics covered: solving by factoring · the quadratic formula · the discriminant · completing the square · the vertex · nonlinear systems · equations with radicals and fractions

Section 1 — Solving by Factoring

The whole method rests on one idea, the **zero product property**:

> If two things multiply to give zero, at least one of them **is** zero.

So if $(x-3)(x+5) = 0$, then either $x - 3 = 0$ or $x + 5 = 0$, giving $x = 3$ or $x = -5$.

This only works against zero. If $(x-3)(x+5) = 7$ you can say nothing about the individual brackets — plenty of pairs multiply to 7. So the first step is always to get everything on one side, with **zero on the other**.

The signs flip: a factor of $(x-3)$ gives a solution of $x = +3$.

Q1 — Basic

If $(x-4)(x+7)=0$, what are all possible values of x?

A) $(-7,\ -4)$

B) $(-7,\ 4)$

C) $(4,\ 7)$

D) $(-4,\ 7)$

Q2 — Medium

What are the solutions of $x^2-5x-14=0$?

A) $(-7,\ 2)$

B) $(-2,\ 7)$

C) $(2,\ 7)$

D) $(-14,\ 1)$

Q3 — Medium

A model's two break-even points are the solutions of $x^2+3x=10$, where x is a quantity the analyst may vary. What is the sum of the solutions of $x^2+3x=10$?

A) 3

B) -3

C) -10

D) 7

Section 2 — The Quadratic Formula and the Discriminant

When factoring does not come within about twenty seconds, use the formula. It always works.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

The part under the root has a name and a job of its own. The **discriminant**:

$$\Delta=b^2-4ac$$

It tells you **how many real solutions** there are, without solving:

$b^2 - 4ac$	**Real solutions**	**On a graph**
positive	**two**	crosses the x-axis twice
zero	**one**	just touches the x-axis
negative	**none**	never reaches the x-axis

Questions asking "for what value of k does this have exactly one solution" are always discriminant questions. Set $b^2 - 4ac = 0$ and solve.

Two cautions on using the formula:

- b **carries its own sign.** In $x^2 - 5x + 6$, $b = -5$, so $-b = +5$.
- $-4ac$ **with a negative** c **becomes positive.** In $x^2 + 2x - 8$, $-4(1)(-8) = +32$.

Q4 — Medium

What are the solutions of $2x^2 + 3x - 2 = 0$?

A) $\left(-1,\ \frac{3}{2}\right)$

B) $\left(-\frac{1}{2},\ 2\right)$

C) $(-2,\ 2)$

D) $\left(-2,\ \frac{1}{2}\right)$

Q5 — Hard

A designer needs the parabola $y = x^2 + kx + 16$ to just touch the x-axis rather than cross it. The constant k is known to be positive. What is the value of k?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q6 — Medium

A path is modelled by $y = x^2 - 6x + c$, and the design requires that it never reaches the x-axis at all, so $x^2 - 6x + c = 0$ has no real solutions. For which value of c does the equation have **no** real solutions?

A) 5

B) 8

C) 9

D) 10

Section 3 — Vertex Form and the Turning Point

The graph of a quadratic is a **parabola** — a symmetric U shape. Its turning point is the **vertex**, and questions about maximum or minimum values are always vertex questions.

Two ways to find it:

From the formula. The vertex sits at

$$x = -\frac{b}{2a}$$

then substitute that x back to get the y-value. This is the fastest route and it always works.

From vertex form. If the quadratic is written

$$y = a(x - h)^2 + k$$

the vertex is (h, k) — read straight off. Note the **sign flip on** h: $y = (x - 3)^2 + 2$ has its vertex at $(3, 2)$, and $y = (x + 3)^2 + 2$ at $(-3, 2)$.

Which way it opens, and so whether the vertex is a minimum or a maximum:

- $a > 0$ — opens upward, vertex is the **minimum**
- $a < 0$ — opens downward, vertex is the **maximum**

The parabola is symmetric about the vertical line through its vertex, so the two x-intercepts are always the same distance from it — which means the vertex's x is the **average of the two roots**.

Q7 — Medium

A cost model is written as $y = 2(x-3)^2 + 5$. The graph of that equation has its vertex at which point?

A) $(-3,\ 5)$

B) $(3,\ 5)$

C) $(3,\ -5)$

D) $(2,\ 3)$

Q8 — Medium

A manufacturer's cost, in thousands of dollars, is modelled by $y = x^2 - 8x + 3$, where x is the number of units produced in hundreds. The manufacturer wants the production level at which the cost is least. At what value of x does the minimum occur?

A) -8

B) -4

C) 4

D) 8

Q9 — Hard

A quantity is modelled by $y = x^2 + 6x + 1$, and the analyst needs the lowest value the model ever takes. What is the minimum value of $y = x^2 + 6x + 1$?

A) -8

B) -3

C) 1

D) 10

Section 4 — Nonlinear Systems and Other Equations

A **nonlinear system** is one straight line and one curve, or two curves. Solve by substitution: rearrange the linear equation to "$y = \ldots$", put it into the other, and you are left with a quadratic in x.

Geometrically you are finding where a line meets a parabola, so expect **two** intersection points, sometimes one, sometimes none — and the discriminant tells you which before you solve.

Two other equation types appear often enough to be worth their own warning:

- **Radical equations.** Isolate the root and square both sides. **Squaring can create solutions that do not work**, so every answer must be checked in the original equation.
- **Equations with the variable in a denominator.** Multiply through by the denominator, but discard any answer that would make a denominator zero.

Q10 — Medium

A straight path and a curved one are modelled by the system $\begin{cases} y = x^2 - 3 \\ y = 2x \end{cases}$, which has two solutions -- the two points at which the paths meet. What is the sum of the two x-values at which they meet?

A) -3

B) -2

C) 2

D) 3

Q11 — Hard

What is the solution to $\sqrt{2x+7} = x - 4$?

A) 1

B) $(1,\ 9)$

C) $(-1,\ 9)$

D) 9

Q12 — Medium

What is the solution to $\dfrac{12}{x} + 1 = \dfrac{8}{x} + 3$?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Section 5 — Mixed Practice

Factoring, the formula, the discriminant and the vertex, shuffled.

Q13 — Basic

What are the solutions of $x^2 - 49 = 0$?

A) $(-7,\ 7)$

B) 7

C) $(-49,\ 49)$

D) $(0,\ 49)$

Q14 — Medium

An analyst needs the product of the two solutions of $3x^2 - 12x + 9 = 0$. What is the product of the solutions?

A) 4

B) 3

C) 9

D) -4

Q15 — Medium

A quantity satisfies $x^2 = 5x + 24$. What is the largest solution of $x^2 = 5x + 24$?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q16 — Medium

How many real solutions does $2x^2 + 3x + 5 = 0$ have?

A) 0

B) 1

C) 2

D) 4

Q17 — Hard

The graph of $y = x^2 - 2x - 15$ crosses the x-axis at two points. What is the x-coordinate of the vertex?

A) -2

B) -1

C) 1

D) 2

Q18 — Medium

A ball is thrown straight upward from ground level, and its height in metres after t seconds is modelled by $h = -5t^2 + 20t$. After how many seconds does it return to the ground?

A) 0

B) 2

C) 20

D) 4

Q19 — Hard

The graphs of $y = x^2 + 4$ and $y = 4x + c$, where c is a constant, intersect at exactly one point. What is the value of c?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q20 — Medium

The function f is defined by $f(x) = (x+6)(x-2)$. For what value of x does f reach its minimum?

A) -2

B) 2

C) -6

D) 6

Q21 — Medium

The equation $x^2 + 6x - 7 = 0$ can be rewritten in the completed-square form $(x+a)^2 = b$, where a and b are constants. What is the value of b?

A) 7

B) 9

C) 16

D) 3

Q22 — Medium

A quadratic is known only by where its graph meets the x-axis: it has roots $x = -4$ and $x = 6$. Which equation could it be?

A) $x^2 - 2x - 24 = 0$

B) $x^2 + 2x - 24 = 0$

C) $x^2 - 10x + 24 = 0$

D) $x^2 + 24x - 2 = 0$

Q23 — Hard

The graph of $y = x^2 - 4x - 5$ is a parabola opening upward, and it crosses the x-axis at $x = -1$ and $x = 5$. For which values of x is $y < 0$?

A) $-1 \leq x \leq 5$

B) $x < -1$ or $x > 5$

C) $-5 < x < 1$

D) $-1 < x < 5$

Q24 — Medium

A line and a curve are modelled by $y = x^2$ and $y = 3x + 4$, and the system has two solutions -- the two points where the graphs meet. What is the sum of the two x-values?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q25 — Hard

A ball's height in metres is $h = -4.9t^2 + 19.6t$ after t seconds. At what time does it reach its greatest height?

A) 1

B) 2

C) 4

D) $\frac{1}{4}$

Q26 — Medium

What are the solutions of $3x^2 - 12x = 0$?

A) $(0,\ -4)$

B) 4

C) $(0,\ 4)$

D) $(-4,\ 4)$

Q27 — Hard

An engineer needs to know whether the model $y = x^2 + 2x + 5$ ever reaches zero. For how many values of x does the graph cross the x-axis?

A) 0

B) 1

C) 2

D) 3

Q28 — Medium

What are the solutions of $(x - 3)^2 = 25$?

A) 8

B) $(-5,\ 5)$

C) $(-8,\ 8)$

D) $(-2,\ 8)$

Answers and Explanations

Section 1 — Solving by Factoring

Q1 — Basic — Answer: B

Topic: Reading solutions off a factored quadratic

Two things multiply to zero, so at least one is zero. Set each bracket to zero in turn:

$$\begin{aligned} x - 4 = 0 \quad &\Longrightarrow \quad x = 4 \\ x + 7 = 0 \quad &\Longrightarrow \quad x = -7 \end{aligned}$$

So $x = 4$ or $x = -7$.

Notice the **sign flip**: the bracket says -4 and the solution is $+4$. Solving $x - 4 = 0$ means adding 4 to both sides, so the sign changes. Getting this backwards is what the other three options are made of.

Check both in the original: $(4-4)(4+7) = 0 \times 11 = 0$ ✓ and $(-7-4)(-7+7) = -11 \times 0 = 0$ ✓

Why each wrong option is wrong:

- **A,** $(-7,\ -4)$ — made both negative, flipping only one of the two signs.
- **C,** $(4,\ 7)$ — copied the numbers straight out of the brackets with no sign change at all.
- **D,** $(-4,\ 7)$ — flipped both signs but in the wrong direction.

> **Takeaway:** A factor $(x - a)$ gives the solution $x = a$ — the sign flips. Then substitute both back; each should make one bracket exactly zero.

Q2 — Medium — Answer: B

Topic: Factoring to solve

Find two numbers that multiply to -14 and add to -5.

The constant is negative, so one number is positive and one negative. The pairs multiplying to -14: 1 and -14 (adds to -13), -1 and 14 (adds to 13), 2 and -7 (adds to -5 ✓).

$$x^2 - 5x - 14 = (x+2)(x-7) = 0$$

So $x = -2$ or $x = 7$.

Check: $(-2)^2 - 5(-2) - 14 = 4 + 10 - 14 = 0$ ✓ and $49 - 35 - 14 = 0$ ✓

Why each wrong option is wrong:

- **A,** $(-7,\ 2)$ — the right pair of sizes with the signs swapped. Those come from $(x+7)(x-2) = x^2 + 5x - 14$, which has the middle term $+5x$.
- **C,** $(2,\ 7)$ — both positive, which would need a $+14$ constant.
- **D,** $(-14,\ 1)$ — a pair multiplying to -14 that does not add to -5.

> **Takeaway:** Multiply to c, add to b — both conditions, every time. When you have a candidate pair, check the sum before you write anything down.

Q3 — Medium — Answer: B

Topic: Rearranging before factoring

Get zero on one side first — the factoring method needs it:

$$x^2 + 3x - 10 = 0$$

Two numbers multiplying to -10 and adding to 3: that is 5 and -2.

$$(x+5)(x-2) = 0 \quad \Longrightarrow \quad x = -5 \text{ or } x = 2$$

Their sum is $-5 + 2 = -3$.

There is a shortcut worth knowing. For $ax^2 + bx + c = 0$:

$$\text{sum of solutions} = -\frac{b}{a} \qquad \text{product of solutions} = \frac{c}{a}$$

Here $-\frac{3}{1} = -3$ ✓ and the product would be $\frac{-10}{1} = -10$. When a question asks only for the sum or the product, this answers it without solving at all.

Why each wrong option is wrong:

- **A,** 3 — the sign of the sum. It is $-\frac{b}{a}$, and $b = 3$.
- **C,** -10 — the **product** of the solutions, not the sum.
- **D,** 7 — added the two numbers ignoring their signs.

> **Takeaway:** Move everything to one side before factoring. And remember sum $= -\frac{b}{a}$, product $= \frac{c}{a}$ — they answer a whole family of questions in one line.

Section 2 — The Quadratic Formula and the Discriminant

Q4 — Medium — Answer: D

Topic: Using the quadratic formula

Here $a = 2$, $b = 3$, $c = -2$. Work out the discriminant first:

$$b^2 - 4ac = 3^2 - 4(2)(-2) = 9 + 16 = 25$$

Positive, so there are two solutions — and 25 is a perfect square, so they will be tidy. Now the formula:

$$x = \frac{-3 \pm \sqrt{25}}{2(2)} = \frac{-3 \pm 5}{4}$$

$$x = \frac{-3+5}{4} = \frac{2}{4} = \frac{1}{2} \qquad x = \frac{-3-5}{4} = \frac{-8}{4} = -2$$

Check the smaller one: $2(4) + 3(-2) - 2 = 8 - 6 - 2 = 0$ ✓

Note $-4ac = -4(2)(-2) = +16$: two negatives made it positive, which is what turned 9 into 25 rather than into -7.

Why each wrong option is wrong:

- **B,** $\left(-\frac{1}{2},\ 2\right)$ — both solutions with their signs flipped, from using $+b$ instead of $-b$ on the top.
- **C,** $(-2,\ 2)$ — solved $2x^2 - 8 = 0$, dropping the middle term.
- **A,** $\left(-1,\ \frac{3}{2}\right)$ — an arithmetic slip inside the root. Substituting $x = -1$ gives $2 - 3 - 2 = -3$, not 0.

> **Takeaway:** Compute $b^2 - 4ac$ first — it tells you how many solutions to expect and warns you if the arithmetic has gone wrong. Watch the sign of c: $-4ac$ with negative c adds.

Q5 — Hard — Answer: 8

Topic: The discriminant and exactly one solution

"Exactly one real solution" means the discriminant is **zero**:

$$b^2 - 4ac = 0$$

Here $a = 1$, $b = k$, $c = 16$:

$$k^2 - 4(1)(16) = 0 \quad \Longrightarrow \quad k^2 = 64 \quad \Longrightarrow \quad k = \pm 8$$

The question says k is positive, so $k = 8$.

Check: $x^2 + 8x + 16 = (x+4)^2$, which is zero only at $x = -4$ — one solution ✓

Notice what "exactly one solution" always means structurally: the quadratic is a **perfect square**. That gives you a second route to the same answer — the middle coefficient of a perfect square is twice the root of the constant, and $2\sqrt{16} = 8$.

This is a grid-in. Enter 8.

> **Takeaway:** One solution ⇒ discriminant zero ⇒ perfect square. All three say the same thing, and each gives you a way in. Watch for a question restricting k to positive values — $k^2 = 64$ has two answers and the grid takes one.

Q6 — Medium — Answer: D

Topic: The discriminant and no real solutions

No real solutions means the discriminant is **negative**:

$$b^2 - 4ac < 0$$

With $a = 1$ and $b = -6$:

$$36 - 4c < 0 \quad \Longrightarrow \quad 36 < 4c \quad \Longrightarrow \quad c > 9$$

Of the options, only $c = 10$ is greater than 9.

Run the four values through $36 - 4c$ to see the whole picture:

c	$36 - 4c$	**Solutions**
5	$+16$	two
8	$+4$	two
9	0	exactly one
10	-4	**none**

Option C is the one to watch. $c = 9$ makes the discriminant exactly zero, which is **one** solution, not none. That boundary is where these questions are always set.

> **Takeaway:** None ⇒ negative discriminant. Zero ⇒ exactly one. The option sitting exactly at the boundary is always offered, so work out which side of it you need before choosing.

Section 3 — Vertex Form and the Turning Point

Q7 — Medium — Answer: B

Topic: Reading the vertex from vertex form

Compare with $y = a(x-h)^2 + k$, whose vertex is (h, k):

$$y = 2(x - \mathbf{3})^2 + \mathbf{5}$$

So $h = 3$ and $k = 5$, and the vertex is $(3, 5)$.

The sign flip is the whole question. The bracket reads $x - 3$, so $h = +3$. Only k is read off unchanged.

Check it directly. At $x = 3$ the squared term is zero, giving $y = 5$ — the smallest y can be, since a square is never negative and $a = 2$ is positive. So $(3, 5)$ is the minimum ✓ At $x = 2$ and $x = 4$, $y = 7$ — higher on both sides ✓

Why each wrong option is wrong:

- **A,** $(-3,\ 5)$ — kept the minus from the bracket. $(x - 3)$ means $h = 3$.
- **C,** $(3,\ -5)$ — flipped k as well. Only h flips.
- **D,** $(2,\ 3)$ — read the leading coefficient and h as if they were the coordinates.

> **Takeaway:** $y = a(x-h)^2 + k$ has vertex (h, k) — the h flips sign, the k does not. Substituting $x = h$ confirms it in one step.

Q8 — Medium — Answer: C

Topic: The vertex from standard form

The vertex of $y = ax^2 + bx + c$ is at

$$x = -\frac{b}{2a}$$

Here $a = 1$ and $b = -8$:

$$x = -\frac{-8}{2(1)} = \frac{8}{2} = 4$$

The two minus signs cancel, which is where option B goes wrong.

Check by symmetry. Factor the quadratic's roots or just test either side: at $x = 3$, $y = 9 - 24 + 3 = -12$; at $x = 4$, $y = 16 - 32 + 3 = -13$; at $x = 5$, $y = 25 - 40 + 3 = -12$. Lower at 4 than on either side, and symmetric about it ✓

Why each wrong option is wrong:

- **A,** -8 — read b off the equation with no formula applied.
- **B,** -4 — divided but dropped the leading minus, so the two negatives did not cancel.
- **D,** 8 — took $-b$ and forgot to divide by $2a$.

> **Takeaway:** $x = -\frac{b}{2a}$, and b brings its own sign with it. A negative b gives a positive vertex x.

Q9 — Hard — Answer: A

Topic: The minimum value itself

Find where the minimum happens, then what it is. Those are two different questions and the options here punish confusing them.

$$x = -\frac{b}{2a} = -\frac{6}{2} = -3$$

Now substitute to get the **value**:

$$y = (-3)^2 + 6(-3) + 1 = 9 - 18 + 1 = -8$$

Check either side: at $x = -4$, $y = 16 - 24 + 1 = -7$; at $x = -2$, $y = 4 - 12 + 1 = -7$. Both higher ✓

Why each wrong option is wrong:

- **B,** -3 — the x-coordinate of the vertex, which is *where* the minimum is, not what it is.
- **C,** 1 — the constant term, which is y at $x = 0$, not at the vertex.
- **D,** 10 — substituted $+3$ instead of -3.

> **Takeaway:** "Where is the minimum" wants x; "what is the minimum" wants y. Find $x = -\frac{b}{2a}$, then substitute — and read the question again to see which of the two numbers it asked for.

Section 4 — Nonlinear Systems and Other Equations

Q10 — Medium — Answer: C

Topic: Where a line meets a parabola

Both equations give y, so set them equal:

$$x^2 - 3 = 2x$$

Move everything to one side:

$$x^2 - 2x - 3 = 0$$

Factor — two numbers multiplying to -3 and adding to -2, so -3 and 1:

$$(x-3)(x+1) = 0 \quad \Longrightarrow \quad x = 3 \text{ or } x = -1$$

Their sum is $3 + (-1) = 2$.

Or use the shortcut: sum of roots $= -\frac{b}{a} = -\frac{-2}{1} = 2$ ✓

Check one point fully: at $x = 3$, the parabola gives $9 - 3 = 6$ and the line gives 6 ✓ They do meet there.

Why each wrong option is wrong:

- **A,** -3 — read the -3 out of the original parabola.
- **B,** -2 — dropped a sign in $-\frac{b}{a}$.
- **D,** 3 — gave one root rather than the sum of both.

> **Takeaway:** Two equations both equal to y — set them equal to each other, collect on one side, and solve the quadratic. Then check whether the question wants the roots, their sum, or the points.

Q11 — Hard — Answer: D

Topic: A radical equation, with a check that matters

The root is already alone, so square both sides:

$$2x + 7 = (x-4)^2 = x^2 - 8x + 16$$

Collect:

$$0 = x^2 - 10x + 9 = (x-1)(x-9)$$

So the squared equation gives $x = 1$ or $x = 9$. **Now check both in the original**, which is not optional here:

$$\begin{aligned} x = 9: &\quad \sqrt{18+7} = \sqrt{25} = 5, \text{ and } 9 - 4 = 5 \quad ✓ \\ x = 1: &\quad \sqrt{2+7} = \sqrt{9} = 3, \text{ but } 1 - 4 = -3 \quad \text{no} \end{aligned}$$

$x = 1$ fails. The square root symbol means the **positive** root, so the left side can never be negative — but squaring destroys that information, because 3^2 and $(-3)^2$ are both 9. A solution created this way is called **extraneous**, and discarding it is part of the method, not an afterthought.

So the only solution is $x = 9$.

Why each wrong option is wrong:

- **A,** 1 — kept the extraneous root and discarded the real one.
- **B,** $(1,\ 9)$ — kept both, skipping the check entirely. This is the option most people pick.
- **C,** $(-1,\ 9)$ — a sign slip in the factoring, then kept both.

> **Takeaway:** Squaring both sides can invent solutions. **Always** substitute back into the original radical equation. A quick filter: if the side without the root comes out negative, that value cannot work.

Q12 — Medium — Answer: 2

Topic: An equation with the variable in a denominator

Gather the fractions on one side and the numbers on the other:

$$\frac{12}{x} - \frac{8}{x} = 3 - 1 \quad\Longrightarrow\quad \frac{4}{x} = 2$$

Multiply both sides by x:

$$4 = 2x \quad\Longrightarrow\quad x = 2$$

Check in the original: $\frac{12}{2} + 1 = 7$ and $\frac{8}{2} + 3 = 7$ ✓

And check the denominators: $x = 2$ is not zero, so the answer is valid. With a variable on the bottom, an answer of $x = 0$ would have to be thrown away no matter how correctly it was derived.

This is a grid-in. Enter 2.

> **Takeaway:** With the same denominator on both sides, combine the fractions first — it is quicker than clearing them. Then confirm your answer does not make any denominator zero.

Section 5 — Mixed Practice

Q13 — Basic — Answer: A

Topic: A quadratic with no middle term

With no x term, take square roots — and take **both**:

$$x^2 = 49 \implies x = \pm 7$$

Or factor as a difference of squares: $(x+7)(x-7) = 0$.

B kept only the positive root, which is the mistake this question exists for. **C** never took the root. **D** factored as if the constant were attached to an x.

> **Takeaway:** $x^2 = k$ has **two** answers, $+\sqrt{k}$ and $-\sqrt{k}$. The $\pm$ is not decoration.

Q14 — Medium — Answer: B

Topic: The product of the solutions

For $ax^2 + bx + c = 0$, the product of the solutions is $\frac{c}{a}$:

$$\frac{9}{3} = 3$$

Check the long way. Divide through by 3: $x^2 - 4x + 3 = 0$, which factors as $(x-1)(x-3)$, giving $x = 1$ and $x = 3$. Their product is 3 ✓ and their sum is 4, which is option A.

A is the sum, $-\frac{b}{a} = 4$. **C** read c without dividing by a. **D** has the sum's sign wrong.

> **Takeaway:** Sum $= -\frac{b}{a}$, product $= \frac{c}{a}$. Both need the division by a — forgetting it is what makes option C tempting.

Q15 — Medium — Answer: 8

Topic: Solving a quadratic that needs rearranging

Move everything to one side:

$$x^2 - 5x - 24 = 0$$

Two numbers multiplying to -24 and adding to -5: -8 and 3.

$$(x-8)(x+3) = 0 \implies x = 8 \text{ or } x = -3$$

The largest is 8. Check: $64 = 40 + 24$ ✓

This is a grid-in. Enter 8.

> **Takeaway:** Zero on one side before factoring, always. Then read whether the question wants the largest, the smallest, the sum or both.

Q16 — Medium — Answer: A

Topic: Counting solutions from the discriminant

Compute the discriminant:

$$b^2 - 4ac = 3^2 - 4(2)(5) = 9 - 40 = -31$$

Negative, so there are **no real solutions**. The parabola sits entirely above the x-axis and never crosses it.

B is the zero-discriminant case. **C** assumes two, which is the usual number but not guaranteed.

> **Takeaway:** Compute $b^2 - 4ac$ before solving. Negative means none — and it saves you from grinding through a formula that will produce a root of a negative number.

Q17 — Hard — Answer: C

Topic: The vertex of a parabola from its intercepts

Two routes, and they agree.

By formula: $x = -\frac{b}{2a} = -\frac{-2}{2} = 1$.

By symmetry: factor to find the crossings. $x^2 - 2x - 15 = (x-5)(x+3)$, so the graph crosses at $x = 5$ and $x = -3$. A parabola is symmetric, so the vertex sits exactly **halfway between** its two x-intercepts:

$$\frac{5+(-3)}{2} = \frac{2}{2} = 1 \checkmark$$

The second route is worth having. When a question gives you the two roots or the two crossings, averaging them is faster than the formula and harder to get wrong.

A read b directly. **B** dropped the leading minus. **D** forgot to divide.

> **Takeaway:** The vertex is halfway between the two x-intercepts. Average the roots — no formula, no sign errors.

Q18 — Medium — Answer: D

Topic: A quadratic in a real situation

"On the ground" means $h = 0$:

$$-5t^2 + 20t = 0$$

Factor out the common $-5t$:

$$-5t(t-4) = 0 \quad \Longrightarrow \quad t = 0 \text{ or } t = 4$$

Both are real moments: $t = 0$ is when it was thrown, $t = 4$ is when it lands. The question asks when it **returns**, so $t = 4$.

Check: at $t = 3$, $h = -45 + 60 = 15$ m — still in the air ✓ At $t = 4$, $h = -80 + 80 = 0$ ✓

A is the launch. **B** is the highest point, at $t = -\frac{20}{2(-5)} = 2$ — worth noting that the peak is exactly halfway between the two ground times. **C** read a coefficient.

> **Takeaway:** In a height model, $h = 0$ gives **two** times: the start and the landing. Read which one the question wants, and remember the peak is halfway between them.

Q19 — Hard — Answer: 0

Topic: A system of a line and a parabola with one intersection

Set the two expressions equal, since both give y:

$$x^2 + 4 = 4x + c$$

Collect everything on one side:

$$x^2 - 4x + (4 - c) = 0$$

"Exactly one point" means this quadratic has exactly one solution, so its discriminant is zero:

$$(-4)^2 - 4(1)(4-c) = 0$$
$$16 - 16 + 4c = 0 \implies 4c = 0 \implies c = 0$$

Check: with $c = 0$ the equation is $x^2 - 4x + 4 = (x-2)^2 = 0$, whose only solution is $x = 2$. At $x = 2$ the parabola gives 8 and the line gives 8 — they touch at $(2, 8)$ ✓

A line meeting a parabola at exactly one point is a **tangent**, and "tangent to" in a question is the same instruction as "exactly one intersection": set the discriminant to zero.

This is a grid-in. Enter `0`.

> **Takeaway:** Line and curve intersecting once ⇒ set them equal, collect, and set the discriminant of the result to zero. "Tangent" means the same thing.

Q20 — Medium — Answer: A

Topic: Reading a quadratic's roots from its factored form

The factored form gives the roots immediately: $x = -6$ and $x = 2$. The vertex is halfway between them:

$$\frac{-6+2}{2} = \frac{-4}{2} = -2$$

Check: at $x = -2$, $f = (4)(-4) = -16$. At the roots $f = 0$, so -2 is indeed lower ✓

B and **C** are the roots themselves — where f is zero, not where it is least. **D** read a bracket without the sign flip.

> **Takeaway:** Roots from the brackets, vertex halfway between. Where a parabola is **zero** and where it is **least** are different questions.

Q21 — Medium — Answer: C

Topic: Completing the square

Move the constant across, then complete the square. Halve the middle coefficient and square it: $\left(\frac{6}{2}\right)^2 = 9$.

$$x^2 + 6x = 7$$
$$x^2 + 6x + 9 = 7 + 9$$
$$(x+3)^2 = 16$$

So $a = 3$ and $b = 16$.

Whatever you add to the left must be added to the right — that is why b is $7 + 9$ and not either number alone.

A forgot to add the 9 to the right as well. **B** gave the 9 by itself. **D** gave a.

> **Takeaway:** Halve, square, add to **both** sides. Then read whether the question wants a or b.

Q22 — Medium — Answer: A

Topic: A quadratic from its roots

Roots -4 and 6 come from factors $(x+4)$ and $(x-6)$ — the signs flip. Multiply out:

$$(x+4)(x-6) = x^2 - 6x + 4x - 24 = x^2 - 2x - 24$$

Check both: $16 + 8 - 24 = 0$ ✓ and $36 - 12 - 24 = 0$ ✓

A faster check uses the shortcuts. The sum of the roots is $-4 + 6 = 2$, and sum $= -\frac{b}{a}$, so $b = -2$. The product is -24, and product $= \frac{c}{a}$, so $c = -24$. That gives $x^2 - 2x - 24$ in one line.

B has the middle sign wrong. **C** used roots 4 and 6.

> **Takeaway:** Root r gives factor $(x - r)$ — the sign flips. Or build it from sum $= -\frac{b}{a}$ and product $= \frac{c}{a}$.

Q23 — Hard — Answer: D

Topic: A quadratic inequality read off the graph

The parabola opens **upward** (the x^2 coefficient is positive), so it dips below the axis **between** its two crossings and sits above outside them.

$y < 0$ is the dip:

$$-1 < x < 5$$

The inequality is strict, so the crossings themselves are excluded — at $x = -1$ and $x = 5$ the value is exactly 0, not less than 0.

Test three points: at $x = 0$, $y = -5$ ✓ below. At $x = 6$, $y = 7$ ✓ above. At $x = -1$, $y = 0$ ✓ on the axis.

B is the answer to $y > 0$. **C** flipped the signs of the roots.

> **Takeaway:** Upward parabola: negative **between** the roots, positive outside. Sketch it, or test one point in the middle — that settles which region you want.

Q24 — Medium — Answer: 3

Topic: A nonlinear system

Set them equal:

$$x^2 = 3x + 4 \quad \Longrightarrow \quad x^2 - 3x - 4 = 0 \quad \Longrightarrow \quad (x-4)(x+1) = 0$$

So $x = 4$ or $x = -1$, and the sum is 3.

Or read it straight off: sum of roots $= -\frac{b}{a} = -\frac{-3}{1} = 3$, with no factoring at all. Enter 3.

> **Takeaway:** Two equations equal to y — set them equal and collect on one side. If only the sum is wanted, $-\frac{b}{a}$ answers it immediately.

Q25 — Hard — Answer: B

Topic: Interpreting a quadratic model

The greatest height is the vertex, at

$$t = -\frac{b}{2a} = -\frac{19.6}{2(-4.9)} = \frac{19.6}{9.8} = 2 \text{ seconds}$$

Check either side: $h(1) = 14.7$, $h(2) = 19.6$, $h(3) = 14.7$ ✓ Highest at 2.

C, 4 seconds, is when the ball lands — and notice the peak sits exactly halfway between launch at $t = 0$ and landing at $t = 4$, which is a useful check in itself.

> **Takeaway:** Greatest height means the vertex, $t = -\frac{b}{2a}$. With a negative a the parabola opens downward, so the vertex is a maximum — and it falls halfway between the two ground times.

Q26 — Medium — Answer: C

Topic: A quadratic with a common factor

Factor out the common $3x$:

$$3x(x-4)=0 \quad \Longrightarrow \quad x=0 \text{ or } x=4$$

B is the trap this question exists for. Dividing both sides by x gives $3x-12=0$ and $x=4$ — but dividing by x **assumes x is not zero**, and throws away the root $x=0$. Never divide an equation by the variable; factor instead.

> **Takeaway:** Take the common factor out; do not divide by x. Dividing by something that might be zero silently deletes a solution.

Q27 — Hard — Answer: A

Topic: The number of intersections

Crossing the x-axis means $y=0$, so this asks how many real solutions $x^2+2x+5=0$ has. The discriminant:

$$b^2-4ac=4-20=-16$$

Negative, so **none**. The parabola never reaches the axis.

Confirm it from the shape. The vertex is at $x=-\frac{2}{2}=-1$, where $y=1-2+5=4$. The parabola opens upward, so 4 is its **lowest** point — and the whole curve sits at or above $y=4$, comfortably clear of the axis.

> **Takeaway:** Crossings of the x-axis are real roots, so the discriminant counts them. Finding the vertex confirms the answer from a different direction.

Q28 — Medium — Answer: D

Topic: Solving by taking roots

Take the square root of both sides, keeping **both** signs:

$$x - 3 = \pm 5$$

$$x = 3 + 5 = 8 \qquad x = 3 - 5 = -2$$

Check both: $(8 - 3)^2 = 25$ ✓ and $(-2 - 3)^2 = (-5)^2 = 25$ ✓

A kept one root. **B** stopped at ± 5 without undoing the -3.

> **Takeaway:** $\pm$ on the root, then undo whatever else the bracket does. Both steps, in that order.

Chapter 5 — Functions, Graphs and Exponential Growth

This chapter finishes **Advanced Math**. Chapter 4 was about *solving* nonlinear equations; this one is about *reading* them — what a function does, what its graph looks like, and how to move between the two.

Two ideas carry most of the chapter.

A function is a rule with an input and an output. $f(x)$ does not mean f times x. It means "the output of the rule f when the input is x". Reading it as multiplication is the single most common beginner error here.

Linear adds, exponential multiplies. A linear model goes up by the same *amount* each step. An exponential model goes up by the same *percentage* each step — it multiplies by a fixed factor. Telling them apart is worth several marks on every paper.

Topics covered: function notation · evaluating and composing · reading graphs · transformations · exponential growth and decay · identifying a model from a table

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Section 1 — Function Notation

$f(x)$ is the output when the input is x. To evaluate $f(3)$, replace every x in the rule with 3.

If $f(x) = x^2 + 2x$, then

$$f(3) = 3^2 + 2(3) = 9 + 6 = 15$$

Some vocabulary the test uses, in plain terms:

It says	It means
$f(2) = 7$	put in 2, get out 7 — so the graph passes through $(2, 7)$
the **zero** of f	an x with $f(x) = 0$ — an x-intercept
the **domain**	the inputs allowed
the **range**	the outputs possible
$f(g(x))$	do g **first**, then feed the result into f

That last one is worth its own warning. **Composition works inside out.** In $f(g(2))$, evaluate $g(2)$ first, and put *that* number into f. Doing them the other way round gives a different answer, and both answers are always offered.

Q1 — Basic

If $f(x) = 3x^2 - 4x + 1$, what is the value of $f(-2)$?

A) -19

B) 5

C) -3

D) 21

Q2 — Medium

If $f(x) = 2x + 1$ and $g(x) = x^2$, what is the value of $f(g(3))$?

A) 19

B) 13

C) 49

D) 36

Q3 — Medium

A conversion is carried out by the linear function f, defined by $f(x) = 5x - 8$. On one occasion the output was recorded as 27, but the input that produced it was not written down. If $f(a) = 27$, what is the value of a?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Section 2 — Reading and Transforming Graphs

Changing a function's rule moves its graph in predictable ways. The one that catches everybody is the horizontal shift, because **it goes the opposite way to what the sign suggests**.

The rule becomes	The graph
$f(x) + k$	moves **up** k
$f(x) - k$	moves **down** k
$f(x + k)$	moves **left** k — opposite to the sign
$f(x - k)$	moves **right** k — opposite to the sign
$-f(x)$	flips **vertically**, over the x-axis
$f(-x)$	flips **horizontally**, over the y-axis
$af(x), a > 1$	stretches vertically — taller

Why does $f(x - 3)$ move the graph *right*? Because to get the same output you now need an input 3 larger. Whatever x used to do the job, $x + 3$ does it now. The change is **inside** the bracket, so it affects the input, and inputs move opposite to the sign.

The reliable way to settle any transformation question: **track one point**. Pick a point you know is on the original graph and work out where it lands.

Q4 — Medium

The graph of $y = f(x)$ records a measurement over time and is known to pass through the point $(2, 5)$. A colleague redraws it as $y = f(x - 3)$. Through which point must the redrawn graph pass?

A) $(-1,\ 5)$

B) $(5,\ 5)$

C) $(2,\ 8)$

D) $(2,\ 2)$

Q5 — Medium

The graph of $y = f(x)$ has a minimum at the point $(1, -4)$. The whole model is then raised by 6 units to give $y = f(x) + 6$. What are the coordinates of the minimum of the new graph?

A) $(1,\ 2)$

B) $(7,\ -4)$

C) $(1,\ -10)$

D) $(-5,\ -4)$

Q6 — Hard

The graph of $y = f(x)$ passes through the point $(-3, 7)$. The model is then reflected in the x-axis to give $y = -f(x)$. Through which point must the reflected graph pass?

A) $(3,\ 7)$

B) $(3,\ -7)$

C) $(7,\ -3)$

D) $(-3,\ -7)$

Section 3 — Exponential Growth and Decay

An exponential model **multiplies** by a fixed factor each period, where a linear model adds a fixed amount.

$$y = a \cdot b^x$$

- a is the **starting value**, the amount at $x = 0$.
- b is the **growth factor** — what you multiply by each period.

Turning a percentage into a factor is where the marks are:

The situation	**b is**
grows by 8%	1.08
grows by 50%	1.5
falls by 8%	0.92
falls by 30%	0.70
doubles	2
halves	0.5

The rule: **increase means $1+r$, decrease means $1-r$.** A factor of 1.08 is not "8%" — it is "100% of what you had, plus 8% more".

Growth means $b>1$. Decay means $0<b<1$. If a model has $b=0.85$ the quantity is shrinking by 15% each period, not growing by 85%.

Q7 — Medium

A laboratory colony starts with 400 bacteria and grows by 15% each hour, compounding from one hour to the next. The technician wants a single function giving the population at any whole number of hours. Which function gives the population after h hours?

A) $400\left(\frac{3}{20}\right)^h$

B) $400\left(\frac{23}{20}\right)^h$

C) $\frac{3h}{20}+400$

D) $400\left(\frac{17}{20}\right)^h$

Q8 — Medium

A car bought new is modelled by $V=24000(0.88)^t$, where V is its value in dollars and t is the number of years since it was purchased. An owner is trying to explain to a friend what the model says. Which statement is best supported by the model?

A) The car loses 88% of its value each year

B) The car loses $88 each year

C) The car loses 12% of its value each year

D) The car gains 88% of its value each year

Q9 — Hard

The table shows values of a function.

x	0	1	2	3
y	5	15	45	135

Which type of model fits, and what is the value at $x = 4$?

A) Exponential, and $y = 405$

B) Linear, and $y = 175$

C) Exponential, and $y = 270$

D) Linear, and $y = 145$

Q10 — Medium

A population of 800 organisms doubles every 5 years. The study runs for 20 years in total. What is the population after 20 years?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Section 4 — Mixed Practice

Q11 — Basic

If $g(x) = \dfrac{x+6}{2}$, what is $g(-4)$?

A) -1

B) 1

C) 5

D) -5

Q12 — Medium

A function is defined by $f(x) = 2^x - 8$. For what value of x is $f(x) = 0$?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q13 — Medium

The graph of $y = f(x)$ passes through the point $(4, 3)$. The model is then stretched vertically by a factor of 2 to give $y = 2f(x)$. Through which point must the stretched graph pass?

A) $(8,\ 3)$

B) $(8,\ 6)$

C) $(4,\ 6)$

D) $(2,\ 3)$

Q14 — Hard

An investment is modelled by $A = 2000(1.05)^n$, where A is its value in dollars after n years. Which statement about the model is true?

A) The investment grows by $105 each year

B) The investment grows by 1.05% each year

C) The initial investment was $1.05

D) The investment grows by 5% each year

Q15 — Medium

The table gives values of the function f.

x	0	1	2	3
$f(x)$	7	4	1	-2

Which equation defines f?

A) $y = 7x - 3$

B) $y = 4 - 3x$

C) $y = 7 - 3x$

D) $y = 3x + 7$

Q16 — Medium

The graph of $y = f(x)$ crosses the x-axis at $x = -2$ and $x = 5$, and has no other x-intercepts. Which statement must be true?

A) $f(-2) = 0$ and $f(5) = 0$

B) $f(0) = -2$ and $f(0) = 5$

C) The graph passes through $(0, -2)$ and $(0, 5)$

D) $f(-2) = 5$

Q17 — Medium

Four descriptions of growing quantities are given below. Which of the following describes a quantity that grows exponentially?

A) A salary that rises by $2,000 every year

B) A tank filling at 5 litres a minute

C) A taxi fare of $3 plus $2 a kilometre

D) A population that rises by 3% every year

Q18 — Hard

A colony of organisms triples in number every day, and after 4 days of growth the count stands at 810. The researcher needs the size of the colony at the moment the observation began. How many were there at the start?

A) 270

B) 10

C) $\frac{135}{2}$

D) 30

Q19 — Medium

If $f(x) = 2x - 1$, what is the value of $f(f(3))$?

A) 5

B) 11

C) 9

D) 25

Q20 — Hard

The graph of $y = f(x)$ is shifted 4 units to the right and 2 units down to give a new model. Which equation describes the new graph?

A) $y = f(x - 4) - 2$

B) $y = f(x + 4) - 2$

C) $y = f(x - 4) + 2$

D) $y = f(x - 2) - 4$

Q21 — Hard

Two investments both start at $1,000. Investment A adds $100 each year. Investment B grows by 8% each year. Which statement is true?

A) A is worth more than B in every year

B) B is worth more than A in every year

C) They are equal in every year

D) A leads at first, but B overtakes it eventually

Q22 — Medium

A function is defined by $f(x) = \dfrac{x+3}{x-5}$. For which value of x is the function undefined?

A) -3

B) 5

C) 3

D) -5

Q23 — Hard

A quantity decaying exponentially is measured as 200 at time 0 and as 50 at time 2. The reading taken at time 1 was lost. What is its value at time 1?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q24 — Medium

Two models of the same quantity are given by $f(x) = 4x - 1$ and $g(x) = x + 8$. For which value of x does $f(x) = g(x)$?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Answers and Explanations

Section 1 — Function Notation

Q1 — Basic — Answer: D

Topic: Evaluating a function

Replace every x with -2, using brackets so the signs survive:

$$f(-2) = 3(-2)^2 - 4(-2) + 1$$

Take it one piece at a time:

$$(-2)^2 = 4,\ \text{so}\ 3(4) = 12$$
$$-4(-2) = +8$$
$$f(-2) = 12 + 8 + 1 = 21$$

Two sign points do all the damage here. $(-2)^2$ is $+4$, because a negative times a negative is positive. And -4×-2 is $+8$, for the same reason. A negative input often produces a larger answer than you expect.

Why each wrong option is wrong:

- **A,** -19 — both sign slips at once: $-12 - 8 + 1$.
- **B,** 5 — squared correctly but lost the minus in the middle term, computing $12 - 8 + 1$. Subtracting a negative **adds**.
- **C,** -3 — squared -2 as -4, giving $-12 + 8 + 1$. The bracket matters: $(-2)^2 = 4$, while $-2^2 = -4$.

> **Takeaway:** Substitute inside brackets, then simplify. A negative squared is positive, and subtracting a negative adds.

Q2 — Medium — Answer: A

Topic: Composition of functions

Work from the **inside out**. The inner function is g:

$$g(3) = 3^2 = 9$$

Now put that 9 into f:

$$f(9) = 2(9) + 1 = 19$$

Why each wrong option is wrong:

- **C,** 49 — did f first: $f(3) = 7$, then $g(7) = 49$. That is $g(f(3))$, a different question — and it is offered every single time.
- **B,** 13 — found 2×9 and forgot the $+1$.
- **D,** 36 — multiplied $f(3)$ and $g(3)$ together. Composition is not multiplication.

> **Takeaway:** $f(g(x))$ means g first. Work outward from the innermost bracket, exactly as with ordinary arithmetic.

Q3 — Medium — Answer: 7

Topic: Working backwards from an output

$f(a) = 27$ means the output is 27, so set the rule equal to 27 and solve for the input:

$$5a - 8 = 27 \quad \Longrightarrow \quad 5a = 35 \quad \Longrightarrow \quad a = 7$$

Check: $f(7) = 35 - 8 = 27$ ✓

This is the reverse of Q1. There you were given the input and asked for the output; here you are given the output and asked for the input. Notice which one the notation is telling you: the number **inside** the brackets is the input, the number the function **equals** is the output.

This is a grid-in. Enter 7.

> **Takeaway:** $f(a) = k$ means "set the rule equal to k and solve". The number inside the brackets is always the input.

Section 2 — Reading and Transforming Graphs

Q4 — Medium — Answer: B

Topic: A horizontal shift

The change is **inside** the bracket, so it is a horizontal shift — and inside changes go the opposite way to the sign. $f(x-3)$ moves the graph **right** 3.

Track the point. Let $g(x) = f(x-3)$. We know $f(2) = 5$, so we want the input that makes the bracket equal 2:

$$x - 3 = 2 \quad \Longrightarrow \quad x = 5$$

Then $g(5) = f(5-3) = f(2) = 5$. So g passes through $(5, 5)$.

The y-value does not change at all — a horizontal shift moves points sideways.

Why each wrong option is wrong:

- **A,** $(-1,\ 5)$ — moved left, following the minus sign. Inside the bracket, the sign lies.
- **C,** $(2,\ 8)$ — moved up 3, which is what $f(x) + 3$ would do. Outside the bracket affects the output.
- **D,** $(2,\ 2)$ — moved down 3.

> **Takeaway:** Inside the bracket → horizontal → **opposite** to the sign. Outside → vertical → as the sign says. Track a single known point and the question answers itself.

Q5 — Medium — Answer: A

Topic: Combining transformations

The $+6$ is **outside** the bracket, so it changes the output: every point moves **up** 6. The x-coordinate is untouched.

$$(1, -4) \longrightarrow (1, -4 + 6) = (1, 2)$$

The minimum stays a minimum — shifting a graph up does not change its shape, only its position.

Why each wrong option is wrong:

- **B,** $(7,\ -4)$ and **D,** $(-5,\ -4)$ — moved sideways. A change outside the bracket never moves a graph sideways.
- **C,** $(1,\ -10)$ — subtracted 6 instead of adding. $-4 + 6 = 2$, not -10.

> **Takeaway:** Outside the bracket moves the graph vertically, in the direction the sign says. Add to y, leave x alone.

Q6 — Hard — Answer: D

Topic: A reflection

The minus is **outside** the function, so it acts on the **output**. Every y-value changes sign; the x-values stay where they are.

$$(-3, 7) \longrightarrow (-3, -7)$$

This is a reflection over the x-axis: the graph is flipped upside down.

Compare with $y = f(-x)$, where the minus is applied to the input. That reflects over the y-axis and would send $(-3, 7)$ to $(3, 7)$ — which is option A, and it is offered precisely because the two reflections are easy to swap.

	Flips	**Point $(-3, 7)$ becomes**
$-f(x)$	over the x-axis	$(-3, -7)$
$f(-x)$	over the y-axis	$(3, 7)$

Why each wrong option is wrong:

- **A,** $(3,\ 7)$ — that is $f(-x)$, the other reflection.
- **B,** $(3,\ -7)$ — flipped both coordinates, which is neither transformation.
- **C,** $(7,\ -3)$ — swapped x and y, which is what an inverse does.

> **Takeaway:** The minus outside negates the output; the minus inside negates the input. Ask which one the minus is attached to before deciding which axis it flips over.

Section 3 — Exponential Growth and Decay

Q7 — Medium — Answer: B

Topic: Building an exponential model

Growth by a percentage each period is exponential, so the shape is $y = a \cdot b^h$.

The starting value is $a = 400$. Growing by 15% means each hour the population becomes 100% + 15% = 115% of what it was, so

$$b = 1.15$$

$$P = 400(1.15)^h$$

Check the first hour: $400 \times 1.15 = 460$, which is $400 + 60$, and 60 is 15% of 400 ✓ After a second hour: $460 \times 1.15 = 529$. Note the second hour added 69, not 60 — that is the difference between multiplying and adding, and it is what makes the model exponential.

Why each wrong option is wrong:

- **A,** $400\left(\frac{3}{20}\right)^h$ — used 0.15 as the factor. That would multiply the colony by 0.15 each hour, wiping out 85% of it hourly.
- **C,** $\frac{3h}{20} + 400$ — a **linear** model, adding 0.15 each hour. It also adds a fraction of a bacterium per hour, which should look wrong.
- **D,** $400\left(\frac{17}{20}\right)^h$ — used $1 - 0.15$, which is the decay factor. The colony is growing.

> **Takeaway:** Growth by r means multiply by $1 + r$ each period. Check by computing the first period by hand — it should be the starting value plus exactly r of it.

Q8 — Medium — Answer: C

Topic: Reading a decay factor

The factor is 0.88, which is less than 1, so this is **decay**. But 0.88 is what **remains**, not what is lost:

$$1 - 0.88 = 0.12$$

So the car loses **12%** of its value each year, keeping 88%.

Check with real numbers. After one year:

$$24000 \times 0.88 = 21120$$

The drop is $24000 - 21120 = 2880$, and $\frac{2880}{24000} = 0.12$ ✓

Why each wrong option is wrong:

- **A, The car loses 88% of its value each year** — read the factor as the loss. Losing 88% a year would leave the car worth \$2880 after one year, not \$21120.
- **B, The car loses \$88 each year** — a fixed dollar loss would be a **linear** model. The actual drop is \$2880 in the first year and less in the second, because 12% of a smaller number is smaller.
- **D, The car gains 88% of its value each year** — a factor below 1 always shrinks.

> **Takeaway:** The factor is what **survives**. Subtract it from 1 to get the percentage lost. And a fixed *percentage* change is exponential; a fixed *amount* is linear.

Q9 — Hard — Answer: A

Topic: Telling linear from exponential in a table

Test both possibilities against the table.

Is it linear? Look at the differences: $15 - 5 = 10$, $45 - 15 = 30$, $135 - 45 = 90$. Not constant, so **not linear**.

Is it exponential? Look at the ratios: $\frac{15}{5} = 3$, $\frac{45}{15} = 3$, $\frac{135}{45} = 3$. Constant, so **yes** — it multiplies by 3 each step.

The model is $y = 5 \cdot 3^x$, and the next value is

$$135 \times 3 = 405$$

Why each wrong option is wrong:

- **B** and **D** — call it linear, which the differences rule out immediately.
- **C, Exponential, and** $y = 270$ — recognised the exponential pattern and then doubled instead of tripling.

> **Takeaway:** Check the **differences** first: constant means linear. If they are not, check the **ratios**: constant means exponential. Two divisions settle which kind of model you are looking at.

Q10 — Medium — Answer: 12800

Topic: An exponential model with a period other than 1

Do not multiply by 2 twenty times. Work out **how many doubling periods** fit into 20 years:

$$\frac{20}{5} = 4 \text{ periods}$$

Each period doubles, so four periods multiply by $2^4 = 16$:

$$800 \times 16 = 12800$$

Written as a model, that is $P = 800 \cdot 2^{t/5}$, and the $t/5$ is exactly this "how many periods" count.

Check by stepping: $800 \to 1600$ (5 years) $\to 3200$ (10) $\to 6400$ (15) $\to 12800$ (20) ✓

This is a grid-in. Enter `12800`. Five characters, which is exactly the grid's limit for a positive answer.

> **Takeaway:** When the period is not 1, divide the time by the period length to count the periods. The exponent is a count of periods, never the raw time.

Section 4 — Mixed Practice

Q11 — Basic — Answer: B

Topic: Function notation with a negative input

$$g(-4) = \frac{-4+6}{2} = \frac{2}{2} = 1$$

A got the numerator's sign wrong. **C** ignored the minus on the input. **D** divided the -4 by 2 before adding the 6.

> **Takeaway:** Do everything on the top before dividing. The fraction bar groups the whole numerator.

Q12 — Medium — Answer: 3

Topic: Finding a zero of a function

Set the rule to zero:

$$2^x - 8 = 0 \quad \Longrightarrow \quad 2^x = 8$$

Now ask: 2 to what power gives 8? Since $2^3 = 8$, the answer is $x = 3$.

This is a grid-in. Enter 3.

> **Takeaway:** A "zero" of a function is an input making the output 0. With a power, rewrite both sides with the same base and compare exponents.

Q13 — Medium — Answer: C

Topic: A vertical stretch

The 2 multiplies the **output**, so every y-value doubles and the x-values stay put:

$$(4, 3) \longrightarrow (4, 6)$$

A and **B** changed x, which only happens when the change is inside the bracket. **D** halved it.

> **Takeaway:** A number multiplying $f(x)$ from outside stretches the graph vertically. Outside acts on y; inside acts on x.

Q14 — Hard — Answer: D

Topic: Interpreting a model's parameters together

The factor is 1.05. Subtract 1 to get the rate:

$$1.05 - 1 = 0.05 = 5\%$$

So the investment grows by **5% each year**, starting from $2000.

Check: year one gives $2000 \times 1.05 = 2100$, a gain of $100 — and $100 is 5% of $2000 ✓ Year two gives $2100 \times 1.05 = 2205$, a gain of $105. The gain **grows**, which is what compounding means.

A is that first-year figure misread as fixed — and notice option A's $105 is the *second* year's gain, not the first. **B** read the factor as a percentage directly. **C** read the factor as the starting amount, which is the 2000.

> **Takeaway:** In $a \cdot b^n$, a is the start and $b - 1$ is the rate. An exponential gain is never a fixed amount — that is precisely what separates it from a linear one.

Q15 — Medium — Answer: C

Topic: A function given by a table

Check the **differences** first: $4 - 7 = -3$, $1 - 4 = -3$, $-2 - 1 = -3$. They are constant, so the function is **linear** and the rate is -3.

The starting value is $f(0) = 7$, which is the constant term. So

$$f(x) = -3x + 7$$

Test every row: $f(2) = -6 + 7 = 1$ ✓

Why each wrong option is wrong:

- **A,** $y = 7x - 3$ — swapped the two numbers, using 7 as the rate and -3 as the start. At $x = 0$ it gives -3, not 7.
- **B,** $y = 4 - 3x$ — took $f(1) = 4$ as the starting value. The start is at $x = 0$.
- **D,** $y = 3x + 7$ — right starting value, wrong sign on the rate. The outputs are falling, so the rate is negative.

> **Takeaway:** With a table, take the differences. Constant differences mean linear; the difference **is** the rate, and $f(0)$ is the constant term.

Q16 — Medium — Answer: A

Topic: Reading a graph's key features

An x-intercept is a point where the graph meets the x-axis, and on that axis $y = 0$. So "crosses at $x = -2$" means the **input** -2 gives the **output** 0:

$$f(-2) = 0 \qquad f(5) = 0$$

Why each wrong option is wrong:

- **B,** $f(0) = -2$ **and** $f(0) = 5$ — swaps input and output, and is impossible besides: a function gives each input exactly one output, so $f(0)$ cannot be both -2 and 5.
- **C, The graph passes through** $(0, -2)$ **and** $(0, 5)$ — describes y-intercepts, and for the same reason there can only ever be one of those.

- **D,** $f(-2) = 5$ — invents a relationship between the two intercepts.

> **Takeaway:** x-intercept means output zero: $f(a) = 0$. The number given is the **input**. And a function has at most one y-intercept, ever.

Q17 — Medium — Answer: D

Topic: Which graph is exponential

The test is simple: does the quantity go up by the same **amount** each step, or by the same **percentage**?

- Same amount added → **linear**
- Same percentage, or same multiplying factor → **exponential**

A, B and C all add a fixed amount per unit: $2,000 a year, 5 litres a minute, $2 a kilometre. All linear.

D rises by 3% *of whatever it currently is*, so the amount added grows as the population grows. That is exponential.

Put numbers on it. A population of 40,000 rising 3% a year gains 1,200 in the first year and 1,236 in the second — a bigger gain each time, from the same percentage.

> **Takeaway:** "Per year", "a minute", "a kilometre" attached to a **fixed amount** means linear. A **percentage** each period means exponential.

Q18 — Hard — Answer: B

Topic: Finding a starting value from a later one

Four days of tripling multiplies by $3^4 = 81$. So

$$81 \times \text{start} = 810 \quad \Longrightarrow \quad \text{start} = 10$$

Check forwards: $10 \to 30 \to 90 \to 270 \to 810$ ✓ Four steps.

Why each wrong option is wrong:

- **A,** 270 — divided by 3 once, which undoes one day, not four. Notice 270 is the count after **three** days, so it is a real number from the sequence and the wrong one.
- **C,** $\frac{135}{2}$ — used $3 \times 4 = 12$ rather than 3^4. Repeated multiplication is a power, not a product.
- **D,** 30 — divided by 27, undoing three days.

> **Takeaway:** Working backwards through n periods means dividing by b^n, not by b or by bn. Then step forwards to check.

Q19 — Medium — Answer: C

Topic: A function of a function's value

Inside first:

$$f(3) = 2(3) - 1 = 5$$

Then feed 5 back in:

$$f(5) = 2(5) - 1 = 9$$

A stopped after one application. **D** squared instead of applying f again.

> **Takeaway:** $f(f(x))$ means apply f, then apply it again to the result — not square it.

Q20 — Hard — Answer: A

Topic: A transformation described in words

Two separate changes, and they behave differently.

Right 4 is horizontal, so it goes **inside** the bracket, and inside changes run opposite to the sign. Right means -4: $f(x-4)$.

Down 2 is vertical, so it goes **outside**, and outside changes follow the sign. Down means -2.

$$y = f(x-4) - 2$$

Test it on a graph you know. Take $f(x) = x^2$, whose lowest point is $(0, 0)$. The new graph is $(x - 4)^2 - 2$, whose lowest point is at $x = 4$ with value -2 — right 4 and down 2, exactly as asked ✓

Why each wrong option is wrong:

- **B,** $y = f(x+4) - 2$ — $f(x+4)$ moves the graph **left**.
- **C,** $y = f(x-4) + 2$ — moves up rather than down.
- **D,** $y = f(x-2) - 4$ — swapped the two numbers, moving right 2 and down 4.

> **Takeaway:** Inside the bracket, horizontal and opposite to the sign. Outside, vertical and as the sign reads. Test with $y = x^2$ if you are unsure — its lowest point makes the move obvious.

Q21 — Hard — Answer: D

Topic: Comparing growth over time

Work out a few years of each.

Year	A (adds $100)	B (grows 8%)
0	1000	1000
1	1100	1080
3	1300	≈1260
10	2000	≈2159
30	4000	≈10,063

A is ahead early, because $100 is more than 8% of $1,000. But B's yearly gain **grows** — 8% of a bigger number each time — while A's stays at $100 forever. So B eventually passes A and then pulls away.

This is the general rule, and it is worth knowing: **exponential growth always overtakes linear growth eventually**, no matter how small the percentage or how large the fixed amount. It just may take a while.

> **Takeaway:** Linear gains are fixed; exponential gains grow. Whenever a question compares the two over a long enough period, the exponential one wins in the end.

Q22 — Medium — Answer: B

Topic: The domain of a function

A fraction is undefined only when its **denominator** is zero. Dividing by zero has no meaning; a zero on top is perfectly fine.

$$x - 5 = 0 \quad \Longrightarrow \quad x = 5$$

A used the numerator. At $x = -3$ the function is defined and equals 0 — a real value, not an undefined one.

> **Takeaway:** Undefined means the bottom is zero. Set the denominator to zero and solve; ignore the top entirely.

Q23 — Hard — Answer: 100

Topic: Building an exponential model from two points

Exponential means multiplying by the same factor each step. Two steps take 200 down to 50, so

$$200 \times b^2 = 50 \quad \Longrightarrow \quad b^2 = \tfrac{1}{4} \quad \Longrightarrow \quad b = \tfrac{1}{2}$$

One step from 200 gives

$$200 \times \tfrac{1}{2} = 100$$

Check: $200 \to 100 \to 50$ ✓

There is a neat way to see this. In an exponential sequence, each term is the **geometric mean** of its neighbours — the square root of their product:

$$\sqrt{200 \times 50} = \sqrt{10\,000} = 100$$

Compare that with a *linear* sequence, where the middle term would be the ordinary average, $\frac{200+50}{2} = 125$. The two are different, and which one applies depends entirely on which kind of model you were given.

This is a grid-in. Enter `100`.

> **Takeaway:** In an exponential model the middle of three evenly spaced values is their geometric mean; in a linear one it is their ordinary average. Check which model the question named.

Q24 — Medium — Answer: 3

Topic: Where two functions are equal

"$f(x) = g(x)$" means set the two rules equal:

$$4x - 1 = x + 8 \quad \Longrightarrow \quad 3x = 9 \quad \Longrightarrow \quad x = 3$$

Check: $f(3) = 11$ and $g(3) = 11$ ✓ The two lines cross at $(3, 11)$.

This is the same idea as a system of equations, and the same idea as "where do the graphs meet" — three ways of describing one question. The built-in graphing calculator answers it in seconds: graph both and read the intersection.

This is a grid-in. Enter `3`.

> **Takeaway:** "Where are two functions equal" means set the rules equal to each other. It is also exactly what an intersection point is, so it can be graphed instead.

Chapter 6 — Ratios, Percentages, Data and Probability

This is **Problem-Solving and Data Analysis**, about 15% of the SAT Math section.

It is the most *readable* part of the test and the easiest to lose marks on, because the mathematics is simple and the questions are long. Almost everything here is arithmetic you have known for years, wrapped in a paragraph about a survey, a recipe, or a scatterplot. The work is in reading carefully and keeping track of what each number means.

A calculator is permitted on every question, so none of these are testing whether you can divide. They are testing whether you divided the *right two numbers*.

Two habits pay for themselves throughout this chapter:

1. **Write the units on every number.** "12 dollars per hour" and "12 hours per dollar" are different, and one of them is always an option.
2. **Ask what the answer should look like before you compute.** Bigger or smaller? A percentage or an amount? Roughly what size? A rough expectation catches most wrong answers instantly.

Topics covered: ratio and proportion · unit rates and conversion · percentages, increase and decrease · mean, median and range · standard deviation · scatterplots and lines of best fit · probability and two-way tables · sampling and inference

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Section 1 — Ratio and Proportion

A **ratio** compares quantities. A **proportion** says two ratios are equal:

$$\frac{a}{b} = \frac{c}{d}$$

which you solve by cross-multiplying: $ad = bc$.

The trap is **part versus whole**. If a class has boys and girls in the ratio $3 : 5$, then:

- boys to girls is $\frac{3}{5}$
- boys to **everyone** is $\frac{3}{8}$, because there are $3 + 5 = 8$ parts altogether

Read carefully which one is wanted. Both are always offered.

Setting up a proportion works whenever a relationship stays constant. Keep the same quantity on top of both fractions — miles over hours on the left means miles over hours on the right.

Q1 — Basic

In a bag, the ratio of red to blue marbles is 4 : 7. If there are 44 marbles altogether, how many are red?

A) 4

B) 11

C) 28

D) 16

Q2 — Medium

A biscuit recipe is written for a fixed batch size and uses 3 cups of flour for every 8 biscuits it makes. A baker needs to scale the recipe up for an order of 20 biscuits. How many cups of flour are needed for 20 biscuits?

A) $\frac{160}{3}$

B) $\frac{8}{3}$

C) 15

D) $\frac{15}{2}$

Q3 — Medium

A tap fills a tank at 250 millilitres per second. How many litres does it deliver in 4 minutes? (1 litre = 1000 millilitres)

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Section 2 — Percentages

Every percentage question becomes easy once you turn the percentage into a **multiplier**:

The words	The multiplier
20% of	$\times 0.20$
increase by 20%	$\times 1.20$
decrease by 20%	$\times 0.80$
what percent is a of b	$\frac{a}{b} \times 100$

Three things the test relies on:

- **Increase then decrease by the same percentage does not return you to the start.** Up 10% then down 10% is $\times 1.1 \times 0.9 = \times 0.99$ — a 1% loss overall.
- **Percentage change is always measured against the original**, not the new value.
- **Working backwards** from a final amount means **dividing** by the multiplier, not subtracting the percentage.

Q4 — Basic

A jacket in a shop is priced at $80. At the start of the new season the shop raises the price of every item in that range by 15%. What is the new price of the jacket?

A) 12

B) 68

C) 95

D) 92

Q5 — Medium

A coat is offered in a sale at 20% off its original price, and after the reduction a customer pays $96 for it. What was the original price?

A) 115.2

B) 116

C) 76.8

D) 120

Q6 — Hard

A share price rises by 25% one year and falls by 20% the next. Over the two years, the price has

A) 1

B) $\frac{21}{20}$

C) $\frac{19}{20}$

D) $\frac{9}{20}$

Section 3 — One-Variable Data

Measure	What it is	Careful of
Mean	total ÷ how many	dragged by extreme values
Median	the middle value, once sorted	**sort first**
Mode	the most common value	there can be several, or none
Range	largest − smallest	a spread, not a centre
Standard deviation	how spread out the values are	never negative

Two facts the SAT tests repeatedly:

An outlier moves the mean far more than the median. A single very large value drags the mean up but shifts the median by at most one place. So if a question says a distribution is skewed, or mentions one unusually large value, the median is the more representative measure.

Standard deviation measures spread only. Two data sets can have the same mean and completely different standard deviations. Values bunched close together give a small standard deviation; values spread far apart give a large one. You will never be asked to calculate one — only to compare two sets and say which is larger.

Q7 — Basic

A researcher recorded six readings during a trial and wrote them down in the order they were taken: $7, 3, 11, 5, 9, 3$. What is the median of the data set?

A) $\frac{19}{3}$

B) 6

C) 3

D) 8

Q8 — Medium

A quality inspector's data set is $12, 14, 15, 13, 16$. A sixth reading of 60 is then recorded. Which statement about the effect of that value is true?

A) The mean and median both increase by the same amount

B) The mean increases much more than the median

C) The median increases much more than the mean

D) Neither the mean nor the median changes

Q9 — Medium

Two laboratories each recorded five measurements of the same quantity. Set P is $20, 20, 20, 20, 20$ and set Q is $10, 15, 20, 25, 30$. Which statement is true?

A) P has the greater standard deviation

B) They have equal standard deviations

C) Q has the greater standard deviation

D) Standard deviation cannot be compared without calculating it

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Section 4 — Two-Variable Data and Probability

Scatterplots and lines of best fit. A line of best fit is a linear model drawn through a cloud of points. Its slope is a rate of change in context, and its y-intercept is the predicted value at $x = 0$. Reading a value *inside* the data range is reliable; predicting far outside it is not, and the test sometimes asks you to say so.

Probability on this test is almost always counting:

$$P(\text{event}) = \frac{\text{outcomes you want}}{\text{outcomes altogether}}$$

The work is deciding what "altogether" means. In a two-way table, the denominator is whichever total the question restricts you to — the grand total, a row total, or a column total. Getting that wrong is the mistake the wrong options are built from.

Q10 — Medium

A survey of 200 people recorded whether they cycle and whether they own a car.

	Owns a car	No car	Total
Cycles	30	50	80
Does not cycle	90	30	120
Total	120	80	200

One person is chosen at random from those who cycle. What is the probability that they own a car?

A) $\frac{3}{20}$

B) $\frac{3}{8}$

C) $\frac{1}{4}$

D) $\frac{2}{5}$

Q11 — Medium

The line of best fit for a scatterplot of study hours x against test score y is $y = 8x + 42$. What does the 8 represent?

A) The predicted increase in score for each extra hour studied

B) The predicted score for a student who does not study

C) The number of students in the study

D) The number of hours the average student studied

Q12 — Hard

A researcher surveys 300 randomly selected students at a university and finds that 42% support a proposed change. Which conclusion is best supported?

A) Exactly 42% of all students at the university support the change

B) 42% of all students in the country support the change

C) It is plausible that close to 42% of students at that university support the change

D) No conclusion can be drawn, because only 300 students were asked

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Section 5 — Mixed Practice

Q13 — Basic

A quality report needs one figure expressed as a percentage of another: 40 items out of a batch of 250 were set aside for inspection. What percent of 250 is 40?

A) 16

B) 6.25

C) 210

D) 62.5

Q14 — Medium

An office printer runs at a steady rate, producing 3 pages every 4 seconds without pausing between jobs. The office manager needs the output over a full minute. How many pages does it produce in one minute?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q15 — Medium

A teacher recorded five results and found that their mean was 18. Four of the five values were 12, 20, 15 and 25, and the fifth was lost. What is the fifth number?

A) 13

B) 90

C) 18

D) 72

Q16 — Hard

In a town, 60% of residents own a bicycle. Of those bicycle owners, 25% ride daily. What percent of all residents ride daily?

A) 25

B) 15

C) 85

D) 35

Q17 — Medium

The table shows how 400 people travel to work.

Method	Car	Bus	Train	Cycle
People	180	96	84	40

What percentage travel by bus?

A) 96

B) 24

C) $\frac{25}{6}$

D) 76

Q18 — Medium

A car's fuel use is recorded.

Distance (km)	0	50	100	150
Fuel left (litres)	60	55	50	45

How many litres does the car use per kilometre?

A) 5

B) 10

C) $\frac{2}{5}$

D) $\frac{1}{10}$

Q19 — Medium

A bag has 3 red and 5 blue counters. One counter is drawn and **not** replaced, then a second is drawn. What is the probability that both are red?

A) $\frac{9}{64}$

B) $\frac{3}{4}$

C) $\frac{3}{28}$

D) $\frac{3}{8}$

Q20 — Hard

A class of 20 students averages 68 on a test. A second class of 30 students averages 78. What is the average across both classes?

A) 74

B) 73

C) 146

D) 78

Q21 — Medium

A tap fills a container at a steady rate of 3 litres per minute and is left running unattended. The volume delivered is wanted over a period of 2 hours. How many litres does it deliver in 2 hours?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q22 — Hard

A researcher wants to know the average number of hours students at a large school sleep. She surveys 40 students from the school's basketball team. What is the main problem with this study?

A) The sample is too small to say anything

B) Hours of sleep cannot be averaged

C) She should have surveyed exactly 100 students

D) The sample was not chosen at random from all students

Q23 — Medium

A shop reduces a coat by 20%, then takes a further 10% off the reduced price. What single percentage reduction is this equivalent to?

A) 30

B) 28

C) 72

D) 2

Q24 — Hard

A scatterplot of 12 points has a line of best fit $y = -2x + 30$. Which is the best interpretation of the slope?

A) The 12 points all lie on the line

B) y is always 2 less than x

C) On average, y falls by 2 for each 1 that x rises

D) The largest value of y is 30

Q25 — Medium

Concrete is mixed with cement, sand and stone in the ratio $1 : 2 : 4$. If 12 kg of sand is used, what is the total mass of the mixture, in kilograms?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q26 — Hard

The table shows the number of siblings reported by 21 students.

Siblings	0	1	2	3
Students	4	9	6	2

What is the median number of siblings?

A) 1

B) $\frac{9}{7}$

C) 2

D) $\frac{21}{2}$

Section 6 — Samples, Margins of Error and Claims

The test asks a small number of questions about what a study is entitled to conclude. They involve almost no calculation, and they are among the easiest marks on the paper once you know the three rules.

1. A margin of error gives a range. If a survey estimates 64% with a margin of error of 4%, the plausible values run from $64 - 4 = 60\%$ to $64 + 4 = 68\%$. The estimate sits in the middle; the margin reaches out both ways.

The wording matters: it is **likely** that the true value lies in that range — never certain, and never exactly the estimate.

2. A bigger random sample gives a smaller margin of error. Asking more people does not change the estimate's direction, it narrows the range around it. Nothing else on this test changes the margin.

3. A conclusion may only reach as far as the sampling did. Two separate limits, and questions test both:

What was done	What may be concluded
Randomly sampled from **group X**	something about **group X**, and nothing beyond it
Observed a relationship	that the two things are **associated**
Randomly assigned a treatment	that the treatment **caused** the difference

That last row is the whole of "observational study versus experiment". If nobody assigned anything, no cause may be claimed — however strong the pattern.

Every option that says "proves", "exactly", or "causes" is suspect. The right answers use "likely", "plausible", "suggests" and "approximately".

Q27 — Medium

A poll of randomly selected voters estimates that 47% support a proposal, with an associated margin of error of 3%. Which is the most appropriate conclusion?

A) Exactly 47% of all voters support the proposal

B) It is likely that between 44% and 50% of all voters support the proposal

C) Between 44% and 50% of the voters polled support the proposal

D) At least 50% of all voters support the proposal

Q28 — Medium

A researcher repeats a survey, using the same method but selecting 4,000 people at random instead of 400. Compared with the first survey, the second is most likely to have

A) a larger margin of error

B) the same margin of error

C) a smaller margin of error

D) no margin of error at all

Q29 — Hard

Researchers recorded the sleep and the test scores of 500 randomly selected students, and found that students who slept longer tended to score higher. Which conclusion is best supported?

A) There is an association between hours of sleep and test scores among these students

B) Sleeping longer causes higher test scores

C) Higher test scores cause students to sleep longer

D) No relationship exists between sleep and test scores

Q30 — Medium

A random sample of 200 members of a large gym were asked how often they attend. The results are used to estimate attendance for the whole gym. Which statement best describes the limitation of this study?

A) The sample was too small to support any estimate

B) The estimate applies to every gym in the country

C) Random selection makes the estimate unreliable

D) The estimate applies to that gym's members, not to gym-goers generally

Q31 — Medium

The table shows five data points.

x	1	2	3	4	5
y	12	19	29	41	52

Which is the most appropriate model for the relationship?

A) Increasing and roughly linear

B) Decreasing and roughly linear

C) Constant

D) Increasing and exponential, doubling each step

Q32 — Hard

A line of best fit for a scatterplot of house size x (square metres) against price y (thousands) is $y = 3x + 40$. The data covers houses from 50 to 200 square metres. Which prediction is least reliable?

A) A 60 square metre house costs about 220 thousand

B) A 120 square metre house costs about 400 thousand

C) A 200 square metre house costs about 640 thousand

D) A 900 square metre house costs about 2,740 thousand

Q33 — Medium

A line of best fit is $y = 2x + 5$. For the data point $(4, 15)$, how does the actual value compare with the predicted value?

A) -2

B) 2

C) 13

D) 15

Q34 — Hard

Two hundred volunteers with headaches were **randomly assigned** to receive either a new painkiller or a placebo. Those given the painkiller reported relief significantly more often. Which conclusion is best supported?

A) The painkiller is associated with relief, but no cause can be claimed

B) The painkiller will relieve every headache

C) For people like these volunteers, the painkiller caused more relief than the placebo

D) No conclusion is possible, because only 200 people took part

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Q35 — Hard

A vehicle's speed increases at a rate of 8 metres per second squared. What is this rate in kilometres per second squared?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q36 — Hard

A storage tank has developed a slow leak, losing 3 litres per minute at a steady rate that shows no sign of changing. The maintenance report needs that figure expressed per day. What is the rate in litres per day?

A) 180

B) 72

C) 1440

D) 4320

Section 7 — Comparing and Transforming Data Sets

The test shows two distributions — as dot plots, histograms or lists — and asks you to compare them, or it changes every value in a set and asks what happened. Both are answered by thinking about **centre** and **spread** separately.

Adding the same number to every value slides the whole distribution along. Every measure of *centre* moves by that amount; every measure of *spread* stays exactly the same, because nothing has moved relative to anything else.

Measure	Add k to every value	Multiply every value by k
Mean	increases by k	multiplied by k
Median	increases by k	multiplied by k
Range	**unchanged**	multiplied by k
Standard deviation	**unchanged**	multiplied by k

That "unchanged" is the whole point, and it is worth seeing why: if every value moves 56 to the right, the gaps between them are exactly what they were.

Comparing two spreads by eye. You are never asked to calculate a standard deviation. You are asked which set is more spread out — so look at how tightly the values cluster around the middle. Values bunched near the centre give a small standard deviation; values pushed out to the extremes give a large one.

Q37 — Medium

Data set B is made by adding 56 to each value in data set A. How do the median and the range of B compare with those of A?

A) The median and the range are both 56 greater

B) The median is 56 greater; the range is unchanged

C) The median is unchanged; the range is 56 greater

D) The median and the range are both unchanged

Q38 — Medium

Two classes each recorded 10 values.

Class A: $6, 7, 7, 8, 8, 8, 9, 9, 9, 10$ Class B: $1, 2, 4, 6, 8, 9, 11, 12, 13, 15$

Both have a mean of 8.1. Which statement is true?

A) Class A has the greater standard deviation

B) The standard deviations are equal, because the means are equal

C) Class B has the greater standard deviation

D) The standard deviation cannot be compared without calculating it

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Q39 — Hard

The table groups 23 integers.

Interval	10–19	20–29	30–39	40–49
Frequency	4	7	9	3

Which interval contains the median?

A) 10–19

B) 20–29

C) 30–39

D) 40–49

Q40 — Hard

Every value in a data set is multiplied by 3. What happens to the mean and to the range?

A) Both are multiplied by 3

B) Both are unchanged

C) The mean is multiplied by 3; the range is unchanged

D) The mean is unchanged; the range is multiplied by 3

Answers and Explanations

Section 1 — Ratio and Proportion

Q1 — Basic — Answer: D

Topic: Part-to-whole from a ratio

The ratio has $4 + 7 = 11$ parts in total. So red marbles are $\frac{4}{11}$ of everything:

$$\frac{4}{11} \times 44 = 4 \times 4 = 16$$

Check: red 16, blue $\frac{7}{11} \times 44 = 28$, and $16 + 28 = 44$ ✓ And $16 : 28$ simplifies to $4 : 7$ ✓

The quick way: 11 parts share 44 marbles, so **one part is 4 marbles**. Red is 4 parts, so $4 \times 4 = 16$. Finding the value of one part first works on every ratio question and is worth making automatic.

Why each wrong option is wrong:

- **A,** 4 — the ratio number, not a count of marbles.
- **C,** 28 — the blue marbles.
- **B,** 11 — divided the total by 4 instead of by 11.

> **Takeaway:** Add the ratio parts to get the whole, then find the value of **one part**. Everything else is multiplication after that.

Q2 — Medium — Answer: D

Topic: Setting up a proportion

Keep the same quantity on top of both fractions — cups over biscuits, both sides:

$$\frac{3 \text{ cups}}{8 \text{ biscuits}} = \frac{c}{20 \text{ biscuits}}$$

Cross-multiply:

$$8c = 60 \quad \Longrightarrow \quad c = \frac{60}{8} = \frac{15}{2} = 7.5$$

Check the size before accepting it. 20 biscuits is a bit more than twice 8, so the flour should be a bit more than twice 3 — around 7 or 8. 7.5 fits ✓

Why each wrong option is wrong:

- **A,** $\frac{160}{3}$ — set the fractions up upside down on one side.
- **B,** $\frac{8}{3}$ — the rate of biscuits per cup, which is not what was asked.
- **C,** 15 — doubled 3 to get 6 and then adjusted wrongly, or stopped at 60 divided by 4.

> **Takeaway:** Same units on top on both sides, then cross-multiply. Estimate the answer's size first — it eliminates the inverted-ratio option immediately.

Q3 — Medium — Answer: 60

Topic: Converting units

Two conversions, done one at a time.

First, minutes to seconds, because the rate is per second:

$$4 \text{ minutes} = 4 \times 60 = 240 \text{ seconds}$$

Now the volume in millilitres:

$$240 \times 250 = 60\,000 \text{ ml}$$

Finally millilitres to litres:

$$\frac{60\,000}{1000} = 60 \text{ litres}$$

Check the direction of each conversion by asking whether the number should get bigger or smaller. Seconds are smaller than minutes, so there are **more** of them — multiply. Litres are bigger than millilitres, so there are **fewer** of them — divide. Getting a conversion upside down changes the answer by a factor of 3600 here, so it is never a close call.

This is a grid-in. Enter `60`.

> **Takeaway:** Convert one unit at a time and check each direction by asking whether the count should grow or shrink. Smaller unit, bigger number.

Section 2 — Percentages

Q4 — Basic — Answer: D

Topic: Percentage increase

An increase of 15% means the new price is 115% of the old:

$$80 \times 1.15 = 92$$

Or find the increase and add it: 15% of 80 is 12, and $80 + 12 = 92$ ✓

Why each wrong option is wrong:

- **A,** 12 — the size of the increase, not the new price.
- **B,** 68 — decreased by 15%.
- **C,** 95 — added $15. A percentage of 80 is not 15.

> **Takeaway:** "Increase by $r\%$" means multiply by $1 + \frac{r}{100}$. One multiplication, no addition step to forget.

Q5 — Medium — Answer: D

Topic: Working backwards from a percentage

The discount means the customer paid 80% of the original:

$$0.80 \times \text{original} = 96$$

To undo a multiplication, **divide**:

$$\text{original} = \frac{96}{0.80} = 120$$

Check: 20% of 120 is 24, and $120 - 24 = 96$ ✓

The tempting move is to add 20% to $96, and it does not work. 20% of 96 is 19.20, not 24, because the discount was 20% of the **larger** original price. That is why working backwards requires division, not a second percentage.

Why each wrong option is wrong:

- **A,** 115.2 — added 20% to the sale price, which is the mistake just described.
- **B,** 116 — added $20.
- **C,** 76.8 — took another 20% off.

> **Takeaway:** To undo a percentage change, **divide** by the multiplier. Adding the same percentage back always undershoots, because it is a percentage of a smaller number.

Q6 — Hard — Answer: A

Topic: Successive percentage changes

The options give the overall multiplier: 1 means the price is unchanged.

Explanation

Multiply the two multipliers together. Do **not** add the percentages.

$$1.25 \times 0.80 = 1.00$$

The price ends exactly where it started.

Follow \$100 through to see why. Up 25%: $100 \to 125$. Down 20%: 20% of 125 is 25, so $125 \to 100$. The same percentage applied to a bigger number is a bigger amount, which is exactly what makes the two changes cancel here.

Why each wrong option is wrong:

- **B,** $\frac{21}{20}$ — added the percentages, $+25 - 20 = +5$. Percentage changes never add.
- **C,** $\frac{19}{20}$ — subtracted in the other direction.
- **D,** $\frac{9}{20}$ — treated the two as a combined 55% loss.

> **Takeaway:** Successive percentage changes **multiply**. Test with \$100 — it takes ten seconds and settles the question completely.

Section 3 — One-Variable Data

Q7 — Basic — Answer: B

Topic: Median of a data set

Sort the values first — this is the step people skip:

$$3,\ 3,\ 5,\ 7,\ 9,\ 11$$

There are six values, an even number, so there is no single middle one. The median is the **average of the middle two**, which are 5 and 7:

$$\frac{5+7}{2}=6$$

Why each wrong option is wrong:

- **A,** $\frac{19}{3}$ — the mean, $\frac{38}{6}=\frac{19}{3}$.
- **C,** 3 — the mode, the most frequent value.
- **D,** 8 — averaged the two middle values of the **unsorted** list.

> **Takeaway:** Sort, then find the middle. Even count → average the middle two. And check which measure the question named; all four are offered.

Q8 — Medium — Answer: B

Topic: The effect of an outlier

Work out both, before and after.

Before: the values sorted are $12, 13, 14, 15, 16$. The mean is $\frac{70}{5}=14$ and the median is the middle value, 14.

After adding 60: the values are $12, 13, 14, 15, 16, 60$. The mean is

$$\frac{130}{6}\approx 21.7$$

and the median is the average of the middle two, $\frac{14+15}{2}=14.5$.

So the mean jumped by about 7.7 while the median moved by 0.5.

The reason is structural. The mean uses the **size** of every value, so one huge number pulls hard. The median only uses **position**, so an extra value at the top shifts the middle by half a place regardless of how extreme it is. Replace the 60 with 6000 and the median would still be 14.5.

Why each wrong option is wrong:

- **A** — they change by very different amounts.
- **C** — backwards. The median is the *resistant* measure.
- **D** — both change; the median just changes very little.

> **Takeaway:** Outliers drag the mean and barely move the median. When a question mentions one extreme value, or calls a distribution skewed, the median is the measure that represents it fairly.

Q9 — Medium — Answer: C

Topic: Comparing standard deviations

Both sets have a mean of 20. But standard deviation is not about the centre — it is about how far the values sit **from** the centre.

Every value in P *is* 20, so nothing deviates at all. P's standard deviation is **zero** — the smallest it can ever be.

Q's values sit 10, 5, 0, 5 and 10 away from the mean. There is real spread, so its standard deviation is clearly greater.

You never have to compute one on this test. Look at how tightly the values cluster: bunched means small, spread out means large.

A has it backwards. **B** ignores the spread entirely. **D** is wrong because comparison by inspection is exactly what the test wants here.

> **Takeaway:** Standard deviation measures spread, not centre. Identical values give zero. Two sets can share a mean and differ completely in standard deviation.

Section 4 — Two-Variable Data and Probability

Q10 — Medium — Answer: B

Topic: Probability from a two-way table

Read the restriction: **"from those who cycle"**. That fixes the pool. You are no longer choosing from all 200 people — only from the 80 who cycle.

Of those 80, the number who own a car is 30:

$$P = \frac{30}{80} = \frac{3}{8}$$

The phrase that sets the denominator is worth hunting for in every one of these questions. "Of those who cycle", "among car owners", "given that" — each one narrows the pool to a single row or column.

Why each wrong option is wrong:

- **A,** $\frac{3}{20}$ — used the grand total, 200. That answers "what is the probability a randomly chosen **person** both cycles and owns a car".

- **C,** $\frac{1}{4}$ — used 120, the car-owner total. That answers the reversed question: given they own a car, do they cycle?
- **D,** $\frac{2}{5}$ — the probability of being a cyclist at all.

> **Takeaway:** Find the phrase that restricts the group; its total is your denominator. "Of those who X" means the X row, not the whole table.

Q11 — Medium — Answer: A

Topic: Reading a line of best fit

The 8 multiplies x, so it is the **slope** — the change in y for each 1 added to x. In context: each extra hour of study predicts 8 more points.

Check: $f(3) = 66$ and $f(4) = 74$, a rise of 8 ✓

B describes the 42, the value at $x = 0$. **C** and **D** are quantities the model says nothing about — a line of best fit describes a relationship, not the size of the sample.

Note the word **predicted**. A line of best fit does not promise that an extra hour gives exactly 8 more marks; it describes the trend across the data.

> **Takeaway:** In $y = mx + b$ fitted to data, m is "per one more x" and b is "at $x = 0$". Check the units: a score per hour cannot be a number of students.

Q12 — Hard — Answer: C

Topic: Inference from a sample

Two rules decide every question of this kind.

Who was sampled? The survey drew from students **at that university**. A random sample supports conclusions about the population it was drawn from, and no further. Nothing here says anything about students elsewhere.

How precise can it be? A sample gives an *estimate*, not an exact figure. The right language is "approximately", "it is plausible that", "close to" — never "exactly".

Option C respects both. It stays inside the university and it hedges the number.

Why each wrong option is wrong:

- **A** — says **exactly**. A sample of 300 cannot pin down the whole university's opinion to the decimal.
- **B** — generalises beyond the population sampled. Students at one university are not a random sample of the country.
- **D** — too pessimistic. 300 randomly selected people support a reasonable estimate; that is the entire purpose of random sampling.

> **Takeaway:** A random sample supports an *estimate* about the population it came from. Reject options that say "exactly", and reject options that reach beyond the group actually sampled.

Section 5 — Mixed Practice

Q13 — Basic — Answer: A

Topic: Finding a percentage

"What percent is a of b" means $\frac{a}{b} \times 100$, with the **part on top**:

$$\frac{40}{250} \times 100 = 16$$

Check the size: 40 is well under half of 250, so the answer must be well under 50% ✓

B divided the wrong way round. **C** subtracted. **D** is the inverted division scaled up.

> **Takeaway:** Part over whole, then times 100. If the answer comes out above 100% when the part is smaller than the whole, the division is upside down.

Q14 — Medium — Answer: 45

Topic: A unit rate

Find the rate per second first:

$$\frac{3 \text{ pages}}{4 \text{ seconds}} = 0.75 \text{ pages per second}$$

Then a minute is 60 seconds:

$$0.75 \times 60 = 45$$

Check by scaling instead: 60 seconds is 15 lots of 4 seconds, and each lot gives 3 pages, so $15 \times 3 = 45$ ✓ Two routes, same answer.

This is a grid-in. Enter 45.

> **Takeaway:** Reduce to a rate per one unit, then scale up. Or find how many whole blocks of the given time fit — often cleaner, and it avoids decimals.

Q15 — Medium — Answer: C

Topic: Mean from a total

Turn the mean into a **total**. Five numbers with mean 18 must add to

$$5 \times 18 = 90$$

The four given numbers add to $12 + 20 + 15 + 25 = 72$. So the fifth is

$$90 - 72 = 18$$

Check: $\frac{12+20+15+25+18}{5} = \frac{90}{5} = 18$ ✓

That the answer happens to equal the mean is a coincidence of these numbers, not a rule.

A subtracted wrongly. **B** is the total of all five. **D** is the total of the four.

> **Takeaway:** Mean questions become easy the moment you convert to a total: total $=$ mean $\times$ count.

Q16 — Hard — Answer: B

Topic: A percentage of a percentage

The 25% applies only to bicycle owners, not to everyone. So take a percentage of a percentage — which means **multiply**:

$$0.60 \times 0.25 = 0.15 = 15\%$$

Follow 1000 residents through to see it. 60% own bicycles, so 600 people. A quarter of those ride daily: $0.25 \times 600 = 150$. And 150 out of 1000 is **15%** ✓

A reported the 25% as though it applied to everyone. **C** added the percentages, **D** subtracted them — neither operation means anything here.

> **Takeaway:** "Of those..." signals a percentage of a subgroup, so multiply the two. Testing with a round population of 100 or 1000 makes it concrete in seconds.

Q17 — Medium — Answer: B

Topic: A percentage of a total from a table

$$\frac{96}{400} \times 100 = 24$$

Check the size: 96 is a bit under a quarter of 400 ✓ And a quick sanity check on the whole table — $180 + 96 + 84 + 40 = 400$, so nothing is missing.

A gave the raw count. **D** answered the opposite question.

> **Takeaway:** Part over whole, times 100. Add the table up first; if it does not reach the stated total, you have misread a row.

Q18 — Medium — Answer: D

Topic: Reading a rate from a table

Every 50 km uses 5 litres, so per kilometre:

$$\frac{5 \text{ litres}}{50 \text{ km}} = 0.1 \text{ litres per km}$$

Check the units the question asked for — **litres per kilometre**. Option B, 10, is kilometres per litre: the same relationship the other way up, and a perfectly sensible number that answers a different question.

A gave the fuel per 50 km without dividing.

> **Takeaway:** Write the units on your answer and compare them with the units the question named. "Per" tells you which quantity goes on the bottom.

Q19 — Medium — Answer: C

Topic: Probability of "at least one"

The first draw: 3 red out of 8, so $\frac{3}{8}$.

The counter is **not replaced**, so for the second draw there are only 2 reds left out of 7 counters:

$$\frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$$

The words "not replaced" are doing all the work. **A** ignored them and used $\frac{3}{8}$ twice, which would be right *with* replacement.

> **Takeaway:** "And then" means multiply. Check whether the first item goes back: without replacement, **both** the top and the bottom of the second fraction drop.

Q20 — Hard — Answer: A

Topic: A weighted average

You cannot average two averages unless the groups are the same size — and here they are not. Go back to **totals**.

$$20 \times 68 = 1360 \qquad 30 \times 78 = 2340$$
$$\text{combined average} = \frac{1360 + 2340}{50} = \frac{3700}{50} = 74$$

The plain average of 68 and 78 would be 73. The true answer is **pulled towards 78**, because more students sat in the class that scored 78. That pull is the whole point of the question.

> **Takeaway:** Averages combine through totals, never by averaging the averages. The answer always leans towards the larger group.

Q21 — Medium — Answer: 360

Topic: Converting a rate between units

Convert to the unit the rate uses. Two hours is $2 \times 60 = 120$ minutes, so

$$3 \times 120 = 360 \text{ litres}$$

Check the direction: minutes are smaller than hours, so there are **more** of them — multiply. Enter `360`.

> **Takeaway:** Convert time into the rate's own unit first. Smaller unit, bigger number.

Q22 — Hard — Answer: D

Topic: Evaluating a statistical claim

The problem is **who** was asked, not how many. A basketball team is not a random slice of the school: its members train, may keep different hours, and are selected for something. Conclusions from it apply to the team, not the school.

Forty people chosen **at random** from the whole school would support a reasonable estimate. Forty chosen from one team does not, however carefully the average is worked out.

A blames the size, which is not the fault here. **C** invents a magic number; there is no required sample size.

> **Takeaway:** A sample supports conclusions about the group it was **randomly drawn from**. Ask who was left out before you ask how many were asked.

Q23 — Medium — Answer: B

Topic: Percentage of a percentage

Multiply the multipliers:

$$0.80 \times 0.90 = 0.72$$

So 72% of the price remains, and the reduction is

$$100 - 72 = 28\%$$

Follow \$100: down 20% to \$80, then 10% off \$80 is \$8, leaving \$72. A \$28 reduction ✓

A added the percentages, which is the mistake the question exists for — the second 10% is taken off a *smaller* price, so it is worth less than 10% of the original. **C** gave what remains rather than what was taken off.

> **Takeaway:** Successive percentage changes multiply. Then read whether the question wants the amount remaining or the amount removed.

Q24 — Hard — Answer: C

Topic: Reading a scatterplot's fit

A slope is a rate: for each 1 added to x, the fitted y changes by -2. Because this is a line of **best fit** and not an exact rule, the honest wording is "on average".

A overstates it — a line of best fit passes near the points, not through them; if it did, it would not need to be a line of *best fit*. **B** describes $y = x - 2$, a different relationship entirely. **D** confuses the intercept with a maximum: 30 is the fitted value at $x = 0$, and the line goes above 30 for negative x.

> **Takeaway:** For a line of best fit, the slope is an average rate of change and the intercept is the fitted value at zero. Neither is a promise about any individual data point.

Q25 — Medium — Answer: 42

Topic: Ratio with a total to find

Sand is 2 parts and weighs 12 kg, so **one part is 6 kg**.

The whole mixture is $1 + 2 + 4 = 7$ parts:

$$7 \times 6 = 42 \text{ kg}$$

Check: cement 6, sand 12, stone 24, total 42 ✓ and $6 : 12 : 24$ simplifies to $1 : 2 : 4$ ✓ Enter 42.

> **Takeaway:** Find the value of one part first. Every ratio question becomes multiplication after that.

Q26 — Hard — Answer: A

Topic: Median from a frequency table

There are 21 students, so the median is the **11th** value when they are lined up in order.

Count along: the first 4 students report 0 siblings (positions 1–4), the next 9 report 1 (positions 5–13). Position 11 falls inside that block, so the median is **1**.

B is the mean, $\frac{27}{21}$. **C** took the middle *column* of the table rather than the middle *student* — the table's columns are values, not people. **D** halved the number of students, which finds the position, not the value at it.

> **Takeaway:** With a frequency table, find which **position** is the middle, then count along the frequencies to see which value sits there. The middle column is not the median.

Section 6 — Samples, Margins of Error and Claims

Q27 — Medium — Answer: B

Topic: Reading a margin of error

The margin of error reaches out both ways from the estimate:

$$47 - 3 = 44 \qquad 47 + 3 = 50$$

So the plausible range is 44% to 50%, and the conclusion is about **all voters**, not just the ones polled — that is the whole purpose of taking a random sample.

Why each wrong option is wrong:

- **A** — says **exactly**. A sample never pins down a population precisely; that is why a margin of error is quoted at all.
- **C** — applies the range to the people polled. Their support is a measured fact, not an estimate; the uncertainty is about everyone else.
- **D** — 50% is the extreme top of the range, not a floor. The poll is at least as consistent with 44%.

> **Takeaway:** Estimate ± margin gives the range, and the claim is about the whole population. Reject any option saying "exactly" — and check whether a boundary value is being quoted as a minimum.

Q28 — Medium — Answer: C

Topic: What makes a margin of error smaller

A larger random sample gives a better estimate, and "better" here means a **narrower** range around it — a smaller margin of error.

A has it backwards. **B** ignores the sample size, which is the one thing that changed. **D** overreaches: only surveying every single person would remove the uncertainty, and even a sample of 4,000 leaves some.

Note what does **not** change the margin: the estimate itself, the wording of the question, or how the results are presented. On this test, sample size is the lever.

> **Takeaway:** Bigger random sample, smaller margin of error. Never zero unless everyone was asked.

Q29 — Hard — Answer: A

Topic: Observational study versus experiment

Nobody **assigned** anyone an amount of sleep. The researchers recorded what students already did, which makes this an **observational study**, and an observational study can establish that two things go together — an **association** — but not that one causes the other.

Both **B** and **C** claim a direction of cause, and the study cannot separate them. Nor can it rule out some third thing affecting both: a student who is ill, or working long hours, might sleep less *and* score lower for reasons that have nothing to do with sleep itself.

D contradicts the finding. A relationship was observed; it just cannot be called causal.

To support a causal claim, the researchers would have had to **randomly assign** students to different amounts of sleep. That is what makes something an experiment rather than an observation, and it is the only design on this test that licenses the word "cause".

> **Takeaway:** Observed → association. Randomly assigned → cause. If nobody assigned anything, reject every option containing "causes".

Q30 — Medium — Answer: D

Topic: How far a conclusion may be generalised

The sample was drawn at random **from one gym's members**, so it supports conclusions about that gym's members and no further.

A blames the size. Two hundred people chosen at random support a perfectly reasonable estimate — the number is not the problem here.

B is the error the question is testing: stretching the conclusion past the population that was sampled.

C has it backwards. Random selection is what makes the estimate trustworthy; a sample of whoever happened to be there on a Monday morning would not be.

> **Takeaway:** A random sample supports conclusions about the group it was drawn from. Ask "who could have been chosen?" — that group, and only that group, is what the estimate describes.

Q31 — Medium — Answer: A

Topic: Choosing a model for two-variable data

Look at the differences between consecutive y values:

$$7, \quad 10, \quad 12, \quad 11$$

They rise, and they are roughly the same size each time — so the relationship increases at a roughly constant rate. That is **linear**, allowing for the scatter real data always has.

B has the direction wrong; y climbs throughout. **C** would need the values to stay put. **D** would mean multiplying by 2 each step: $12, 24, 48, 96, 192$. The actual value at $x = 5$ is 52, nowhere near 192.

Remember the distinction from Chapter 5: constant **differences** mean linear, constant **ratios** mean exponential. Here $\frac{19}{12} \approx 1.6$ but $\frac{52}{41} \approx 1.3$, so the ratios are not constant.

> **Takeaway:** Differences roughly constant → linear. Ratios roughly constant → exponential. Real data will never be exact; look for which is *closer* to constant.

Q32 — Hard — Answer: D

Topic: Predicting inside and outside the data

Every one of these four numbers is what the model gives — check them and they all come out right. So the arithmetic is not what separates them.

What separates them is **where the prediction sits**. The data covers houses from 50 to 200 square metres:

- A, B and C ask about 60, 120 and 200 — all inside that range, or at its edge. Predicting inside the data is what a line of best fit is for.

- D asks about **900** square metres, four and a half times the largest house ever measured. Nothing in the data says the relationship continues out there, and in reality it almost certainly does not.

Predicting beyond the data is called **extrapolation**, and it is the least reliable thing you can do with a model. The test asks about it directly.

> **Takeaway:** A model is trustworthy inside the range of the data it was built from. A prediction far outside that range is the least reliable one, even when the arithmetic is perfect.

Q33 — Medium — Answer: B

Topic: Comparing actual values with predicted ones

The model predicts

$$y = 2(4) + 5 = 13$$

The actual value is 15, so the actual is **2 above** the prediction:

$$15 - 13 = 2$$

A point sitting above the line of best fit means the model **under**-predicted it. Below the line means it over-predicted.

A subtracted predicted minus actual, which reverses the sign and so reverses the meaning. **C** and **D** each give one of the two values rather than the gap between them.

> **Takeaway:** Substitute the x into the model to get the prediction, then subtract it from the actual value. Above the line is positive; below is negative.

Q34 — Hard — Answer: C

Topic: An experiment, where a causal claim is allowed

This is the other side of Q29, and the difference is one word: **assigned**.

Because the researchers randomly assigned who got the painkiller and who got the placebo, the two groups were alike in every other respect on average. So when one group did better, the treatment is what explains it. That is an **experiment**, and an experiment does license a causal claim.

Why the other options fail:

- **A** would be right for an *observational* study — if the researchers had merely recorded which volunteers happened to take a painkiller. They did not; they assigned it.

- **B** overreaches in the other direction. "Significantly more often" is not "every time"; the study compares rates, not guarantees.
- **D** dismisses a perfectly workable study. Two hundred randomly assigned participants support a conclusion.

Note the careful wording of the correct option: **"for people like these volunteers"**. The volunteers were not a random sample of everybody, so the causal claim holds for the kind of people who took part. Random *assignment* licenses cause; random *selection* licenses generalising to a wider population. This study had the first and not the second.

> **Takeaway:** Random assignment → cause may be claimed. Random selection → the result may be generalised to the population sampled. They are different things, and a question will give you one, the other, or neither.

Q35 — Hard — Answer: .008 or 0.008

Topic: A conversion with a squared unit

The unit is metres **per second squared** — metres divided by (seconds × seconds). Only the *metres* are being changed; the seconds are the same on both sides, so they are left alone.

$$\frac{8\text{ m}}{\text{s}^2} \times \frac{1\text{ km}}{1000\text{ m}} = \frac{8}{1000} = 0.008\text{ km/s}^2$$

Check the direction: a kilometre is bigger than a metre, so the same rate is a **smaller number** of kilometres ✓

The "squared" is what makes this look harder than it is. It attaches to the seconds, and the seconds are not being converted — so it never enters the arithmetic at all. Had the question asked for metres per **minute** squared, the squared would matter and you would multiply by $60^2 = 3600$, not by 60.

This is a grid-in. Enter `.008` or `0.008`. Both fit the five-character limit; `0.008` uses exactly five.

> **Takeaway:** In a compound unit, convert one part at a time and leave the rest alone. A squared unit is squared only when *that* unit is the one being changed.

Q36 — Hard — Answer: D

Topic: A two-step conversion

Two conversions, one at a time.

Minutes to hours:

$$3 \times 60 = 180 \text{ litres per hour}$$

Hours to a day:

$$180 \times 24 = 4320 \text{ litres per day}$$

Check the direction at each step: a day is much longer than a minute, so the number must get much **bigger** ✓

A stopped after one step, **B** used 24 without the 60, and **C** worked out how many minutes are in a day and forgot to multiply by the leak rate — a number that is correct about time and says nothing about litres.

> **Takeaway:** Chain conversions one unit at a time, checking after each step whether the number should grow or shrink. Most wrong answers here are correct partial results.

Section 7 — Comparing and Transforming Data Sets

Q37 — Medium — Answer: B

Topic: Adding a constant to every value

Adding the same amount to every value **slides** the whole distribution. The middle value slides with it, so the median rises by 56.

The range is largest minus smallest. Both ends move up by 56, so the difference between them does not change at all:

$$(\text{largest} + 56) - (\text{smallest} + 56) = \text{largest} - \text{smallest}$$

Try it on five values: $3, 5, 8, 9, 14$ has median 8 and range 11. Adding 56 gives $59, 61, 64, 65, 70$ — median 64, which is $8 + 56$ ✓ and range $70 - 59 = 11$, unchanged ✓

The same argument covers the standard deviation, which is also a measure of spread: sliding everything changes no gap between any two values.

> **Takeaway:** Adding a constant shifts every measure of **centre** and leaves every measure of **spread** alone. Multiplying by a constant scales both.

Q38 — Medium — Answer: C

Topic: Comparing the spread of two distributions

Same mean, different spread — which is exactly what standard deviation is for.

Class A's values run from 6 to 10, a span of 4, and most sit at 8 or 9. Class B runs from 1 to 15, a span of 14, with values pushed out to both extremes.

B is far more spread out, so B has the greater standard deviation.

B's option confuses centre with spread: equal means say nothing about how tightly the values cluster. **D** is wrong because comparing by inspection is precisely what the test expects — you are never asked to compute one.

> **Takeaway:** Look at how tightly the values cluster around the middle. Equal means with different clustering is the standard setup for this question.

Q39 — Hard — Answer: C

Topic: The median from a grouped frequency table

With 23 values, the median is the **12th** when they are lined up in order.

Count along the intervals, keeping a running total:

Interval	Frequency	Running total
10–19	4	4
20–29	7	**11**
30–39	9	20
40–49	3	23

After two intervals you have reached only the 11th value, so the 12th is in the next one: **30–39**.

B is the trap: 20–29 is the interval where the count is closest to halfway, but it stops one short. Run the total to 12, not to "about half".

> **Takeaway:** Find which **position** is the middle, then run the frequencies until you pass it. The interval you are in when you pass it holds the median.

Q40 — Hard — Answer: A

Topic: Multiplying every value in a data set

Multiplying stretches the distribution rather than sliding it, so **both** the centre and the spread scale.

Try $2, 4, 5, 9$: the mean is 5 and the range is 7. Tripling gives $6, 12, 15, 27$ — mean 15, which is 3×5 ✓ and range $27 - 6 = 21$, which is 3×7 ✓

Contrast this with Q37. **Adding** slides everything and leaves the gaps alone, so spread is unchanged. **Multiplying** stretches the gaps too, so spread scales with everything else. Knowing which operation was applied decides the answer.

> **Takeaway:** Add → centre moves, spread unchanged. Multiply → both scale. The two questions look alike and have different answers.

Chapter 7 — Geometry and Trigonometry

About **15%** of the SAT Math section — the smallest of the four areas, which surprises most people. It is worth saying plainly: **geometry is not where most of the marks are.** If your study time is limited, Chapters 1 to 5 pay better.

The good news is that the reference sheet carries the area and volume formulas, so nothing here needs memorising except the things it leaves out. Those are:

- **SOHCAHTOA** and everything trigonometric
- The equation of a **circle**
- The **distance** and **midpoint** formulas
- Angle facts — parallel lines, triangles, the lot

The directions page also gives you a fact worth using: **figures are drawn to scale unless the question says otherwise.** So when a figure is provided, you can estimate. If an angle looks like about 60°, an option offering 120° is probably wrong. That is not a substitute for working, but it eliminates options fast.

Topics covered: angles and parallel lines · triangles and similarity · right triangles and Pythagoras · special right triangles · SOHCAHTOA · circles, arcs and sectors · the circle equation · area, volume and scaling

Section 1 — Angles and Triangles

The facts everything else is built on:

Fact	
Angles on a straight line	add to 180°
Angles around a point	add to 360°
Vertically opposite angles	are equal
Angles in a triangle	add to 180°
Angles in a quadrilateral	add to 360°
An exterior angle of a triangle	equals the two opposite interior angles added

When a line crosses two **parallel** lines:

- **Corresponding** angles (same position at each crossing) are equal
- **Alternate** angles (the Z shape) are equal
- **Co-interior** angles (the C shape) add to 180°

For triangles specifically:

- **Isosceles**: two equal sides, and the angles opposite them are equal too.
- **Equilateral**: all sides equal, every angle 60°.
- The **largest angle** is opposite the **longest side**, always.

Q1 — Basic

A surveyor measures two of the three angles of a triangular plot of land and records them as 47° and 68°. What is the measure of the third angle?

A) 115

B) 245

C) 25

D) 65

Q2 — Medium

A roof truss is built as triangle ABC with $AB = AC$, making it isosceles, and the angle at the apex A measures 40°. What is the measure of angle B?

A) 40

B) 100

C) 140

D) 70

Q3 — Medium

Lines ℓ and m are parallel and are crossed by a third line. One of the angles formed measures 118°. Which of the following must be the measure of the co-interior (same-side interior) angle?

A) 118

B) 62

C) 32

D) 242

Section 2 — Right Triangles

Pythagoras is on the reference sheet:

$$c^2 = a^2 + b^2$$

where c is the **hypotenuse** — the side opposite the right angle, always the longest. Putting a shorter side in c's place is the standard error.

Two triangles appear so often they are worth recognising instantly. Both are on the reference sheet, but recognising them saves the algebra:

Triangle	Sides in the ratio
45°–45°–90°	$s : s : s\sqrt{2}$
30°–60°–90°	$x : x\sqrt{3} : 2x$

For the 30–60–90: the shortest side faces 30°, the $x\sqrt{3}$ faces 60°, and the hypotenuse — twice the shortest — faces the right angle.

Two whole-number triples worth memorising because the test reuses them: **3–4–5** and **5–12–13**, along with their multiples (6–8–10, 9–12–15).

Q4 — Basic

A rectangular gate is braced by a diagonal strut, forming a right triangle whose two legs measure 9 and 12 units. What is the length of the hypotenuse?

A) 21

B) $3\sqrt{7}$

C) 15

D) $\frac{21}{2}$

Q5 — Medium

A support bracket is cut as a 30°–60°–90° triangle. The side opposite the 30° angle has length 7. What is the length of the hypotenuse?

A) $7\sqrt{3}$

B) 14

C) $\frac{7}{2}$

D) $7\sqrt{2}$

Q6 — Medium

A ramp is modelled by right triangle ABC with the right angle at C. The sloping edge AB measures 13 units and the side BC, which is opposite angle A, measures 5. What is the value of $\sin A$?

A) $\frac{5}{13}$

B) $\frac{12}{13}$

C) $\frac{5}{12}$

D) $\frac{13}{5}$

Section 3 — Circles

On the reference sheet:

$$A = \pi r^2 \qquad C = 2\pi r$$

Not on it, and needed:

$$\textbf{Circle equation: } (x-h)^2 + (y-k)^2 = r^2$$

with centre (h, k) and radius r. Two things to watch: the **signs flip** — $(x-3)^2$ means the centre is at $x = 3$ — and the right-hand side is r^2, not r. If it says 25, the radius is 5.

Arcs and sectors are just fractions of the whole circle. A slice with a central angle of θ degrees takes $\frac{\theta}{360}$ of everything:

$$\text{arc length} = \frac{\theta}{360} \times 2\pi r \qquad \text{sector area} = \frac{\theta}{360} \times \pi r^2$$

You do not need to memorise those two — work out the fraction and multiply.

Q7 — Medium

A radio transmitter's coverage area is described in the xy-plane by $(x-4)^2 + (y+3)^2 = 25$. What are its centre and radius?

A) Centre $(-4, 3)$, radius 25

B) Centre $(4, -3)$, radius 5

C) Centre $(4, -3)$, radius 25

D) Centre $(-4, 3)$, radius 5

Q8 — Medium

A circular flower bed of radius 6 metres is to be divided into wedge-shaped planting sections. One section is a sector with a central angle of 60° measured at the centre of the bed. What is the area of that sector?

A) 36π

B) 2π

C) 6π

D) 12π

Q9 — Hard

A circular boundary is drawn with its centre at $(2, 5)$, and it is known to pass through the point $(2, 9)$ on its edge. The radius is the distance between those two points. What is the radius?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Section 4 — Area, Volume and Scaling

The area and volume formulas are all on the reference sheet, so the questions are never about recall. They are about **which** formula, and about what happens when a shape is scaled.

Scaling is the part worth learning, because it is not obvious:

> If every length is multiplied by k, then **area** is multiplied by k^2 and **volume** by k^3.

Double every side of a cube and its surface area quadruples while its volume goes up **eight** times. This is the single most tested idea in this section.

Q10 — Medium

A cylindrical water tank has a base of radius 3 metres and stands 10 metres tall. What is its volume?

A) 90π

B) 30π

C) 60π

D) 900π

Q11 — Hard

A spherical balloon is inflated until its radius is three times what it was before. By what factor does its volume increase?

A) 3

B) 9

C) 6

D) 27

Section 5 — Mixed Practice

Q12 — Basic

Three angles meet at a single point on a straight line. Two of them measure 52° and 71°. What is the third?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q13 — Medium

Two triangles are similar. The sides of the smaller are 4, 6 and 8. The longest side of the larger is 20. What is the perimeter of the larger triangle?

A) 18

B) 45

C) 30

D) $\frac{45}{2}$

Q14 — Medium

A triangular sail has a base measuring 14 units along its foot and a perpendicular height of 9 units from that base to the opposite corner. What is its area?

A) 126

B) 23

C) 63

D) $\frac{63}{2}$

Q15 — Hard

A ladder leans against a wall, making a 65° angle with the ground. The foot of the ladder is 2.5 metres from the wall. Which expression gives the length of the ladder in metres?

A) $\dfrac{2.5}{\cos 65^\circ}$

B) $2.5\cos 65^\circ$

C) $\dfrac{2.5}{\sin 65^\circ}$

D) $2.5\tan 65^\circ$

Q16 — Medium

A circular pond is known only by the distance around its edge, which measures 16π units. What is the area of the circle?

A) 16π

B) 256π

C) 32π

D) 64π

Q17 — Medium

In a triangle, the two interior angles not adjacent to an exterior angle measure 52° and 61°. What is the exterior angle?

A) 67

B) 180

C) 113

D) $\frac{113}{2}$

Q18 — Medium

A quadrilateral panel has three of its interior angles measured on site as 95°, 78° and 114°. What is the fourth?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q19 — Medium

A rectangle 12 cm by 5 cm has a semicircle attached along one 12 cm side. What is the perimeter of the whole shape, in centimetres?

A) $6\pi + 34$

B) $6\pi + 22$

C) $22 + 12\pi$

D) $22 + 36\pi$

Q20 — Hard

A circular arena is drawn on a plan with a diameter running between the endpoints $(1, 2)$ and $(7, 10)$. The radius is half the distance between those two points. What is the radius of the circle?

A) 5

B) 10

C) $(4,\ 6)$

D) $\sqrt{10}$

Q21 — Medium

A closed cardboard box measures 4 by 3 by 2 units. What is its surface area?

A) 24

B) 9

C) 26

D) 52

Q22 — Hard

In a triangle, a line parallel to the base cuts the other two sides. It divides one side into lengths 4 (top) and 6 (bottom). If the base is 20, how long is the parallel line?

A) $\frac{40}{3}$

B) 8

C) 12

D) 16

Q23 — Medium

In a right triangle, one acute angle A satisfies $\sin A = \frac{5}{13}$, and B is the other acute angle. What is $\cos B$?

A) $\frac{5}{13}$

B) $\frac{12}{13}$

C) $\frac{13}{5}$

D) $\frac{5}{12}$

Q24 — Hard

A cylindrical tank is redesigned so that its radius is doubled while its height is left exactly as it was. By what factor does its volume change?

A) 2

B) 8

C) 4

D) 6

Q25 — Medium

A circular table top is specified by its diameter, which measures 14 units across. What is its area?

A) 196π

B) 49π

C) 14π

D) 7π

Q26 — Hard

A rectangular field measures 9 by 12 units, and a path is to run straight from one corner to the opposite one. What is the length of the diagonal?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Answers and Explanations

Section 1 — Angles and Triangles

Q1 — Basic — Answer: D

Topic: Angles in a triangle

The three angles of a triangle add to 180°:

$$180 - 47 - 68 = 65$$

Check: $47 + 68 + 65 = 180$ ✓

Why each wrong option is wrong:

- **A,** 115 — added the two given angles instead of subtracting them from 180.
- **B,** 245 — used 360°, which is the total for a quadrilateral or the angles around a point.
- **C,** 25 — assumed a right angle that the question never mentions.

> **Takeaway:** Triangle angles total 180°; quadrilateral angles total 360°. Add your three angles at the end as a check — it costs nothing.

Q2 — Medium — Answer: D

Topic: An isosceles triangle

$AB = AC$ makes the triangle **isosceles**, so the two angles *opposite* those equal sides are equal. The sides AB and AC meet at A, so the equal angles are B and C.

The three angles total 180°, and A takes 40°, leaving 140° shared equally between B and C:

$$\frac{180 - 40}{2} = \frac{140}{2} = 70$$

Check: $40 + 70 + 70 = 180$ ✓

Why each wrong option is wrong:

- **A,** 40 — that would make it equilateral, and an equilateral triangle's angles are all 60°, not 40°.
- **B,** 100 — assumed B and C were 40° each.
- **C,** 140 — found the 140° that B and C share and forgot to halve it.

> **Takeaway:** Equal sides give equal **opposite** angles. Subtract the odd angle from 180 and halve what is left — and identify which two angles are the equal pair before you do.

Q3 — Medium — Answer: B

Topic: Parallel lines cut by a transversal

Co-interior angles — the pair inside the parallel lines and on the same side of the crossing line, forming a C shape — are **supplementary**: they add to 180°.

$$180 - 118 = 62$$

Check: $118 + 62 = 180$ ✓

It is worth keeping the three relationships apart, because all three angles exist in every one of these figures:

Pair	Shape	Relationship
Corresponding	F	equal
Alternate	Z	equal
Co-interior	C	add to 180°

Why each wrong option is wrong:

- **A,** 118 — that is the corresponding or alternate angle, which *is* 118°. The question asked for the co-interior one.
- **C,** 32 — subtracted from 90.
- **D,** 242 — subtracted from 360.

> **Takeaway:** Two of the three parallel-line relationships give equal angles and one gives a total of 180°. Identify which pair the question names before choosing.

Section 2 — Right Triangles

Q4 — Basic — Answer: C

Topic: Pythagoras

$$c^2 = 9^2 + 12^2 = 81 + 144 = 225$$
$$c = \sqrt{225} = 15$$

This is the 3–4–5 triangle scaled by 3: $9 = 3 \times 3$, $12 = 3 \times 4$, so the hypotenuse is $3 \times 5 = 15$. Recognising it skips the arithmetic entirely.

Check the size: the hypotenuse must be longer than either leg but shorter than their sum. $12 < 15 < 21$ ✓

Why each wrong option is wrong:

- **A,** 21 — added the legs. That is the upper limit the hypotenuse can never reach, not the answer.
- **B,** $3\sqrt{7}$ — subtracted, which is what you do when the **hypotenuse** is given and you want a leg.
- **D,** $\frac{21}{2}$ — averaged them.

> **Takeaway:** Squares add, then take the root. Learn 3–4–5 and 5–12–13 with their multiples — they appear constantly and turn the question into recognition.

Q5 — Medium — Answer: B

Topic: A special right triangle

The sides are in the ratio $x : x\sqrt{3} : 2x$, where x faces the 30° angle.

Here $x = 7$, so:

- opposite 30°: 7
- opposite 60°: $7\sqrt{3} \approx 12.1$
- hypotenuse: $2 \times 7 = 14$

The hypotenuse is **twice** the shortest side — the single most useful fact about this triangle.

Check with Pythagoras: $7^2 + (7\sqrt{3})^2 = 49 + 147 = 196 = 14^2$ ✓

Why each wrong option is wrong:

- **A,** $7\sqrt{3}$ — the side opposite 60°.

- **C,** $\frac{7}{2}$ — halved rather than doubled. The hypotenuse is the longest side, so it cannot be smaller than a leg.
- **D,** $7\sqrt{2}$ — the 45–45–90 ratio, which belongs to the other special triangle.

> **Takeaway:** In a 30–60–90, the hypotenuse is twice the shortest side, and the shortest side faces the 30°. Check any answer against "the hypotenuse is the longest side".

Q6 — Medium — Answer: A

Topic: SOHCAHTOA

SOH CAH TOA:

$$\sin = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan = \frac{\text{opposite}}{\text{adjacent}}$$

The right angle is at C, so the hypotenuse is $AB = 13$.

For angle A, the **opposite** side is the one that does not touch A — that is $BC = 5$.

$$\sin A = \frac{5}{13}$$

The third side is $AC = \sqrt{169 - 25} = 12$, so this is the 5–12–13 triangle.

Why each wrong option is wrong:

- **B,** $\frac{12}{13}$ — $\cos A = \frac{12}{13}$, using the adjacent side.
- **C,** $\frac{5}{12}$ — $\tan A = \frac{5}{12}$.
- **D,** $\frac{13}{5}$ — upside down. Sine and cosine are always at most 1, because the hypotenuse is the longest side. **Any option greater than 1 for a sine or cosine can be eliminated immediately.**

> **Takeaway:** Identify the hypotenuse first, then which side is opposite the angle named. And remember sin and cos can never exceed 1 — it eliminates an option for free.

Section 3 — Circles

Q7 — Medium — Answer: B

Topic: Reading the centre and radius of a circle

Compare with $(x-h)^2+(y-k)^2=r^2$.

$$(x-\mathbf{4})^2+(y-(-\mathbf{3}))^2=25$$

The y bracket reads $(y+3)$, which is $(y-(-3))$, so $k=-3$. The signs **flip**: what you read is the negative of the coordinate.

Centre: $(4,-3)$.

The right-hand side is $r^2=25$, so

$$r=\sqrt{25}=5$$

Check: the point $(9,-3)$ is 5 units right of the centre. Substituting gives $25+0=25$ ✓

Why each wrong option is wrong:

- **A** — flipped the signs the wrong way **and** took 25 as the radius.
- **C** — right centre, but read r^2 as r.
- **D** — right radius, wrong centre.

> **Takeaway:** Signs flip for the centre; take the square root for the radius. Both mistakes are offered separately and together, so check both before choosing.

Q8 — Medium — Answer: C

Topic: A sector of a circle

A 60° sector is $\frac{60}{360}=\frac{1}{6}$ of the circle.

The whole circle's area is

$$\pi r^2=\pi(6)^2=36\pi$$

So the sector is

$$\frac{1}{6}\times 36\pi=6\pi$$

Why each wrong option is wrong:

- **A,** 36π — the whole circle, with the fraction never applied.

- **B,** 2π — the **arc length**, $\frac{1}{6} \times 2\pi(6) = 2\pi$. An arc is a length; a sector is an area. Check the units the question asks for.
- **D,** 12π — used $2\pi r^2$, which is not a formula for anything.

> **Takeaway:** Find the fraction $\frac{\theta}{360}$, then multiply by whichever whole-circle quantity you want — area for a sector, circumference for an arc.

Q9 — Hard — Answer: 4

Topic: A circle equation given the centre and a point

The radius is the distance from the centre to any point on the circle. Use the distance formula — which is Pythagoras in disguise, and is **not** on the reference sheet:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(2-2)^2 + (9-5)^2} = \sqrt{0+16} = 4$$

Here the two points share an x-coordinate, so they sit on a vertical line and you can just count: from $y = 5$ to $y = 9$ is 4.

This is a grid-in. Enter 4.

> **Takeaway:** The radius is the distance from the centre to any point on the circle. When two points share a coordinate, subtract the other one — no formula needed.

Section 4 — Area, Volume and Scaling

Q10 — Medium — Answer: A

Topic: Volume of a cylinder

From the reference sheet, $V = \pi r^2 h$:

$$V = \pi(3)^2(10) = \pi \times 9 \times 10 = 90\pi$$

Why each wrong option is wrong:

- **B,** 30π — used r instead of r^2.
- **C,** 60π — that is $2\pi rh$, the curved surface **area**. A volume and an area are different quantities; check what the question asked for.

- **D,** 900π — squared $r \times h$ together.

> **Takeaway:** Square the radius, not the height. The formula is on the sheet — copy it before substituting rather than working from memory.

Q11 — Hard — Answer: D

Topic: Scaling a solid

Volume depends on the **cube** of a length. The formula is $V = \frac{4}{3}\pi r^3$, so replacing r with $3r$:

$$\frac{4}{3}\pi(3r)^3 = \frac{4}{3}\pi \cdot 27r^3 = 27 \times \left(\frac{4}{3}\pi r^3\right)$$

The volume is multiplied by **27**.

The 3 inside the bracket gets cubed along with the r — that is the whole question. Try it with numbers if it feels abstract: a sphere of radius 1 has volume $\frac{4}{3}\pi$; radius 3 gives 36π; and $36 \div \frac{4}{3} = 27$ ✓

Why each wrong option is wrong:

- **A,** 3 — the factor for **lengths**.
- **B,** 9 — the factor for **areas**, which is 3^2.
- **C,** 6 — no rule gives 6 here.

> **Takeaway:** Lengths $\times k$, areas $\times k^2$, volumes $\times k^3$. Decide which kind of quantity is being asked about, then apply the right power.

Section 5 — Mixed Practice

Q12 — Basic — Answer: 57

Topic: Angles on a straight line

Angles on a straight line add to 180°:

$$180 - 52 - 71 = 57$$

Check: $52 + 71 + 57 = 180$ ✓

This is a grid-in. Enter 57.

> **Takeaway:** Straight line, 180°. Around a point, 360°. Add your answer back in as a check.

Q13 — Medium — Answer: B

Topic: Similar triangles

Similar means the same shape at a different size, so **every** length is multiplied by the same factor. Find it from the pair of sides you know:

$$k = \frac{20}{8} = 2.5$$

The smaller perimeter is $4 + 6 + 8 = 18$. Perimeter is a length, so it scales by the same k:

$$18 \times 2.5 = 45$$

Check by scaling each side: $10, 15, 20$, which sum to 45 ✓

A gave the smaller triangle's perimeter. **C** scaled only one side. **D** used a wrong factor.

> **Takeaway:** Find the scale factor from a matching pair, then apply it to whatever length is asked for. Perimeter scales like a length — no squaring.

Q14 — Medium — Answer: C

Topic: Area of a triangle

$$A = \frac{1}{2}bh = \frac{1}{2}(14)(9) = 63$$

A forgot the $\frac{1}{2}$ — the most common slip with this formula. **B** added. **D** halved twice.

> **Takeaway:** The half is part of the formula, not an optional flourish. It is on the reference sheet; copy it out before substituting.

Q15 — Hard — Answer: A

Topic: Trigonometry with a real situation

Draw it. The ladder is the **hypotenuse**, the ground distance of 2.5 m is **adjacent** to the 65° angle, and the wall is opposite.

Adjacent and hypotenuse means cosine:

$$\cos 65^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{2.5}{L}$$

Rearranging for L:

$$L = \frac{2.5}{\cos 65^\circ}$$

Sanity-check without computing anything. The ladder is the hypotenuse, so it must be **longer** than 2.5 m. Cosine of an acute angle is between 0 and 1, so *dividing* by it makes the number bigger ✓ while *multiplying* by it would make it smaller — which eliminates options B and D immediately.

B and **D** both produce something shorter than 2.5 m. **C** uses the opposite side, which is the wall's height, not the given distance.

> **Takeaway:** Label hypotenuse, opposite and adjacent before choosing a ratio. Then check the size: the hypotenuse is the longest side, so any expression making it smaller is wrong.

Q16 — Medium — Answer: D

Topic: Area of a circle from its circumference

Work back to the radius first:

$$2\pi r = 16\pi \quad \Longrightarrow \quad r = 8$$

Now the area:

$$A = \pi r^2 = \pi(8)^2 = 64\pi$$

A used $r = 4$. **B** treated 16 as the radius. **C** applied a circumference formula.

> **Takeaway:** Circumference to area always goes through the **radius**. Find r, then use it — never move between the two formulas directly.

Q17 — Medium — Answer: C

Topic: An exterior angle of a triangle

An exterior angle equals the **sum of the two opposite interior angles**:

$$52 + 61 = 113$$

Check it the long way. The third interior angle is $180 - 113 = 67$, and the exterior angle sits on a straight line with it: $180 - 67 = 113$ ✓ Both routes agree, which is exactly why the shortcut is safe.

A gave that third interior angle — a real number in the figure, and the wrong one.

> **Takeaway:** Exterior angle = the two opposite interior angles added. It saves a step, and the straight-line route confirms it.

Q18 — Medium — Answer: 73

Topic: Angles in a quadrilateral

A quadrilateral's angles total 360°:

$$360 - 95 - 78 - 114 = 73$$

Check: $95 + 78 + 114 + 73 = 360$ ✓ Enter 73.

Using 180° here would give -107, and a negative angle should stop you at once.

> **Takeaway:** Triangle 180°, quadrilateral 360°. A negative answer means you used the wrong total.

Q19 — Medium — Answer: B

Topic: The perimeter of a composite shape

Walk around the outside. The 12 cm side the semicircle sits on is **inside** the shape now, so it is not part of the perimeter. What you walk is:

$$12 + 5 + 5 = 22 \text{ cm of straight edge}$$

plus the curve. The semicircle has diameter 12, so radius 6, and half a circumference is

$$\tfrac{1}{2} \times 2\pi(6) = 6\pi$$

Total: $22 + 6\pi$ cm.

A counted the joined side twice. **C** used a whole circumference; it is a semicircle. **D** used πr^2, which is an area.

> **Takeaway:** For a composite perimeter, trace the outline with your finger. Any edge that ends up inside the shape is not part of it.

Q20 — Hard — Answer: A

Topic: A circle's equation from a diameter

The distance between the two endpoints is the **diameter**:

$$d = \sqrt{(7-1)^2 + (10-2)^2} = \sqrt{36+64} = \sqrt{100} = 10$$

The radius is half of it:

$$r = 5$$

(The centre, if a later part asks, is the midpoint $(4, 6)$ — which is option C.)

B stopped at the diameter, which is the most common slip here.

> **Takeaway:** Endpoints of a **diameter** give twice the radius. Compute the distance, then halve it — and use the midpoint for the centre.

Q21 — Medium — Answer: D

Topic: Surface area of a rectangular solid

A box has three pairs of matching faces:

$$4 \times 3 = 12, \qquad 4 \times 2 = 8, \qquad 3 \times 2 = 6$$

Each appears twice:

$$2(12 + 8 + 6) = 2(26) = 52$$

A gave the volume, 24 — a different quantity in different units. **C** is the commonest error: three faces counted instead of six.

> **Takeaway:** Three distinct faces, each appearing twice. Doubling at the end is the step that gets forgotten.

Q22 — Hard — Answer: B

Topic: Similar triangles inside one figure

A line parallel to the base makes a **smaller triangle similar to the whole**. The small triangle's side is the top piece, 4; the whole triangle's matching side is the entire length, $4 + 6 = 10$.

So the scale factor is $\frac{4}{10}$, and

$$\text{parallel line} = \frac{4}{10} \times 20 = 8$$

The trap is comparing 4 to 6. Those are the two **pieces** of one side; the similar triangle's side is 4 compared with the **whole** side, 10.

> **Takeaway:** In a "parallel line inside a triangle" figure, compare the small triangle's side with the **whole** side, never with the remaining piece.

Q23 — Medium — Answer: A

Topic: Trigonometry of complementary angles

The two acute angles of a right triangle add to 90°, so they are **complementary** — and for complementary angles,

$$\sin A = \cos B$$

The reason is that the side **opposite** A is the side **adjacent** to B: one side, two descriptions. So $\cos B = \frac{5}{13}$, with no calculation at all.

B is $\cos A$, which uses the other side.

> **Takeaway:** In a right triangle, sin of one acute angle equals cos of the other. The opposite side of one angle is the adjacent side of the other.

Q24 — Hard — Answer: C

Topic: Volume with a changed dimension

$V = \pi r^2 h$, and only r changes:

$$\pi(2r)^2 h = \pi \cdot 4r^2 \cdot h = 4 \times (\pi r^2 h)$$

The volume is multiplied by **4**, not 8 — because the height did not change. The radius appears **squared**, so doubling it multiplies by 2^2.

B is the answer to "all three dimensions doubled", where the factor would be $2^3 = 8$. Read which dimensions actually change.

> **Takeaway:** Apply the power that the changing dimension carries in the formula. r is squared in a cylinder, so doubling r quadruples the volume.

Q25 — Medium — Answer: B

Topic: Area of a circle from its diameter

The radius is half the diameter: $r = 7$.

$$A = \pi r^2 = 49\pi$$

A used 14 as the radius, which gives four times too much — an area error of exactly 2^2, for the reason in the previous question. **C** is the circumference.

> **Takeaway:** Halve the diameter before doing anything. A question that gives the diameter is testing that step.

Q26 — Hard — Answer: 15

Topic: A right triangle inside a rectangle

A diagonal cuts the rectangle into two right triangles, with the sides as legs:

$$d = \sqrt{81 + 144} = \sqrt{225} = 15$$

This is the 3–4–5 triangle scaled by 3. Check the size: the diagonal must be longer than either side but shorter than their sum, and $12 < 15 < 21$ ✓ Enter 15.

> **Takeaway:** A rectangle's diagonal is the hypotenuse of a right triangle with the sides as legs. Watch for the familiar triples.

Chapter 8 — Student-Produced Responses

Seven questions in every Math module have **no options at all**. You work the answer out and enter it yourself. College Board calls them student-produced responses; most people call them **grid-ins**.

They are worth their own chapter for three reasons, and none of them is that the mathematics is harder.

They are a quarter of the section. Across the eight official practice tests examined, 112 of the 432 Math questions were grid-ins — 26%.

They sit in fixed places. In all sixteen modules checked, without a single exception, the grid-ins were questions

6, 7, 13, 14, 20, 21, 27

So the last question of every module is a grid-in, and so are the two before each third of the way through. When you reach question 6 and there are no options, it is not a misprint.

They take away your safety nets. Two techniques that carry you through the rest of the test simply do not exist here:

- You cannot **work backwards from the options**, because there are none.
- You cannot **guess usefully**. On a multiple-choice question a blind guess is worth a 1-in-4 chance; here it is worth almost nothing.

What replaces them is **checking your own answer by substitution**, and that habit is what this chapter is really training.

Topics covered: the entry rules · which forms are accepted · rounding and truncating · negative answers · questions with two valid answers · checking your own work

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Section 1 — The Rules for Entering an Answer

These come from the test's own instructions page. Every one of them costs a mark if broken, on a question you may have solved perfectly.

Length. Up to **5 characters** for a positive answer, **6** for a negative one — the minus sign counts as a character.

Fractions that do not fit. Write the decimal instead.

Decimals that do not fit. Truncate or round at the fourth digit. Either is accepted, which is worth knowing: you do not have to decide which the test prefers.

Mixed numbers are not allowed. $3\frac{1}{2}$ must be entered as `7/2` or `3.5`. Entering `3 1/2` scores **zero** — it is read as thirty-one halves.

No symbols. No percent sign, no comma, no dollar sign. One thousand two hundred and fifty is `1250`, never `1,250`. Forty per cent is `40` or `.4` depending on what the question asked for — never `40%`.

Several forms are usually accepted

This is the part almost nobody is taught, and it removes a lot of anxiety. If the answer is $\frac{1}{4}$, then `1/4`, `0.25` and `.25` are **all** accepted. You do not have to guess which form is wanted.

Here are real accepted answers from official answer keys, and what each shows:

The key accepts	What it tells you
`0.25` or `1/4`	fraction or decimal, your choice
`29/3`, `9.666` or `9.667`	truncating **and** rounding are both accepted
`361/8`, `45.12` or `45.13`	the fraction is too long, so decimals are the way in
`15` or `-5`	two genuinely **different** correct answers — enter either

The habit to build

When a fraction fits in five characters, enter the fraction. It cannot be wrong through rounding. Convert to a decimal only when the fraction is too long, and then give four digits.

Q1 — Basic

A length is divided into two equal parts, and the two parts together measure 7 units, so each part x satisfies $2x = 7$. What is the value of x?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q2 — Basic

A quantity is divided into three equal shares totalling 1 unit, so each share x satisfies $3x = 1$. What is the value of x?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q3 — Medium

A reading x satisfies $4x + 19 = 7$. What is the value of x?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q4 — Medium

A model's break-even points are the solutions of $x^2 - 6x - 27 = 0$. What is one possible value of x?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Section 2 — Checking Without Options

With no options to test, the check has to come from you. Three methods, in order of how often they apply.

Substitute back into the original. The most reliable. Put your answer into the question's own equation and confirm both sides match. This catches sign errors, which are the most common mistake and the hardest to spot by re-reading.

Check the size against the situation. A word problem's answer has to make sense as a real quantity. A negative number of people, 2.6 text messages, a ladder shorter than its distance from the wall — each of these should stop you.

Re-read the final sentence. Grid-ins ask for a specific thing, and the most expensive error on the whole test is solving correctly and reporting the wrong quantity. You found x; did it want x, or $2x$, or y?

Budget about ten seconds for the check. On a question you have spent ninety seconds on, that is a good trade.

Q5 — Medium

A conversion satisfies $\dfrac{3x - 5}{2} = 8$. What is the value of x?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q6 — Medium

Two quantities satisfy $2x + y = 17$, and in this case the second is known to be $y = 5$. The question asks not for x itself but for $4x$. What is the value of $4x$?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q7 — Hard

A shop buys a chair from its supplier for $60 and sells it at a 35% profit on that cost. What is the selling price, in dollars?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q8 — Hard

A rectangle has area 45 and width 8. What is its length?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Section 3 — Mixed Practice

Every question here is a grid-in, as they would appear at positions 6, 7, 13, 14, 20, 21 and 27. Work each one on paper, then check it before entering.

Q9 — Medium

Two quantities satisfy $5x + 2y = 31$, and the second is known to be $y = 3$. The question asks for the other one. What is the value of x?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q10 — Medium

A survey covered 150 respondents, and 24% of them gave a particular answer. What number is 24% of 150?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q11 — Medium

A cost is modelled by the function $f(x) = x^2 - 10x + 3$, whose graph is a parabola with a minimum. The question asks for the input at which that minimum occurs. At what value of x does it occur?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q12 — Hard

A quantity satisfies $\frac{2}{3}x + 4 = 9$. What is the value of x?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q13 — Medium

A right triangle has a hypotenuse measuring 26 units and one leg measuring 10 units. What is the length of the other leg?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q14 — Hard

A bacterial culture starts with 250 cells and triples in number every 2 hours. The observation runs for 6 hours in total. How many cells are there after 6 hours?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q15 — Hard

A measurement satisfies $|2x - 5| = 11$. The question restricts the answer to the **positive** value of x. What is that value?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Answers and Explanations

Section 1 — The Rules for Entering an Answer

Q1 — Basic — Answer: 7/2 or 3.5

Topic: An answer that must not be written as a mixed number

$$x = \frac{7}{2}$$

Now the part this question is really about. $\frac{7}{2}$ is $3\frac{1}{2}$, and you must **not** enter it that way. `3 1/2` is read as $\frac{31}{2}$, which is 15.5, and scores zero.

Two acceptable entries:

- `7/2` — the improper fraction, 3 characters
- `3.5` — the decimal, 3 characters

Both fit and both are accepted. The fraction is the safer habit because it is exact.

> **Takeaway:** Never enter a mixed number. Improper fraction or decimal — those are the only two forms the grid understands.

Q2 — Basic — Answer: 1/3 or .333 or 0.333

Topic: A repeating decimal that must be truncated or rounded

$$x = \frac{1}{3} = 0.3333\ldots$$

The decimal never ends, so it cannot be entered in full. Three valid entries:

- `1/3` — exact, 3 characters. **This is the best answer.**
- `.333` — 4 characters, truncated at the fourth digit
- `0.333` — 5 characters, the same value with a leading zero

What would **not** work is `0.33`, which is rounded too early, or `0.3333333`, which is too long for the grid.

Notice that the exact fraction sidesteps the entire question of how many digits to give. That is why "enter the fraction when it fits" is the rule to build a habit around.

> **Takeaway:** When a decimal repeats, enter the fraction. If you must use a decimal, fill the grid: four digits, truncated or rounded — both are accepted.

Q3 — Medium — Answer: -3

Topic: A negative answer

$$4x = 7 - 19 = -12 \implies x = -3$$

Check: $4(-3) + 19 = -12 + 19 = 7\,\checkmark$

Negative answers are perfectly acceptable. Enter `-3` — two characters, and the grid allows **six** for a negative answer, so there is plenty of room.

Do not be tempted to drop the minus sign because the answer "looks wrong". Of the 112 grid-in answers across the eight official tests examined, four were negative. They are uncommon but they are real.

> **Takeaway:** Enter the minus sign. It counts as one of your six characters and the answer is wrong without it.

Q4 — Medium — Answer: 9 or -3

Topic: A question with two correct answers

Factor: two numbers multiplying to -27 and adding to -6 are -9 and 3.

$$(x-9)(x+3) = 0 \implies x = 9 \text{ or } x = -3$$

Both are correct. The question says "**one possible value**", and the grid takes one answer, so enter **either** `9` or `-3`.

This is a real pattern in the official keys — one of them literally lists `15; -5` as the accepted answers, meaning both were valid. Do not waste time deciding which one the test "wants". Enter the first one you found.

Watch the wording, though. If a question instead says "what is the **positive** solution", or "the least value of x", then only one answer scores.

> **Takeaway:** "One possible value" means either answer scores. Enter whichever you found first and move on — but re-read the question in case it restricted you to one of them.

Section 2 — Checking Without Options

Q5 — Medium — Answer: 7

Topic: Substituting back to catch a sign error

Multiply both sides by 2 first, to clear the fraction:

$$3x - 5 = 16$$

Then:

$$3x = 21 \quad \Longrightarrow \quad x = 7$$

Now check, which is the whole point of this question:

$$\frac{3(7)-5}{2} = \frac{21-5}{2} = \frac{16}{2} = 8 \checkmark$$

The common wrong answer here is $x = \frac{11}{3}$, from adding 5 to 8 before multiplying by 2 — that is, treating the fraction bar as though it only covered the $3x$. Substituting catches it instantly: $\frac{11-5}{2} = 3$, not 8.

> **Takeaway:** The fraction bar groups **everything** above it. Multiply the whole side by the denominator first, then substitute your answer back.

Q6 — Medium — Answer: 24

Topic: Reading which quantity is wanted

Substitute $y = 5$:

$$2x + 5 = 17 \quad \Longrightarrow \quad 2x = 12 \quad \Longrightarrow \quad x = 6$$

But the question asks for $4x$, not x:

$$4x = 24$$

There is a faster route. You reached $2x = 12$ along the way, and $4x$ is simply double that: $4x = 24$. You never needed x at all.

On a multiple-choice question, 6 and 12 would both be offered. Here there are no options to warn you — which is exactly why re-reading the last line matters more on grid-ins than anywhere else.

> **Takeaway:** Read the final sentence again before entering. Then look for whether the quantity wanted is a simple multiple of something you already have.

Q7 — Hard — Answer: 81

Topic: An answer whose size should be checked

A 35% profit means the selling price is 135% of the cost:

$$60 \times 1.35 = 81$$

Check the size, which is the habit this question is for. The shop makes a profit, so the selling price **must be more than $60**. It is ✓

Two wrong answers this eliminates immediately:

- `21` — the profit itself, not the selling price. Less than 60, so it fails the check.
- `39` — from 60×0.65, taking 35% *off*. Also less than 60.

Neither survives a single glance at whether the number should be bigger or smaller than 60.

Enter `81`. Not `$81` — the dollar sign is not allowed.

> **Takeaway:** Before entering, ask whether the answer should be bigger or smaller than the number you started from. It is the fastest check there is, and it needs no arithmetic.

Q8 — Hard — Answer: 45/8 or 5.625

Topic: A long fraction that must become a decimal

$$\text{length} = \frac{\text{area}}{\text{width}} = \frac{45}{8} = 5.625$$

Both entries work here, and it is worth seeing why:

- `45/8` — 4 characters, exact, fits comfortably
- `5.625` — 5 characters, exactly at the limit, and exact as well

This decimal happens to terminate, so nothing is lost either way. Compare with the official key that accepted `361/8`, `45.12` **or** `45.13`: there the decimal is 45.125, which is 6 characters and does not fit, so it had to be cut to four digits — and both cutting and rounding were accepted.

> **Takeaway:** Divide, then check the character count. A fraction that fits is always safe; a decimal is only safe when it fits **and** terminates within the grid.

Section 3 — Mixed Practice

Q9 — Medium — Answer: 5

Topic: A system, answered for one variable

$$5x + 6 = 31 \implies 5x = 25 \implies x = 5$$

Check: $25 + 6 = 31$ ✓

> **Takeaway:** Substitute the known value first, then solve. Check by putting both numbers back into the original.

Q10 — Medium — Answer: 36

Topic: A percentage answer entered without its sign

$$0.24 \times 150 = 36$$

Check the size: 24% is about a quarter, and a quarter of 150 is about 37 ✓

Enter 36. Not 36% — the question asked *what number*, and percent signs are not accepted in the grid regardless.

> **Takeaway:** "What number is r% of b" means multiply. Estimate first with a friendly percentage to confirm the size.

Q11 — Medium — Answer: 5

Topic: The vertex of a parabola

$$x = -\frac{b}{2a} = -\frac{-10}{2} = 5$$

Check either side: $f(4) = 16 - 40 + 3 = -21$, $f(5) = 25 - 50 + 3 = -22$, $f(6) = 36 - 60 + 3 = -21$. Lowest at 5 ✓

The question asked *where* the minimum occurs, so the answer is 5, not -22.

> **Takeaway:** $x = -\frac{b}{2a}$, and read whether the question wants where the minimum is or what it is.

Q12 — Hard — Answer: 15/2 or 7.5

Topic: An answer that needs the fraction form

$$\frac{2}{3}x = 5 \quad \Longrightarrow \quad x = 5 \times \frac{3}{2} = \frac{15}{2}$$

Check: $\frac{2}{3} \times \frac{15}{2} = 5$, and $5 + 4 = 9$ ✓

Enter `15/2` or `7.5`. **Not** `7 1/2`.

> **Takeaway:** To undo a fraction, multiply by its reciprocal. Then choose a legal form — improper fraction or decimal.

Q13 — Medium — Answer: 24

Topic: A geometry answer

The hypotenuse is given, so **subtract**:

$$a^2 = 26^2 - 10^2 = 676 - 100 = 576 \quad \Longrightarrow \quad a = 24$$

Check: $100 + 576 = 676$ ✓ This is the 5–12–13 triangle doubled.

Check the size too: a leg must be shorter than the hypotenuse, and $24 < 26$ ✓

> **Takeaway:** Given the hypotenuse, subtract. Given two legs, add. Then confirm your leg is shorter than the hypotenuse.

Q14 — Hard — Answer: 6750

Topic: An exponential model

Count the periods rather than the hours:

$$\frac{6}{2} = 3 \text{ periods}$$

Each triples, so the factor is $3^3 = 27$:

$$250 \times 27 = 6750$$

Check by stepping: $250 \to 750$ (2 hrs) $\to 2250$ (4 hrs) $\to 6750$ (6 hrs) ✓

> **Takeaway:** Divide the time by the period length to count the periods. The exponent counts periods, never hours.

Q15 — Hard — Answer: 8

Topic: A question restricting you to one of two solutions

Split into two cases:

$$2x - 5 = 11 \implies x = 8 \qquad 2x - 5 = -11 \implies x = -3$$

Both are genuine solutions. But the question says **positive**, so only $x = 8$ scores.

Contrast this with Q4, where "one possible value" made either answer acceptable. The difference is one word, and it is worth reading twice — an absolute-value grid-in almost always has two solutions, and the question decides which of them you may enter.

> **Takeaway:** Solve both cases, then let the wording pick. "One possible value" takes either; "positive", "least" or "greatest" takes exactly one.

Chapter 9 — Two Full Practice Papers

Two complete Math sections, built to the real shape:

- **Two modules of 27 questions**, done one after the other
- **Grid-ins at questions 6, 7, 13, 14, 20, 21 and 27** in each module — the positions they occupied in all sixteen official modules examined
- A mix of topics weighted the way the test weights them: about 35% Algebra, 35% Advanced Math, 15% Problem-Solving and Data Analysis, 15% Geometry and Trigonometry
- Difficulty rising through each module, roughly

Save these for the final weeks. They are most useful once you have worked through the chapters, because a mock taken too early mostly measures what you have not studied yet.

How to sit one

Give yourself **35 minutes per module**, which is the real timing. Do Module 1, take a short break, then Module 2. Use a calculator — you are allowed one on every question.

Work on paper. Write the answer down for every question, including ones you guess: there is no penalty for a wrong answer, and a blank scores the same as a wrong one.

Do not check answers between modules.

Scoring

Count how many of the 54 you got right. That is your **raw score**. This table converts it to the 200–800 Math section scale, and it is the real conversion table published with official practice test 10 — not an estimate.

Raw score	Math section score
0	200
3	210–220
6	220–250
9	250–310
12	310–350
15	340–380
18	370–410
21	390–430
24	410–450
27	440–480
30	470–510
33	490–550
36	530–590
39	560–620
42	600–660
45	640–700
48	680–740
50	730–770
52	770–790
54	800

The score is a **range** because this simplified conversion is slightly less precise than the one the real test uses. Treat the middle of the range as your estimate.

Two things the table tells you that are worth reading off it. **Every question is worth roughly the same**, so the cheapest marks are the easy ones you rushed. And the **middle of the range is where the gains are largest**: moving from 27 right to 39 right is about 120 points, while moving from 45 to 51 is a similar gain for far more work.

Paper 1 — Module 1

27 questions · 35 minutes · calculator permitted

Q1 — Basic

A cyclist's distance from home, in kilometres, is given by the expression $6x - 4$, where x is the number of hours since the ride began. The cyclist is 26 kilometres from home. What is the value of x?

A) $\frac{11}{3}$

B) 5

C) $\frac{15}{2}$

D) 30

Q2 — Basic

A regional council surveyed 240 households about recycling and found that 30% of them separate glass from other waste. The council plans to write to each of those households with information about a new collection service. How many households will receive the letter?

A) 8

B) 210

C) 270

D) 72

Q3 — Basic

A biologist recorded the length of a seedling on two occasions. After 1 week the seedling was 2 centimetres long, and after 5 weeks it was 14 centimetres long. Treating length as a linear function of time, what is the rate of growth, in centimetres per week?

A) $\frac{8}{3}$

B) $\frac{1}{3}$

C) -3

D) 3

Q4 — Basic

The function f is defined by $f(x) = x^2 - 3x$, where $f(x)$ is the profit, in hundreds of dollars, from selling x hundred units of a product. What is the value of $f(5)$?

A) -5

B) 15

C) 25

D) 10

Q5 — Medium

A manufacturer models the area of a rectangular panel, in square centimetres, as the product $(x - 4)(x + 6)$, where x is a length in centimetres. Before the panel can be costed, the expression must be written without brackets. Which of the following expressions is equivalent to it for every value of x?

A) $x^2 + 10x - 24$

B) $x^2 - 2x - 24$

C) $x^2 - 24$

D) $x^2 + 2x - 24$

Q6 — Medium

A water tank is being filled. The volume in the tank, in litres, is given by $\frac{x}{4} + 3$, where x is the number of seconds since filling began. After how many seconds does the tank hold 11 litres?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q7 — Medium

A teacher recorded the scores of four students on a recent class test and set them out in the table below. One script was mislaid before its score could be entered.

Student	Amara	Ben	Chen	Dara
Score	18	24	19	?

The mean of all four scores is known to be 21. What was the fourth score?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q8 — Medium

At a school fair, the number of adult tickets sold, y, was 2 fewer than three times the number of child tickets, x, so that $y = 3x - 2$. The organisers also found that $2x + y = 13$. What is the value of x?

A) $\frac{11}{5}$

B) $\frac{13}{5}$

C) 7

D) 3

Q9 — Medium

In a computer science calculation, the number of operations required is given by the expression $\frac{(x^3)^4}{x^5}$, where x is a positive integer describing the size of the input. Before the expression can be compared with another algorithm's, it must be written as a single power of x. Which of the following is equivalent to it?

A) x^2

B) x^7

C) x^{12}

D) x^{60}

Q10 — Medium

The ratio of cats to dogs at a shelter is $5 : 3$. There are 48 animals in total. How many are dogs?

A) 3

B) 30

C) 16

D) 18

Q11 — Medium

A rectangle's area, in square metres, is given by $x^2 - 2x - 15$, where x is its width in metres. For what width is the area equal to zero square metres? Give the greatest such value of x.

A) -3

B) 3

C) -5

D) 5

Q12 — Medium

A rectangular garden plot is 8 metres wide and 15 metres long. A gardener wants to run a straight path from one corner of the plot to the opposite corner. What is the length of that path, in metres?

A) 23

B) $\sqrt{161}$

C) $\frac{23}{2}$

D) 17

Q13 — Medium

A machine is calibrated to fill bottles to 7 millilitres, and a bottle is rejected if its contents differ from 7 millilitres by exactly 4 millilitres or more. A bottle containing x millilitres satisfies $|x - 7| = 4$. What is the greatest possible value of x?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q14 — Medium

A bottling plant runs one machine that fills 45 bottles every 3 minutes, and the supervisor needs to plan how many crates to have ready at the end of each shift segment. Assuming the machine continues at the same rate and does not stop, how many bottles will it fill in 8 minutes?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q15 — Medium

A small business's profit, in thousands of dollars, is a linear function of the number of units it sells. Two of its values were recorded in the table below.

Units sold, x	**2**	**5**
Profit, y	3	15

The y-intercept of the graph is the profit when nothing at all is sold, which for a business with fixed costs to cover may well be negative. What is the y-intercept of that graph?

A) 3

B) -5

C) 11

D) 5

Q16 — Medium

An ecologist has been monitoring a population of 500 fish in a lake and estimates that the population grows by 20% during each year, with the growth compounding from one year to the next. Assuming the rate continues, what will the population be at the end of 2 years?

A) 600

B) 700

C) 620

D) 720

Q17 — Medium

An engineer simplifies the expression $\dfrac{x^2 - 16}{x + 4}$ that arises in a calculation, where $x \neq -4$. Which of the following expressions is equivalent to it for every allowed value of x?

A) -4

B) $x + 4$

C) $x^2 - 4$

D) $x - 4$

Q18 — Medium

A school recorded which of its 120 students study Spanish and which study history, and set the counts out in the two-way table below.

	Studies history	Does not study history	Total
Studies Spanish	45	25	70
Does not study Spanish	20	30	50
Total	65	55	120

A student who studies Spanish is chosen at random. What is the probability that the student also studies history?

A) $\frac{3}{8}$

B) $\frac{9}{14}$

C) $\frac{7}{12}$

D) $\frac{5}{14}$

Q19 — Hard

A car rental company charges a fixed amount to hire a vehicle and a further amount for each mile driven. The total cost, in dollars, of hiring a car and driving it m miles is given by the function $C = 45 + 0.30m$. Which of the following is the best interpretation of the number 0.30 in this context?

A) The fixed rental fee

B) The cost of each additional mile

C) The total cost of a 30-mile trip

D) The number of miles included free

Q20 — Hard

A caterer buys x trays of sandwiches and y trays of pastries. Two orders are recorded: the first satisfies $4x + 3y = 27$ and the second satisfies $4x - y = 7$. What is the value of y?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q21 — Hard

In the equation $x^2 + 10x + c = 0$, c is a constant. The graph of the related function $y = x^2 + 10x + c$ touches the x-axis at exactly one point. What is the value of c?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q22 — Hard

A circular sprinkler is installed in a field. The set of points it can reach is described in the xy-plane by the equation $(x + 2)^2 + (y - 5)^2 = 36$, where distances are measured in metres. What is the radius, in metres, of the region the sprinkler reaches?

A) 6

B) 36

C) 18

D) $(-2,\ 5)$

Q23 — Hard

An architect is preparing two scale models of the same building, one larger than the other. The two models are similar solids, and the areas of their corresponding faces are in the ratio $9 : 25$. What is the ratio of their corresponding side lengths?

A) $\frac{9}{25}$

B) $\frac{3}{5}$

C) $\frac{81}{625}$

D) $\frac{5}{3}$

Q24 — Hard

Two functions are defined by $f(x) = x + 3$ and $g(x) = 2x$. An engineer applies f to an input and then applies g to the result. What is the value of $g(f(4))$?

A) 11

B) 8

C) 24

D) 14

Q25 — Hard

A lift holds at most 500 kg. An operator weighing 75 kg loads boxes of 35 kg each. What is the greatest number of boxes that can be carried in one trip?

A) 14

B) 13

C) 15

D) 12

Q26 — Hard

A ramp meets the ground at an acute angle θ, forming a right triangle with the ground and a vertical support. For this angle, $\cos\theta = \frac{3}{5}$. What is the value of $\sin\theta$?

A) $\frac{3}{5}$

B) $\frac{4}{5}$

C) $\frac{5}{3}$

D) $\frac{3}{4}$

Q27 — Hard

After a 25% increase, a subscription costs $60 a year. What was the price before the increase, in dollars?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Paper 1 — Module 2

27 questions · 35 minutes · calculator permitted

Take a short break before starting. On the real test the second module's difficulty depends on how you did in the first; here it simply continues.

Q28 — Basic

A diver's depth below the surface, in metres, is modelled by the expression $9 - 2x$, where x is the number of minutes since the ascent began. At what value of x is the diver 1 metre below the surface?

A) -4

B) 5

C) 4

D) -5

Q29 — Basic

A delivery service charges a fixed handling fee plus an amount for each kilometre driven. The total charge, in dollars, is $f(x) = 4x + 7$, where x is the distance in kilometres. What is the value of $f(0)$, and what does it represent?

A) 0

B) 7

C) 4

D) 11

Q30 — Basic

A school is laying artificial turf on a rectangular play area that measures 12 metres in length and 7 metres in width. The turf is sold by the square metre. How many square metres are needed to cover the area exactly once?

A) 19

B) 38

C) 42

D) 84

Q31 — Medium

A shop's daily takings, in dollars, are given by the expression $3(2x - 5) + 4x$, where x is the number of hours the shop is open. Which of the following expressions is equivalent to it for every value of x?

A) $10x - 15$

B) $10x - 5$

C) $6x - 11$

D) $6x - 1$

Q32 — Medium

On a scale drawing, a length of x centimetres represents a real distance in the same ratio as 10 is to 4. This relationship is written $\frac{x}{6} = \frac{10}{4}$. What is the value of x?

A) $\frac{12}{5}$

B) $\frac{5}{2}$

C) 15

D) $\frac{20}{3}$

Q33 — Medium

The height above the ground of a thrown ball, in metres, is zero when $x^2 + 4x - 21 = 0$, where x is the time in seconds since it was thrown. Since time cannot be negative, only a positive solution is meaningful. What is the value of x?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q34 — Medium

A retailer reduced the price of a jacket at the end of the season. Before the reduction the jacket cost \$80, and afterwards it cost \$68. By what percent was the price reduced?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q35 — Medium

In the xy-plane, a road is drawn as the line $y = -3x + 8$. A second road is drawn parallel to the first. What is the slope of the second road?

A) 3

B) -3

C) $\frac{1}{3}$

D) 8

Q36 — Medium

A packing crate has a rectangular base and vertical sides, measuring 5 metres by 4 metres by 3 metres. A supplier needs to know how much space the crate encloses in order to quote for shipping. What is the volume of the crate, in cubic metres?

A) 60

B) 12

C) 94

D) 47

Q37 — Medium

A researcher recorded the number of birds visiting a feeder on each of six consecutive mornings. The counts are listed in the table below in the order they were taken.

Morning	Mon	Tue	Wed	Thu	Fri	Sat
Birds	4	9	2	11	6	8

What is the median of these values?

A) $\frac{20}{3}$

B) 6

C) 7

D) 8

Q38 — Medium

In a calculation involving the intensity of a signal, a physicist writes the factor 5^{-2}. What is the value of this expression?

A) -25

B) -10

C) 10

D) $\frac{1}{25}$

Q39 — Medium

Two rental firms price a job differently. The first charges according to $y = 4x + 1$ and the second according to $y = kx - 3$, where x is the number of hours and k is a constant. For what value of k do the two firms never charge the same amount?

A) -4

B) 4

C) 1

D) -3

Q40 — Hard

A carpenter buys x boxes of screws and y boxes of nails for two separate jobs. The first job's order satisfies $5x + 2y = 26$ and the second job's order satisfies $3x - 2y = 6$, where both quantities are counted in boxes. What is the value of x?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q41 — Hard

The cost per unit of manufacturing an item, in dollars, is modelled by the function $f(x) = x^2 + 6x + 11$, where x is measured in thousands of units produced. The function has a minimum value. What is that minimum value?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q42 — Hard

A paediatric clinic recorded the age and height of a group of children and drew a line of best fit through the resulting scatterplot. The line has equation $y = 6x + 78$, where x is age in years and y is height in centimetres. Which of the following is the best interpretation of the number 78 in this context?

A) The predicted height at age 0

B) The predicted growth each year

C) The number of children measured

D) The oldest age in the study

Q43 — Hard

A laboratory is monitoring a sample that decays to 85% of its mass during each hour that passes, beginning from a mass of 400 grams. The technician wants a function that gives the mass remaining after any whole number of hours. Which function gives the mass, in grams, after h hours?

A) $400\left(\frac{23}{20}\right)^h$

B) $400 - \frac{17h}{20}$

C) $400\left(\frac{17}{20}\right)^h$

D) $400\left(\frac{3}{20}\right)^h$

Q44 — Hard

Two readings from a linear model were recorded in the table below, where x is measured in minutes from a chosen reference time.

x	-1	**3**
y	4	16

What is the y-intercept of the line through those two points?

A) 4

B) 3

C) 7

D) 1

Q45 — Hard

In an engineering formula the quantity $\sqrt{3x+1}$ is required to equal 4. Solving the resulting equation gives candidate values, and any value that does not satisfy the original equation must be discarded. What is the solution to $\sqrt{3x+1} = 4$?

A) 5

B) $\frac{4}{3}$

C) $\frac{16}{3}$

D) $\frac{17}{3}$

Q46 — Hard

A landscape designer is edging a curved section of a circular garden bed of radius 9 metres. The curved section corresponds to a central angle of 40° measured at the centre of the circle, and the edging is sold by the metre. What is the length of that curved section, in metres?

A) 9π

B) 2π

C) 4π

D) 18π

Q47 — Hard

A charity received two donations on the same day. Together the two amounts came to 54 units of currency, and the larger donation exceeded the smaller by 12 units. The treasurer needs to record each amount separately in the accounts. What is the smaller of the two donations?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q48 — Hard

A designer needs the expression $x^2 - 14x + c$ to be a perfect square, so that it can be written as the square of a single bracket. What value must the constant c take?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q49 — Hard

An $800 investment earns 25% interest each year, and the interest is compounded annually so that each year's interest is calculated on the balance at the start of that year rather than on the original deposit. What is the value of the investment after 2 years?

A) 1000

B) 1200

C) 1250

D) 2000

Q50 — Hard

The graph of the function $y = f(x)$ models the height of a wave and passes through the point $(3, 8)$. The whole graph is then redrawn as $y = f(x) - 5$. Through which point must the new graph pass?

A) $(3,\ 3)$

B) $(-2,\ 8)$

C) $(8,\ 3)$

D) $(3,\ 13)$

Q51 — Hard

A bag used in a probability demonstration holds counters that are identical apart from their colour. The counts are recorded in the table below.

Colour	Red	Blue	Green	Total
Number of counters	5	7	8	20

A student draws one counter at random without looking into the bag. What is the probability that the counter drawn is **not** green?

A) $\frac{2}{5}$

B) $\frac{2}{3}$

C) $\frac{1}{3}$

D) $\frac{3}{5}$

Q52 — Hard

The perimeter P of a rectangular field is related to its length l and its width w by the formula $P = 2l + 2w$. A surveyor knows the perimeter and the length and needs the width. Which expression gives w in terms of P and l?

A) $P - 2l$

B) $\frac{P}{2} - l$

C) $\frac{P}{2} - 2l$

D) $\frac{P}{2} + l$

Q53 — Hard

Line ℓ has equation $y = \dfrac{4}{7}x - 5$. What is the slope of a line perpendicular to ℓ?

A) $\frac{4}{7}$

B) $\frac{7}{4}$

C) $-\frac{4}{7}$

D) $-\frac{7}{4}$

Q54 — Hard

A landscaper is marking out a rectangular flower bed. Its length is 3 metres more than twice its width, and the total distance around the outside of the bed is 42 metres. What is the width of the flower bed, in metres?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Paper 2 — Module 1

27 questions · 35 minutes · calculator permitted

Sit this one at least a week after Paper 1, and mark Paper 1 first — a mock is only useful if you have acted on what the last one told you.

Q55 — Basic

A courier charges a fixed collection fee plus an amount for each parcel. For one job the total charge, in dollars, satisfies $5x + 12 = 47$, where x is the number of parcels collected. What is the value of x?

A) $\frac{59}{5}$

B) 7

C) 35

D) $\frac{47}{5}$

Q56 — Basic

A factory's daily output, in units, is given by the expression $7x - 2(x - 5)$, where x is the number of machines running. Which of the following expressions is equivalent to it for every value of x?

A) $5x + 10$

B) $5x - 10$

C) $9x - 10$

D) $5x - 5$

Q57 — Basic

A tank is draining. The volume remaining, in litres, is modelled by the line $y = -2x + 9$, where x is the number of minutes since draining began. At what value of y does this line cross the y-axis?

A) -2

B) $\frac{9}{2}$

C) 9

D) 0

Q58 — Basic

A tailor charges a fitting fee and then an amount per metre of fabric. For one order the amount owed, in dollars, satisfies $7x - 9 = 33$, where x is the number of metres used. What is the value of x?

A) $\frac{24}{7}$

B) 6

C) 42

D) $\frac{33}{7}$

Q59 — Medium

A projectile is at ground level at the times that satisfy $x^2 - 9x + 20 = 0$, where x is measured in seconds. What is the sum of the two solutions of this equation?

A) -9

B) 9

C) 20

D) 1

Q60 — Medium

A town's population rises from 4500 to 5400. By what percent did it rise?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q61 — Medium

A colony of bacteria triples in each observation period. The population is found to have grown by a factor of 81. What is the value of x?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q62 — Medium

A market stall sells x bags of apples and y bags of pears. On one day the stall recorded $3x + y = 14$, and on another it recorded $x - y = 2$. What is the value of x?

A) 4

B) 2

C) 16

D) $\frac{7}{2}$

Q63 — Medium

A cylindrical water butt has a radius of 5 decimetres and a height of 4 decimetres. A gardener wants to know how much water it holds when full. What is the volume of the water butt, in cubic decimetres?

A) 20π

B) 40π

C) 100π

D) 400π

Q64 — Medium

The function g is defined by $g(x) = 2x^2 + 1$, where $g(x)$ gives the energy, in joules, stored by a spring displaced x centimetres from its resting position. What is the value of $g(-3)$?

A) -17

B) 37

C) -35

D) 19

Q65 — Medium

A caterer's budget satisfies $3x + 4y = 24$, where x is the number of trays of sandwiches and y the number of trays of fruit. The caterer wants to express the number of fruit trays in terms of the number of sandwich trays. Which expression gives y in terms of x?

A) $6 - \frac{3x}{4}$

B) $\frac{3x}{4} - 6$

C) $\frac{3x}{4} + 6$

D) $24 - 3x$

Q66 — Medium

A surveyor measures two of the three interior angles of a triangular plot of land and finds them to be 35° and 95°. What is the measure, in degrees, of the third angle of the plot?

A) 50

B) 130

C) 230

D) 40

Q67 — Medium

A microbiologist is growing a culture that begins with 60 cells and doubles in number every 3 hours, and the growth continues at that rate without interruption. The culture is left in the incubator for 12 hours. How many cells are there at the end of that period?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q68 — Medium

A print shop is enlarging a rectangular poster so that both its length and its width are doubled, keeping the same shape. The manager needs to know how much more ink the larger poster will require. By what factor does the area increase?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q69 — Medium

A printing job satisfies the relationship $5x - 2y = 20$, where x is the number of colour pages and y the number of black-and-white pages. Which expression gives y in terms of x?

A) $\frac{5x}{2} - 10$

B) $10 - \frac{5x}{2}$

C) $\frac{5x}{2} + 10$

D) $5x - 20$

Q70 — Medium

A bag used for a classroom demonstration holds 4 white marbles and 11 black marbles, identical apart from colour. A pupil draws a single marble at random without looking into the bag. What is the probability that the marble drawn is white?

A) $\frac{4}{15}$

B) $\frac{4}{11}$

C) $\frac{11}{15}$

D) $\frac{1}{4}$

Q71 — Medium

A linear function models the cost, in dollars, of running a machine for x hours. Four of its values were recorded in the table below.

Hours, x	**1**	**2**	**3**	**4**
Cost, $f(x)$	9	13	17	21

The slope of a linear function is the change in its output divided by the change in its input, and it is the same between any two points. What is the slope of the function?

A) 12

B) 3

C) 4

D) $\frac{1}{4}$

Q72 — Hard

An architect needs the expression $x^2 + 12x + k$ to factor as the square of a single bracket, so that a design calculation simplifies. What value must the constant k take?

A) 6

B) 12

C) 24

D) 36

Q73 — Hard

A thermostat keeps a room within a range described by $-4 \leq 2x - 6 < 8$, where x is the temperature in degrees Celsius. Which of the following gives all possible values of x?

A) $1 \leq x < 7$

B) $1 < x \leq 7$

C) $-1 \leq x < 7$

D) $1 \leq x \leq 7$

Q74 — Hard

A cinema sells adult tickets at $14 each and child tickets at $9 each, and every ticket sold is one or the other. On one evening the cinema sold 60 tickets in total and took $690 at the box office. How many child tickets were sold that evening?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q75 — Hard

Plan A costs $30 a month plus $2 per gigabyte. Plan B costs $50 a month with no per-gigabyte charge. At how many gigabytes do the two plans cost the same?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q76 — Hard

Two functions are defined by $f(x) = x - 4$ and $g(x) = x^2$. A calculation applies g to an input and then applies f to the result. What is the value of $f(g(3))$?

A) 1

B) 5

C) -1

D) 13

Q77 — Hard

A mobile phone mast covers a circular region described in the xy-plane by $(x-7)^2+(y+1)^2=49$, where distances are measured in kilometres. What are the coordinates of the centre of that region?

A) $(-7,\ 1)$

B) $(7,\ 1)$

C) $(7,\ -1)$

D) $(-7,\ -1)$

Q78 — Hard

A delivery van can carry at most 900 kg. The driver weighs 80 kg and each crate weighs 60 kg. Which inequality gives n, the number of crates that can be carried?

A) $60n \leq 900$

B) $60n + 80 \geq 900$

C) $60n - 80 \leq 900$

D) $60n + 80 \leq 900$

Q79 — Hard

Two classes sat the same test, and their results are summarised in the table below. Both classes reached the same average, but their scores are spread very differently around it.

Class	Mean	Lowest score	Highest score
A	70	68	72
B	70	45	95

Which statement about the two classes is true?

A) Class A has the greater standard deviation

B) Class B has the greater standard deviation

C) The standard deviations are equal, because the means are equal

D) Standard deviation cannot be judged from a range

Q80 — Hard

Two competing services quote for the same work. The first charges according to $y = 6x - 1$ and the second according to $y = kx + 5$, where x is the number of units ordered and k is a constant. For what value of k is there no order size at which the two quotes are equal?

A) -6

B) 5

C) 6

D) -1

Q81 — Hard

The graph of $y = x^2 + bx + 7$, where b is a constant, models the profit in thousands of dollars of a small business over its first few months of trading. The graph is known to pass through the point $(2, 1)$. What is the value of the constant b?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Paper 2 — Module 2

27 questions · 35 minutes · calculator permitted

The last module. Work steadily, and keep ten seconds at the end of each grid-in to substitute your answer back.

Q82 — Basic

A weather balloon's altitude, in hundreds of metres, is given by $8 - 3x$, where x is the number of minutes since it began to descend. At what value of x is the reading equal to 23?

A) 5

B) $\frac{31}{3}$

C) -5

D) $-\frac{31}{3}$

Q83 — Basic

Five less than four times a number x is equal to 31. Which equation represents this?

A) $5 - 4x = 31$

B) $4x - 5 = 31$

C) $4x - 20 = 31$

D) $4x + 5 = 31$

Q84 — Basic

A café's profit, in dollars, is given by the expression $4(2x + 3) - 5$, where x is the number of hours the café stays open past midday. Which of the following expressions is equivalent to it for every value of x?

A) $8x + 7$

B) $8x + 17$

C) $8x - 2$

D) $2x + 7$

Q85 — Basic

In the xy-plane, the depth of a lake at a distance x metres from the shore is modelled by the line $y = 7 - 2x$. What is the slope of this line?

A) 7

B) 2

C) $\frac{1}{2}$

D) -2

Q86 — Medium

A city taxi firm charges a fixed amount when the journey begins and then a further amount for every mile travelled, so that the total fare in dollars is $C = 4 + 2.5m$ for a journey of m miles. Which of the following is the best interpretation of the number 4 in this context?

A) The fixed charge before any distance is travelled

B) The cost of each mile

C) The number of miles included in the base fare

D) The total cost of a 4-mile journey

Q87 — Medium

The height of a drone above a launch pad returns to zero at the times satisfying $x^2 - 11x + 30 = 0$, where x is measured in seconds after launch. The operator is interested only in the earlier of the two times. What is that value of x?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q88 — Medium

If $\frac{2x}{5} - 3 = 7$, what is the value of x?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q89 — Medium

In a physics calculation the quantity $(2x^4)^3$ appears, where x is a positive real number. Which of the following expressions is equivalent to it for every value of x?

A) $2x^{12}$

B) $8x^{12}$

C) $6x^{12}$

D) $8x^7$

Q90 — Medium

A club sells x standard memberships and y premium memberships. The number of premium memberships is 5 more than the number of standard ones, so $y = x + 5$, and the club's income satisfies $3x + 2y = 25$. What is the value of y?

A) 8

B) 3

C) 5

D) $\frac{25}{2}$

Q91 — Medium

A triangular sail has a base of 18 metres and a perpendicular height of 11 metres. A sailmaker needs the area in order to order the right amount of cloth. What is the area of the sail, in square metres?

A) 198

B) 29

C) 99

D) $\frac{99}{2}$

Q92 — Medium

A component is accepted if the reading $3x + 2$ differs from zero by exactly 14 units, so that $|3x + 2| = 14$. What is the sum of all possible values of x?

A) 4

B) $-\frac{16}{3}$

C) 14

D) $-\frac{4}{3}$

Q93 — Medium

An accountant is depreciating a machine and models its value by $V = 12000(0.9)^t$ dollars, where t is the number of years since it was bought. The accountant needs to explain the model to a colleague. Which of the following is the best interpretation of the number 0.9 in this context?

A) The machine loses 90% of its value each year

B) The machine loses 10% of its value each year

C) The machine gains 90% of its value each year

D) The machine is worth $0.90 after t years

Q94 — Hard

A machine operates safely while $5 < 3x - 1 \leq 20$, where x is the pressure in bars. What is the greatest integer value of x for which the machine still operates safely?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q95 — Hard

In the equation $x^2 - 8x + c = 0$, c is a constant. The graph of the related function touches the x-axis at exactly one point. What is the value of c?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q96 — Hard

In the xy-plane, a path is drawn as the line $y = \frac{3}{4}x - 2$. A second path is drawn perpendicular to the first. What is the slope of the second path?

A) $\frac{3}{4}$

B) $\frac{4}{3}$

C) $-\frac{3}{4}$

D) $-\frac{4}{3}$

Q97 — Hard

A stationery shop sells pens at \$3 each and notebooks at \$7 each, and every item sold is one or the other. A customer buys 10 items altogether and pays \$54 in total. How many notebooks did the customer buy?

A) 4

B) 6

C) 10

D) 18

Q98 — Hard

The graph of the function $y = f(x)$ describes the depth of water in a harbour over the course of a day and passes through the point $(5, 2)$. A colleague redraws the whole graph as $y = f(x + 3)$ in order to compare it with a second harbour's readings. Through which point must the redrawn graph pass?

A) $(2,\ 2)$

B) $(8,\ 2)$

C) $(5,\ 5)$

D) $(5,\ -1)$

Q99 — Hard

A conservation group models the number of a bird species in a region by $P = 8000(0.94)^t$, where t is the number of years since monitoring began and P is the estimated population. The group is preparing a summary for the public. Which of the following statements is best supported by the model?

A) The population falls by 94% each year

B) The population falls by 800 each year

C) The population falls by 6% each year

D) The population grows by 6% each year

Q100 — Hard

A technician simplifies the expression $\dfrac{3x^2 - 12}{x - 2}$ that arises in a calculation, where $x \neq 2$. Which of the following expressions is equivalent to it for every allowed value of x?

A) $3x + 6$

B) $3x - 6$

C) $3x^2 - 6$

D) $3x + 2$

Q101 — Hard

A workshop makes a profit only when $7x - 4 > 3x + 20$, where x is the number of items produced in a day. What is the least integer number of items that yields a profit?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q102 — Hard

A drone is at ground level at the times satisfying $x^2 - 9x + 14 = 0$, where x is measured in seconds after launch. What is the sum of the two solutions of this equation?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Q103 — Hard

One gym charges a membership fee of \$25 a month and a further \$4 for each class attended, while a second gym charges a flat \$65 a month with no charge per class. A prospective member attends the same number of classes each month. At how many classes a month do the two gyms cost the same?

A) $\frac{65}{4}$

B) 10

C) $\frac{45}{2}$

D) 16

Q104 — Hard

The daily running cost of a generator, in dollars, is modelled by $y = x^2 - 12x + 40$, where x is the number of hours it runs. The graph of this function has a minimum. What is that minimum value?

A) 4

B) 6

C) 40

D) -6

Q105 — Hard

The concentration of a chemical, in parts per million, is modelled by $f(x) = x^2 - 6x + 11$, where x is the number of hours since mixing. The function has a minimum. What is that minimum value?

A) 2

B) 3

C) 11

D) -3

Q106 — Hard

A builder is installing a wheelchair ramp that rises at an angle of 12° to the horizontal ground, and the sloping surface of the ramp measures 9 metres from end to end. Building regulations require the vertical rise to be recorded. Which expression gives that height, in metres?

A) $9\sin 12^\circ$

B) $9\cos 12^\circ$

C) $\dfrac{9}{\sin 12^\circ}$

D) $9\tan 12^\circ$

Q107 — Hard

A model requires the value of x for which $\sqrt{x+6} = x$, where the square root denotes the non-negative root. What is the solution to $\sqrt{x+6} = x$?

A) -2

B) 1

C) 3

D) 6

Q108 — Hard

A groundskeeper is marking out a rectangular practice area whose length is twice its width, and the total distance around the outside of the area comes to 54 metres. The groundskeeper needs the area of the rectangle in order to order enough surfacing material. What is the area, in square metres?

There are no options — work the answer out and enter it yourself, in the form the grid accepts.

Answers and Explanations

Paper 1 — Module 1

Q1 — Basic — Answer: B

Topic: Solving a linear equation

$$6x = 30 \implies x = 5$$

Check: $30 - 4 = 26$ ✓ **A** subtracted on the wrong side, **C** used the wrong coefficient, **D** stopped one step early.

> **Takeaway:** Add before dividing, and finish the last step.

Q2 — Basic — Answer: D

Topic: Percentage of a number

$$0.30 \times 240 = 72$$

A divided, **B** subtracted 30, **C** added 30. A percentage *of* something is always a multiplication.

> **Takeaway:** "r% of b" means $\frac{r}{100} \times b$.

Q3 — Basic — Answer: D

Topic: Slope from two points

$$m = \frac{14 - 2}{5 - 1} = \frac{12}{4} = 3$$

B is upside down, **C** has the sign flipped, **A** added.

> **Takeaway:** Rise over run, same order top and bottom.

Q4 — Basic — Answer: D

Topic: Evaluating a function

$$f(5) = 25 - 15 = 10$$

B and **C** each computed one term and stopped. **A** slipped a sign.

> **Takeaway:** Substitute into every term, then combine.

Q5 — Medium — Answer: D

Topic: Expanding brackets

$$x^2 + 6x - 4x - 24 = x^2 + 2x - 24$$

B has $-2x$; the $+6$ is bigger than the -4, so the middle is positive. **C** skipped the middle. **A** multiplied rather than added them.

> **Takeaway:** Four products, then collect the middle two.

Q6 — Medium — Answer: 32

Topic: A linear equation with a fraction

$$\frac{x}{4} = 8 \quad \Longrightarrow \quad x = 32$$

Check: $8 + 3 = 11$ ✓ Enter 32.

> **Takeaway:** Subtract before multiplying up.

Q7 — Medium — Answer: 23

Topic: Mean from a total

The four must total $4 \times 21 = 84$. The three given total 61, so the fourth is $84 - 61 = 23$. Enter 23.

> **Takeaway:** Turn a mean into a total straight away.

Q8 — Medium — Answer: D

Topic: A system by substitution

$$2x + (3x - 2) = 13 \implies 5x = 15 \implies x = 3$$

Then $y = 7$. Check both: $y = 9 - 2 = 7$ ✓ and $6 + 7 = 13$ ✓ **C** gave y.

> **Takeaway:** Substitute, solve, then check which variable was asked for.

Q9 — Medium — Answer: B

Topic: Exponent rules

$(x^3)^4 = x^{12}$, then dividing subtracts: $x^{12-5} = x^7$.

A added instead of multiplying at the first step. **C** skipped the division. **D** multiplied everything.

> **Takeaway:** Power of a power multiplies; division subtracts.

Q10 — Medium — Answer: D

Topic: Ratio

$5 + 3 = 8$ parts, so one part is $48 \div 8 = 6$. Dogs are 3 parts: $3 \times 6 = 18$.

Check: cats $= 30$, and $18 + 30 = 48$ ✓ **B** gave the cats.

> **Takeaway:** Add the parts, find one part, then multiply.

Q11 — Medium — Answer: D

Topic: Solving a quadratic by factoring

$(x - 5)(x + 3) = 0$, so $x = 5$ or $x = -3$. The greatest is 5.

A gave the smaller. **B** and **C** flipped the signs.

> **Takeaway:** Factor, then read which root the question wants.

Q12 — Medium — Answer: D

Topic: Pythagoras

$$64 + 225 = 289 \implies c = 17$$

The 8–15–17 triple. **A** added the legs, **B** subtracted.

> **Takeaway:** Squares add, then take the root.

Q13 — Medium — Answer: 11

Topic: An absolute-value equation

$x - 7 = 4$ gives $x = 11$; $x - 7 = -4$ gives $x = 3$. The greatest is 11. Enter `11`.

> **Takeaway:** Two cases, then read the wording.

Q14 — Medium — Answer: 120

Topic: A unit rate

$45 \div 3 = 15$ bottles a minute, so $15 \times 8 = 120$. Enter `120`.

> **Takeaway:** Reduce to a per-one rate, then scale.

Q15 — Medium — Answer: B

Topic: The equation of a line

$$3 = 4(2) + b = 8 + b \implies b = -5$$

Check: at $x = 2$, $8 - 5 = 3$ ✓ **C** added instead of subtracting.

> **Takeaway:** Substitute the point into $y = mx + b$ and solve for b.

Q16 — Medium — Answer: D

Topic: Exponential growth

$$500 \times 1.2 = 600, \qquad 600 \times 1.2 = 720$$

A did one year only. **B** added 20% of the *original* twice, which ignores compounding — the second year's 20% is of 600, not 500. **C** worked out the second year's growth correctly, 120, and then added it to 500 instead of 600.

> **Takeaway:** Percentage growth compounds: multiply, do not add.

Q17 — Medium — Answer: D

Topic: Simplifying a rational expression

$$\frac{(x+4)(x-4)}{x+4} = x - 4$$

B kept the wrong factor. **C** and **A** cancelled terms, not factors.

> **Takeaway:** Factor first, cancel whole brackets only.

Q18 — Medium — Answer: B

Topic: A two-way table

The pool is restricted to Spanish students, so the denominator is 70:

$$P = \frac{45}{70} = \frac{9}{14}$$

A used all 120. **D** counted the other 25.

> **Takeaway:** "Chosen from those who..." fixes the denominator.

Q19 — Hard — Answer: B

Topic: Interpreting a model

The number multiplied by m is the rate: 30 cents per mile. **A** describes the 45. **C** would be $54.

> **Takeaway:** In $b + mx$, the multiplied number is the rate — check the units.

Q20 — Hard — Answer: 5

Topic: A system solved by elimination

Both have $4x$, so subtract: $4y = 20$, giving $y = 5$. Then $x = 3$.

Check: $12 + 15 = 27$ ✓ and $12 - 5 = 7$ ✓ Enter 5.

> **Takeaway:** Matching coefficients, subtract. Bracket the second equation so the double negative is visible.

Q21 — Hard — Answer: 25

Topic: The discriminant

One solution means $b^2 - 4ac = 0$:

$$100 - 4c = 0 \implies c = 25$$

Then $x^2 + 10x + 25 = (x + 5)^2$ ✓ Enter 25.

> **Takeaway:** Exactly one solution ⇒ discriminant zero ⇒ perfect square.

Q22 — Hard — Answer: A

Topic: A circle equation

The right side is $r^2 = 36$, so $r = 6$. The centre is $(-2, 5)$, which is option D — a different question.

> **Takeaway:** Take the square root of the right-hand side.

Q23 — Hard — Answer: B

Topic: Similar triangles

Areas scale by the **square** of the length ratio, so take square roots:

$$\sqrt{\frac{9}{25}} = \frac{3}{5}$$

A left the area ratio unchanged. **C** squared instead of rooting. **D** took the root and then wrote it upside down.

> **Takeaway:** Lengths k, areas k^2, volumes k^3. Go backwards with a root.

Q24 — Hard — Answer: D

Topic: A composite function

Inside first: $f(4) = 7$. Then $g(7) = 14$.

A is $f(g(4)) = 11$, the other order — always offered.

> **Takeaway:** $g(f(x))$ means f first. Work outward from the inner bracket.

Q25 — Hard — Answer: D

Topic: An inequality in context

$$35n + 75 \leq 500 \quad \Longrightarrow \quad 35n \leq 425 \quad \Longrightarrow \quad n \leq 12.1$$

You cannot carry part of a box, so round **down** to 12.

Check: $12 \times 35 + 75 = 495 \leq 500$ ✓ and $13 \times 35 + 75 = 530 > 500$ ✓

A forgot the operator. **B** rounded up — a count of physical objects always rounds down against a maximum.

> **Takeaway:** Include everything that adds to the load, then round **down**.

Q26 — Hard — Answer: B

Topic: Trigonometry

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{3}{5}$, so the adjacent side is 3 and the hypotenuse 5. The third side is $\sqrt{25-9} = 4$, and that is the opposite side:

$$\sin\theta = \frac{4}{5}$$

C is greater than 1, which no sine can be.

> **Takeaway:** Draw the 3–4–5 triangle from the ratio given, then read off the one you need.

Q27 — Hard — Answer: 48

Topic: Working backwards through a percentage

The new price is 125% of the old:

$$1.25 \times \text{old} = 60 \quad\Longrightarrow\quad \text{old} = \frac{60}{1.25} = 48$$

Check: 25% of 48 is 12, and $48 + 12 = 60$ ✓

Taking 25% *off* \$60 gives \$45, not \$48 — which is exactly why working backwards needs division, not a second percentage. Enter 48.

> **Takeaway:** To undo a percentage change, divide by the multiplier.

Paper 1 — Module 2

Q28 — Basic — Answer: C

Topic: Solving a linear equation

$$-2x = -8 \quad\Longrightarrow\quad x = 4$$

Check: $9 - 8 = 1$ ✓ **A** kept the minus after dividing two negatives.

> **Takeaway:** A negative divided by a negative is positive.

Q29 — Basic — Answer: B

Topic: Reading a linear function

$$f(0) = 4(0) + 7 = 7$$

A assumed an input of 0 gives an output of 0. **C** gave the slope.

> **Takeaway:** $f(0)$ is the y-intercept — the constant term.

Q30 — Basic — Answer: D

Topic: Area of a rectangle

$$A = 12 \times 7 = 84$$

B is the perimeter. **C** halved, which belongs to a triangle.

> **Takeaway:** Area multiplies; perimeter adds. Rectangles have no halving step.

Q31 — Medium — Answer: A

Topic: An equivalent expression

$$3(2x - 5) = 6x - 15, \qquad 6x - 15 + 4x = 10x - 15$$

B left the -5 undistributed. **C** dropped the x from $4x$.

> **Takeaway:** Distribute across every term, then collect like with like.

Q32 — Medium — Answer: C

Topic: A proportion

Cross-multiply: $4x = 60$, so $x = 15$.

Check: $\frac{15}{6} = 2.5$ and $\frac{10}{4} = 2.5$ ✓

> **Takeaway:** Cross-multiply, then check both fractions come to the same decimal.

Q33 — Medium — Answer: 3

Topic: A quadratic solved by factoring

Two numbers multiplying to -21 and adding to 4: that is 7 and -3.

$$(x+7)(x-3)=0 \quad \Longrightarrow \quad x=-7 \text{ or } x=3$$

The condition $x>0$ selects 3. Enter 3.

> **Takeaway:** Solve both roots, then let the stated condition choose.

Q34 — Medium — Answer: 15

Topic: Percentage change

The fall is $12. Percentage change is measured against the **original**:

$$\frac{12}{80}\times 100 = 15$$

Using 68 as the base would give 17.6%, which is the standard wrong answer. Enter 15.

> **Takeaway:** Percentage change divides by the **original** value, always.

Q35 — Medium — Answer: B

Topic: Parallel lines

Parallel lines have equal slopes, so the slope is -3.

C flipped and negated, which gives a **perpendicular** slope, $\frac{1}{3}$. **D** read the intercept.

> **Takeaway:** Parallel copies the slope exactly, sign included.

Q36 — Medium — Answer: A

Topic: Volume of a rectangular prism

$$V = 5 \times 4 \times 3 = 60$$

C is the surface area. Check what the question asked for — a volume is in cubic units, a surface area in square ones.

> **Takeaway:** Volume multiplies all three dimensions.

Q37 — Medium — Answer: C

Topic: Median with an even count

Sorted: $2, 4, 6, 8, 9, 11$. Six values, so average the middle two:

$$\frac{6+8}{2} = 7$$

A is the mean. **B** and **D** picked one middle value without averaging.

> **Takeaway:** Sort first. Even count means average the middle pair.

Q38 — Medium — Answer: D

Topic: A negative exponent

$$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

A negative exponent gives a small **positive** number, never a negative one.

> **Takeaway:** Negative exponent means reciprocal, not negative.

Q39 — Medium — Answer: B

Topic: A system with no solution

No solution means parallel: equal slopes, different intercepts. So $k = 4$, and the intercepts 1 and -3 differ ✓

C matched intercepts, which would make the lines *cross*, not miss.

> **Takeaway:** Parallel and distinct. Check the intercept after matching the slope.

Q40 — Hard — Answer: 4

Topic: A system solved by elimination

The y terms are $+2y$ and $-2y$, so **adding** the equations removes them:

$$8x = 32 \quad \Longrightarrow \quad x = 4$$

Then $5(4) + 2y = 26$ gives $y = 3$. Check both: $20 + 6 = 26$ ✓ and $12 - 6 = 6$ ✓ Enter 4.

> **Takeaway:** Opposite signs on a variable, add the equations. Then check both originals.

Q41 — Hard — Answer: 2

Topic: The vertex of a parabola

$$x = -\frac{6}{2} = -3, \qquad f(-3) = 9 - 18 + 11 = 2$$

Check: $f(-4) = 16 - 24 + 11 = 3$ and $f(-2) = 4 - 12 + 11 = 3$ — higher on both sides ✓

The question asked for the minimum **value**, which is the y, not the -3. Enter 2.

> **Takeaway:** Find $x = -\frac{b}{2a}$, substitute, and check whether the question wants where or what.

Q42 — Hard — Answer: A

Topic: Interpreting a scatterplot model

The constant is the value at $x = 0$. **B** describes the 6. The model says nothing about sample size or age range.

> **Takeaway:** Constant = value at zero; coefficient = rate. Check the units.

Q43 — Hard — Answer: C

Topic: An exponential decay model

"Decays **to** 85%" gives the factor directly: $b = 0.85$.

$$M = 400(0.85)^h$$

D used 0.15, the fraction lost. **B** is linear.

> **Takeaway:** "Decays to 85%" means $b = 0.85$; "decays **by** 15%" means the same thing. Read which word is used.

Q44 — Hard — Answer: C

Topic: A line through two points

$$m = \frac{16 - 4}{3 - (-1)} = \frac{12}{4} = 3$$

So $y = 3x + b$. Substitute $(-1, 4)$: $4 = -3 + b$, giving $b = 7$.

Check with the other point: $3(3) + 7 = 16$ ✓

A read a coordinate off a given point; **B** gave the slope.

> **Takeaway:** Slope first, then substitute either point for b — and confirm with the point you did not use.

Q45 — Hard — Answer: A

Topic: A radical equation

Square both sides:

$$3x + 1 = 16 \quad \Longrightarrow \quad 3x = 15 \quad \Longrightarrow \quad x = 5$$

Check in the original: $\sqrt{3(5)+1} = \sqrt{16} = 4 \checkmark$ — and with radicals the check is compulsory, because squaring can invent solutions.

B added 1 to both sides rather than squaring. **C** forgot to subtract the 1, and **D** added it instead of subtracting.

> **Takeaway:** Square, solve, then substitute back into the original.

Q46 — Hard — Answer: B

Topic: An arc of a circle

$\frac{40}{360} = \frac{1}{9}$ of the circle. The whole circumference is $2\pi(9) = 18\pi$, so the arc is

$$\frac{1}{9} \times 18\pi = 2\pi$$

A is the sector **area**, a different quantity — an arc is a length. **C** used 18 as the radius, which is the diameter. **D** is the whole circumference, with the fraction never applied.

> **Takeaway:** Fraction of the circle, times the circumference for an arc, times the area for a sector.

Q47 — Hard — Answer: 21

Topic: A system in context

Adding the two statements gives $2x = 66$, so the larger is 33 and the smaller is $54 - 33 = 21$.

Check: $21 + 33 = 54 \checkmark$ and $33 - 21 = 12 \checkmark$ Enter 21.

> **Takeaway:** The smaller of two numbers is $\frac{\text{sum} - \text{difference}}{2}$.

Q48 — Hard — Answer: 49

Topic: A perfect square

A perfect square is $(x+a)^2 = x^2 + 2ax + a^2$. Here $2a = -14$, so $a = -7$ and

$$c = a^2 = 49$$

Check: $(x-7)^2 = x^2 - 14x + 49$ ✓ Enter 49.

> **Takeaway:** Halve the middle coefficient, then square it.

Q49 — Hard — Answer: C

Topic: Compound percentage

$$800 \times 1.25 = 1000, \qquad 1000 \times 1.25 = 1250$$

A did one year only. **B** used **simple** interest — 25% of the original \$800 twice, which is \$400 rather than \$450. Compounding earns more, because the second year's interest is on \$1000, not \$800. **D** grew for one year and then doubled.

> **Takeaway:** Compound interest multiplies by the factor once per period. Compounding always beats simple interest over more than one period, and the gap is the whole point of the question.

Q50 — Hard — Answer: A

Topic: Function transformation

The -5 is outside the bracket, so it moves the graph **down** 5: $(3,8) \to (3,3)$.

B moved sideways, which only happens inside the bracket.

> **Takeaway:** Outside the bracket moves vertically, in the direction the sign says.

Q51 — Hard — Answer: D

Topic: Probability from a table

There are 20 counters, of which $5 + 7 = 12$ are not green:

$$\frac{12}{20} = \frac{3}{5}$$

Or take the complement: $1 - \frac{8}{20} = \frac{12}{20}$ ✓

A answered the opposite question.

> **Takeaway:** "Not" means subtract from 1, or count the ones you want directly. Both should agree.

Q52 — Hard — Answer: B

Topic: Solving for a variable in terms of others

$$2w = P - 2l \quad \Longrightarrow \quad w = \frac{P - 2l}{2}$$

A skipped the division. **C** divided only the P.

> **Takeaway:** Whatever you divide by must divide **every** term.

Q53 — Hard — Answer: D

Topic: A perpendicular slope

Flip **and** negate: $-\frac{7}{4}$. Check: $\frac{4}{7} \times -\frac{7}{4} = -1$ ✓

B and **C** each did one half of the job.

> **Takeaway:** Negative reciprocal is two steps. Multiply the two slopes; you should get exactly -1.

Q54 — Hard — Answer: 6

Topic: Putting several steps together

Let the width be w. Then the length is $2w + 3$, and the perimeter is

$$2(2w + 3) + 2w = 42$$
$$4w + 6 + 2w = 42 \quad \Longrightarrow \quad 6w = 36 \quad \Longrightarrow \quad w = 6$$

Check: width 6, length 15, perimeter $2(15+6)=42$ ✓

The question asked for the **width**, not the length — and with no options to warn you, re-reading the final line is the whole safety net. Enter 6.

> **Takeaway:** Define the letter, build the equation, solve, then check which of the two quantities was asked for. This is the last question of the module; do not rush it.

Paper 2 — Module 1

Q55 — Basic — Answer: B

Topic: Solving a linear equation

$$5x = 35 \qquad \Longrightarrow \qquad x = 7$$

Check: $35 + 12 = 47$ ✓ **C** stopped one step early.

> **Takeaway:** Subtract, then divide, then finish.

Q56 — Basic — Answer: A

Topic: Simplifying an expression

$$-2(x-5) = -2x + 10, \qquad 7x - 2x + 10 = 5x + 10$$

B kept the constant negative — but $-2 \times -5 = +10$.

> **Takeaway:** A minus before a bracket flips every sign inside it.

Q57 — Basic — Answer: C

Topic: Reading a graph's intercept

On the y-axis, $x=0$, so $y=9$.

B is the x-intercept, found by setting $y=0$.

> **Takeaway:** y-axis means $x=0$. Each axis zeroes the *other* variable.

Q58 — Basic — Answer: B

Topic: Solving a two-step linear equation

$$7x = 42 \quad \Longrightarrow \quad x = 6$$

Check: $42 - 9 = 33$ ✓ **A** subtracted where it should have added; **C** stopped a step early.

> **Takeaway:** Undo the subtraction first, then the multiplication — and finish.

Q59 — Medium — Answer: B

Topic: A quadratic with two roots

Sum of roots $= -\frac{b}{a} = 9$. (The roots are 4 and 5.)

C is the product, $\frac{c}{a} = 20$.

> **Takeaway:** Sum $= -\frac{b}{a}$, product $= \frac{c}{a}$.

Q60 — Medium — Answer: 20

Topic: A percentage increase

The rise is 900, measured against the **original** 4500:

$$\frac{900}{4500} \times 100 = 20$$

Using 5400 as the base would give 16.7%, the standard wrong answer. Enter `20`.

> **Takeaway:** Percentage change divides by the original.

Q61 — Medium — Answer: 4

Topic: An exponent equation

Write 81 as a power of 3, so both sides share a base:

$$81 = 3 \times 3 \times 3 \times 3 = 3^4$$

The equation becomes $3^x = 3^4$. With the same base on both sides, the exponents must be equal:

$$x = 4$$

Enter 4.

> **Takeaway:** Rewrite both sides over a common base, then match the exponents. The powers of 2, 3 and 5 are the ones this test reuses, so knowing them to the fifth power or so is worth the memorising.

Q62 — Medium — Answer: A

Topic: A system by elimination

The y terms are $+y$ and $-y$, so **add** the equations: $4x = 16$, giving $x = 4$. Then $y = 2$.

Check: $12 + 2 = 14$ ✓ and $4 - 2 = 2$ ✓

B gave y. **C** stopped at $4x = 16$. **D** added only the left sides, using $4x = 14$.

> **Takeaway:** Opposite signs, add the equations.

Q63 — Medium — Answer: C

Topic: Volume of a cylinder

$$V = \pi r^2 h = \pi(25)(4) = 100\pi$$

A used r rather than r^2; **B** is a surface area.

> **Takeaway:** Square the radius, not the height.

Q64 — Medium — Answer: D

Topic: Function notation

$$g(-3) = 2(9) + 1 = 19$$

$(-3)^2 = 9$, not -9. **A** made that error.

> **Takeaway:** Square inside brackets: a negative squared is positive.

Q65 — Medium — Answer: A

Topic: Solving for one variable in terms of another

$$4y = 24 - 3x \quad \Longrightarrow \quad y = \frac{24 - 3x}{4}$$

Check at $x = 4$: the original gives $12 + 4y = 24$, so $y = 3$; the answer gives $\frac{24-12}{4} = 3$ ✓

D never divided by 4.

> **Takeaway:** Whatever divides must divide **every** term. Check with one value.

Q66 — Medium — Answer: A

Topic: Angles in a triangle

$$180 - 35 - 95 = 50$$

Check: $35 + 95 + 50 = 180$ ✓

B added the two given angles. **C** used 360°, the total for a quadrilateral. **D** assumed a right angle the question never mentions.

> **Takeaway:** Triangle angles total 180°. Add all three back as a check.

Q67 — Medium — Answer: 960

Topic: An exponential model

$\frac{12}{3} = 4$ periods, so the factor is $2^4 = 16$:

$$60 \times 16 = 960$$

Check by stepping: $60 \to 120 \to 240 \to 480 \to 960$ ✓ Enter `960`.

> **Takeaway:** Divide the time by the period length to count the periods.

Q68 — Medium — Answer: 4

Topic: Scaling an area

$$(2l)(2w) = 4lw$$

Both dimensions double, so the area is multiplied by $2^2 = 4$. Try it: a 3×5 rectangle has area 15; a 6×10 one has area 60 ✓ Enter `4`.

> **Takeaway:** Lengths $\times k$, areas $\times k^2$, volumes $\times k^3$.

Q69 — Medium — Answer: A

Topic: Solving for a variable in terms of another

$$-2y = 20 - 5x \quad \Longrightarrow \quad y = \frac{5x - 20}{2}$$

Check at $x = 4$: the original gives $20 - 2y = 20$, so $y = 0$; and the answer gives $\frac{20-20}{2} = 0$ ✓

D never divided by 2.

> **Takeaway:** Whatever divides must divide every term. Check with one value.

Q70 — Medium — Answer: A

Topic: Probability

There are 15 marbles altogether, 4 of them white: $\frac{4}{15}$.

B used 11 as the denominator, which is the count of the *other* colour, not the total.

> **Takeaway:** The denominator is the total number of outcomes, not the number of the ones you do not want.

Q71 — Medium — Answer: C

Topic: Slope from a table

$$m = \frac{21 - 9}{4 - 1} = \frac{12}{3} = 4$$

A and **B** each gave one of the two changes without dividing.

> **Takeaway:** Slope is a change divided by a change, not either one alone.

Q72 — Hard — Answer: D

Topic: A perfect-square trinomial

Halve the middle coefficient and square it: $\left(\frac{12}{2}\right)^2 = 36$.

Check: $(x + 6)^2 = x^2 + 12x + 36$ ✓ **A** halved and forgot to square.

> **Takeaway:** Halve, then square. Two steps.

Q73 — Hard — Answer: A

Topic: A compound inequality

Add 6 to all three parts: $2 \leq 2x < 14$. Divide by 2: $1 \leq x < 7$.

Each end keeps the sign it started with — the left inclusive, the right strict.

Check: at $x = 1$, the middle is -4, and $-4 \leq -4$ is true ✓ At $x = 7$, the middle is 8, and $8 < 8$ is false ✓

> **Takeaway:** Do every step to all three parts and carry each end's sign through. Test both endpoints in the original.

Q74 — Hard — Answer: 30

Topic: A word problem with two unknowns

Let a and c be the two counts:

$$a + c = 60 \qquad 14a + 9c = 690$$

Substituting $a = 60 - c$:

$$14(60 - c) + 9c = 690 \implies 840 - 5c = 690 \implies c = 30$$

Check: 30 adults and 30 children give $420 + 270 = 690$ ✓ Enter `30`.

> **Takeaway:** One equation counts the items, the other counts the money. Then read which count the question asked for.

Q75 — Hard — Answer: 10

Topic: A model compared across two options

$$30 + 2g = 50 \implies 2g = 20 \implies g = 10$$

Below 10 GB plan A is cheaper; above it, plan B. Enter `10`.

> **Takeaway:** "Cost the same" means set the two expressions equal.

Q76 — Hard — Answer: B

Topic: Composite functions

Inside first: $g(3) = 9$. Then $f(9) = 5$.

A is $g(f(3)) = 1$, the other order.

> **Takeaway:** $f(g(x))$ means g first — work outward.

Q77 — Hard — Answer: C

Topic: A circle's centre

$(x - h)^2 + (y - k)^2 = r^2$ has centre (h, k), and the signs **flip**. $(x - 7)$ gives $h = 7$; $(y + 1)$ is $(y - (-1))$, so $k = -1$.

> **Takeaway:** Read the bracket, then flip the sign. Both coordinates.

Q78 — Hard — Answer: D

Topic: A linear inequality in context

The load is the crates **plus the driver**, and "at most 900" means $\leq$:

$$60n + 80 \leq 900$$

A left the driver out, **B** points the wrong way, **C** subtracts a weight that adds.

> **Takeaway:** "At most" is $\leq$. Include everything that adds to the load.

Q79 — Hard — Answer: B

Topic: Interpreting standard deviation

Standard deviation measures **spread**, not centre. Class A's scores sit within 4 marks of each other; class B's span 50. B is far more spread out.

C confuses the two ideas: equal means say nothing about spread.

> **Takeaway:** Same mean, different spread is entirely possible — that is what standard deviation exists to describe.

Q80 — Hard — Answer: C

Topic: A system with no solution

No solution means parallel: equal slopes, different intercepts. So $k = 6$, and the intercepts -1 and 5 differ ✓

B matched intercepts, which would make the lines cross rather than miss.

> **Takeaway:** Parallel **and** distinct. Check the intercept after matching the slope.

Q81 — Hard — Answer: -5

Topic: Putting several ideas together

Substitute the point into the equation:

$$1 = 2^2 + 2b + 7 = 4 + 2b + 7 = 11 + 2b$$

$$2b = -10 \implies b = -5$$

Check: $4 - 10 + 7 = 1$ ✓

This answer is negative. Enter `-5`, minus sign included — it counts as one of the six characters a negative answer is allowed.

> **Takeaway:** A point on a graph satisfies its equation. Substitute both coordinates and solve for the unknown constant.

Paper 2 — Module 2

Q82 — Basic — Answer: C

Topic: Solving a linear equation

$$-3x = 15 \implies x = -5$$

Check: $8 + 15 = 23$ ✓ **A** dropped the minus sign.

> **Takeaway:** Subtract first, then divide — and carry the sign through.

Q83 — Basic — Answer: B

Topic: Translating a sentence into an equation

"Four times a number" is $4x$; "five less than" that is $4x - 5$.

$$4x - 5 = 31$$

A reversed the order — "five less than y" is $y - 5$, never $5 - y$. **D** added.

> **Takeaway:** "Less than" subtracts **and** reverses the order.

Q84 — Basic — Answer: A

Topic: Expanding a bracket

$$4(2x + 3) = 8x + 12, \qquad 8x + 12 - 5 = 8x + 7$$

C left the 3 undistributed.

> **Takeaway:** Distribute to every term inside, then combine the constants.

Q85 — Basic — Answer: D

Topic: The slope of a line

Rewrite as $y = -2x + 7$. The slope is the number multiplying x: -2.

A read the intercept. **B** lost the minus.

> **Takeaway:** Put the equation in $y = mx + b$ order before reading anything off.

Q86 — Medium — Answer: A

Topic: Reading a linear model

Set $m = 0$: the fare is already $4 before any distance. That is the fixed charge.

B describes the 2.5. **D** would be $14.

> **Takeaway:** Substitute zero to see what the constant means, and check the units — dollars cannot be miles.

Q87 — Medium — Answer: 5

Topic: A quadratic solved by factoring

Two numbers multiplying to 30 and adding to -11: -5 and -6.

$$(x-5)(x-6)=0 \quad\Longrightarrow\quad x=5 \text{ or } x=6$$

The condition $x<6$ selects 5. Enter 5.

> **Takeaway:** Find both roots, then let the stated condition pick one.

Q88 — Medium — Answer: 25

Topic: A linear equation with a fraction

Add 3 to both sides:

$$\frac{2x}{5}=10$$

Multiply by 5, then divide by 2:

$$2x=50 \quad\Longrightarrow\quad x=25$$

Check: $\frac{50}{5}-3=10-3=7\checkmark$ Enter 25.

> **Takeaway:** Clear the constant first, then the fraction. Substituting back is your only safety net on a grid-in.

Q89 — Medium — Answer: B

Topic: Exponent rules

$$2^3=8, \qquad (x^4)^3=x^{12}$$

so $(2x^4)^3=8x^{12}$.

C multiplied the coefficient by 3 rather than cubing it.

> **Takeaway:** The outside power applies to the coefficient too — and it cubes it, not triples it.

Q90 — Medium — Answer: A

Topic: A system by substitution

$$3x + 2(x+5) = 25 \quad \Longrightarrow \quad 5x + 10 = 25 \quad \Longrightarrow \quad x = 3$$

Then $y = 3 + 5 = 8$.

Check: $9 + 16 = 25$ ✓ **B** stopped at x.

> **Takeaway:** Substitute, solve, then finish by finding the variable asked for.

Q91 — Medium — Answer: C

Topic: Area of a triangle

$$A = \tfrac{1}{2}(18)(11) = 99$$

A forgot the $\frac{1}{2}$ — the usual slip.

> **Takeaway:** The half is part of the formula. It is on the reference sheet; copy it before substituting.

Q92 — Medium — Answer: D

Topic: An absolute-value equation

$$3x + 2 = 14 \Longrightarrow x = 4 \qquad 3x + 2 = -14 \Longrightarrow x = -\tfrac{16}{3}$$

Their sum is $4 - \frac{16}{3} = -\frac{4}{3}$.

A and **B** each kept one case only.

> **Takeaway:** Absolute value gives two cases, and "sum of all possible values" needs both of them.

Q93 — Medium — Answer: B

Topic: Reading an exponential model

The factor is what **remains**: $1 - 0.9 = 0.1$, so 10% is lost each year.

Check: $12000 \times 0.9 = 10800$, a drop of 1200, which is 10% of 12000 ✓

A read the factor as the loss.

> **Takeaway:** The factor is what survives. Subtract from 1 for the percentage lost.

Q94 — Hard — Answer: 7

Topic: A three-part inequality

Add 1 to all three parts: $6 < 3x \leq 21$. Divide by 3: $2 < x \leq 7$.

The greatest integer is 7, and the right end is inclusive so 7 is allowed.

Check: at $x = 7$ the middle is 20, and $20 \leq 20$ ✓ At $x = 8$ it is 23, which is too big ✓ Enter 7.

> **Takeaway:** Every step to all three parts. Then check whether the end you want is inclusive.

Q95 — Hard — Answer: 16

Topic: The discriminant

One solution means $b^2 - 4ac = 0$:

$$64 - 4c = 0 \quad \Longrightarrow \quad c = 16$$

Then $x^2 - 8x + 16 = (x - 4)^2$ ✓ Enter 16.

> **Takeaway:** One solution ⇒ discriminant zero ⇒ perfect square.

Q96 — Hard — Answer: D

Topic: Perpendicular lines

Flip **and** negate: $-\frac{4}{3}$. Check: $\frac{3}{4} \times -\frac{4}{3} = -1$ ✓

B and **C** each did one half of the job.

> **Takeaway:** Negative reciprocal means both steps. Multiply the two slopes; you should get exactly -1.

Q97 — Hard — Answer: B

Topic: A system built from a situation

Let p be pens and n notebooks:

$$p + n = 10 \qquad 3p + 7n = 54$$

Substituting $p = 10 - n$: $30 - 3n + 7n = 54$, so $4n = 24$ and $n = 6$.

Check: 4 pens and 6 notebooks give $12 + 42 = 54$ ✓

A is the pen count — both come out, and the question named notebooks.

> **Takeaway:** One equation counts the items, the other the money. Then read which count was asked for.

Q98 — Hard — Answer: A

Topic: A function transformation

The change is **inside** the bracket, so it is horizontal and goes **opposite** to the sign: $f(x+3)$ moves the graph **left** 3.

Track the point: we want the input making the bracket equal 5, so $x + 3 = 5$ and $x = 2$. Then $(2, 2)$.

B moved right, following the plus sign.

> **Takeaway:** Inside the bracket, opposite to the sign. Track one known point.

Q99 — Hard — Answer: C

Topic: An exponential decay factor

The factor 0.94 is what **remains**, so $1 - 0.94 = 0.06$ is lost: **6% a year**.

Check: $8000 \times 0.94 = 7520$, a drop of 480, and $\frac{480}{8000} = 0.06$ ✓

A read the factor as the loss. **B** would be a linear model.

> **Takeaway:** The factor is what survives. Subtract from 1 for the percentage lost.

Q100 — Hard — Answer: A

Topic: Simplifying a rational expression

Take out the 3, then use the difference of two squares:

$$3x^2 - 12 = 3(x^2 - 4) = 3(x+2)(x-2)$$
$$\frac{3(x+2)(x-2)}{x-2} = 3(x+2)$$

Check at $x = 0$: the original is $\frac{-12}{-2} = 6$, and $3(2) = 6$ ✓

> **Takeaway:** Common factor, then the pattern, then cancel whole brackets.

Q101 — Hard — Answer: 7

Topic: An inequality with the variable on both sides

Subtract $3x$: $4x - 4 > 20$. Add 4: $4x > 24$. Divide by 4, which is positive, so nothing flips:

$$x > 6$$

The inequality is **strict**, so $x = 6$ does not qualify and the least integer is 7.

Check: at $x = 7$, $45 > 41$ ✓ At $x = 6$, $38 > 38$ is false ✓ Enter 7.

> **Takeaway:** Move the variable to the side that keeps it positive, then read the sign: $>$ excludes the boundary, so the answer is the next integer up.

Q102 — Hard — Answer: 9

Topic: The sum of a quadratic's solutions

Two numbers multiplying to 14 and adding to -9: -2 and -7.

$$(x-2)(x-7)=0 \quad\Longrightarrow\quad x=2 \text{ or } x=7$$

Their sum is 9. Or read it straight off: sum $= -\frac{b}{a} = 9$, with no factoring at all. Enter 9.

> **Takeaway:** Sum of roots $= -\frac{b}{a}$, product $= \frac{c}{a}$. When only the sum is wanted, that is the whole question.

Q103 — Hard — Answer: B

Topic: A linear model compared to a fixed cost

$$25+4n=65 \quad\Longrightarrow\quad 4n=40 \quad\Longrightarrow\quad n=10$$

Check: $25+40=65$ ✓ **A** ignored the monthly fee.

> **Takeaway:** "Cost the same" sets the two expressions equal. Subtract the fixed part before dividing.

Q104 — Hard — Answer: A

Topic: The vertex of a parabola

$$x=-\frac{-12}{2}=6, \qquad y=36-72+40=4$$

Check: $f(5)=25-60+40=5$ and $f(7)=49-84+40=5$ — higher on both sides ✓

B gave *where* the minimum is; the question asked *what* it is.

> **Takeaway:** Find $x=-\frac{b}{2a}$, then substitute. Read which of the two numbers is wanted.

Q105 — Hard — Answer: A

Topic: The minimum of a quadratic

$$x = -\frac{-6}{2} = 3, \qquad f(3) = 9 - 18 + 11 = 2$$

Check either side: $f(2) = 3$ and $f(4) = 3$, both higher ✓

B gives *where* the minimum is; the question asked *what* it is.

> **Takeaway:** Find $x = -\frac{b}{2a}$, then substitute. Read which of the two numbers is wanted.

Q106 — Hard — Answer: A

Topic: Trigonometry in context

The ramp is the **hypotenuse** and the height is **opposite** the 12° angle. Opposite and hypotenuse means sine:

$$\sin 12^\circ = \frac{h}{9} \quad \Longrightarrow \quad h = 9 \sin 12^\circ$$

Sanity-check the size: the height must be **less** than the 9 m ramp. Sine of an acute angle is under 1, so multiplying shrinks it ✓ Option C divides, giving about 43 m — a ramp 9 m long cannot rise 43 m.

> **Takeaway:** Label hypotenuse, opposite and adjacent, then pick the ratio. Check the answer's size against the geometry.

Q107 — Hard — Answer: C

Topic: A radical equation with an extraneous solution

Square both sides to clear the root:

$$x + 6 = x^2$$

Bring everything to one side and factor:

$$x^2 - x - 6 = 0 \quad \Longrightarrow \quad (x-3)(x+2) = 0$$

So the squared equation has two solutions, $x = 3$ and $x = -2$. **Both must now be tested in the original equation**, because squaring turns a false statement into a true one — $-2 \neq 2$, but $(-2)^2 = 2^2$.

At $x = 3$: $\sqrt{3+6} = \sqrt{9} = 3$ ✓

At $x = -2$: $\sqrt{-2+6} = \sqrt{4} = 2$, and $2 \neq -2$ ✗

The square-root symbol means the **non-negative** root, so it can never equal a negative number. $x = -2$ is extraneous — it was manufactured by the squaring.

$$x = 3$$

Why each wrong option is wrong:

- **A,** -2 — kept the extraneous root without testing it. This is the whole point of the question: squaring is not a reversible step.
- **B,** 1 — added the two roots of the squared equation. The question asks for the solution, not for a sum.
- **D,** 6 — doubled the right-hand side instead of squaring the left, solving $x + 6 = 2x$.

> **Takeaway:** whenever you square both sides, every solution you find is only a candidate. Substitute each one back into the **original** equation. A square root is never negative, so an equation like $\sqrt{\cdot} = x$ has no solution where $x < 0$.

Q108 — Hard — Answer: 162

Topic: Several steps together

Let the width be w; the length is $2w$. The perimeter:

$$2(2w) + 2w = 6w = 54 \quad \Longrightarrow \quad w = 9$$

So the width is 9 and the length 18. The question asks for the **area**:

$$18 \times 9 = 162$$

Check the perimeter: $2(18 + 9) = 54$ ✓

Two traps in one question. The first is stopping at $w = 9$; the second is giving the perimeter-shaped answer. With no options to warn you, re-read the last line. Enter `162`.

> **Takeaway:** Solve for the unknown, then do the extra step the question actually asks for. This is the last question of the paper — read it twice.

Chapter 10 — The Challenge Set

Twenty questions, all of them harder than anything the SAT is likely to ask you.

That is deliberate. Every other chapter in this book is calibrated to the real test. This one is not: it is calibrated to the *edge* of what the test's own content can be made to do. If you can work through these, the real thing will feel narrow.

What makes these different. A normal SAT question tests one idea and tells you, more or less, which idea it is. These do neither. Most of them need two or three ideas joined together, and none of them announces which. Several have a first answer that is wrong for a reason you will only see if you check — an extraneous root, a case that collapses, a ratio applied to the wrong quantity.

Nothing here is off-syllabus. There is no calculus, no matrices beyond what Chapter 6 covers, nothing you have not already met. The difficulty is entirely in the combination and in the traps, not in unfamiliar content. That is also true of the hardest real SAT questions, which is why practising this way transfers.

How to use it. Attempt a question with a pen and no time limit before you look at anything. If you get it out in under two minutes you were probably supposed to. If you get stuck, the thing to look for is usually the second condition — the clause you read past.

A warning about the wrong options. In this chapter every wrong option is a value you can actually arrive at by making one specific, reasonable mistake. None of them are padding. Getting a "nice" answer here is not evidence you are right.

Topics covered: parameters that make systems degenerate · tangency · extraneous roots · exponential models from two points · absolute value equations with two cases · circles and distance · similarity and area · complementary angles · composition · compound percentage change · the factor theorem · conditional probability · simultaneous constraints

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Section 1 — When a Parameter Changes Everything

Several of the hardest questions on the test hide a constant in the coefficients and ask what value it must take. The move is always the same: write the condition the question describes as an equation *in that constant*, then solve it. The trap is almost always that the equation you get has more roots than the question allows.

Q1 — Brutal

In the system of equations below, k is a constant.

$$\begin{cases} kx + 4y = 10 \\ 9x + ky = 15 \end{cases}$$

The system has no solution. What is the value of k?

A) -6

B) 6

C) 0

D) 36

Q2 — Brutal

In the xy-plane, the graph of $y = x^2 + kx + 9$ and the line $y = 3x + 5$ intersect at exactly one point, where k is a constant. What is the sum of all possible values of k?

A) -1

B) 7

C) 6

D) -6

Q3 — Brutal

A population is modelled by $f(x) = ab^x$, where a and b are constants and $b > 0$. It is known that $f(2) = 18$ and $f(5) = 486$. What is the value of $f(0)$?

A) 6

B) 2

C) 3

D) 27

Section 2 — Answers That Are Not Solutions

Squaring both sides, multiplying by a denominator and taking a root can all create values that satisfy the *equation you wrote* but not the *equation you started with*. On a hard question that extra value is always offered as an option.

Q4 — Brutal

What is the sum of all values of x that satisfy $|2x - 5| = |x + 4|$?

A) $\frac{26}{3}$

B) 9

C) $\frac{1}{3}$

D) $\frac{28}{3}$

Q5 — Brutal

What is the value of x that satisfies $\sqrt{x + 7} = x - 5$?

A) 9

B) 2

C) 11

D) 18

Section 3 — Two Ideas at Once

The rest of the chapter is questions that need something from one topic and something from another. There is no signal in the wording about which two.

Q6 — Brutal

In the xy-plane, a circle has equation $x^2 + y^2 - 10x + 6y + 18 = 0$. What is the shortest distance from the origin to the circle?

A) $4 + \sqrt{34}$

B) $\sqrt{34}$

C) $-4 + \sqrt{34}$

D) 4

Q7 — Brutal

Two similar triangles have areas in the ratio $9 : 25$. The perimeter of the smaller triangle is 24 centimetres. What is the perimeter of the larger triangle, in centimetres?

A) $\frac{200}{3}$

B) 40

C) $\frac{72}{5}$

D) 120

Q8 — Brutal

In a right triangle, $\sin(x^\circ) = \cos((3x - 10)^\circ)$, where x is an acute angle measure. What is the value of x?

A) 50

B) 20

C) $\frac{45}{2}$

D) 25

Q9 — Brutal

Let $f(x) = x^2$ and $g(x) = x + 3$. For what value of x does $f(g(x)) = g(f(x))$?

A) -1

B) 1

C) -3

D) -2

Q10 — Brutal

The price of an item is increased by 20% and the new price is then decreased by 20%. The final price is what percentage of the original price?

A) 104

B) 100

C) 96

D) 80

Q11 — Brutal

The polynomial $p(x) = 2x^3 + ax^2 - 5x + b$ is divisible by both $(x - 1)$ and $(x + 2)$, where a and b are constants. What is the value of $a + b$?

A) 1

B) 3

C) 2

D) 6

Q12 — Brutal

The mean of the 20 scores in a class is 72. One score was recorded as 35 when it should have been 75. What is the correct mean?

A) 76

B) $\frac{303}{4}$

C) 72

D) 74

Q13 — Brutal

The table shows the results of an examination for 100 candidates.

	Passed	**Failed**	**Total**
Morning session	45	15	60
Evening session	28	12	40
Total	**73**	**27**	**100**

One candidate is selected at random from those who **passed**. What is the probability that this candidate sat the morning session?

A) $\frac{45}{73}$

B) $\frac{3}{4}$

C) $\frac{9}{20}$

D) $\frac{73}{100}$

Q14 — Brutal

What is the value of x that satisfies $4^{x+1} = 8^{x-1}$?

A) -1

B) 3

C) 5

D) 2

Q15 — Brutal

The numbers x, y and z satisfy $x + y = 7$, $y + z = 9$ and $x + z = 12$. What is the value of x?

A) 2

B) 5

C) 7

D) 14

Q16 — Brutal

The function $f(x) = x^2 - 6x + c$ has a minimum value of 4, where c is a constant. What is the value of c?

A) 22

B) 4

C) -5

D) 13

Q17 — Brutal

A tap can fill a tank in 6 hours. A drain can empty the full tank in 9 hours. If both are open from the moment the tank is empty, how many hours does it take to fill?

A) 18

B) $\frac{18}{5}$

C) 15

D) 3

Q18 — Brutal

The inequality $x^2 - 6x + c > 0$ is true for **all** real values of x, where c is a constant. What is the least possible integer value of c?

A) 3

B) 9

C) 10

D) 36

Q19 — Brutal

A quarter circle of radius 10 is drawn inside a square of side 10, centred on one corner of the square. What is the area of the region inside the square but outside the quarter circle?

A) $100 - 100\pi$

B) $100 - 25\pi$

C) $100 - 50\pi$

D) 25π

Q20 — Brutal

Tank A holds 240 litres and is draining at 15 litres per minute. Tank B holds 60 litres and is filling at 5 litres per minute. After how many minutes do the two tanks hold the same volume?

A) 18

B) 105

C) 12

D) 9

Answers and Explanations

Section 1 — When a Parameter Changes Everything

Q1 — Brutal — Answer: A

Topic: A parameter that makes a system have no solution

Two straight lines fail to meet in exactly one situation: they are parallel and they are not the same line. Both halves matter here, and the second half is the whole question.

Parallel first. Two equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ describe parallel lines when the coefficients are in proportion:

$$\frac{k}{9} = \frac{4}{k} \quad \Longrightarrow \quad k^2 = 36 \quad \Longrightarrow \quad k = 6 \text{ or } k = -6$$

Now test each one, which is the step that gets skipped.

If $k = 6$ the system is $6x + 4y = 10$ and $9x + 6y = 15$. Multiply the first by $\frac{3}{2}$: $9x + 6y = 15$. That is the second equation exactly. The two equations are the *same line*, so every point on it is a solution — infinitely many, not none.

If $k = -6$ the system is $-6x + 4y = 10$ and $9x - 6y = 15$. Multiply the first by $-\frac{3}{2}$: $9x - 6y = -15$. The left sides now agree and the right sides do not, $-15 \neq 15$. Parallel, distinct, no solution.

So $k = -6$, option -6.

Why each wrong option is wrong:

- **B,** 6 — solved $k^2 = 36$ and took the positive root. This is the coincident case: it gives *infinitely many* solutions, which is the opposite of what was asked. Proportional coefficients only make lines parallel; whether they are the *same* line depends on the constants on the right, and nothing about $k^2 = 36$ checks those.
- **C,** 0 — took both roots and added them, reading "what is the value of k" as "the sum of the possible values". Only one value survives the test, so there is nothing to sum.
- **D,** 36 — solved as far as $k^2 = 36$ and stopped. A very common way to lose a hard question: the algebra was right and the last step was never taken.

> **Takeaway:** "No solution" is two conditions, not one — parallel *and* distinct. Whenever a proportionality gives you two candidate values, substitute both back and look at the constants. One of them is usually the coincident case, and it is usually the one that looks nicer.

Q2 — Brutal — Answer: C

Topic: A line tangent to a parabola

"Intersect at exactly one point" means the equation you get by setting the two expressions equal has exactly one solution.

$$x^2 + kx + 9 = 3x + 5 \quad \Longrightarrow \quad x^2 + (k-3)x + 4 = 0$$

A quadratic has exactly one solution when its discriminant is zero:

$$(k-3)^2 - 4(1)(4) = 0 \quad \Longrightarrow \quad (k-3)^2 = 16 \quad \Longrightarrow \quad k - 3 = \pm 4$$

So $k = 7$ or $k = -1$, and their sum is 6 — option 6.

Note what happened to the constant: the 9 and the 5 combined into a 4 before the discriminant was taken. Taking the discriminant of the original $x^2 + kx + 9$ is the single most common way to lose this question.

Why each wrong option is wrong:

- **B,** 7 — found both values correctly and then reported only $k = 7$. The question asks for the sum, and a question that asks for a sum is telling you there is more than one value.
- **A,** -1 — the same slip the other way round.
- **D,** -6 — took $k - 3 = \pm 4$ and wrote $k = -3 \pm 4$, giving 1 and -7, which sum to -6. The 3 moves across the equals sign and changes sign as it goes.

> **Takeaway:** Tangency is a discriminant question in disguise. Set the two expressions equal, collect everything on one side **first**, and only then reach for $b^2 - 4ac$ — using the coefficients of the combined equation, never the original curve.

Q3 — Brutal — Answer: B

Topic: An exponential model from two data points

Divide one reading by the other. The a cancels, which is the whole reason to start there:

$$\frac{f(5)}{f(2)} = \frac{ab^5}{ab^2} = b^3 = \frac{486}{18} = 27$$

So $b = 3$. Now use either reading to get a:

$$f(2) = a \cdot 3^2 = 9a = 18 \quad \Longrightarrow \quad a = 2$$

And $f(0) = a \cdot b^0 = a \cdot 1 = 2$, option 2.

The last line is the point of the question. In *any* model of the form ab^x, $f(0)$ **is** a — the starting value — because $b^0 = 1.

Why each wrong option is wrong:

- **A,** 6 — computed $a \cdot b = 6$, which is $f(1)$, not $f(0)$. Off by one step down the sequence.
- **C,** 3 — reported the growth factor b instead of the initial value a. These are the two constants in the model and it is easy to hand back the wrong one under time pressure.
- **D,** 27 — stopped at $b^3 = 27$ without taking the cube root, and without going on to find a at all.

> **Takeaway:** For $f(x) = ab^x$, dividing two readings kills a and hands you b; dividing the *exponents* tells you what power of b you have. And $f(0) = a$ always — if a question asks for the value at zero, it is asking for a.

Section 2 — Answers That Are Not Solutions

Q4 — Brutal — Answer: D

Topic: An equation with absolute values on both sides

Two absolute values are equal when the things inside are equal, **or** when one is the negative of the other. Both cases have to be worked.

Case 1: $2x - 5 = x + 4$, so $x = 9$.

Case 2: $2x - 5 = -(x + 4) = -x - 4$, so $3x = 1$ and $x = \frac{1}{3}$.

Both check: at $x = 9$, $|13| = |13|$; at $x = \frac{1}{3}$, $\left|-\frac{13}{3}\right| = \left|\frac{13}{3}\right|$. Neither is extraneous here, which is worth noticing — with absolute values on *both* sides both cases usually survive, unlike the one-sided kind.

The sum is $9 + \frac{1}{3} = \frac{28}{3}$, option $\frac{28}{3}$.

Why each wrong option is wrong:

- **B,** 9 — solved the first case and stopped. A single bar equation almost always has two solutions; a question asking for their *sum* is confirming it.
- **C,** $\frac{1}{3}$ — solved the second case only.

- **A,** $\frac{26}{3}$ — found both and then subtracted rather than added.

> **Takeaway:** $|A| = |B|$ gives two equations, $A = B$ and $A = -B$. Write both down before solving either — the second is the one that gets forgotten, and it is the one with the awkward fraction.

Q5 — Brutal — Answer: A

Topic: A radical equation with an extraneous root

Square both sides. That is the only way to get at x, and it is also what creates the problem.

$$x + 7 = (x-5)^2 = x^2 - 10x + 25 \implies x^2 - 11x + 18 = 0$$
$$(x-2)(x-9) = 0 \implies x = 2 \text{ or } x = 9$$

Now check both in the original equation, which squaring has quietly changed.

At $x = 2$: the left side is $\sqrt{9} = 3$; the right side is $2 - 5 = -3$. A square root sign means the **non-negative** root, so the left side can never be negative. $3 \neq -3$, and $x = 2$ is not a solution of the equation that was asked.

At $x = 9$: left side $\sqrt{16} = 4$, right side $9 - 5 = 4$. ✓

So $x = 9$, option 9.

Why each wrong option is wrong:

- **B,** 2 — the extraneous root. It solves the squared equation perfectly and fails the original. Squaring turns -3 into 9 just as it turns 3 into 9, so it cannot tell the two apart; only substituting back can.
- **C,** 11 — added the two roots. There is only one solution, so there is nothing to add.
- **D,** 18 — multiplied them, which is the constant term of the quadratic and not a solution of anything.

> **Takeaway:** Every time you square both sides you must substitute back. The extraneous root appears exactly when the two sides had opposite signs, and it will always be one of the options.

Section 3 — Two Ideas at Once

Q6 — Brutal — Answer: C

Topic: A circle from its general equation, and a distance

Get the circle into standard form by completing the square in both variables:

$$x^2 - 10x + y^2 + 6y = -18$$
$$(x-5)^2 - 25 + (y+3)^2 - 9 = -18$$
$$(x-5)^2 + (y+3)^2 = 16$$

So the centre is $(5, -3)$ and the radius is $\sqrt{16} = 4$.

The distance from the origin to the **centre** is

$$\sqrt{5^2 + (-3)^2} = \sqrt{34} \approx 5.83$$

That is more than the radius, so the origin lies outside the circle. The nearest point of the circle sits on the line joining the origin to the centre, one radius back from the centre:

$$\sqrt{34} - 4$$

which is option $-4 + \sqrt{34}$, about 1.83.

Why each wrong option is wrong:

- **B,** $\sqrt{34}$ — the distance to the centre. The centre is not on the circle; every point of the circle is 4 units away from it.
- **A,** $4 + \sqrt{34}$ — went one radius **past** the centre instead of stopping short of it. That is the *furthest* point of the circle from the origin, not the nearest.
- **D,** 4 — the radius, which is the distance from the centre to the circle, not from the origin.

> **Takeaway:** For a point outside a circle, nearest distance is $d_{\text{centre}} - r$ and furthest is $d_{\text{centre}} + r$. Complete the square first — a general-form equation tells you nothing until you do.

Q7 — Brutal — Answer: B

Topic: Similar figures: area ratio against length ratio

This is the one fact the question rests on: **areas of similar figures scale as the square of the length ratio.** So if areas are in the ratio $9 : 25$, lengths are in the ratio $\sqrt{9} : \sqrt{25} = 3 : 5$.

Perimeter is a length, so it scales by $\frac{5}{3}$:

$$24 \times \frac{5}{3} = 40$$

option 40.

Why each wrong option is wrong:

- **A,** $\frac{200}{3}$ — scaled the perimeter by the *area* ratio $\frac{25}{9}$, giving $\frac{200}{3} \approx 66.7$. This is the mistake the question is built to catch: the ratio you are given is not the ratio you may apply.
- **C,** $\frac{72}{5}$ — used $\frac{3}{5}$ instead of $\frac{5}{3}$ and made the larger triangle smaller than the smaller one. Always sanity-check the direction.
- **D,** 120 — multiplied by 5 and never divided by 3, using one half of the ratio as though it were the whole scale factor.

> **Takeaway:** Lengths scale by k, areas by k^2, volumes by k^3. Given an area ratio, take the square root **before** you touch any length.

Q8 — Brutal — Answer: D

Topic: Complementary angles in a trigonometric equation

The identity to reach for is $\sin(\theta) = \cos(90^\circ - \theta)$: sine and cosine agree exactly when their angles are **complementary**, adding to 90°.

So the two angles here must add to 90:

$$x + (3x - 10) = 90 \quad \Longrightarrow \quad 4x - 10 = 90 \quad \Longrightarrow \quad 4x = 100 \quad \Longrightarrow \quad x = 25$$

Check: $\sin(25^\circ) = \cos(65^\circ)$, and $3(25) - 10 = 65$. ✓ Option 25.

Why each wrong option is wrong:

- **B,** 20 — wrote $x + 3x + 10 = 90$, flipping the sign of the -10 when moving it. Solving gives $x = 20$, and $3(20) - 10 = 50$, but $25 + 50 \neq 90$ — the check would have caught it.
- **C,** $\frac{45}{2}$ — dropped the -10 and solved $4x = 90$, giving 22.5.
- **A,** 50 — used 180° instead of 90°. Angles that sum to 180° are *supplementary*, which is the relationship between sin and sin, not between sin and cos.

> **Takeaway:** $\sin A = \cos B$ means $A + B = 90$. It does not mean $A = B$, and it does not mean $A + B = 180$. Write the sum equation immediately and then just solve a linear equation.

Q9 — Brutal — Answer: A

Topic: Composition in both orders

Composition is not commutative, and this question is about how far apart the two orders are.

$$f(g(x)) = (x+3)^2 = x^2 + 6x + 9$$
$$g(f(x)) = x^2 + 3$$

Set them equal:

$$x^2 + 6x + 9 = x^2 + 3$$

The x^2 cancels — which is why a question that looks quadratic is linear:

$$6x + 9 = 3 \implies 6x = -6 \implies x = -1$$

Check: $f(g(-1)) = f(2) = 4$ and $g(f(-1)) = g(1) = 4$. ✓ Option -1.

Why each wrong option is wrong:

- **B,** 1 — moved the 9 across as $+9$ and solved $6x = 6$. The 9 is on the left, so subtracting it from both sides makes the right side $3 - 9 = -6$.
- **C,** -3 — solved $g(x) = 0$. A natural thing to do with a composition question and not what was asked.
- **D,** -2 — expanded $(x+3)^2$ as $x^2 + 3x + 9$, halving the middle term. The middle term of $(x+a)^2$ is $2ax$, not ax.

> **Takeaway:** Write out both compositions in full before comparing them. When the outer function is a square, expand it properly — $(x+3)^2$ has a $6x$, and losing it turns a solvable question into a different one.

Q10 — Brutal — Answer: C

Topic: Compound percentage change

The two percentages are taken of **different amounts**, which is the entire question. Work with a multiplier for each step rather than adding and subtracting percentages.

Take the original price as 100.

An increase of 20% multiplies by 1.20: $100 \times 1.20 = 120$.

A decrease of 20% multiplies by 0.80 — and it is applied to 120, not to 100: $120 \times 0.80 = 96$.

So the final price is 96% of the original, option 96. The item is 4% cheaper than it started, even though the two percentages were equal.

The reason is that 20% of 120 is 24, while 20% of 100 is only 20. The decrease is taken from a bigger number, so it is bigger.

Why each wrong option is wrong:

- **B,** 100 — assumed $+20\%$ and -20% cancel. They only would if both were percentages *of the same amount*, and they never are in a sequence like this.
- **A,** 104 — got the size of the change right and its direction wrong. A rise followed by an equal fall always leaves you **below** where you started, never above.
- **D,** 80 — applied the decrease to the original price and ignored the increase entirely.

> **Takeaway:** Chain percentage changes as multipliers: $1.20 \times 0.80 = 0.96$. Never add or subtract the percentages themselves, and remember the answer to "up then down by the same percent" is always a net loss.

Q11 — Brutal — Answer: B

Topic: The factor theorem with two unknowns

The factor theorem says $(x - r)$ divides $p(x)$ exactly when $p(r) = 0$. Two factors give two equations, and two equations are what two unknowns need.

From $(x - 1)$, substitute $x = 1$:

$$2 + a - 5 + b = 0 \quad \Longrightarrow \quad a + b = 3$$

From $(x + 2)$, substitute $x = -2$ — note the root is -2, not 2:

$$2(-8) + 4a + 10 + b = 0 \quad \Longrightarrow \quad 4a + b = 6$$

Subtract the first from the second: $3a = 3$, so $a = 1$ and then $b = 2$.

$a + b = 3$, option 3.

There is a shortcut worth seeing: the first equation **is** $a + b = 3$, so the question was answered before the second factor was ever used. The second factor is there to make you do work you did not need.

Why each wrong option is wrong:

- **A,** 1 — solved the system correctly and handed back a.
- **C,** 2 — handed back b.

- **D,** 6 — reported $4a + b$, the left side of the second equation, rather than $a + b$.

> **Takeaway:** $(x + 2)$ is a factor when $p(-2) = 0$. The sign flips, and it is the most reliable way to lose this question. And read what is actually being asked — sometimes the first line of working already contains it.

Q12 — Brutal — Answer: D

Topic: A mean after one value is corrected

Go back to the total. A mean is a total divided by a count, and the count has not changed — only the total has.

$$\text{original total} = 20 \times 72 = 1440$$

One score of 35 has to come out and one of 75 has to go in, so the total changes by $75 - 35 = 40$:

$$\text{correct total} = 1440 - 35 + 75 = 1480$$

$$\text{correct mean} = \frac{1480}{20} = 74$$

option 74. Equivalently, the mean rises by $\frac{40}{20} = 2$.

Why each wrong option is wrong:

- **B,** $\frac{303}{4}$ — added the new score without removing the old one, giving a total of 1515 over 20 scores. There are still only 20 scores; one of them changed value.
- **C,** 72 — assumed correcting one value cannot move the mean. It moves it by the change divided by the count.
- **A,** 76 — divided the change of 40 by 10 instead of 20. The count is the number of scores, not half of it.

> **Takeaway:** For any "one value was recorded wrongly" question, convert the mean to a total, adjust the total by the **difference**, and divide by the unchanged count.

Q13 — Brutal — Answer: A

Topic: Conditional probability from a two-way table

"Selected at random from those who passed" changes the population. You are no longer choosing from 100 candidates — you are choosing from the 73 who passed. That number is the denominator.

Of those 73, the ones who sat in the morning number 45. So

$$P(\text{morning} \mid \text{passed}) = \frac{45}{73}$$

option $\frac{45}{73}$, about 0.616.

The reliable method: find the phrase that restricts who is being chosen from, take **that** total as the denominator, and count within it.

Why each wrong option is wrong:

- **B,** $\frac{3}{4}$ — $\frac{45}{60}$ is $P(\text{passed} \mid \text{morning})$, the chance a *morning candidate* passed. Same cell, different total, and it answers the question the other way round. Conditional probabilities are not symmetric, and this is the standard trap.
- **C,** $\frac{9}{20}$ — $\frac{45}{100}$ is the probability of picking someone who is both a morning candidate and a pass, out of everyone. It ignores the restriction entirely.
- **D,** $\frac{73}{100}$ — $\frac{73}{100}$ is simply the probability of passing.

> **Takeaway:** In "given that", the given event supplies the denominator. Underline the conditioning phrase, write its total down first, then look for the numerator inside it.

Q14 — Brutal — Answer: C

Topic: An exponential equation with different bases

The bases are different, so nothing can be compared yet. Both are powers of 2, which is the way in:

$$4^{x+1} = (2^2)^{x+1} = 2^{2x+2} \qquad 8^{x-1} = (2^3)^{x-1} = 2^{3x-3}$$

Now the bases match, so the exponents must be equal:

$$2x + 2 = 3x - 3 \quad \Longrightarrow \quad x = 5$$

Check: $4^6 = 4096$ and $8^4 = 4096$. ✓ Option 5.

Why each wrong option is wrong:

- **B,** 3 — treated the bases as multipliers and solved $4(x+1) = 8(x-1)$, reaching $x = 3$. An exponent is not a coefficient.
- **A,** -1 — distributed the 3 as $3x + 3$ instead of $3x - 3$. The exponent is $x - 1$, so both of its terms are multiplied by 3, sign included.
- **D,** 2 — rewrote 8 as 2^3 but left the 4 alone, comparing $x + 1$ with $3(x - 1)$. Both sides have to be converted before exponents can be equated.

> **Takeaway:** Same base first, then equate exponents. Write $(2^2)^{x+1} = 2^{2(x+1)}$ out in full — the bracket is what stops the 2 being forgotten.

Q15 — Brutal — Answer: B

Topic: Three equations solved by adding them

Adding all three equations is far faster than substituting, and it is the move the question is built around. Each variable appears in exactly two of the three equations, so adding gives

$$2(x+y+z) = 7+9+12 = 28 \quad \Longrightarrow \quad x+y+z = 14$$

Now each variable is the total minus the equation that does **not** contain it. The equation without x is $y+z=9$, so

$$x = 14 - 9 = 5$$

option 5. The same trick gives $y = 14 - 12 = 2$ and $z = 14 - 7 = 7$, and $5 + 2 = 7$ checks out. ✓

Why each wrong option is wrong:

- **A,** 2 — subtracted the wrong equation and produced y.
- **C,** 7 — produced z. Note 7 also appears in the question as the first right-hand side, which makes it feel familiar.
- **D,** 14 — stopped at $x+y+z=14$.

> **Takeaway:** When every variable appears the same number of times, add all the equations at once. Each unknown is then the grand total minus the equation it is missing from, with no substitution at all.

Q16 — Brutal — Answer: D

Topic: The minimum value of a quadratic

A parabola opening upwards takes its minimum at the vertex, which sits at

$$x = -\frac{b}{2a} = -\frac{-6}{2} = 3$$

The minimum *value* is the function evaluated there:

$$f(3) = 3^2 - 6(3) + c = 9 - 18 + c = c - 9$$

Setting that equal to 4 gives $c = 13$, option 13.

Check by completing the square: $x^2 - 6x + 13 = (x-3)^2 + 4$, whose least value is 4 at $x = 3$. ✓

Why each wrong option is wrong:

- **B,** 4 — took c to be the minimum value. c is the y-intercept, $f(0)$ — the two coincide only when the vertex sits on the y-axis.
- **C,** -5 — wrote $f(3) = c + 9$, losing a sign when combining $9 - 18$.
- **A,** 22 — computed $-6(3) = -18$ but forgot the $3^2 = 9$, giving $c - 18 = 4$.

> **Takeaway:** Vertex x from $-\frac{b}{2a}$, then substitute for the vertex y. Completing the square gives both at once and is worth doing as a check: $(x-p)^2 + q$ has minimum q at $x = p$.

Q17 — Brutal — Answer: A

Topic: Rates working against each other

Rates add and subtract; times do not. Convert each device to a rate first — the fraction of the tank it handles in one hour.

The tap fills $\frac{1}{6}$ of the tank per hour; the drain removes $\frac{1}{9}$ per hour. They oppose each other, so the net rate is a subtraction:

$$\frac{1}{6} - \frac{1}{9} = \frac{3}{18} - \frac{2}{18} = \frac{1}{18}$$

The tank gains $\frac{1}{18}$ of itself each hour, so filling all of it takes

$$1 \div \frac{1}{18} = 18 \text{ hours}$$

option 18. It is slower than the tap alone, which it has to be — a sanity check worth ten seconds.

Why each wrong option is wrong:

- **B,** $\frac{18}{5}$ — added the rates instead of subtracting. That is the answer to "two taps both filling", and it gives 3.6 hours — *faster* than either device alone, which is impossible when one of them empties the tank.
- **C,** 15 — added the two times. Times cannot be added; only rates can.
- **D,** 3 — subtracted the two times. The same error in the other direction.

> **Takeaway:** Turn every "takes n hours" into "$\frac{1}{n}$ per hour" before doing anything else. Combine the rates, invert once at the end, and check the answer falls on the sensible side of the faster device.

Q18 — Brutal — Answer: C

Topic: A quadratic that is positive for every x

The parabola opens upwards, so it lies above the axis everywhere exactly when it never touches the axis — that is, when it has **no real roots**. No real roots means a negative discriminant:

$$b^2 - 4ac < 0 \implies 36 - 4c < 0 \implies c > 9$$

The inequality is strict, so $c = 9$ is not allowed: at $c = 9$ the expression is $(x-3)^2$, which equals 0 at $x = 3$ and so is not > 0 for *all* x.

The least integer strictly greater than 9 is 10, option 10.

Why each wrong option is wrong:

- **B,** 9 — solved $36 - 4c = 0$ and took the boundary. The boundary is the single value that fails, because it puts a repeated root exactly on the axis.
- **A,** 3 — took a square root that was not called for, reaching 3.
- **D,** 36 — computed $b^2 = 36$ and reported it.

> **Takeaway:** "Positive for all x" with an upward parabola means discriminant < 0, strictly. Whenever an inequality is strict, test the boundary separately — it is offered as an option precisely because it so nearly works.

Q19 — Brutal — Answer: B

Topic: A shaded region built from a square and a circle

Two areas, subtracted.

The square has area $10^2 = 100$. A quarter circle of radius 10 is one fourth of a full circle of that radius:

$$\frac{1}{4}\pi r^2 = \frac{1}{4}\pi(10)^2 = 25\pi$$

What is left is

$$100 - 25\pi$$

option $100 - 25\pi$, about $100 - 78.5 = 21.5$.

The sanity check matters here: the answer must be positive and smaller than the square. Any option that comes out negative can be discarded on sight.

Why each wrong option is wrong:

- **A,** $100 - 100\pi$ — subtracted a whole circle of radius 10, area $100\pi \approx 314$, from a square of area 100. That is roughly -214: a negative area, impossible and visible without any calculation at all.
- **C,** $100 - 50\pi$ — used a half circle. The shape is bounded by two sides of the square meeting at a corner, and a corner is a right angle — one quarter of a full turn.
- **D,** 25π — gave the quarter circle itself: the region removed, not the region left.

> **Takeaway:** A sector's fraction is its angle over 360°, and a square's corner is 90°, so a quarter. On any shaded-region question, check the sign and the rough size of your answer against the picture before choosing.

Q20 — Brutal — Answer: D

Topic: Two linear models meeting

Write a model for each tank as a function of the time t in minutes. One is falling, so its rate is negative; the other is rising.

$$A(t) = 240 - 15t \qquad B(t) = 60 + 5t$$

Set them equal:

$$240 - 15t = 60 + 5t \quad \Longrightarrow \quad 180 = 20t \quad \Longrightarrow \quad t = 9$$

option 9. Check: $A(9) = 240 - 135 = 105$ and $B(9) = 60 + 45 = 105$. ✓

The 20 is the important number. The tanks close the 180-litre gap between them at a combined 20 litres per minute, because one is losing while the other gains — the rates **add**, even though the signs in the two models differ.

Why each wrong option is wrong:

- **B,** 105 — solved correctly and then reported the shared volume, 105 litres, instead of the time. Both numbers are in the working; only one was asked for.
- **C,** 12 — divided 240 by 20, ignoring the 60 litres already in tank B. The gap to close is $240 - 60 = 180$, not 240.
- **A,** 18 — used $15 - 5 = 10$ for the combined rate, giving 18 minutes. Subtracting is right when both tanks move the same way; here they move towards each other, so the gap closes at the **sum** of the rates.

Takeaway: Two linear models meet where their expressions are equal. Watch the sign of each rate as you build them, and when you have finished, read the question again to see whether it wants the time or the value.